Turkish Journal of Electrical Engineering & Computer Sciences
http://journals.tubitak.gov.tr/elektrik/
Research Article
Turk J Elec Eng & Comp Sci
(2016) 24: 4508 – 4516

doi:10.3906/elk-1405-116

Optimal placing of PMUs in a constrained grid: an approach

Jyoti BHONSLE[1,*], **Anjali JUNGHARE**[2]
[1]Department of Electrical Engineering, Priyadarshini Institute of Engineering and Technology, Nagpur, India
[2]Department of Electrical Engineering, Visvesvaraya National Institute of Technology, Nagpur, India

Received: 19.05.2014 • **Accepted/Published Online:** 11.08.2015 • **Final Version:** 20.06.2016

Abstract: Synchronized phasor measurement units (PMUs) are replacing conventional measuring meters in modern power networks. The paradigm of grid monitoring has shifted to a wider area. This offers better monitoring, protection, and control of the overall power grid. This paper presents an approach for optimal placing of PMUs in constrained power grids using the binary integer programming technique. The proposed approach deals with four constraints: 1) observability of network, two cases of fully observable and partially observable networks are investigated; 2) conventional measurements, three cases of zero injection bus and injection measurement, conventional power flow measurement, and both injection and power flow measurement are considered; 3) failure of a single PMU or communication line; and 4) rate of failure of PMUs or communication line, two cases of low and high are considered. The proposed approach is tested on an IEEE 14-bus system, a 21-bus 400-kV real power system, and an IEEE 30-bus system. This study will help the power system planner to design an economical, efficient, and reliable monitoring system.

Key words: Binary integer programming technique, constrained power system, failure of a single PMU, failure of a single communication line, optimal PMU placement

1. Introduction

Phasor measurement units (PMUs) were introduced in the early 1980s and quickly became a popular technology. Various applications of PMUs are in a developing and operational phase around the world [1]. A PMU uses synchronized signals from global positioning system (GPS) satellites. It measures voltage, currents, and frequency at the bus.

Several PMU placement algorithms have been proposed by researchers and reported in the literature. PMU placement solution sets for network observability using mathematical and heuristics algorithms are cited in [2–16]. Numerous heuristic algorithms are explored for the optimal placement of PMUs in power systems, such as the bisecting search and simulated-annealing algorithm [2], tailored nondominated sorting genetic algorithm [3], tree search method [4], immunity genetic algorithm [5], binary imperialistic competition algorithm [6], and the improved tabu search [7]. Mathematical algorithms proposed for the PMU placement problem are based on integer nonlinear programming [8], integer linear programming [9–16], and integer quadratic programming. In the literature, PMU placement is analyzed for constraints such as a mixed measurement set, single PMU outage or single line outage, multistage PMU placement, etc. Authors discussed the reliability evaluation of electric power systems, as in [17,18]. A reliability-based PMU placement solution was presented in [19]. In [20],

*Correspondence: jsbhonsle.piet@gmail.com

the authors proposed a multicriteria decision-making technique for the strategic placement of PMUs in power systems.

Here an approach for optimal placing of a PMU in a constrained grid using the binary integer programming technique is proposed. Four constraints are chosen for investigation: 1) observability of network: case a) fully observable network, case b) partially observable network; 2) conventional measurements: case a) consideration of zero injection bus and injection measurement, case b) consideration of conventional power flow measurement, case c) both injection and power flow measurement; 3) failure of a single PMU or communication line; and 4) rate of failure of PMUs or communication line: case a) low, case b) high.

The paper is organized as follows. Section 2 explains the basic rules for the PMU placement problem. A brief PMU placement formulation is presented in Section 3. The case studies are presented in Section 4. The results and conclusion are presented in Section 5.

2. Basic rules for PMU placement problem

A PMU is placed at a bus capture, the bus voltage phasor is placed at its associated bus, and the current phasors are placed along all branches that are incident to that bus. Three basic rules can be utilized for PMU placement as follows [15]:

1) If the voltage phasor and current phasor at one end of a branch are known, the voltage phasor at the other end of the branch can be calculated using Ohm's law.

