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Threshold optimization according to the restricted Bayes criterion in decentralized detection problems

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Abstract: In this paper, the restricted Bayes approach is studied in a decentralized detection problem. All decisions on which the hypothesis is true are made by local sensors through conditionally independent observations. Then these decisions are transmitted to the fusion center for the final decision. In the conventional approach, all thresholds of local sensors and the fusion center are considered as deterministic variables and optimized according to the given criterion for given test statistics of local sensors and the fusion center. In this paper, it is shown that setting thresholds as random variables instead of deterministic ones can improve the performance according to the restricted Bayes criterion. It is proved that optimal random thresholds are dependent on each other, and the probability density function of each one consists of at most two point masses. Two methods for the implementation of this scheme are proposed. A necessary and sufficient condition for improvability of the conventional approach through replacing optimal deterministic thresholds by optimal random ones is derived. Finally, theoretical results are investigated through simulations.

Key words: Decentralized detection, random threshold, restricted Bayes

1. Introduction

The decentralized detection problem was first presented in [1], and has been studied extensively in recent years A limited capacity of the wireless channel is one of the most important issues raised in decentralized detection problems In [2], the decentralized detection problem is studied in a binary hypothesis-testing framework under a limitation on the capacity of wireless channel, over which the maximum transmission rate is specified by R bits of information per unit time It is shown that using R identical binary sensors is asymptotically the optimal strategy, where the number of observation at the local sensors converges to infinity and observations are modeled as identical and independent Gaussian or exponential random variables As an alternative approach to addressing the limited capacity of the wireless channel, data reduction at the local sensors is considered in terms of the optimal quantizer design according to both Bayesian and Neyman–Pearson criteria [3] In the presence of full uncertainty about a distribution of the additive noise at the local sensors, a universal decentralized scheme is proposed in [4], which is operating under the bandwidth constrained communication channels between sensors and the fusion center The error probability is shown to decay exponentially with a rate that is bounded from below by the noise range for bounded noise and by SNR for unbounded noise.

The frequency of occurrence of one hypothesis happens to be much higher than that of the others under some scenarios in decentralized detection problems [5,6] In those circumstances, the censoring scheme is proposed for sensor networks operating under the limited energy resources and limited wireless channel capacity in order

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to exploit this gap between the frequencies of occurrence of hypotheses [5] Under the censoring scheme, local sensors do not transmit the observation for which the corresponding value of the local likelihood ratio is in the censoring interval, which means that the related observation is not considered as informative and simply discarded [5] The censoring scheme is also studied for sensor networks including randomized decision rules and operating in the presence of uncertainty about the observations at the local sensors [6].

In the collaborative human decision making problem under a binary hypothesis testing framework, thresholds used by the individual human agents to decide on which hypothesis is true are modeled as random variables because of mainly unpredictability and cognitive limitations of humans [7] Decentralized detection schemes find extensive applications areas in defense systems; therefore it is important to develop novel schemes performing better than conventional ones To that end, in this study, thresholds of local sensors and the fusion center are set as random variables to improve the performance over the conventional approach, in which thresholds are deterministic, according to the restricted Bayes criterion in a decentralized detection problem under a binary hypothesis testing framework It turns out that optimum random thresholds are dependent on each other and the probability density function (PDF) of each consists of at most two point masses This case can in fact be considered as dependent randomization of decision rules of local sensors and the fusion center, where each decision rule is selected from a set of deterministic decision rules based on the realization of the associated discrete random variable [8] In our case, the set of deterministic decision rules consists of the decision rules with the same test statistics but different deterministic thresholds Dependent randomization of decision rules is studied in [8] under the Neyman–Pearson framework in a general sense In this study, we focus on threshold randomization, which is a special case of randomization of decision rules, under the restricted Bayes framework Focusing on threshold randomization instead of generic randomization of decision rules allows us to do quantitative and detailed analysis of the proposed scheme To the best of our knowledge, there is no study considering the effects of replacing deterministic thresholds with random ones on decentralized detection under the restricted Bayes framework.

