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Peak-to-average power ratio reduction using backtracking search optimization algorithm in OFDM systems

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Abstract: A peak-to-average power ratio (PAPR) reduction method for an orthogonal frequency division multiplexing (OFDM) system based on a combination of a partial transmit sequence (PTS) scheme with the backtracking search optimization algorithm (BSA) is proposed. The PTS scheme shows good PAPR reduction performance; however, it requires a complex computation to search the set of optimum phase factors, especially for high-speed data transmissions. To minimize the complexity of the search for optimum phase factors, the PTS scheme is combined with the BSA. The BSA is a new and efficient population-based evolutionary search algorithm for real-valued optimization problems. A set of simulations is conducted for the PAPR reduction performance and computational complexity comparisons of the BSA-PTS scheme with different PTS schemes. Simulation results indicate that the BSA-PTS scheme yields an effective PAPR reduction performance with low computational complexity.

Key words: Orthogonal frequency division multiplexing, backtracking search optimization algorithm, partial transmit sequence, peak-to-average power ratio

1. Introduction

Orthogonal frequency division multiplexing (OFDM) systems are widely utilized in digital communication systems because of key features like efficient implementation, high bandwidth efficiency, robustness to frequency selective fading, and narrowband interference [1–4]. Besides these useful advantages, they have some troubles, such as sensitivity to time and frequency synchronization, and high peak-to-average power ratio (PAPR) [5]. High PAPR is an important implementation problem in OFDM systems, especially for systems with large numbers of subcarriers. A high PAPR leads to decreases in the efficiency of radiofrequency power amplifiers and increases in the complexity of analog-to-digital and digital-to-analog converters. Constitutional arrangements and restrictions can be applied to reduce the PAPR; however, these lead to operation of the power amplifier in a nonlinear region and reduce the spectral efficiency of the OFDM system.

To mitigate the high PAPR issue in OFDM systems, several methods, such as active constellation extension [6], interleaving [7], peak windowing [8], tone reservation [9], tone injection [10], coding [11], clipping [12], clipping and filtering [13], selected mapping [14], and partial transmit sequence (PTS) [15–19], have been proposed. All these methods have their own PAPR reduction levels and their own computation algorithms.

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The PTS is the most commonly used one due to its significant PAPR reduction performance without adding any signal distortion.

The PTS scheme consists of the following steps: division of input signal into several disjoint subblocks, transformation of subblocks using inverse fast Fourier transform (IFFT), rotation of subblocks by a set of phase factor, and finally summation of subblocks. The factors could be selected arbitrarily between and 2π to optimize the PAPR of the OFDM system. The selection of optimum phase factors is a complex nonlinear optimization problem and it requires an exhaustive search, especially for systems with large numbers of subblocks. The computational complexity of the optimal phase search is the major drawback of the conventional PTS scheme. To reduce the complexity of the phase search, the PTS scheme has been combined with certain optimization algorithms such as random search [19], particle swarm optimization (PSO) [20], differential evolution (DE) [21], artificial bee colony (ABC) [22], parallel tabu search (parallel-TS) [23], and harmony search (HS) [24]. By the application of these algorithms, PAPR reductions with low computational complexities are successfully obtained. In this paper, PTS is combined with a new evolutionary algorithm, the backtracking search optimization algorithm (BSA), to reduce the PAPR with low computational complexity. The BSA was developed to find useful solutions in optimization problems [25]. It tries to mimic natural evolution using new selection operators with a single control parameter.

The paper is organized as follows: in Section 2, the model of the OFDM system based on a PTS scheme is presented. In Section 3, the BSA for PAPR reduction is explained. Section 4 describes simulation results. In Section 5, a complexity comparison is presented. Finally, conclusions are given in Section 6.

