

Winding temperature prediction in split-winding traction transformer

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Abstract: Incombustible features of resin make multiwinding dry-type transformers suitable for use in traction systems. Nevertheless, due to resin's heat transfer property and the special structure of traction transformers, temperature distribution in traction transformers is undesirable and it is essential to study its thermal behavior. In this research paper, the thermal behavior of traction transformers is modeled using the finite difference approach. After validating the model results (using the experimental results), the temperature distribution in a cast-resin traction transformer with split windings is calculated. The thermal behavior of the split-winding traction transformer is compared to the normal two-winding cast-resin transformer. A traction transformer usually feeds electronic converters; the converter system as a nonlinear load causes harmonics to appear in the winding currents. Thus, the thermal behavior of a traction transformer has been modeled in the presence of harmonic currents and their effects on temperature distribution have been discussed.

Key words: Finite difference method, split-winding, thermal modeling, transformer, traction

1. Introduction

In the past years, traction networks and subways have developed rapidly. The main and most expensive piece of equipment in a traction network is its transformer. The traction transformer that is the aim of this paper is manufactured as seen in the Figure 1 [1] and is usually called a split-winding transformer. The transformer in a traction network should be protected from explosion; thus, it is essential to use transformers with nonflammable insulations. The most applicable kind of these transformers that is usually used in traction systems is the dry-type (cast-resin) transformer [2]. In this transformer low-voltage windings are impregnated and high-voltage windings are casted by resins of class F [3].

The traction transformer has special geometry and its thermal behavior is more important and serious in comparison with the two-winding transformer that only has concentric windings [4–7]. Thus, studying the thermal behavior in dry-type traction transformers is essential.

Some researchers used experimental temperatures to investigate the thermal behavior in dry-type transformers [8–11]. Previously, there have been some mathematical models for analyzing the thermal behavior of ventilated [12,13] and cast-resin [14,15] dry-type transformers with two concentric windings. A study on the dynamic thermal behavior of the two-winding cast-resin transformer was presented in [16]. Unfortunately, no research about thermal modeling of the mentioned traction transformer has been presented. Hence, using the finite difference method (FDM), the thermal performance of this transformer is studied in this research. Thermal modeling results have been verified with the help of experimental data. Using the validated model,

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the temperature distribution in the split-winding traction transformer has been calculated and the results are compared to the normal two-winding transformer.

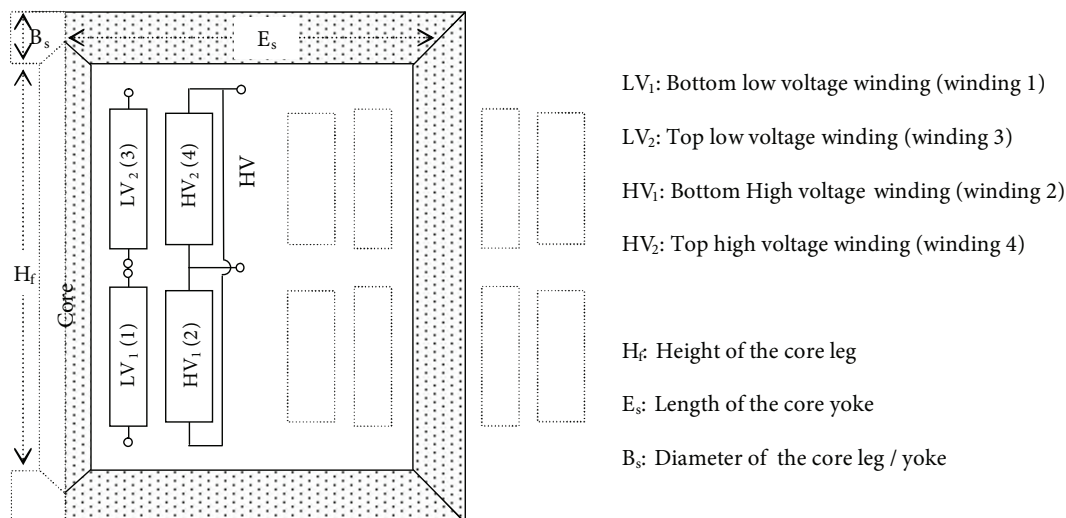


Figure 1. Schematic view of split windings in a traction transformer.

In practice, converters that are connected to the traction transformer's low voltages cause the harmonics to influence the temperature distribution. The effect of the voltage's harmonics on the temperature is negligible, but harmonic currents strongly affect the windings' temperature. Moreover, in this paper, the thermal modeling has been done in the presence of harmonic currents and their effects on the temperature have been studied.