In order to implement optimal random thresholds that are dependent on each other, one way is to allow the fusion center to control all thresholds of local sensors However, this requires extensive communication capacity and increase in the level of centralization opposed to the nature of decentralized detection as stated in [8] for dependent randomization of decision rules There are two alternative ways proposed in [8] for generic implementation of dependent randomization of decision rules under the Neyman–Pearson framework However, they can be easily adapted to our case of dependent randomization of thresholds as follows One of the alternative ways is to make all local sensors and the fusion center implement a predefined sequence of the different sets of thresholds without any communication among themselves The other alternative is to use the common clock in all local sensors and the fusion center to choose which threshold to use.

In this study, we consider the restricted Bayes criterion to handle the uncertainty in the observations of the local sensors. More specifically, the aim of this study is to investigate the effects of setting and accordingly optimizing thresholds as random variables instead of deterministic variables under the restricted Bayes framework, whose aim is to minimize the global probability of error while keeping the worst-case global probability of error below the predefined level [9,10]. We also consider the likelihood ratio as a test statistic for the fusion center, which is a common use in practice. Uncertainty in the transmission of decisions of local sensors to the fusion center is modeled by a binary asymmetric channel (BASC).

To the best of our knowledge, the restricted Bayes approach has never been studied before in decentralized detection problems. Therefore, this study also aims to illustrate the use of this approach in the presence of uncertainty about observations at the local sensors. Along with the illustration of the restricted Bayes approach in sensor networks, it is also shown how to optimize the restricted Bayes criterion by employing dependent random thresholds at the local sensors and the fusion center.

The paper is organized as follows. In Section 2, the problem formulation is given, and derivations for optimum random thresholds are provided. The statistical characterization of the optimum random thresholds is studied along with the calculation of the optimal PDFs of thresholds in Section 3. In Section 4, a sufficient and necessary condition for the improvability of the conventional approach through replacing deterministic thresholds by random ones is provided. In Section 5, a numerical example is studied to investigate theoretical results. Finally, concluding remarks are presented in Section 6.

2. Problem formulation

Consider the decentralized detection problem, in which each local sensor decides on which hypothesis is true; then all decisions of the local sensors are transmitted to the fusion center where the final decision is made. We have N local sensors and the observation vector $\mathbf{x}_i \in \mathrm{R}^K$ at the local sensor i can be expressed under binary hypotheses as follows:

$$\mathcal{H}_0 : \mathbf{x}_i = \mathbf{s}_{i0} + \mathbf{n}_i, \mathcal{H}_1 : \mathbf{x}_i = \mathbf{s}_{i1} + \mathbf{n}_i, \tag{1}$$

where $\mathbf{n}_i$ is the background noise with PDF $p_{\mathbf{n}_i}(\cdot)$. The signals are modeled as random vectors $\mathbf{s}_0 = [\mathbf{s}_{10}^T \mathbf{s}_{20}^T ... \mathbf{s}_{N0}^T]^T$ with the estimated PDF $p_{\mathbf{s}_0}(\cdot)$ under the hypothesis $\mathcal{H}_0$ and $\mathbf{s}_1 = [\mathbf{s}_{11}^T \mathbf{s}_{21}^T ... \mathbf{s}_{N1}^T]^T$ with the estimated PDF $p_{\mathbf{s}_1}(\cdot)$ under the hypothesis $\mathcal{H}_1$. In practice, true PDFs of the signals can be very different from the estimated ones due to estimation errors in obtaining $p_{\mathbf{s}_0}(\cdot)$ and $p_{\mathbf{s}_1}(\cdot)$ [9,10]. In other words, there exists uncertainty in the PDFs of the signals. In the restricted Bayes criterion, the worst-case scenario is also involved to take this uncertainty into account [9,10]. Therefore, in the restricted Bayes framework, the least-favorable PDFs of the signals corresponding to the worst-case scenario are also considered along with the estimated ones [9,10]. Let us denote the least favorable PDFs of the signals under hypotheses $\mathcal{H}_1$ and $\mathcal{H}_0$ with $p_{\mathbf{s}_1}^{\mathbf{ls}}(\cdot)$ and $p_{\mathbf{s}_0}^{\mathbf{ls}}(\cdot)$, respectively. Observations of the local sensors are also assumed to be conditionally independent. It should be noted that the least-favorable PDFs of the signals may be dependent on thresholds, that is, when the thresholds of the local sensors and the fusion center change the least-favorable PDFs may accordingly change in some cases.