2. Model of OFDM system based on PTS scheme

Figure 1 indicates the model of the OFDM system based on the PTS scheme utilized in the computer simulations. Initially, bit sequences coming from the users are interleaved to convert burst errors induced by the channel noise to random errors. The PTS is used to reduce the PAPR of the 16-QAM modulated interleaved signal. The PTS requires side information to obtain the original signal at the receiver. A cycle prefix (CP) is added in the signal and then the signal is amplified by the high power amplifier (HPA) to get rid of the intersymbol interference induced from the communication channel. At the receiver, the CP is extracted from the transmitted signal. The fast Fourier transform (FFT) is realized and then the phase of the original signal is obtained by phase rotation using side information. After 16-QAM_demodulation, each demodulated symbol is conveyed to the original place in the bit sequence by means of the deinterleaver [1].

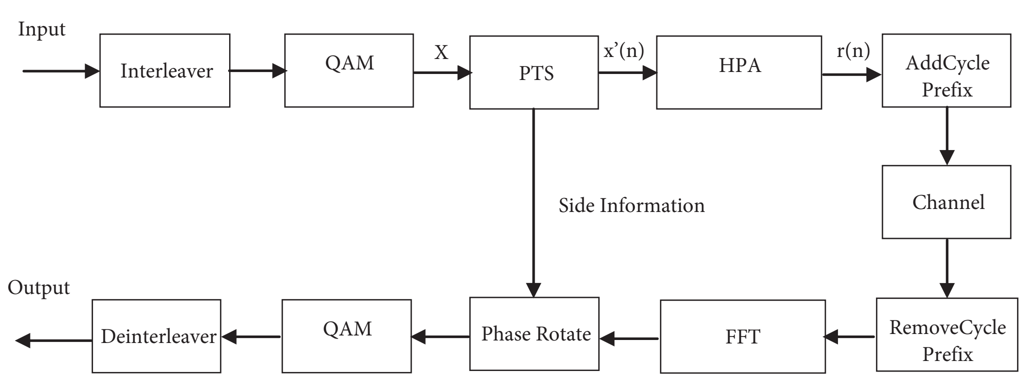

Figure 1. Block diagram for the model of OFDM system based on PTS scheme.

2.1. PAPR of OFDM signal

The discrete-time conducted OFDM signal with N subcarriers is shown by the following equation:

$$x_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j\frac{2\pi nk}{LN}}, k = 1, 2, \cdots, LN-1, \tag{1}$$

where L is the oversampling factor and $L = 4$ is sufficient to get accurate PAPR results. In Eq. (1), $X = [X_0, X_1, \cdots X_{N-1}]^T$ is the input signal vector with each symbol modulated by 16-QAM. The PAPR of OFDM signals is defined as the ratio of the maximum power of the signal to the average power of it.

$$PAPR = 10log_{10}\frac{max_k |x_k|^2}{E\left[|x_k|^2\right]}(dB) \tag{2}$$

The complementary cumulative density function (CCDF) is a common gauge used to express the reduction of PAPR in OFDM systems and it is defined as $CCDF = PrPAPR(x) > PAPR_0$ where $PAPR_0$ is a particular value of PAPR [1].

2.2. The conventional PTS scheme for PAPR reduction

Figure 2 shows the block diagram of the BSA-PTS model. In the PTS, input signal vector X is divided into M disjoint subblocks X_m, where $X_m = [X_{m,0}, X_{m,1}, \cdots, X_{m,N-1}]^T$ for $1 \leq m \leq M$, such that

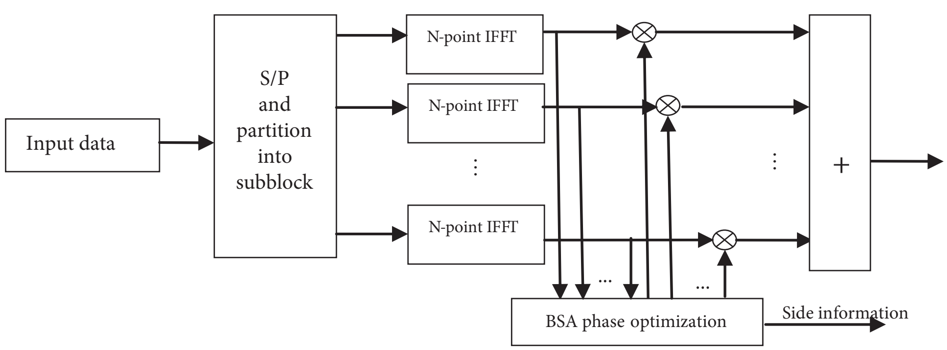

Figure 2. Block diagram of the BSA-PTS model.