Consequently, this paper presents the following novelties:

- A traction transformer with four split windings (two pairs) that are manufactured on each other is studied in this paper. This transformer has a special structure and its thermal behavior is important.
- Finite difference approach is applied to temperature calculation of the traction (split-winding) transformer,
- Temperature distribution analysis of the traction (split-winding) transformer and comparison of the temperatures with the normal two-winding transformer are performed.
- Analysis of the effects of harmonics on the temperature distribution of the traction (split-winding) transformer is also conducted.

2. Thermal modeling using FDM

Figure 2 shows the geometry of the cast-resin dry-type traction transformer. Here, LV and HV symbolize the low- and high-voltage windings respectively. According to the symmetry, heat transfer of the transformer has been taken into two-dimensional (2D) coordinates.

The FDM is a numerical approach based on mathematical discretization of partial differential equations. Using the FDM, continuous problems are studied in a finite number of small time intervals. In the small intervals, it is possible to approximate a problem by approximate expressions. As was shown in [14], the FDM is an efficient, accurate, and easily implemented method for thermal modeling of cast-resin dry-type transformers.

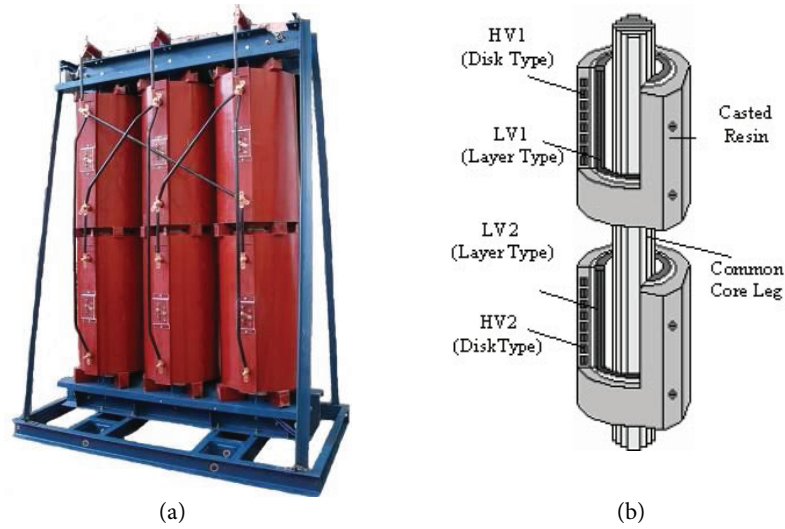


Figure 2. Split-winding cast-resin transformer: a) three-phase and b) single-phase views.

As shown in Figure 3, in the FDM, windings are to be separated into small units, each represented by a node (each node is connected to the others through thermal resistances) [14]. The finite difference technique and extracting nodal equations are based on the rule of energy conversion: “heat generated in the unit is equivalent to heat transferred to other units”. The heat flow among a pair of nodes might be explained as “temperature difference/thermal resistance among nodes”.

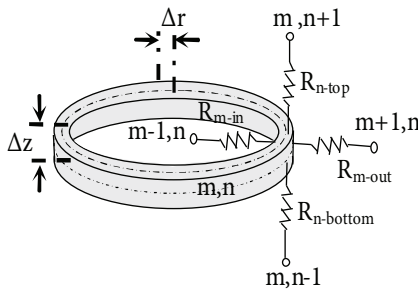


Figure 3. A unit of FDM and its related node [13].

By applying the energy conversion law to the node presented in Figure 3, Eq. (1) can be obtained:

$$\frac{(T_{m,n} - T_{m-1,n})}{R_{m-in}} + \frac{(T_{m,n} - T_{m+1,n})}{R_{m-out}} + \frac{(T_{m,n} - T_{m,n-1})}{R_{n-bottom}} + \frac{(T_{m,n} - T_{m,n+1})}{R_{n-top}} = Q_{m,n}, \quad (1)$$

where R_{m-in} , R_{m-out} , $R_{n-bottom}$, and R_{n-top} are the thermal resistances connecting the node (m, n) and neighboring nodes and $Q_{m,n}$ is the generated heat or losses in the unit. Thermal resistances and losses are explained in the next subsections. In the units belonging to the air (outer surface, the walls of axial and radial air channels, and the bottom and top surfaces), temperature must be replaced by the ambient temperature.

By applying Eq. (1) to all nodes in the windings, the matrix form of the nodal heat transfer equations can be derived. Heat transfer equations depend on temperature [14], so an iterative process has been used to solve them and winding temperatures have been calculated.