2) If the voltage phasors at both ends of the branch are known, the branch current can be calculated.

3) If there is a zero injection bus without a PMU, whose outgoing currents are known except for one, then the unknown outgoing current can be calculated using Kirchhoff's current law.

Measurements directly obtained from PMUs, such as bus voltage phasor and outgoing phasor currents, are referred to as direct measurements. Measurements derived using the above three rules are referred to as pseudomeasurements.

All the network buses must be observed at least once for minimum measurement redundancy. To ensure the complete network observability under a single PMU or communication line failure, measurement redundancy must be increased by placing additional PMUs at strategic locations.

3. PMU placement formulation

The objective of the PMU placement problem is to find the minimum number of PMUs and their locations in order to make the power network topologically observable. The observability of a bus depends on placing the PMU at that bus or at one of its incident buses.

The problem is formulated as in Eq. (1). Minimizing the objective function F gives the solution of the stated problem:

$$\text{Minimize } F = \sum_{i=1}^{n} c_i x_i, \tag{1}$$

subject to the condition given by function f_j. The definition of f_j varies from constraint to constraint:
where f_j is the constraint function at bus j,
c_i is the PMU installation cost at bus i,
x_i is the binary PMU placement variable.

3.1. Observability constraint

For complete network observability, the constraint f_j becomes as in Eq. (2).

$$f_j \geq 1 \tag{2}$$

Constraint function f_j is defined as in Eqs. (3) and (4).

$$f_j = \sum_{j=1}^{n} a_{ij} x_i \tag{3}$$

1, if $i = j$

$$a_{ij} = 1, \text{ if buses } i \text{ and } j \text{ are connected} \tag{4}$$

0, otherwise

where f_j is the observability constraint function at bus j.

a_{ij} is the network binary connectivity matrix.

3.2. Incorporation of zero injection bus and injection measurement

The constraint function for the zero injection bus is formulated as in Eq. (5), whereas Eq. (6) states the presence of the zero injection bus.

$$f_j = \sum_{j=1}^{n} a_{ij} x_i + \sum_{j=1}^{n} a_{ij} z_i y_{ij} x_i \tag{5}$$

$$\sum_{j=1}^{n} a_{ij} y_{ij} = z_i \tag{6}$$

$z_i = 1$, if bus i is a zero injection bus; otherwise, $z_i = 0$.

y_{ij} is an injection measurement matrix formed by entering one for the element of the column corresponding to the branch associated with the zero injection measurement bus, and the rest of the entries are zero.

When no PMU is placed at the zero injection bus, the formulation is as in Eq. (7).

$$z_i x_i = 0 \tag{7}$$

3.3. Incorporation of flow measurement

In a branch equipped with a flow measurement, if one of its terminal buses is observable, the voltage phasor of the other terminal bus can be calculated from the measured flow (active and reactive) and line parameters. Mathematically, in branch $i-j$ with a flow measurement, buses i and j become observable if the PMU is placed at either of the buses. This can reduce the number of required PMUs for the observability of the power system.

Thus constraint function becomes as in Eq. (8):

$$f_j + f_i \geq 1 \tag{8}$$

Inclusion of both nonflow measurement buses and flow measurement buses in the observability constraint is given in Eq. (9).

$$f_{ij} = \sum_{j=1}^{n} a_{ij}x_i + \sum_{j=1}^{n} P_{ij}x_i \tag{9}$$

P_{ij} is a power flow measurement matrix, formed by entering one for the element of the column corresponding to the nonflow measurement bus, followed by entering one for the element of the column corresponding to the flow measurement bus while the rest of the entries are zero.