$$X = \sum_{m=1}^{M} X_m. \tag{3}$$

By LN-point IFFT, subblocks are then converted from the frequency domain to the time domain. The mathematical expression of the block in the time domain is shown by

$$x = IFFT\left\{\sum_{m=1}^{M} X_m\right\} = \sum_{m=1}^{M} IFFT\{X_m\} = \sum_{m=1}^{M} x_m. \tag{4}$$

The purpose of the PTS is to create a weighted combination of the M time-domain partial sequences $\mathbf{x_m}$ by a rotation vector $\mathbf{b} = [b_1 b_2 \cdots b_M]^T$ to reduce the PAPR, which is shown by

$$x^{'}(b) = \sum_{m=1}^{M} b_m x_{m.} \tag{5}$$

In order to reduce the peak power of $\mathbf{x'}$, each partial sequences $\mathbf{x_m}$ should be suitably returned. Letting $b_m = e^{j\emptyset_m}$, where ϕ_m can be selected freely within $[0, 2\pi)$, Eq. (5) can be stated as

$$x^{'}(\Phi) = \sum_{m=1}^{M} e^{j\emptyset_m} x_m, \tag{6}$$

where $x^{'}(\Phi) = \left[x_1^{'}(\Phi), x_2^{'}(\Phi), \cdots, x_{NL}^{'}(\Phi)\right]$. The practical application of the PTS method is to contemplate a rotation phase vector $\mathbf{\Phi}$ that reduces the PAPR. The PTS technique to reduce PAPR is interested in the problem of minimizing $max|\mathbf{x'_k}(\Phi)|$ subject to $0 \leq k \leq NL-1$, and it involves a detailed look for a combinatorial optimization problem, which needs a huge quantity of computations to analyze all possible applicant rotation phase vectors [1].

3. PTS for PAPR reduction using backtracking search optimization algorithm

3.1. Backtracking search optimization algorithm

The BSA is a recently developed evolutionary algorithm for real-valued optimization problems and it was developed by Civicioglu [25]. It has been shown that the BSA has some enhancements over other evolutionary algorithms, such as reduction in sensitivity to control parameters and an increase in convergence performance. The BSA also has only one control parameter used to control mutation in the crossover step. The genetic operators of selection, mutation, and crossover are used in the BSA to find the best values of trial individuals by evolving the trial individuals. This evolution is carried out in five steps: initialization, selection-I, mutation, crossover, and selection-II.

A population (P) in which the individuals are initialized randomly between the upper and lower constraints is generated in the initialization step. The individuals of population can be written as

$$P_{ij} \sim rand\left(low_j, up_j\right) \tag{7}$$

for $i = 1, 2, \ldots.., N$ and $j = 1, 2, \ldots., D$, where N is the population size, D is the number of parameters to be optimized, and low_j and up_j are the predefined lower and upper limits of the problem. The BSA sets the population of a randomly selected iteration as the historical population ($oldP$). The initial values of the historical population are determined using

$$oldP_{i,j} \sim rand\left[low_j, up_j\right]. \tag{8}$$

In the selection-I step, a historical population ($oldP$) is determined and the BSA has an option to redefine the historical population ($oldP$) at the beginning of each iteration using

$$\text{If } a < b \text{ then } oldP \xleftarrow{\text{update}} P\,|a, b \sim rand\,(0,1)\,. \tag{9}$$

The order of individuals of this historical population is altered randomly using Eq. (10).

$$oldP \xleftarrow{\text{update}} \text{permutting}\,(oldP) \tag{10}$$

In the mutation step, a mutant is generated as the initial form of a trial population (T) with the help of the historical population ($oldP$) and the population (P). It can be expressed as

$$Mutant = P + F \times (oldP - P)\,, \tag{11}$$

where F is the control parameter for the amplitude of search direction. According to fitness values, the individuals of the mutation step are manipulated by the relevant individuals of the population (P) in the crossover step. In the selection-II step, the fitness values of individuals on the same order for the trial population (T) and the population (P) are compared; if the fitness values of individuals of the trial function (T) are better than the population (P), they are used to update the individuals of the population (P) [25].