The fusion center and all local sensors employ fixed test statistics. Let us denote the test statistic at the local sensor i with $\mathcal{T}_i(\cdot)$ and at the fusion center with $\mathcal{L}(\cdot)$. In the conventional approach, thresholds of the fusion center and local sensors are considered as deterministic variables, and optimized according to the given criterion [8,11], which is the restricted Bayes criterion in this study.

Let us denote the threshold of the local sensor i with η_i and the threshold of the fusion center with τ. In this study, while a random variable is denoted in bold font, its realization is depicted without bold font. We define the random vector η with the PDF $p_\eta(\cdot)$ consisting of all thresholds of the local sensors: $\eta = [\eta_1 \eta_2 \cdots \eta_N]^T$.

The decision rule at the local sensor i is denoted with ϕ_i, where $\phi_i(x_i) = 1$ if $\mathcal{T}_i(x_i) \geq \eta_i$, otherwise $\phi_i(x_i) = 0$. The observation received at the fusion center from the local sensor i is denoted with $\mathbf{u}_i$, where $u_i \in \{0, 1\}$. Define the random vector $\mathbf{u}$ consisting of all observations received at the fusion center from the local sensors: $\mathbf{u} = [\mathbf{u}_1 \mathbf{u}_2 \cdots \mathbf{u}_N]^T$. The fusion center makes a final decision based on the observation $\mathbf{u}$, that is, $\phi(u) = 1$ if $\mathcal{L}(u) \geq \tau$, otherwise $\phi(u) = 0$, where $\phi(\cdot)$ is the decision rule employed at the fusion center.

A binary asymmetric channel (BASC) is assumed, and the crossover probabilities are defined as $c_{0i} = p(\mathbf{u}_i = 1 | \phi_i(x_i) = 0)$ and $c_{1i} = p(\mathbf{u}_i = 0 | \phi_i(x_i) = 1)$ for $i = 1, 2 \ldots, N$.

When the signal and thresholds of local sensors are given, observations ($\mathbf{u}_i$ for $i = 1, 2..., N$) at the fusion center are independent from each other. Accordingly, we have the following:

$$p(u|\eta, s_k, \mathcal{H}_k) = \prod_{i=1}^{N} p(u_i|\eta_i, s_{ik}, \mathcal{H}_k). \tag{2}$$

The probability of local sensor i deciding on $\mathcal{H}_1$ when the threshold and the signal are given is denoted with $F_{ik}(\eta_i, s_{ik}) = p(\phi_i(x_i) = 1|\eta_i, s_{ik}, \mathcal{H}_k)$. Then $p(u_i|\eta_i, s_{ik}, \mathcal{H}_k)$ can be expressed as follows:

$$p(\mathbf{u}_i = 1|\eta_i, s_{ik}, \mathcal{H}_k) = (1 - c_{1i})F_{ik}(\eta_i, s_{ik}) + c_{0i}(1 - F_{ik}(\eta_i, s_{ik})) \tag{3}$$

$$p(\mathbf{u}_i = 0|\eta_i, s_{ik}, \mathcal{H}_k) = (1 - c_{0i})(1 - F_{ik}(\eta_i, s_{ik})) + c_{1i}F_{ik}(\eta_i, s_{ik}) \tag{4}$$

The PDFs of the observations received at the fusion center for the expected and the worst-case scenarios can be calculated as follows:

$$p(u|\mathcal{H}_k) = \int_{\mathrm{RN}} \int_{\mathrm{R}^K} p(u|\eta, s_k \mathcal{H}_k) p_{s_k}(s_k) p_\eta(\eta) ds_k d\eta$$

$$p^{ls}(u|\mathcal{H}_k) = \int_{\mathrm{R}^N} \int_{\mathrm{R}^K} p(u|\eta, s_k \mathcal{H}_k) p^{ls}_{s_k}(s_k) p_\eta(\eta) ds_k d\eta$$