3.4. Incorporation of conventional measurement (both flow and injection)

Zero injection bus or injection measurement and flow measurement are combined and the new constraint function obtained is as in Eq. (10).

$$f_j = \sum_{j=1}^{n} a_{ij}x_i + \sum_{j=1}^{n} a_{ij}z_i y_{ij}x_i + \sum_{j=1}^{n} P_{ij}x_i \tag{10}$$

3.5. Failure of a single PMU or communication line

A PMU and communication line may fail depending on their failure rates [17,18]. The constraint function of Eq. (2) is modified with the inclusion of failure of a PMU or communication line to Eq. (11).

$$f_j \geq 2 \tag{11}$$

f_j can be obtained from Eqs. (3) and (10) for complete network observability using PMU-based measurement and both PMU- and conventional-based measurement, respectively. The constraint of Eq. (11) ensures that the network bus will remain observable under failure of a single PMU or communication line. $f_j \geq 2$ states that the bus needs a minimum of two observability sources or monitoring devices.

4. Case studies

The above PMU placement formulation is tested on the IEEE 14-bus system, a 21-bus 400-kV real power system, and the IEEE 30-bus system using a binary integer programming technique in MATLAB. The 21-bus 400-kV real system is part of the Maharashtra grid, which comprises 6 generators and 33 transmission lines, of which one is a triple circuit line, twelve are double circuit lines, and the remaining lines are single circuit lines for transmitting electrical power. Generation from a private company is not considered in the analysis. Table 1 gives the topology and details of the grid.

The PMUs to be placed are of (measurement) M-class. It is assumed that they have a sufficient number of measurement channels as required at a given bus to capture voltage and current phasors. Their costs are assumed to be 1 p.u.

4.1. Optimal PMU placement

To decide the optimal placement of the PMUs, four constraints are chosen as mentioned in Section 1. These are: 1) observability of network, 2) conventional measurements, 3) failure of a single PMU or communication line, and 4) rate of failure of PMUs or communication line. Accordingly, eight cases are investigated, as shown in the Figure, and are mentioned below:

Table 1. Topology and details of the grid.

System	Configuration			Meter placed for power flow measurement in branches
	No. of buses	No. of branches	Zero injection buses (bus location)	
IEEE 14-bus system	14	20	1 (7)	-
21-bus 400-KV real power system	21	33	1 (18)	3–15
IEEE 30-bus system	30	41	6 (6, 9, 22, 25, 27, 28)	3–4, 12–14, 16–17, 18–19