3.2. Phase factor optimization by the BSA

In the BSA, the generation of an initial population with a predefined number of chromosomes and genes is the starting point of the solution. The numbers of chromosomes and genes are physically interpreted as the size of the population and number of subblocks. The values of genes are associated with the phase factors of the PTS scheme. The initial values of genes for the initial population are defined as

$$\Phi_i = (\Phi_h - \Phi_l)\,u + \Phi_l i = 1, 2, \ldots, N, \tag{12}$$

where Φ_h and Φ_l refer to the highest and lowest values in the variable range, and u refers to random values between and 1. The values used for Φ_h and Φ_l are and 2π. The PAPR values are computed using values of genes of the old population to compute the search direction, which defines the update of the old population by changing the order of genes. This is called first selection. The application of natural evolution operators, mutation and crossover, generates the initial and final forms of the new population. The genes having lower fitness values than the corresponding genes of the old population are used to update the old population and this operation is called second selection. Due to the systematic calculation of the BSA, the potential phase factors Φ are represented with real values. The phase factor values are converted to $\{-1, +1\}$ according to Eq. (13) for the fitness computations [22]:

$$\Phi_i^{'} = \begin{cases} +1, & \text{if } \frac{3\pi}{4} < \Phi_i^{'} < \frac{5\pi}{4} \\ -1, & \text{otherwise} \end{cases}. \tag{13}$$

The flowchart and pseudocode of the BSA to search for better combinations of phase factors are given in Figure 3 and Table 1, respectively.

In this pseudocode, the subscript "best" refers to the index of the global best phase rotation vector in the BSA. P_i and T_i show indexes of the phase rotation vector and trial phase rotation vector in their populations, respectively.

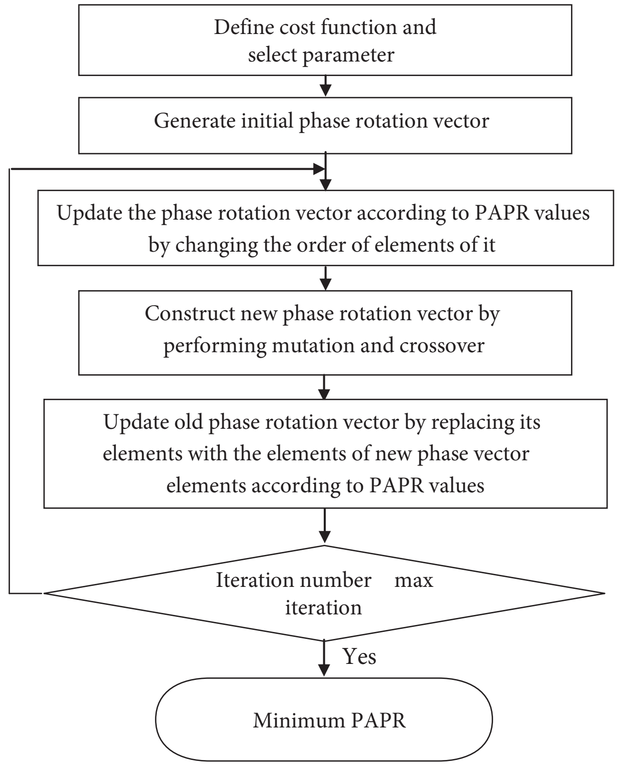

Figure 3. Flowchart of the backtracking search optimization algorithm for phase factors' optimization.