In this study, the fusion center is assumed to use the likelihood ratio as a test statistic. When the thresholds of the local sensors are given, the likelihood ratio can be calculated as follows:

$$\mathcal{L}(u) = \frac{\int_{\mathrm{R}^K}^{p(u|\eta, s_1 \mathcal{H}_1) p_{s_1}(s_1)} ds_1}{\int_{\mathrm{R}^K}^{p(u|\eta, s_0 \mathcal{H}_0) p_{s_0}(s_0)} ds_0} = \frac{G_1(u, \eta)}{G_0(u, \eta)}, \tag{5}$$

where $G_k(u, \eta) = \int_{\mathrm{R}^K}^{p(u|\eta, s_k \mathcal{H}_k) p_{s_k}(s_k)} ds_k$ for $k = 0, 1$. For convenience, let us also define $G^{ls}_k(u, \eta) = \int_{R^K}^{p(u|\eta, s_k \mathcal{H}_k) p^{ls}_{s_k}(s_k)} ds_k$ for $k = 0, 1$.

Since $u_i \in \{0, 1\}$, the size of the set consisting of the possible realizations of $\mathbf{u}$ is 2^N. Therefore, $\mathcal{L}(u)$ can take 2^N different values corresponding to a possible realization of $\mathbf{u}$. Let us arrange the values of $\mathcal{L}(u)$ in ascending order as $l_1, l_2, ..., l_{2^N}$ with corresponding values of u denoted with u^1, $u^2, \ldots,$ u^{2^N}, that is, $l_i = \mathcal{L}(u^i) = \frac{G_1(u^i, \eta)}{G_0(u^i, \eta)}$.

For convenience let us represent all thresholds with the random vector θ having the PDF $p_\theta(\cdot)$, which is defined as $\theta = [\eta^T \tau]^T$. Then probabilities of the fusion center deciding on $\mathcal{H}_1$ given that the true hypothesis is $\mathcal{H}_1$ (the global detection probability) for the expected and the worst-case scenarios can be calculated as follows:

$$P_D(\theta) = \sum_{i=0}^{2^N - 1} P_{D_i}(\theta) \tag{6}$$

$$P^{ls}_D(\theta) = \sum_{i=0}^{2^N - 1} P^{ls}_{D_i}(\theta), \tag{7}$$

where $P_{D_0}(\theta) = I(\tau \leq l_1)$, $P_{D_i}(\theta) = I(l_i < \tau \leq l_{i+1})\left(G_1(u^{i+1},\eta) + G_1(u^{i+2},\eta) + \cdots + G_1(u^{2^N},\eta)\right)$, similarly $P^{ls}_{D_i}(\theta) = I(\tau \leq l_1), P^{ls}_{D_i}(\theta) = I(l_i < \tau \leq l_{i+1})\left(G^{ls}_1(u^{i+1},\eta) + G^{ls}_1(u^{i+2},\eta) + \cdots + G^{ls}_1(u^{2^N},\eta)\right)$ for $i =$ 1, 2,...,(2^N-1) . Here $I(\cdot)$ denotes an indicator function: If the event A is true then $I(A) = 1$, otherwise $I(A) = 0$.

The probabilities of the fusion center deciding on $\mathcal{H}_1$ given that the true hypothesis is $\mathcal{H}_0$ (the global false alarm probability) for the expected and the worst-case scenarios can be computed as follows:

$$P_F(\theta) = \sum_{i=0}^{2^N-1} P_{F_i}(\theta) \tag{8}$$

$$P^{ls}_F(\theta) = \sum_{i=0}^{2^N-1} P^{ls}_{F_i}(\theta), \tag{9}$$

where $P_{F_0}(\theta) = I(\tau \leq l_1), P_{F_i}(\theta) = I(l_i < \tau \leq l_{i+1})\left(G_0(u^{i+1},\eta) + G_0(u^{i+2},\eta) + \cdots + G_0(u^{2^N},\eta)\right)$, similarly $P^{ls}_{F_0}(\theta) = I(\tau \leq l_1)$, $P^{ls}_{Fi}(\theta) = I(l_i < \tau \leq l_{i+1})\left(G^{ls}_0(u^{i+1},\eta) + G^{ls}_0(u^{i+2},\eta) + \cdots + G^{ls}_0(u^{2^N},\eta)\right)$ for $i = 1$, 2,...,(2^N-1).