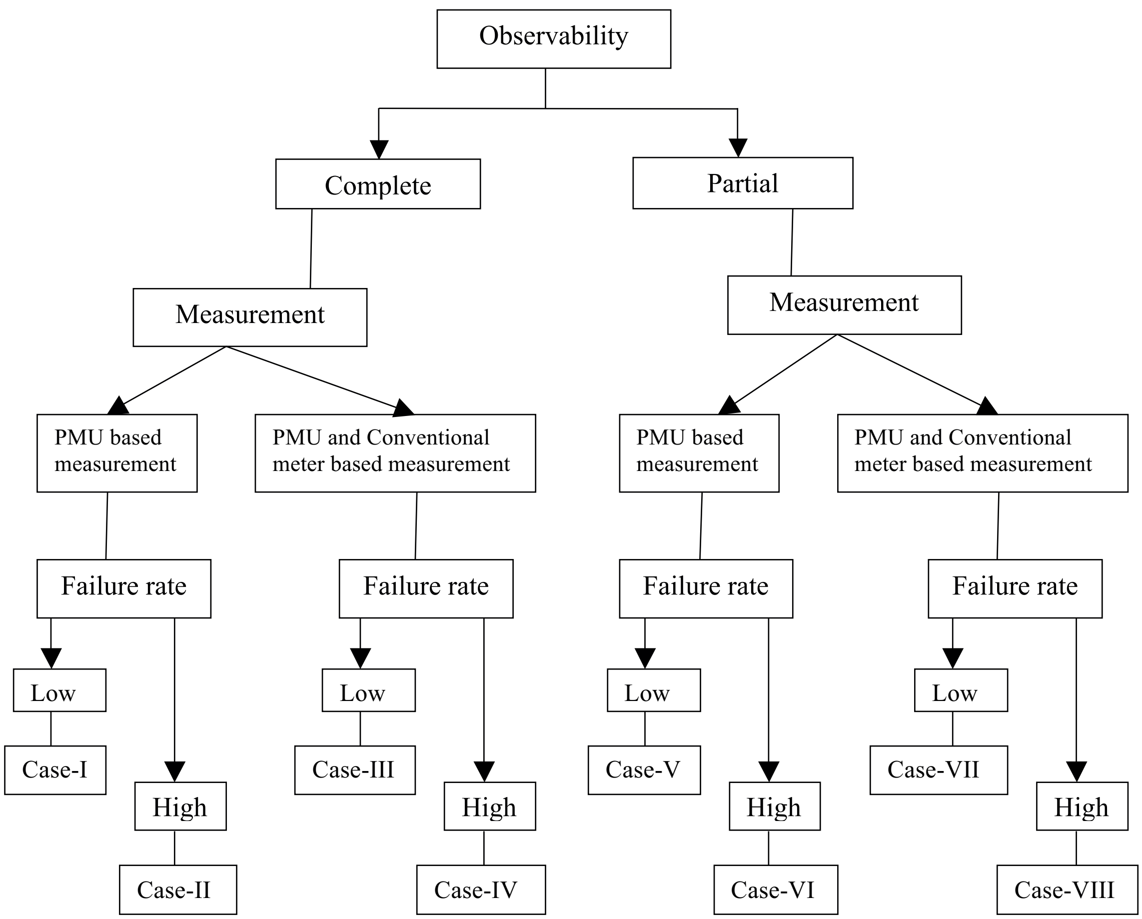

Figure. Eight investigated cases.

Case I: Complete observable network using PMU-based measurement for low failure rate of PMU or communication line.

Case II: Complete observable network using PMU-based measurement for high failure rate of PMU or communication line.

Case III: Complete observable network using PMU and conventional-based measurement for low failure rate of PMU or communication line.

Case IV: Complete observable network using PMU and conventional-based measurement for high failure rate of PMU or communication line.

Case V: Partial observable network (one depth of unobservability) using PMU-based measurement for low failure rate of PMU or communication line.

Case VI: Partial observable network (one depth of unobservability) using PMU-based measurement for high failure rate of PMU or communication line.

Case VII: Partial observable network (one depth of unobservability) using PMU and conventional-based measurement for low failure rate of PMU or communication line.

Case VIII: Partial observable network (one depth of unobservability) using PMU and conventional-based measurement for high failure rate of PMU or communication line.

Considering the IEEE 14-bus system, for Case I each bus is to be observed once, and the failure rate of the PMU or communication line is low. Hence, $f_j \geq 1$. Using Eq. (4), connectivity matrix a_{ij} is formed as in Eq. (12).

$$a_{ij} = \begin{pmatrix} 1\,1\,0\,0\,1\,0\,0\,0\,0\,0\,0\,0\,0\,0 \\ 1\,1\,1\,1\,1\,0\,0\,0\,0\,0\,0\,0\,0\,0 \\ 0\,1\,1\,1\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0 \\ 0\,1\,1\,1\,1\,0\,1\,0\,1\,0\,0\,0\,0\,0 \\ 1\,1\,0\,1\,1\,1\,0\,0\,0\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,1\,1\,0\,0\,0\,0\,1\,1\,1\,0 \\ 0\,0\,0\,1\,0\,0\,1\,1\,1\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0\,1\,1\,0\,0\,0\,0\,0\,0 \\ 0\,0\,0\,1\,0\,0\,1\,0\,1\,1\,0\,0\,0\,1 \\ 0\,0\,0\,0\,0\,0\,0\,0\,1\,1\,1\,0\,0\,0 \\ 0\,0\,0\,0\,0\,1\,0\,0\,0\,1\,1\,0\,0\,0 \\ 0\,0\,0\,0\,0\,1\,0\,0\,0\,0\,0\,1\,1\,0 \\ 0\,0\,0\,0\,0\,1\,0\,0\,0\,0\,0\,1\,1\,1 \\ 0\,0\,0\,0\,0\,0\,0\,0\,1\,0\,0\,0\,1\,1 \end{pmatrix} \tag{12}$$

