Turkish Journal of Electrical Engineering & Computer Sciences

http://journals.tubitak.gov.tr/elektrik/

Research Article

Turk J Elec Eng & Comp Sci
(2017) 25: 1154 – 1162
© TÜBİTAK
doi:10.3906/elk-1511-302

Advanced probabilistic power flow methodology for power systems with renewable resources

Dinh Duong LE[1,2,*], **Nhi Thi Ai NGUYEN**[1,2], **Van Duong NGO**[3], **Alberto BERIZZI**[1]
[1]Department of Energy, Polytechnic University of Milan, Milan, Italy
[2]Department of Electrical Engineering, The University of Danang-University of Science and Technology, Danang, Vietnam
[3]The University of Danang, Danang, Vietnam

Received: 01.12.2015 • **Accepted/Published Online:** 31.03.2016 • **Final Version:** 10.04.2017

Abstract: Renewable resources have added additional uncertainty to power grids. Deterministic power flow does not provide sufficient information for power system calculation and analysis, since all sources of uncertainty are not taken into account. To handle uncertainties PPF has been introduced and used as an efficient tool. In this paper, we present a cumulant-based PPF approach that can account for various sources of uncertainty in power systems with renewable resources such as wind and photovoltaic energy. We also propose the use of a new methodology to estimate probability distribution for wind power output based on measured data. The proposed approach is carried out on a modified IEEE-14 bus test system. Simulation results of the proposed method are then compared with the result obtained by Monte Carlo simulation.

Key words: Probabilistic power flow, uncertainty, renewable resource

1. Introduction

Deterministic power flow (DPF) has been used as a fundamental tool for calculation and analysis of power systems. However, this approach does not take into account any source of uncertainty in power systems, such as load and renewable source uncertainties. In order to take the uncertainties into consideration, probabilistic power flow (PPF) has been introduced and has served as an effective tool for various research areas of power systems.

PPF was first introduced by Borkowska in 1974 [1]. In recent years, many papers have been published in this field. In general, PPF can be classified into three categories: numerical, approximate (e.g., point estimate method), and analytical methods. Monte Carlo simulation (MCS) [2,3] is a typical numerical method. The main issue of MCS is that it uses a large number of solutions from DPF, so it is usually computationally intensive. Its accuracy mostly depends on the number of samples. Conversely, point estimate methods [4–6] use an approximation technique, in which input variables are decomposed into a series of pairs of values and weights, and then the moments of the output variables are calculated as a function of the inputs. In a different way, the analytical approach applies an arithmetic algorithm (e.g., using a convolution technique [7] or cumulant method [8–11]) with probability density functions (PDFs) and/or cumulative distribution functions (CDFs) of input random variables so that PDFs and CDFs of output random variables can be obtained. From the probability

*Correspondence: dinhduong.le@polimi.it

distributions of input random variables and linearized power flow equations, while the cumulant method uses the properties of cumulants, the convolution method convolves all random variables.

Series expansions (such as Edgeworth, Gram–Charlier, or Cornish–Fisher) are widely used in cumulant-based PPF methods to obtain the probability distributions of desired random variables from their cumulants or moments [12–15]. However, if input variables of PPF computation contain distributions far from Gaussian distribution, especially discrete distributions, the accuracy of the computation will be significantly affected. To treat this issue, the Von Mises method [8,16] was adopted, giving a good performance. In particular, in [16], the authors treated discrete and continuous distributions separately: discrete distributions are related to random branch outages, while all input continuous distributions are assumed to have Gaussian distribution. However, in real PPF applications, continuous distributions, including several types of probability distributions representing different sources of uncertainty due to the stochastic nature of the load, renewable resources, etc., need to be taken into account.

In this paper, we present a cumulant-based PPF in which different types of probability distributions, including Gaussian/non-Gaussian and discrete/continuous distributions, can be accounted for. In addition, in order to overcome the difficulty of estimating a probability distribution for power output of a wind farm in reality, we propose the use of clustering methodologies based on measured wind power data.

In Section 2, we present probabilistic modeling of load and renewable power generation. In Section 3, a technique for constructing probability distributions of random variables is provided. In Section 4, the proposed PPF methodology is presented. We perform tests on a modified IEEE 14-bus test system and discuss the results in Section 5. Concluding remarks are given in Section 6.

2. Probabilistic modeling of load and renewable power generation

In power systems, the stochastic nature of a power injection such as a load or renewable power generation can be characterized by a PDF and/or a CDF.

In the literature, a load is usually assumed to have a Gaussian distribution. The power output of PV power generation can be characterized by a generic distribution function such as beta, gamma, or Weibull.

In terms of wind resources, fitting the power output of a wind farm to a common distribution function is very challenging, since the probability distribution regularity of wind power is usually poor. To deal with this difficulty, we propose to use a clustering technique to build a discrete distribution for wind power output based on its measured data as follows.