2.1. Heat transfer in solid parts (windings)

According to the firm structure of windings, the heat transfer method is conduction [17,18]. Thermal resistance related to conduction between two nodes can be described as in Eq. (2):

$$R_{cw} = \frac{Ak_w}{L_c}, \quad (2)$$

where A is the heat transfer cross-section (2π times mean radius of the related nodes), L_c is the length of the heat flow path (distance between the related nodes), and k_w is the winding's thermal conductivity across the heat flow.

2.2. Heat transfer in outer surfaces

Outer surfaces of the split-winding cast-resin transformer might be assumed like vertical plates [14]. Heat transfer in these surfaces can be considered as natural convection and radiation.

Natural convection: In order to avoid the difficulty of convection equations [17,18], here some simple experimental equations are used. By assuming the outer surfaces as the same as infinite walls, the natural convection in these surfaces can be expressed as follows [14]:

$$q_{co}(z) = h_{co}A(T_s - T_{air}), \quad (3)$$

$$R_{co} = \frac{1}{h_{co}A}, \quad (4)$$

$$h_{co}(z) = \frac{Nu_z k_{air}}{Z}, \quad (5)$$

where q_{co} is the heat transferred by convection in the outer surface, h_{co} is the convection coefficient in the outer surface, R_{co} is the thermal resistance due to the convection, Z is the vertical distance, and

$$Nu_z = \left[\frac{4 Pr^2 Gr^*}{36 + 45 Pr} \right]^{\frac{1}{5}}, \quad (6)$$

$$Gr^* = \frac{g\beta\bar{q}''_{co}Z^4}{k_{air}\nu^2}, \quad (7)$$

where Gr^* is the Grashof number, Pr is the Prandtl number, Nu_z is the Nusselt number, and g is acceleration of gravity. k_{air} , ν , and β are the thermal conductivity, the kinematical viscosity, and the volumetric expansion of the air, and they depend on temperature [14].

From Eqs. (4) and (5), it is seen that the thermal resistivity between the outer surfaces and the ambient temperature increases with an increase in height. This will cause a considerable increase in the temperature of the top windings in comparison with the bottom windings.

Radiation: Radiation in outer surfaces of the split-winding dry-type transformer can be expressed as follows [17,18]:

$$q_{ro}(z) = \varepsilon\sigma A(T_s^4 - T_{air}^4) = h_{ro}A(T_s - T_{air}), \quad (8)$$

$$R_{ro} = \frac{1}{h_{ro}A} = \frac{1}{\varepsilon\sigma A(T_s^3 + T_s T_{air}^2 + T_s^2 T_{air} + T_{air}^3)}, \quad (9)$$

where q_{ro} is the radiation heat transferred in the outer surface, h_{ro} is the radiation coefficient in the outer surface, R_{ro} is the thermal resistance due to radiation in the outer surface, ε is the emissivity coefficient, T_s is the surface temperature, T_{air} is the ambient temperature, and σ is the Boltzmann coefficient.

2.3. Heat transfer in air channels between windings

Air channels in the split-winding dry-type transformer can be assumed as vertical channels [14]. Heat transfer in these surfaces might be considered as radiation, conduction, and natural convection. It was shown in [14] that the radiation in the air channels has a negligible effect on heat transfer and so it has been neglected in this research. Conduction and natural convection in vertical ducts of the traction (split-winding) transformer are explained in the following text.

Conduction: Most of the conduction in the air channel may occur between the walls (radial direction) and conduction in the axial direction may be neglected. Consequently, conduction’s thermal resistance in the air channel can be described as in Eq. (10):

$$R_{ca} = \frac{Ak_a}{b}, \tag{10}$$

where k_a is the thermal conductivity of the air.

Natural convection: Similarly, in order to avoid the difficulty of theoretical equations, some simple equations have been used for natural convection. The natural convection in air channels is similar to channels with infinite walls and its relations can be described as follows [14]:

$$q_{cd} = h_{cd}A(T_s - T_{air}), \tag{11}$$

$$R_{cd} = \frac{1}{h_{cd}A}, \tag{12}$$

$$h_{cd}(z) = \frac{Nu_z k_{air}}{b}, \tag{13}$$

$$Nu_z = C_1 (1 + R)^{\frac{1}{6}} \psi_Z^{\frac{1}{3}}, \text{ for } \psi_Z \leq 60^\circ, \tag{14}$$

$$Nu_z = C_2 \left[(1 + R) \left[\frac{1}{\psi_Z} \left(\frac{24}{1 + R} \right)^{\frac{1}{2}} - \frac{9}{70} \right] + \frac{1}{2} \right]^{-1}, \text{ for } \psi_Z \geq 60^\circ, \tag{15}$$

$$\Phi_Z = \frac{\left(\frac{b}{Z}\right) Gr^* Pr}{\left[\left(\frac{b}{L}\right) Gr^* Pr\right]^{\frac{1}{2}}}, \tag{16}$$

$$Gr^* = \frac{g\beta q'' b^4}{k\nu^2}, \tag{17}$$

where q_{cd} is the convection heat transferred from the walls (inner or outer), R_{cd} is the thermal resistance due to the convection in the duct, q'' is the average heat flux of the walls, R is the ratio of heat flux in other wall of duct/heat flux in considered wall, h_{cd} is the convection coefficient of the wall, b is the width of the duct, and L is the total height of the duct. In these equations C_1 and C_2 are constant coefficients that are related to the duct’s width.