Solving Eqs. (1), (2), and (3), x_i = [0 1 0 0 0 1 1 0 1 0 0 0 0 0] is obtained. These state PMUs are placed at bus numbers 2, 6, 7, and 9. By placing a PMU at bus number 2, adjoining bus numbers 1, 3, 4, and 5 become observable. Similarly, adjoining bus numbers 5, 11, 12, and 13 become observable by placing the PMU at bus number 6. Bus numbers 4, 8, and 9 become observable by placing the PMU at bus number 7, and bus numbers 4, 7, 10, and 14 become observable by placing the PMU at bus number 9. Hence, all the network buses are made completely observable, as obtained from the solution set.

For Case VIII, each bus is to be observed twice, and the failure rate of the PMU or communication line is high. Hence, $f_j \geq 2$. Matrix a_{ij} remains the same. Solving Eqs. (1), (2), and (11), x_i = [0 0 0 0 0 0 0 0 0 1 0 0 1 0] is obtained. The state PMUs are placed at bus numbers 10 and 13. Eq. (11) incorporates both the PMU-based measurement and the conventional measurement. Bus number 7 is the injection measurement bus. Bus numbers 4, 8, and 9 are connected to bus number 7. Therefore, they become observable, and the branch currents are also known. Buses 2, 3, 5, 10, and 14 are made observable through pseudomeasurement. They are connected to any of buses 4, 8, and 9. Buses 1, 6, 11, and 13 are connected to any of the 2, 3, 5, 10, and 14, and hence they are unobservable with a depth of one. By placing a PMU at bus number 10, adjoining bus numbers 9 and 11 become observable. Buses 4, 7, 10, 14, and 6 are made observable through pseudomeasurement. They are connected to either of buses 9 and 11. Buses 2, 3, 5, 13, and 12 are connected to any of buses 4, 7, 10, 14, and 6, and hence they are unobservable with a depth of one. Similarly, by placing a PMU at bus number 13, adjoining bus numbers 6, 12, and 14 become observable. Buses 5, 11, and 9 are made observable through pseudomeasurement. They are connected to any of buses 6, 12, and 14. Buses 1 and 2 are connected to any of buses 5, 11, and 9. Hence, they are unobservable with a depth of one. The network is partially observable and the failure rate of a PMU or communication line is high. Therefore, only two PMUs are required. They are placed at bus numbers 10 and 13. Bus numbers 1 and 2 become unobservable in the case of failure of a single PMU or communication line.

Cases I–IV investigate the optimal PMU placement for a complete observable network under different grid constraints. Cases II–IV are solved in a similar way to Case I. Cases V–VIII investigate optimal PMU placement for a partial observable network under different grid constraints. Cases V–VII are solved similarly to Case VIII. Tables 2–4 present the PMU placement for the IEEE 14-bus system, the 21-bus 400-kV real power system, and the IEEE 30-bus system, respectively. Each system is investigated for all eight cases.

Table 2. PMU placements for the IEEE 14-bus system.

System	Cases	Optimum number of PMUs	Location of PMU
IEEE 14-bus system	Case I	4	2, 6, 7, 9
	Case II	9	2, 4, 5, 6, 7, 8, 9, 11, 13
	Case III	3	2, 6, 9
	Case IV	7	2, 4, 5, 6, 9, 10, 13
	Case V	2	4, 6
	Case VI	4	4, 5, 6, 9
	Case VII	1	6
	Case VIII	2	10, 13

Table 3. PMU placements for the 21-bus, 400-kV real power system.