First, using a clustering technique, the wind power data of several years are clustered into distinct groups (clusters). Data belonging to each group are then used to build an impulse for the discrete distribution. Next, the probability of each impulse (each cluster) is computed proportionally to the total number of data. Finally, the discrete distribution of the wind power output is obtained.

Many techniques have been used to perform clustering analysis in the literature [17]. In this paper, for the identification of wind power clusters, we use the k-means algorithm that was presented in detail in [17]. Other clustering techniques can also be adopted.

3. Constructing probability distributions for random variables

In this section, we present a technique developed to enhance cumulant-based PPF methods for taking into account different types of probability distributions in the following.

In [8,16], discrete and continuous distributions of input random variables were separately treated so that the probability distributions of output random variables were obtained through an approximation approach. In order to build a discrete distribution for a random variable with ξ impulses, the Von Mises method uses the first $(2\xi - 1)$ moments of the random variable. The approximation approach was applied in PPF computation [8,16] with the assumption of the continuous part being of Gaussian distribution. Nevertheless, when using PPF for a real power system, several types of distributions (including non-Gaussian distributions) for the continuous part need to be accounted for. For such cases, in the present paper, the continuous part is first approximated by a series expansion method such as Gram–Charlier, and then the probability distribution of output random variables are constructed.

Suppose that an output random variable $\tilde{Y}$ of the PPF computation includes discrete part $\tilde{Y}_d$ and continuous part $\tilde{Y}_c$:

$$\tilde{Y} = \tilde{Y}_d + \tilde{Y}_c. \tag{1}$$

When continuous part $\tilde{Y}_c$ has Gaussian distribution, the PDF and CDF of random variable $\tilde{Y}$ [16] can be calculated as:

$$f_{\tilde{Y}}(x) = \sum_{i=1}^{\xi} p_i \frac{1}{\sqrt{2\pi}\sigma_{\tilde{Y}_c}} e^{-x_{N_i}^2/2}, \tag{2}$$

$$F_{\tilde{Y}}(x) = \sum_{i=1}^{\xi} p_i \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x_{N_i}} e^{-t^2/2} dt, \tag{3}$$

where ξ is the number of impulses used to characterize $\tilde{Y}_d$; x_i and p_i $(i = 1:\xi)$ are abscissas and corresponding probabilities for ξ impulses of $\tilde{Y}_d$; $x_{N_i} = (x - x_i - m_{\tilde{Y}_c})/\sigma_{\tilde{Y}_c}$; and $m_{\tilde{Y}_c}$ and $\sigma_{\tilde{Y}_c}$ are the mean and standard deviation of $\tilde{Y}_c$.

On the contrary, in the case of the continuous part being of non-Gaussian distribution, Gram–Charlier expansion is first adopted to give an approximation for the continuous part.

For Gram–Charlier expansion, the PDF $f_{\tilde{Y}_c}(\cdot)$ and CDF $F_{\tilde{Y}_c}(\cdot)$ of continuous part $\tilde{Y}_c$ with mean $m_{\tilde{Y}_c}$ and standard deviation $\sigma_{\tilde{Y}_c}$ can be written as follows [9,14]:

$$f_{\tilde{Y}_c}(x) = \phi(x_N) + \phi(x_N) \sum_{\eta=1}^{\infty} c_\eta H_\eta(x_N), \tag{4}$$

$$F_{\tilde{Y}_c}(x) = \Phi(x_N) + \phi(x_N) \sum_{\eta=1}^{\infty} c_\eta H_{\eta-1}(x_N), \tag{5}$$

where $x_N = (x - m_{\tilde{Y}_c})/\sigma_{\tilde{Y}_c}$ and coefficients c_η $(\eta \geq 1)$ can be obtained from its cumulants:

$$\begin{cases} c_\eta = 0 & \eta = 1;2 \\ c_\eta = \frac{k_{\tilde{Y}_c \eta}}{\eta! \sigma_{\tilde{Y}_c}^{\eta}} & \eta = 3;4;5 \\ c_\eta = \frac{k_{\tilde{Y}_c \eta}}{\eta! \sigma_{\tilde{Y}_c}^{\eta}} + \frac{1}{2} \sum\limits_{j=3}^{\eta-3} c_j c_{\eta-j} & \eta \geq 6 \end{cases}, \tag{6}$$

and $H_\eta(x_N)$ is the η^{th} order of the so-called Hermite polynomial, computed as:

$$\begin{cases} H_\eta(x_N) = 1 & \eta = 0 \\ H_\eta(x_N) = x_N & \eta = 1 \\ H_\eta(x_N) = x_N H_{\eta-1}(x_N) - (\eta - 1) H_{\eta-2}(x_N) & \eta \geq 2 \end{cases}, \tag{7}$$

where, in Eq. (6), $k_{\tilde{Y}_c^\eta}$ denotes the η^{th} order of cumulant of $\tilde{Y}_c$.