The global error probabilities at the fusion center for the expected and the worst-case scenarios can be calculated as follows:

$$P_E(\theta) = \pi_1(1 - P_D(\theta)) + (1 - \pi_1)P_F(\theta) \tag{10}$$

$$P^{ls}_E(\theta) = \pi_1(1 - P^{ls}_D(\theta)) + (1 - \pi_1)P^{ls}_F(\theta), \tag{11}$$

where π_1 is the prior probability of hypothesis $\mathcal{H}_1$.

In the restricted Bayes criterion, the goal is to minimize the global probability of error corresponding to the expected scenario under the constraint on the global error probability corresponding to the worst-case scenario.

Therefore, in the conventional approach, the following optimization problem is solved to obtain optimum deterministic thresholds:

$$\min_{\theta} P_E(\theta)$$

$$\text{Subject to } P^{ls}_E(\theta) \leq \beta, \tag{12}$$

where β is a predefined parameter determined based on the level of uncertainty [9,10,12]. Let us denote the optimal deterministic thresholds with θ^{opt}; then in the conventional approach the global error probabilities corresponding to the expected and the worst-case scenarios are $P_E(\theta^{opt})$ and $P^{ls}_E(\theta^{opt})$, respectively. The restricted Bayes criterion generalizes the minimax and the Bayes criteria, and includes them as special cases. In (12), as β increases the restricted Bayes criterion converges to the Bayes criterion, and after some value of β the restricted Bayes becomes equivalent to the Bayes criterion [9,10]. Similarly, as β decreases the restricted Bayes criterion converges to the minimax criterion, and at the minimum value of β the restricted Bayes becomes equivalent to the minimax criterion [9,10]. In fact, the minimum value of β is the probability of error when the minimax criterion is employed.

In the case of thresholds being random variables, the aim is to obtain optimum PDFs of the thresholds that minimize the average global probability of error corresponding to the expected scenario while keeping the average global error probability corresponding to the worst-case scenario below the predefined level:

$$\min_{p_\theta(\cdot)} E_\theta\{P_E(\theta)\}$$

$$\text{Subject to } E_\theta\{P_E^{\mathbf{ls}}(\theta)\} \leq \beta. \tag{13}$$

It should be noted that when the value of β is high enough so that the constraint on the average global error probability corresponding to the worst-case scenario becomes ineffective, then the optimization problem in (13) reduces to the minimization of average global probability of error corresponding to the expected scenario, which is the optimization problem of the Bayes criterion. Under this case, replacing deterministic thresholds by random ones is useless since optimal PDFs of random thresholds that minimize the average global error probability corresponding to the expected scenario consist of only one point mass, which means that random thresholds are indeed deterministic ones.

Since instantaneous error probabilities are equivalent to average error probabilities for deterministic thresholds, all error probabilities mentioned throughout the rest of the paper are averaged ones.

3. Characterization and calculation of optimal solution

The following proposition shows that the optimal PDF of each of the thresholds consists of at most two point masses, and the optimal thresholds depend on each other.