Similarly, thermal resistivity of the walls increases in proportion with the height and this causes a higher temperature in the top windings.

2.4. Heat transfer in bottom and top surfaces

Heat transfer in the bottom and top surfaces is unknown and is a combination of radiation and natural convection. Total heat transfer in these surfaces can be described as in Eqs. (18) and (19):

$$q_{t,b} = h_{t,b}A(T_s - T_{air}), \tag{18}$$

$$R_{t,b} = \frac{1}{h_{t,b}A}, \tag{19}$$

where $h_{t,b}$ is the equivalent heat transfer coefficient, $R_{t,b}$ is the equivalent thermal resistance, and $q_{t,b}$ is the heat transferred to the air from the bottom and top surfaces.

It was shown that heat transfer from the bottom and top surfaces can be neglected without any considerable effect on thermal modeling [14].

2.5. Heat transfer in the distance between the bottom and top windings

Heat transfer in this part is a combination of conduction, radiation, and convection; as with the bottom and top surfaces, its behavior is not known completely. Heat transfer from the channel between the bottom and top windings depends on many parameters and, similar to the bottom and top surfaces, it can be neglected.

The only problem may be the amount of air that enters the ducts from this distance and deforms the normal heat transfer mechanism in the air channels. Neglecting the air that enters from this distance can cause the temperatures to be calculated as slightly higher than the actual amounts. Additionally, the distance between the bottom and top windings is usually designed to be very small and sometimes it may be removed for mechanical reasons.

2.6. Heat generation (losses) in the windings

Windings in split-winding dry-type transformers are usually constructed of foil conductors; it is convenient to consider the conductor's cross-section as in Figure 4 (w' times h' and w times h are the dimensions of the conductor with and without insulation).

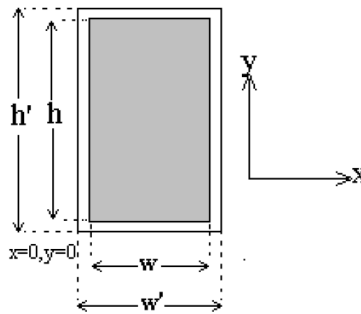


Figure 4. Foil conductor's cross-section.

For this unit, losses might be written as in Eq. (20):

$$P_{total} = (1 + K_{eddy})P_{dc}, \tag{20}$$

where K_{eddy} and P_{dc} are given in [19,20] and they vary with temperature. K_{eddy} is calculated based upon the magnetic field and its accuracy depends on the electromagnetic modeling method. In this research an analytical

method [4] is used for magnetic field calculations. If higher accuracy is needed, the semianalytical calculation introduced in [4] can be used for this purpose. Nodes are chosen in the insulations; thus, each node contains $\frac{1}{4}$ part of conductors (and conductors' losses) adjacent to insulation (Figure 5).

Note that the mentioned split-winding traction transformer can be considered as two pairs of windings that are constructed on each other (axially). Each of these pairs of windings acts as a two-winding cast-resin transformer. Therefore, it is helpful to analyze the temperature distribution of a normal two-winding cast-resin transformer (with concentric windings) and compare its results to a split-winding traction transformer.

3. Thermal analysis of two-winding dry-type transformer with concentric windings

To verify the introduced model, a typical two-winding transformer (800 kVA [14]) is chosen and the measured temperatures are compared to the modeling results on the outer surface of the HV winding (Figure 6). Note that the temperature rise test was carried out in accordance with the simulated loading method as defined by the IEC 60076-11 standard [3,14] and the outer surface temperatures were measured using an infrared gun [14].

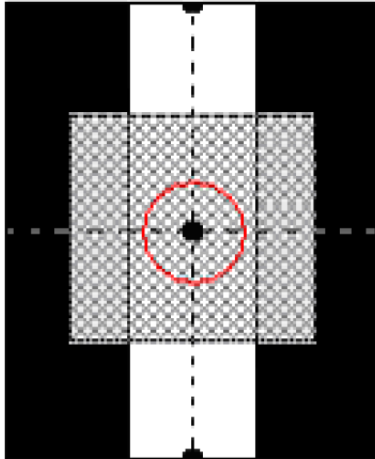


Figure 5. Node losses (includes the adjacent conductors).