Gram–Charlier expansion allows us to approximate the PDF and CDF of a random variable using its cumulants based on Eqs. (4) and (5).

After applying Gram–Charlier expansion for $\tilde{Y}_c$, the PDF and CDF of $\tilde{Y}$ are obtained as follows:

$$f_{\tilde{Y}}(x) = \sum_{i=1}^{\xi} p_i \left[\phi(x_{N_i}) + \phi(x_{N_i}) \sum_{\eta=1}^{\infty} c_\eta H_\eta(x_{N_i}) \right], \tag{8}$$

$$F_{\tilde{Y}}(x) = \sum_{i=1}^{\xi} p_i \left[\Phi(x_{N_i}) + \phi(x_{N_i}) \sum_{\eta=1}^{\infty} c_\eta H_{\eta-1}(x_{N_i}) \right], \tag{9}$$

where $\Phi(\cdot)$ and $\phi(\cdot)$ are the CDF and PDF of the standard normal distribution, respectively.

4. Probabilistic power flow methodology

The basic power flow equations can be expressed in matrix form [12] as:

$$\mathbf{w} = g(\mathbf{x}), \tag{10}$$

$$\mathbf{z} = h(\mathbf{x}), \tag{11}$$

where $\boldsymbol{w}$ is the vector of nodal power injections, $\boldsymbol{x}$ is the vector of state variables, $\boldsymbol{z}$ is the vector of line power flows, $g(\boldsymbol{x})$ are the power flow equations, and $h(\boldsymbol{x})$ are the functions to compute line power flows.

Solving a DPF for the system and then using Taylor series expansion to linearize the above equations around the solution point gives:

$$\Delta\mathbf{x} = \mathbf{S} \mid_{\bar{\mathbf{x}}} \Delta\mathbf{w}, \tag{12}$$

$$\Delta\mathbf{z} = \mathbf{T} \mid_{\bar{\mathbf{x}}} \Delta\mathbf{w}, \tag{13}$$

where $\mathbf{S} \mid_{\bar{\mathbf{x}}}$ is the inverse of the Jacobian matrix and $\mathbf{T} \mid_{\bar{\mathbf{x}}}$ is the sensitivity matrix of power flows with respect to nodal power injections. $\mathbf{S} \mid_{\bar{\mathbf{x}}}$ and $\mathbf{T} \mid_{\bar{\mathbf{x}}}$ are computed at the solution point $\mathbf{x}$-par of the DPF calculation.

For PPF, each element of $\boldsymbol{w}$, $\boldsymbol{x}$, and $\boldsymbol{z}$ is considered as the realization of the random variable associated with each nodal power injection, state variable, and power flow, respectively. Based on the relationships in Eqs. (12) and (13), cumulant-based PPF can be adopted.

The proposed PPF approach is implemented step by step as follows:

Step 1: Solve DPF for the system to obtain the expected value of random state variables $\mathbf{x}$ and the sensitivity matrices $\mathbf{S}\mid_{\bar{\mathbf{x}}}$ and $\mathbf{T}\mid_{\bar{\mathbf{x}}}$ computed at $\mathbf{x}$-par;

Step 2: Compute self and joint cumulants [12,14] of nodal power injections for both discrete and continuous parts:

- *For discrete part*: Compute cumulants of state variables and line power flows using Eqs. (12) and (13); convert cumulants to moments [18], and then use the process presented in Section 3 (partly described in detail in [16]) to compute abscissas x_i and corresponding probabilities p_i;
- *For continuous part*: Compute cumulants of state variables and line power flows using Eqs. (12) and (13), then obtain coefficients c_η using Eq. (6) and Hermite polynomials H_η based on Eq. (7);

Step 3: Obtain PDFs and CDFs of the outputs of interest using Eqs. (8) and (9).

5. Results and discussion

The proposed approach was tested on a modified IEEE 14-bus test system. The single line diagram is presented in Figure 1, while branch, bus, and generator data can be found in [7].