System	Cases	Optimum number of PMUs	Location of PMU
21-bus 400-kV real system	Case I	6	1, 6, 8, 15, 17, 20
	Case II	13	1, 3, 5, 6, 7, 8, 10, 12, 15, 16, 17, 19, 20
	Case III	5	1, 6, 8, 10, 17
	Case IV	12	1, 2, 5, 6, 7, 8, 10, 11, 14, 16, 17, 18
	Case V	3	8, 15, 20
	Case VI	5	3, 6, 8, 15, 18
	Case VII	1	18
	Case VIII	2	7, 18

Table 4. PMU placements for the IEEE 30-bus system.

System	Cases	Optimum number of PMUs	Location of PMU
IEEE 30-bus system	Case I	10	1, 7, 9, 10, 12, 18, 24, 25, 27, 28
	Case II	21	1, 3, 5, 7, 8, 9, 10, 11, 12, 13, 15, 17, 18, 20, 22, 24, 25, 27, 28, 29, 30
	Case III	7	2, 11, 13, 16, 19, 25, 27
	Case IV	17	1, 2, 5, 9, 11, 12, 13, 14, 15, 19, 20, 24, 25, 26, 27, 29, 30
	Case V	4	2, 10, 15, 27
	Case VI	8	1, 6, 7, 10, 12, 19, 24, 27
	Case VII	3	18, 22, 27
	Case VIII	5	8, 10, 18, 24, 27

4.2. Two-stage PMU placement

Currently, the grid is monitored using both conventional meters and PMUs. It is not economically feasible to add the featured PMU in one stage. Therefore, a stage-wise addition of PMUs, based on constraint of failure of a single PMU or communication line, is suggested here. Table 5 provides the optimal PMU placement solution set in two stages for the constrained grid system. In Case I, the network is completely observable and requires four PMUs to be placed at bus numbers 2, 6, 7, and 9. Here the failure rate of the PMU or communication line is negligible. In Case II, the network is completely observable and needs nine PMUs to be placed at bus numbers 2, 4, 5, 6, 7, 8, 9, 11, and 13. Here the failure rate of the PMU or communication line is high. Therefore, the number of PMUs required to make the network completely observable is high. It is observed that the locations resulting from Case I are all included in the locations resulting from Case II. Case II is more severe than Case I. Therefore, it is suggested to place four PMUs at bus numbers 2, 6, 7, and 9, as resulting from Case I in the first stage. This makes the network completely observable. In the case of failure of any one of these PMUs or communication lines, the system becomes unobservable. If, at stage II, five more PMUs are added as obtained from the result of Case II in the remaining locations, i.e. bus numbers 4, 5, 8, 11, and 13, this ensures the complete observability of the network under the condition of failure of any PMU and communication line.

Table 5. Two-stage optimal PMU placement in a constrained system.

System	Stages	No. of PMUs	Locations
IEEE 14-bus system	Stage I	4	2, 6, 7, 9
	Stage II	5	4, 5, 8, 11, 13
IEEE 21-bus, 400-KV real system	Stage I	6	1, 6, 8, 15, 17, 20
	Stage II	7	3, 5, 7, 10, 12, 16, 19
IEEE 30-bus system	Stage I	10	1, 7, 9, 10, 12, 18, 24, 25, 27, 28
	Stage II	11	3, 5, 8, 11, 13, 15, 17, 20, 22, 29, 30

5. Conclusion

This paper presented an approach for optimal placement of PMUs using a binary integer programming technique. The number of PMUs in the network increases with the increase in network complexity and constraints. The PMU placement at the zero injection bus helps to find the optimal solution easily. When the depth of observability is reduced from complete to one depth of unobservability, the number of PMUs is reduced with the trade-off of network security and system reliability. Here a multistage PMU placement for a large power grid is proposed. The optimal number of PMUs is found to be approximately two-thirds of the number of buses depending on network topology. The main contribution here is the stage-wise addition of PMUs based on the constraint of the failure rate of the PMU or communication line. Stage-wise placement of the PMU in the network ensures network security and reliability.