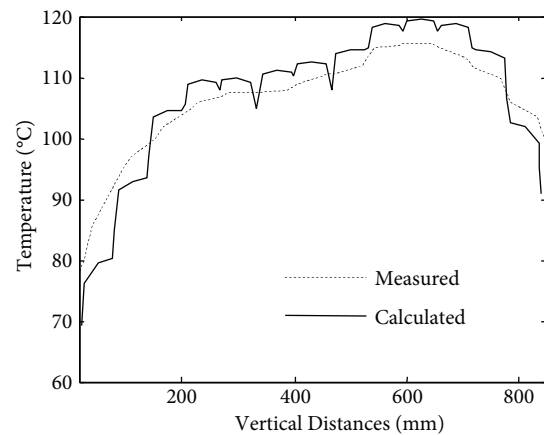


Figure 6. Temperature of the outer surface in a typical 800 kVA transformer.

Table 1 shows the computed average and hottest spot temperatures and compares them with the experimental results. Similarly, the temperature rise test was carried out in accordance with the simulated loading method as defined by IEC 60076-11 and average temperature rises of the windings were measured by gathering the hot and cold resistances of the windings [3,14]. The average temperature rises (AV) in this table are the average of the temperature rises in each winding and the hottest spot temperature rises (HS) are the maximum temperature rises that occur in the windings. LV and HV in this table denote the low- and high-voltage windings (Figure 1).

From Figure 6 and Table 1, it is seen that the FDM has good accuracy in predicting the temperature distribution of the cast-resin transformer (average temperature prediction error is lower than 3%). Additionally, the differences between modeling results and the experiments may be caused by some experimental and modeling insufficiencies [14]. Consequently, the presented mathematical model (based on the FDM) can be employed to analyze the thermal behavior of the cast-resin transformer with different structures (such as the mentioned split-winding traction transformer).

Table 1. Temperature rises in the typical 800 kVA transformer.

	AV: Average temperature rise (°C)		HS: Hottest spot temperature rise (°C)		HS/AV	
	LV	HV	LV	HV	LV	HV
Modeling	86.88	95.8	109.36	118.1	1.26	1.29
Experience	85	93	Not applicable		Not applicable	

Consequently, Figure 7 shows the temperature distribution of the windings in the mentioned two-winding dry-type transformer.

4. Thermal analysis of dry-type (cast-resin) traction transformer with four split windings

After reviewing the thermal behavior of the normal two-winding transformer, in this section temperature distribution of the split-winding dry-type traction transformer is analyzed. The traction (split-winding) transformer usually consists of two split transformers that are positioned on a same core leg. Usually the bottom and top transformers are not the same, but to obtain a clear and obvious conclusion, the top and bottom windings are assumed to be as same as the 800 kVA transformer in Section 3 (this will not cause a problem in studying the traction transformer’s behavior). Applying the nominal currents to the windings, temperature rises were calculated and the results are given in Table 2.

Table 2. Average and hottest spot temperature rises in a typical 1600 kVA split-winding traction transformer.

	AV (°C)		HS (°C)		HS/AV	
	LV	HV	LV	HV	LV	HV
Bottom windings	77.83	88.73	95.92	114.7	1.23	1.29
Top windings	110.57	103.46	150.91	122.37	1.36	1.18
Two-winding transformer (Table 1)	86.88	91.8	109.36	118.01	1.26	1.29

Consequently, temperature distributions in the bottom and top windings are shown in Figure 8.

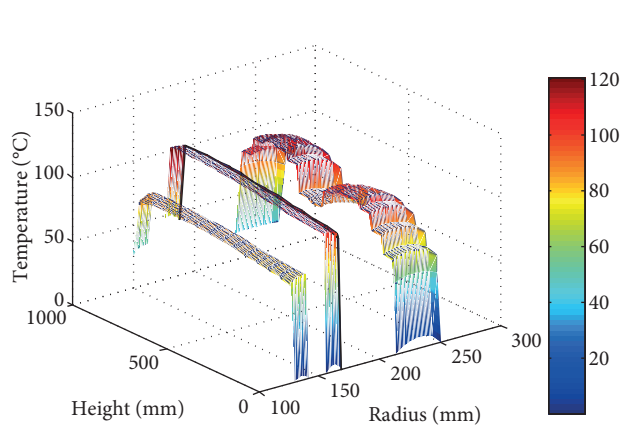


Figure 7. The windings temperature distribution in 800 kVA transformer.