Proposition 1 *Assume that $P_E(\theta)$ and $P_E^{ls}(\theta)$ are continuous functions, and θ belongs to a finite closed set. Then the optimum PDF for θ is in the form of $p_\theta(\theta) = \lambda\delta(\theta - \theta_1) + (1 - \lambda)\delta(\theta - \theta_2)$, where $0 \leq \lambda \leq 1$.*

Proof The proof is similar to the proof of Proposition 1 in [13]. □

We can reformulate (13) by employing the results in Proposition 1:

$$\min_{\{\lambda,\theta_1,\theta_2\}} \lambda P_E(\theta_1) + (1-\lambda)P_E(\theta_2)$$

$$\text{Subject to } \lambda P_E^{ls}(\theta_1) + (1-\lambda)P_E^{ls}(\theta_2) \leq \beta. \tag{14}$$

Techniques for obtaining solution of (14) are extensively studied in [14,15]. In Section V, the particle swarm optimization (PSO) algorithm is used to solve the problem in (14).

Proposition 1 characterizes the optimal PDFs of random thresholds together with the optimum way of implementing them. Specifically, the optimal PDFs consist of at most two point masses, and the optimum way of employing thresholds is to sync them together. To exemplify, for a given time interval T, all local sensors and the fusion center synchronously employ the corresponding thresholds specified by θ_1 during the time interval λT, and they use the thresholds specified by θ_2 in a synchronous manner in the rest part of the time interval (T- λT). According to Proposition 1, the only restriction on the thresholds is them being finite, which is emphasized there by the statement that θ must be confined in finite set, which is already the case in practice. Thanks to the results in Proposition 1, the optimization problem in (13) is reformulated in a manageable form, which has already been well studied.

4. A necessary and sufficient condition for nonimprovability

Next, a necessary and sufficient condition is presented for the nonimprovability of the conventional approach through replacing deterministic thresholds by random ones. To that end, define the auxiliary function $J(t) = \inf(P_E(\theta)|P_E^{ls}(\theta) = t)$. Then we have the following proposition:

Proposition 2 *Conventional approach cannot be improved through replacing deterministic thresholds by random ones if and only if there exist* $\xi \leq 0$ *such that*

$$J(t) \geq (t - \beta)\xi + J(\beta)\forall t \tag{15}$$

Proof The proof is based on the approach in the proof of Proposition 3 in [13]. We only present the sufficiency of the condition due to space limitation. Consider a generic PDF for θ as $p_\theta(\theta) = \lambda\delta(\theta - \theta_1) + (1 - \lambda)\delta(\theta - \theta_2)$; then we have $t_1 = P_E^{ls}(\theta_1)$ and $t_2 = P_E^{ls}(\theta_2)$. Based on the condition in the proposition, we have the following $J(t_1) \geq (t_1 - \beta)\xi + J(\beta)$ and $J(t_2) \geq (t_2 - \beta)\xi + J(\beta)$. Therefore, employing $p_\theta(\theta)$ gives the following relation:

$$P_E(\theta_1) + (1 - \lambda)P_E(\theta_2) \geq \lambda J(t_1) + (1 - \lambda)J(t_2) \geq J(\beta) - \xi(\beta - (\lambda t_1 + (1 - \lambda)t_2)) \geq P_E(\theta^{opt}),$$

because of $\xi \geq 0$ and $\lambda t_1 + (1 - \lambda)t_2 \leq \beta$. □

In Proposition 2, we assume that $P_E^{ls}(\theta^{opt}) = \beta$, which is the case in practice since β is set by the designer based on the uncertainty level. If $J(t)$ is first-order continuously differentiable, we have the relation $\xi = J'(\beta)$.

In some circumstances, deterministic thresholds turn out to be optimal. In those cases, there is no need to engage in the optimization problem in (13). Proposition 2 specifies these circumstances completely, and beforehand gives us certain information about the form of the solution.

5. Numerical results

Consider the decentralized detection problem with two local sensors, and scalar observations at the local sensor i are given as follows:

$$\mathcal{H}_0 : \mathbf{x}_i = \mathbf{n}, \mathcal{H}_1 : \mathbf{x}_i = \mathbf{s} + \mathbf{n}, \tag{16}$$

where $\boldsymbol{s}$ is a random variable with the PDF in the form of $p_{\mathbf{s}}(\mathbf{s}) = 0.5\delta(\mathbf{s} - A) + 0.5\delta(\mathbf{s} + A)$, where $\delta(\cdot)$ is the Dirac delta function and the value of A is estimated based on previous experience. In this model, the signal under $\mathcal{H}_1$ employs binary modulation, namely, binary phase shift keying (BPSK). The background noise $\mathbf{n}$ is symmetric Gaussian mixture with the PDF:

$$p_{\mathbf{n}}(n) = \sum_{i=1}^{M} \omega_i \psi_i(n - \mu_i), \tag{17}$$

where M is the number of Gaussian components in the mixture noise PDF, μ_i is the mean values of the Gaussian components, $\sum_{i=1}^{M} \omega_i = 1$, $\omega_i \geq 0$, $\psi_i(y) = (1/\sqrt{2\pi}\sigma_i)\exp\{-y^2/(2\sigma_i^2)\}$ for $i = 1, \ldots, M$, with σ_i being the standard deviations of the Gaussian components. All parameters are adjusted to make the PDF symmetric around the origin.

The local sensors employ the following test statistics $\mathcal{T}_1(x_1) = x_1^2$ and $\mathcal{T}_2(x_2) = x_2^2$. For this example, we can present $F_{ik}(\eta_i, s_{ik})$ in the closed form expression: $F_{ik}(\eta_i, s_{ik}) = p\left((n+s_{ik})^2 \geq \eta_i | \eta_i, s_{ik}, \mathrm{H}_k\right) = \sum_{m=1}^{M} \omega_m \left(Q\left(\frac{\sqrt{\eta_i}-s_{ik}-\mu_m}{\sigma_m}\right) + Q\left(\frac{\sqrt{\eta_i}+s_{ik}-\mu_m}{\sigma_m}\right)\right)$,

where $s_{i0} = 0$ and $s_{i1} \in \{-A, A\}$, and Q-function is given as $Q(x) = (1/\sqrt{2\pi})\int_x^\infty e^{-t^2/2}dt$. Based on previous experience, A is assumed to be estimated as 5, but it is also assumed to be known for sure that $A \geq 3$. In this case, the estimated PDF for the signal is obtained by inserting 5 for A in $p_{\mathbf{s}}(s)$ since A is estimated as 5, and the least-favorable PDF is obtained by inserting 3 for A in $p_{\mathbf{s}}(s)$ since in the scenario studied in the remaining part of the section the maximum value of means of the gaussian components in the background noise is 2, to which 3 is the closest value A can take. In this example, the least-favorable PDF of the signal is independent from the thresholds for the scenario studied in the remaining part of the section.

In the Figure, the curves of error probability corresponding to the expected scenario versus error probability corresponding to the worst-case scenario are plotted for the case of thresholds being optimal deterministic variables and the case of thresholds being optimal random variables, where $c_{0i} = c_{1i} = 0.01$ for $i = 1, 2$, $M = 2$, $\mu_1 = -\mu_2 = 2$, $\omega_1 = \omega_2 = 0.5$, $\sigma_1 = \sigma_2 = 0.8$, $\pi_1 = 0.82$. The optimal random thresholds improve performance over optimal deterministic ones for $\beta \in (0.18, 0.2378)$, which is also confirmed by Proposition 2. In the cases of $\beta = 0.18$ and $\beta = 0.2378$, the restricted Bayes criterion is equivalent to the minimax and Bayes criteria, respectively. For these cases, using random thresholds does not provide any benefits over deterministic thresholds. It is also interesting to note that the curve corresponding to optimal random thresholds is convex.