Table 1. Pseudocode of the BSA for phase detection.

```
function bsa()
        // Initialization of population of phase rotation vector (P)
        // Initialization of historical phase rotation vector (oldP)
        initialize_population();
        // Evaluation of initial PAPR values of P
        evaluate_fitness(P);
        for iteration = 1:maxiteration
        // Selection-I
                permute_oldP();
                // Generation of population of trial phase rotation vector (T)
                        apply_mutation();
                        apply_crossover();
                // Selection-II
                // Evaluation of initial PAPR values of trial phase rotation vector (T)
                        evaluate_fitness(T);
                        for i = 1:populationSize
                        if (fitnessT_i<fitnessP_i)
                                P_i=T_i
                        end
                        end
                        fitnessP_best = min(fitnessP) | best = {1,2,3,….,populationSize}
        end
end
```

4. Simulation results

Simulations were performed by using the CCDF and bit error rate (BER) criteria to verify the performance of the proposed PAPR reduction scheme for the OFDM system. The system is considered with $N = 256$ subcarriers and 16-QAM modulation. In the PTS optimization, number of phase factor $W = 2$ and subblocks $M = 16$ are chosen. A solid-state power amplifier (SSPA) is used with input back-off factor $IBO = [0, 3, 6, 12]$ dB and smoothness factor $p = 2$. The communication channel is an additive white Gaussian noise (AWGN) channel. The simulation parameters are documented in Table 2.

Table 2. Simulation parameters.

Symbol	Quantity	Value
16-QAM	Modulation method	16
N	Number of subcarriers	256
M	Number of subblocks	16
W	Number of phase factor	2 (±1)
SSPA, linear amp.	Amplifier	
IBO	Input back-off	0, 3, 6,12 dB
P	Smoothness factor	2
AWGN	Channel	
G	Generation	50
P	Population	20

In Figure 4, the variations of PAPR_0 (dB) versus CCDF for the PTS scheme based on the BSA, PSO, DE, and ABC algorithms are given to compare their PAPR reduction performances in OFDM systems. It is shown that PSO yields the worst performance while the BSA yields the best performance for the PAPR reduction of OFDM systems. The PAPR values of the OFDM system are $7.1dB$ for the PSO algorithm, $6.85dB$ for the DE algorithm, $6.65dB$ for the ABC algorithm, and $6.44dB$ for the BSA at $CCDF = 10^{-3}$.

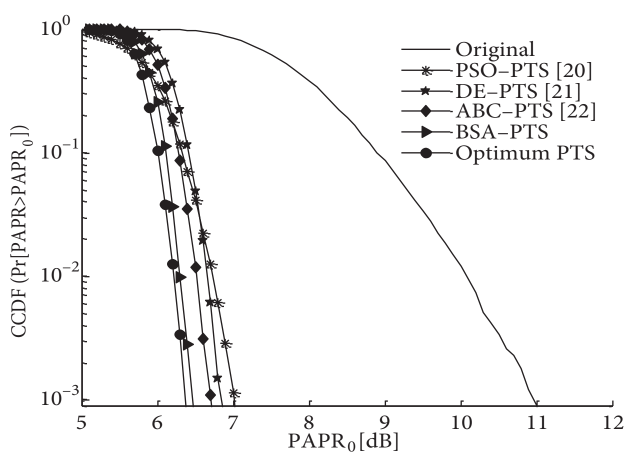

Figure 4. PAPR comparison of PSO-PTS, DE-PTS, ABC-PTS, BSA-PTS, optimum-PTS, and original OFDM signals.

Figure 5 illustrates the CCDFs of PAPR of the PTS scheme based on the BSA, random search (RS), parallel-TS, and HS algorithms and optimum PTS. It is seen that RS yields the worst performance while the BSA yields the best performance for the PAPR reduction of OFDM systems. The PAPR values of the OFDM system are $7.2dB$ for the RS algorithm, $6.62dB$ for the parallel-TS algorithm, $6.59dB$ for the HS algorithm, and $6.44dB$ for the BSA at $CCDF = 10^{-3}$.