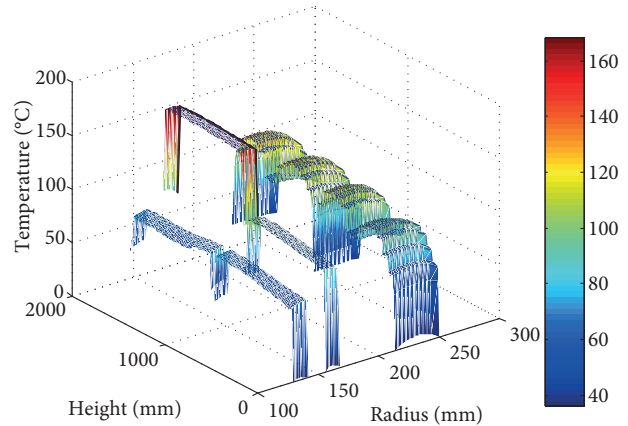


Figure 8. Temperature distribution of the bottom and top windings in a typical 1600 kVA split-winding transformer.

It is clear that the top windings are considerably hotter than in a similar two-winding transformer (Section 3). However, the bottom windings are cooler than when they are used in a normal two-winding transformer. These behaviors must be taken into account by designers of traction transformers.

Someone may ask why the bottom windings are cooler than when they are manufactured as in a normal two-winding transformer. This can be correlated to characteristics of the air channels in Eq. (17) where the heat transfer is proportional to the height of the air channel (L). It is obvious that when two windings are manufactured axially on each other, the height of the ducts will be doubled and thus heat transfer is increased in the bottom windings. In the top windings the local height of the nodes (Z) is increased more than the duct's height (L) and so the heat transfer will be decreased.

The difference between temperatures of the split-winding and the normal transformer (or difference between temperatures of the bottom and top windings in the split-winding transformer) is higher in the low-voltage windings (especially in outer layers of the low-voltage windings). Consequently, the hottest spot (HS) temperature is more dangerous in top low-voltage winding; in this winding HS/AV is about 1.36, which is considerable in comparison with 1.26 in the normal two-winding transformer. A suitable and possible proposal is to decrease the height of the split-winding transformer. It can be shown that decreasing the height of the transformer can modify this phenomenon and will reduce the average (AV) and hottest spot (HS) temperatures of the top windings.

5. Harmonics in the traction transformer and their effects on the temperature distribution

Basically the transformer is designed to work with a 50 Hz frequency. However, the loads in a traction system are usually nonlinear. Nonlinear loads cause high levels of voltage and current harmonics. The most important effect of harmonics in the transformer is their influence on the temperature distribution of the windings. Voltage's harmonics just influence the no-load losses and this can easily be neglected [21]. In this section, only the effects of current harmonics on the temperature distribution of the windings are analyzed.

The traction (split-winding) transformer usually feeds a 12-pulse converter; this converter is a major source of harmonics. If we assume the current as $I_L = I_h \cos(h\omega t)$, harmonics (I_h) related to a traction transformer that feeds a 12-pulse rectifier can be calculated as in Eq. (21) [22–24]. These harmonics are shown in Figure 9.

$$\frac{I_h}{I_1} = \frac{1}{\left(h - \frac{5}{h}\right)^{1.2}}, \tag{21}$$

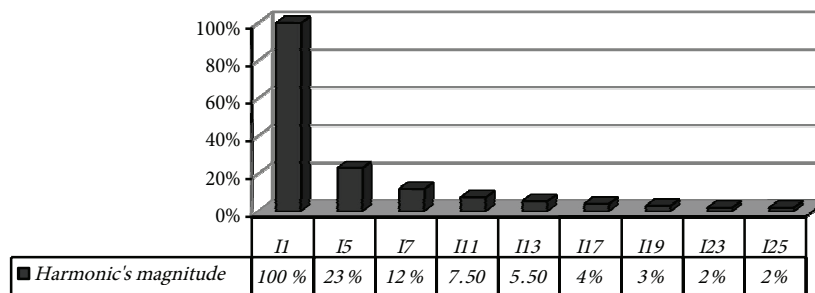


Figure 9. Magnitude of current harmonics proportional to fundamental component in the traction transformer.

where

$$h = \frac{P}{2}k \pm 1, P = 12 \text{ (12 - pulse), and } k = 0, 1, 2, \dots \tag{22}$$

Harmonic currents cause additional losses in windings; harmonics have major effects on the eddy losses. The harmonics may increase the ohmic losses because of increasing the current's rms. Stray losses are negligible in dry-type transformers and here they are assumed to be zero. For a harmonic current, ohmic and eddy current losses can be expressed as in Eqs. (23) and (24) [22–24].