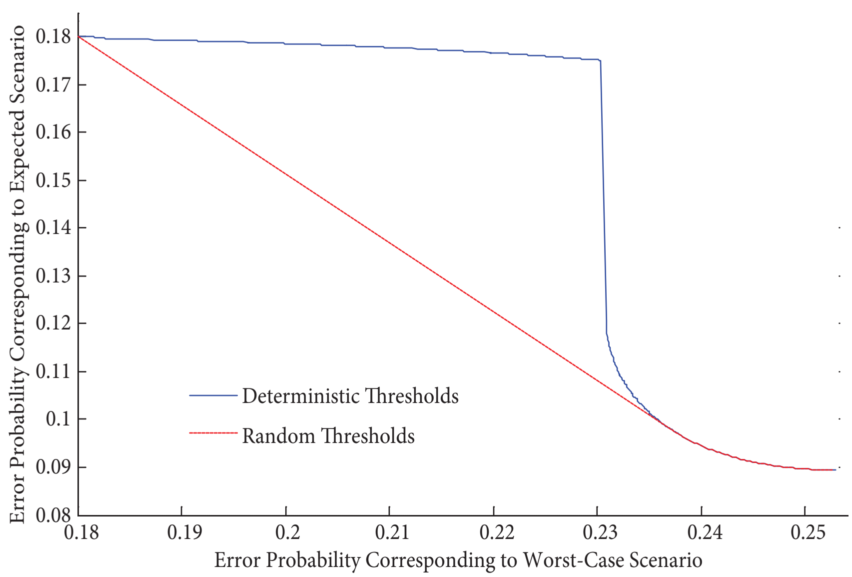

Figure. The curves of error probability corresponding to the expected scenario versus error probability corresponding to the worst-case scenario for the case of using optimal deterministic thresholds and the case of using optimal random thresholds, where $c_{0i} = c_{1i} = 0.01$ for $i = 1,2$, $M = 2$, $\mu_1 = -\mu_2 = 2$, $\omega_1 = \omega_2 = 0.5$, $\sigma_1 = \sigma_2 = 0.8$, $\pi_1 = 0.82$.

In the Table, the optimal PDFs of thresholds are presented for various values of β. From the Table, it is observed that the optimal PDF of thresholds consists of at most two point masses as stated in Proposition 1.

Because of uncertainty issues, the designer sets an upper bound for error performance according to design metrics. Performances higher than the upper bound are not tolerable from the designer's perspective. In our study, the upper bound is an error probability corresponding to the worst-case scenario. Therefore, the designer first sets an upper bound, which is the worst-case error probability in our case, and then aims to optimize the expected error probability, which is true error probability if all estimations turn out to be perfectly correct. In the Figure, it is shown that for some values of the upper bound, which is the worst-case error probability, optimal random thresholds outperform optimal deterministic ones in terms of minimizing the expected error probability, which is the approach adopted in the restricted Bayes criterion.

Table. Optimal PDFS of random thresholds under the scenario in the Figure for various values of β, $p_\theta(\theta) = \lambda\delta(\theta-\theta_1)+(1-\lambda)\delta(\theta-\theta_2)$, where $\theta_1 = [\eta_{11}\,\eta_{12}\,\tau_1]$ and $\theta_2 = [\eta_{21}\,\eta_{22}\,\tau_2]$.

β	λ	η_{11}	η_{12}	τ_1	η_{21}	η_{22}	τ_2
0.20	0.3621	10.4206	10.4376	3.2090	0	0.1320	0.3597
0.21	0.4550	11.3602	8.0927	0.5258	0	2.1927	0.0916
0.22	0.6974	9.9897	9.7801	1.0889	0	0.3333	0.2290
0.23	0.13	0.0866	0	0.4212	9.8691	9.7368	0.4975
0.24	1	9.4285	9.4285	0.7844	-	-	-

6. Concluding remarks

In this paper, the effects of replacing deterministic thresholds of local sensors and the fusion center by random ones have been investigated according to the restricted Bayes criterion. It has been shown that the optimal random thresholds are dependent on each other and contain at most two point masses. Two methods for the implementation of the optimal random thresholds are proposed. A necessary and sufficient condition has been presented to determine when employing the optimal random thresholds outperforms employing the optimal deterministic ones. Through simulations, the effectiveness of the using the optimal random thresholds in place of the optimal deterministic ones has been observed. Employing the absolute worst-case error probability as a performance metric in place of the average worst-case error probability as a more conservative approach will be investigated in future work. Considering the fact that implementing independent randomized rules is much less costly than implementing dependent randomized ones, optimization of independent random thresholds instead of dependent ones can also be investigated in a future study. Effects of adding correlated noises to observations at local sensors can be investigated in other future work.

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