In Figure 6, the variations of CCDFs of PAPR for different generations with $P = 10$ are shown. According to Figure 6, $G = 20$ yields the worst performance while $G = 100$ yields the best performance for OFDM. The PAPR is $6.93dB$, $6.84dB$, $6.8dB$, $6.74dB$, and $6.68dB$ at $CCDF = 10^{-3}$ for $G = 20$, $G = 40$, $G = 60$, $G = 80$, and $G = 100$, respectively.

In Figure 7, the variations of CCDFs of PAPR for OFDM using different populations with $G = 10$ are illustrated. According to Figure 7, the PAPR is $6.82dB$, $6.7dB$, $6.53dB$, $6.48dB$ and $6.46dB$ at $CCDF = 10^{-3}$ for $P = 20$, $P = 40$, $P = 60$, $P = 80$, and $P = 100$, respectively. It is observed that an increment in population yields an increase in PAPR reduction performance.

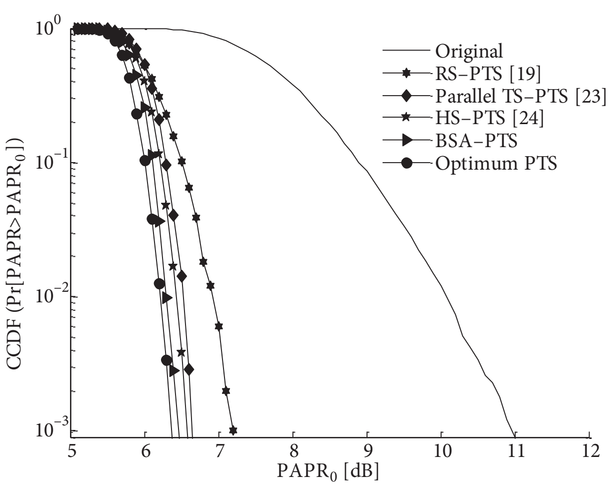

Figure 5. PAPR comparison of BSA-PTS, RS-PTS, parallel-TS-PTS, HS-PTS, optimum-PTS, and original OFDM signals.

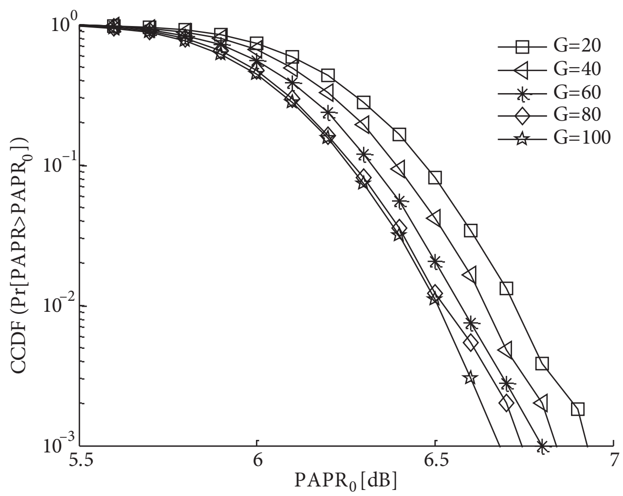

Figure 6. PAPR comparison of BSA-PTS for different generations with $P = 10$.

The BER performance of BSA-PTS is shown in Figure 8. The BER performances of DE-PTS and ABC-PTS are also illustrated to compare them with the BER performance of BSA-PTS. The comparisons are carried out with different IBO values such as $0, 3, 6$, and 12. It is shown that an increase in IBO values results in a decrease in BER values of the system.

5. Complexity comparison

Computational complexity and the reduced PAPR values of the various PTS schemes at $CCDF = 10^{-3}$ are given in Table 3. In the original OFDM signal, the PAPR value is calculated before the PTS optimization and therefore its search number is . Optimum PTS considers all the phase factors and thus it needs $W^{M-1} = 2^{16-1} = 32,768$ search numbers. RS-PTS requires the number of randomly chosen phase factors. In PSO-PTS, the search number is equal to $S \times K = 20 \times 50 = 1000$, where K is the maximum iteration number and S is the size of particle swarm. In ABC-PTS, the search number is equal to $S \times K = 20 \times 50 = 1000$, where K is the maximum iteration number and P is the size of the population. The total search in the parallel-TS-PTS for 20 cycles is $[(maxit \times h) + c] \times cycle = [(11 \times 4) + 6] \times 20 = 1000$, where $maxit$ is the maximum iteration, h is the number of TS working in parallel, and c is the search cost of crossover.