$$P_{dc} = P_{dc0} \sum \lim_{h=1}^n \left(\frac{I_h}{I_0} \right)^2, \tag{23}$$

$$P_{eddy} = P_{eddy0} \sum \lim_{h=1}^n \left(\frac{I_h}{I_0} \right)^2 h^2, \tag{24}$$

where P_{dc0} and P_{eddy0} are the ohmic and eddy current losses where there are no harmonics. Combining Eqs. (23) and (24), total losses due to the harmonic currents can be expressed as in Eq. (25).

$$P_{total} = P_{dc0} \sum \lim_{h=1}^n \left[\left(\frac{I_h}{I_0} \right)^2 (1 + K_{eddy}h^2) \right] \tag{25}$$

Using Eq. (25) and applying it to the introduced thermal model, the winding temperatures of the traction transformer have been calculated as in Table 3 and Figure 10.

Table 3. Temperature rises of the split-winding traction transformer due to the harmonics.

		AV (°C)		HS (°C)		HS/AV	
		LV	HV	LV	HV	LV	HV
Without harmonics	Bottom windings	77.83	88.73	95.92	114.7	1.23	1.29
	Top windings	110.57	103.46	150.91	122.37	1.36	1.18
With harmonics	Bottom windings	87	95.6	107	123.7	1.23	1.29
	Top windings	140	115	190	135.5	1.36	1.18

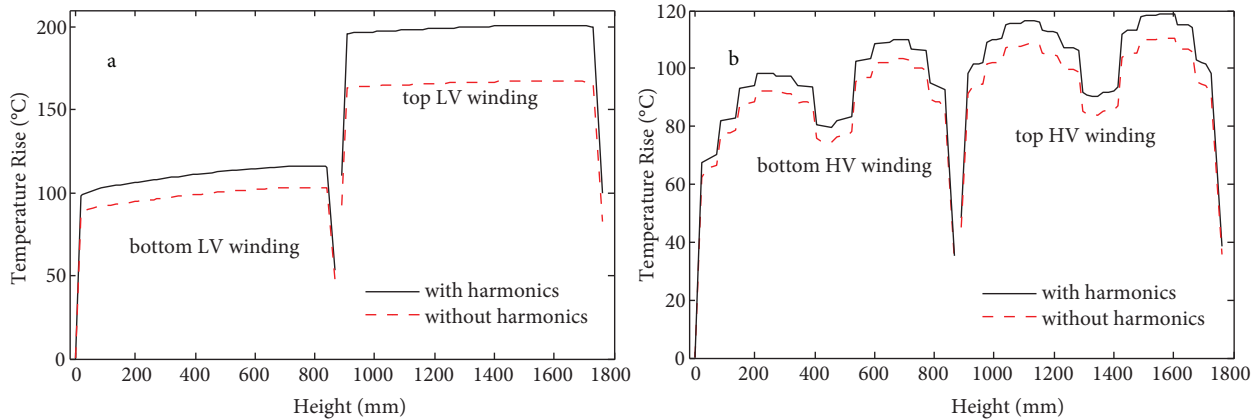


Figure 10. Temperature distribution in the outer surface of a) the low-voltage and b) the high-voltage windings.

Rises in the temperatures due to harmonic currents cannot be neglected in the traction transformer. In some cases, the temperature may reach up to 127% of the temperature without harmonic consideration. The ratio of the hottest spot to the average temperature (HS/AV) is invariant against linear and nonlinear loads and just depends on the transformer's structure.

6. Conclusion

In this paper, a cast-resin traction transformer with split windings was modeled in order to analyze the thermal behavior of windings. The model was validated using a typical 800 kVA cast-resin transformer. The validated thermal model was used to calculate the temperature distribution of a normal transformer with two concentric windings. Afterwards, the thermal behavior of the traction transformer with four split windings was modeled and the temperatures were compared to a normal two-winding transformer.

Reviewing the thermal behavior of the split-winding traction transformer, it was seen that the top windings are considerably hotter than the bottom windings (and hotter than the same two-winding transformer). However, windings in the bottom are cooler than in the same two-winding transformer. This problem is serious in outer layers of the low-voltage windings. Additionally, the hottest spot temperatures of the top low-voltage windings are more dangerous than other parts. From the viewpoint of a designer, this phenomenon must be taken into account in temperature calculations of a split-winding traction transformer. A way for modifying this behavior in the traction transformer is to decrease the height of windings in comparison with their radius.

Finally, the effects of the current's harmonics on the load losses and temperature distribution were discussed. An increase in the temperature due to the harmonics cannot be neglected in the design process of the traction transformer.