This study provides a wide scope for a PMU placement strategy. The power system planner can choose the PMU placement depending on financial availability, network density, and network observability.

References

[1] Ree J, Centeno V, Thorp J, Phadke A. Synchronized phasor measurement applications in power systems. IEEE T Smart Grid 2010; 1: 20-27.

[2] Baldwin T, Mili L, Boisen M, Adapa R. Power system observability with minimal phasor measurement placement. IEEE T Power Syst 1993; 18: 707-715.

[3] Begović M, Milošević B. Nondominated sorting genetic algorithm for optimal phasor measurement placement. IEEE T Power Syst 2003; 18: 69-75.

[4] Nuqui R, Phadke A. Phasor measurement unit placement techniques for complete and incomplete observability. IEEE T Power Deliver 2005; 20: 2381-2388.

[5] Aminifar F, Lucas C, Khodaei A, Fotuhi-Firuzabad M. Optimal placement of phasor measurement units using immunity genetic algorithm. IEEE T Power Deliver 2009; 24: 1014-1020.

[6] Mahari A, Seyedi H. Optimal PMU placement for power system observability using BICA considering measurement redundancy. Int J Elec Power Syst 2013; 103: 78-85.

[7] El-Zonkoly A, El-Safty S, Maher R. Optimal placement of PMUs using improved tabu search for complete observability and out-of-step prediction. Turk J Elec Eng & Comp Sci 2013; 21: 1376-1393.

[8] Kavasseri R, Srinivasan S. Joint placement of phasor and power flow measurements for observability of power systems. IEEE T Power Syst 2011; 26: 1929-1936.

[9] Xu B, Abur A. Observability analysis and measurement placement for systems with PMUs. In: IEEE 2004 Power Systems Conference and Exposition; 10–13 October 2004; New York, NY, USA. New York, NY, USA: IEEE. pp. 943-946.

[10] Rakpenthai C, Premrudeepreechacharn S, Uatrongjit S, Watson N. An optimal PMU placement method against measurement loss and branch outage. IEEE T Power Deliver 2007; 22: 101-107.

[11] Dua D, Dambhare S, Gajbhiye R, Soman S. Optimal multistage scheduling of PMU placement: an ILP approach. IEEE T Power Deliver 2008; 23: 1812-1820.

[12] Gou B. Optimal placement of PMUs by linear programming. IEEE T Power System 2008; 23: 1525-1526.

[13] Gou B. Generalized integer linear programming formulation for optimal PMU placement. IEEE T Power Syst 2008; 23: 1099-1104.

[14] Chakrabarti S, Kyriakides E. Optimal placement of phasor measurement units for power system observability. IEEE T Power Syst 2008; 23: 1433-1440.

[15] Sodhi R, Srivastava S, Singh S. Optimal PMU placement method for complete topological and numerical observability of power system. Int J Elec Power Syst 2010; 80: 1154-1159.

[16] Roy B, Sinha A, Pradhan A. An optimal PMU placement technique for power system observability. Int J Elec Power 2012; 42: 71-77.

[17] Singh C, Wang L. Role of artificial intelligence in the reliability evaluation of electric power systems. Turk J Elec Eng & Comp Sci 2008; 16: 189-200.

[18] Wang Y, Li W, Lu J, Liu H. Evaluating multiple reliability indices of regional networks in wide area measurement system. Int J Elec Power 2009; 79: 1353-1359.

[19] Wang Y, Wang C, Li J, Lin F. Reliability-based incremental PMU placement. IEEE T Power Syst 2014; 29: 2744-2752.

[20] Bhonsle J, Junghare A. Application of MCDM-AHP technique for PMU placement in power system. In: IEEE 2015 International Conference on Computational Intelligence and Communication Technology; 13–14 February 2015; Ghaziabad, India. New York, NY, USA: IEEE. pp. 513-517.