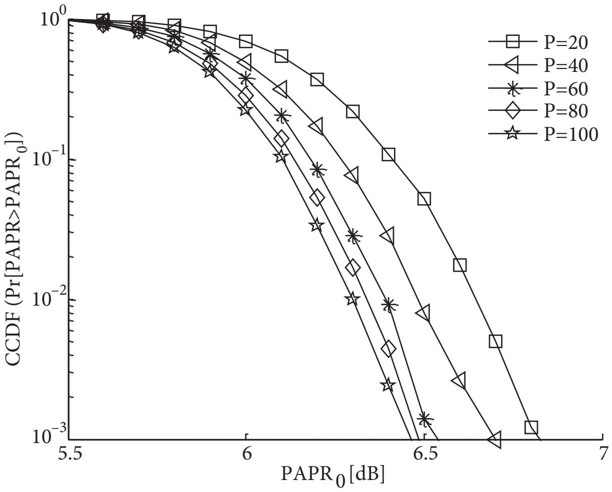

Figure 7. PAPR comparison of BSA-PTS for different populations with $G = 10$.

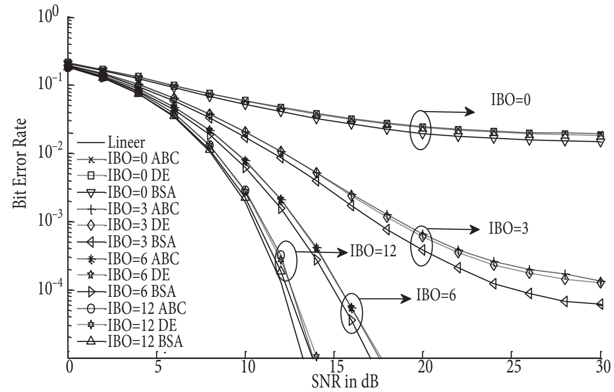

Figure 8. BER comparison of BSA-PTS, ABC-PTS, and DE-PTS for different IBO values.

Table 3. Computational complexity of the different PTS schemes for size of population/particle $P = S = 20$ and maximum generation/iteration $G = K = 50$.

Methods	Number of searches	PAPR [dB]
Original		10.9
RS-PTS	$randomlyselected = 1000$	7.2
PSO-PTS	$S \times K = 20 \times 50 = 1000$	7.1
DE-PTS	$P \times G = 20 \times 50 = 1000$	6.85
ABC-PTS	$S \times K = 20 \times 50 = 1000$	6.65
Parallel-TS-PTS	$[(maxit \times h) + c] \times cycle =$ $[(11 \times 4) + 6] \times 20 = 1000$	6.62
HSA-PTS	$S \times K = 20 \times 50 = 1000$	6.59
BSA-PTS	$P \times G = 20 \times 50 = 1000$	6.44
Optimum-PTS	$W^{M-1} = 2^{16-1} = 32,768$	6.38

6. Conclusion

In this paper, we propose a BSA-PTS technique to reduce the PAPR value with low computational complexity of OFDM systems. The BSA-PTS is compared with HS-PTS, RS-PTS, PSO-PTS, DE-PTS, ABC-PTS, parallel-TS-PTS, and the conventional PTS in respect to CCDF values versus $PAPR_0$. Furthermore the proposed method is also compared with ABC-PTS and DE-PTS for different IBO values in respect to BER values versus signal-to-noise ratio. The simulation results indicate that the proposed method yields the lowest CCDF values versus PAPR_0 and the lowest BER performances according to the compared PTS schemes.

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