References

- [1] Azizian D, Vakilian M, Faiz J, Bigdeli M. Calculating leakage inductances of split-windings in dry-type traction transformers. *ECTI Transactions on Electrical Engineering, Electronics, and Communications* 2012; 10: 99-106.
- [2] Nunn T. A comparison of liquid-filled and dry-type transformer technologies. In: *2000 IEEE-IAS/PCA Cement Industry Technical Conference*; 7–12 May 2000; Salt Lake City, UT, USA.
- [3] IEC. IEC 60076-11. Dry Type Power Transformers. 1st ed. Geneva, Switzerland: IEC, 2004.
- [4] Azizian D, Vakilian M, Faiz J. A new multi-winding traction transformer equivalent circuit for short-circuit performance analysis. *Int T Electr Energy* 2014; 24: 186-202.
- [5] Azizian D. Design Schemes in Traction Transformers. Tehran, Iran: Iran Transformer Research Institute, 2007 (in Persian).
- [6] Chase DD, Grain AN. Split winding transformers. *Transactions of the American Institute of Electrical Engineers* 1934; 53: 914-922.
- [7] IEC. IEC 60310. Railway Applications – Traction Transformers and Inductors on Board Rolling Stock. 3rd ed. Geneva, Switzerland: IEC, 2004.
- [8] Dianchun Z, Jiaxiang Y, Zhenghua W. Thermal field and hottest spot of the ventilated dry-type transformer. In: *IEEE Proceedings of the 6th International Conference on Properties and Applications of Dielectric Materials*; 21–26 June 2000; Xi'an, China.
- [9] Pierce LW. Thermal consideration in specifying dry-type transformers. *IEEE T Ind Appl* 1994; 30: 1090-1098.
- [10] Cho HG, Lee UY, Kim SS, Park YD. The temperature distribution and thermal Stress analysis of pole cast resin transformer for power distribution. In: *IEEE Conference International Symposium on Electrical Insulation*; 7–10 April 2002; Boston, MA, USA.

- [11] Pierce LW. An investigation of the temperature distribution in cast-resin transformer windings. *IEEE T Power Deliver* 1992; 7: 920-926.
- [12] Pierce LW. Predicting hottest spot temperatures in ventilated dry type transformer windings. *IEEE T Power Deliver* 1994; 9: 1160-1172.
- [13] Lee M, Abdullah HA, Jofriet JC, Patel D, Fahrioglu M. Air temperature effect on thermal models for ventilated dry-type transformers. *J Elec Power Sys Res* 2011; 81: 783-789.
- [14] Rahimpour E, Azizian D. Analysis of temperature distribution in cast-resin dry-type transformers. *J Elec Eng* 2007; 89: 301-309.
- [15] Eslamian M, Vahidi B, Eslamian A. Thermal analysis of cast-resin dry-type transformers. *Energ Convers Manage* 2011; 52: 2479-2488.
- [16] Azizian D, Bigdeli M, Fotuhi-Firuzabad M. A dynamic thermal based reliability model of cast-resin dry-type transformers. In: *International Conference on Power System Technology*; 24–28 October 2010; Hangzhou, China.
- [17] Holman JP. *Heat Transfer*. 10th ed. New York, NY, USA: McGraw-Hill, 2009.
- [18] Incropera FP, Dewitt DP. *Introduction to Heat Transfer*. 7th ed. New York, NY, USA: John Wiley & Sons, 2011.
- [19] Dietrich W. Berechnung der Wirbelstromverluste in den Wicklungen von Mehrwicklungstransformatoren. *J ETZ-Archiv* 1998; 10: 309-317 (in German).
- [20] Rahimpour E. *Hochfrequente Modellierung von Transformatoren zur Berechnung der Übertragungsfunktionen*. PhD, University of Stuttgart, Stuttgart, Germany, 2003 (in German).
- [21] Elmoudi AA. *Evaluation of power system harmonic effects on transformers*. PhD, Helsinki University, Helsinki, Finland, 2006.
- [22] Elmoudi AA. *Evaluation of power system harmonic effects on transformers: hot spot calculation and loss of life estimation*. In: *5th International Multi-Conference on Systems, Signals and Devices*; 20–22 July 2008; Amman, Jordan.
- [23] Zeraatparvar A, Sami T, Raheli FA. *Evaluation of performance of distribution transformers supplying non linear loads currents: effect of oversizing*. In: *10th International Conference on Environment and Electrical Engineering*; 8–11 May 2011; Rome, Italy.
- [24] IEC. *IEC Std. 61378-1. Converter Transformers - Part 1: Transformers for Industrial Applications*. 2nd ed. Geneva, Switzerland: IEC, 2011.