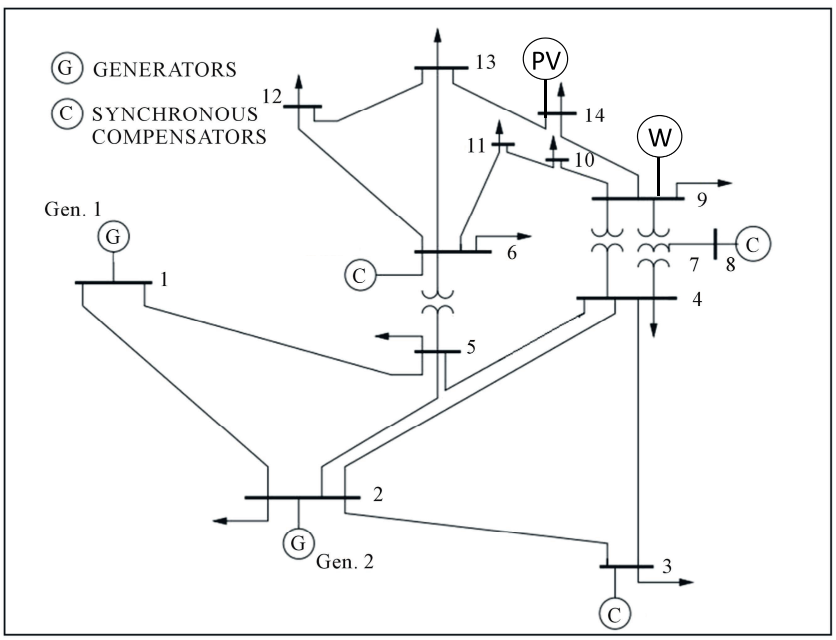

Figure 1. Modified IEEE 14-bus test system.

The load at each bus is modeled by a Gaussian distribution characterized by its mean and standard deviation. Correlation coefficients among loads are given in the Table. Generators at buses 1 and 2 are modeled by binomial distributions with relevant forced outage rates. Detailed information about the above Gaussian and binomial distributions was also given in [7].

Three-year measured hourly wind power (from 1 January 2009 to 31 December 2011) of a real wind farm (rated at 25 MW) in Sicily (Italy) is used to estimate a discrete probability distribution using the method presented in Section 2. The wind farm is assumed to connect to bus 9 of the system as in Figure 1. Figure

2 shows the probability distribution of power output in which the discrete distribution is characterized by 20 impulses (corresponding to 20 clusters).

Table. Correlation coefficient among loads.

Bus	2	3	4	5	6	9	10	11	12	13	14
2	1.00	0.35	0.35	0.35	0.15	0.15	0.15	0.15	0.15	0.15	0.15
3	-	1.00	0.35	0.35	0.15	0.15	0.15	0.15	0.15	0.15	0.15
4	-	-	1.00	0.35	0.15	0.15	0.15	0.15	0.15	0.15	0.15
5	-	-	-	1.00	0.15	0.15	0.15	0.15	0.15	0.15	0.15
6	-	-	-	-	1.00	0.20	0.20	0.20	0.15	0.15	0.15
9	-	-	-	-	-	1.00	0.20	0.20	0.15	0.15	0.15
10	-	-	-	-	-	-	1.00	0.20	0.15	0.15	0.15
11	-	-	-	-	-	-	-	1.00	0.15	0.15	0.15
12	-	-	-	-	-	-	-	-	1.00	0.20	0.20
13	-	-	-	-	-	-	-	-	-	1.00	0.20
14	-	-	-	-	-	-	-	-	-	-	1.00

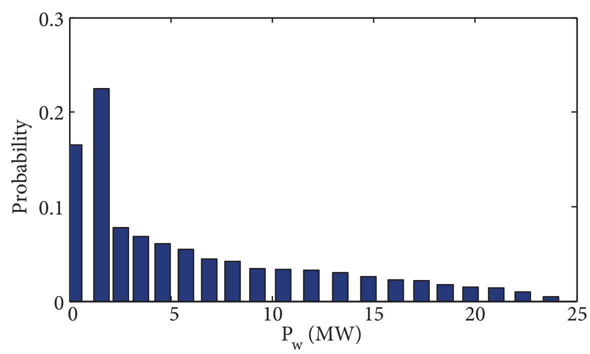

Figure 2. Distribution of wind power output at bus 9.

A PV power generation resource is also added to the system at bus 14 with the installed capacity of 12 MW (see Figure 1). PV power output is assumed to have a beta distribution with parameters computed by using the expected value (9.6 MW) and standard deviation (assumed to be equal to 11% of its rated power) [19].

In the system, the continuous part consists of Gaussian distributions of loads and beta distribution of PV generation, while the discrete part includes binomial distributions of generating units and discrete distribution of wind power. The proposed PPF method can take these distributions into consideration thanks to the construction technique described in Section 3.

The proposed PPF methodology takes into account the first nine self cumulants for the discrete part, the first six self cumulants for the continuous part, and the second order joint cumulants among loads. To assess the accuracy of the proposed PPF approach, a MCS with 10,000 samples has been carried out and taken as a reference.

In order to compare the proposed PPF and the MCS results, ARMS error is calculated [9]:

$$ARMS = \frac{1}{N}\sqrt{\sum_{i=1}^{N}(F_{M_i} - F_{P_i})^2}, \tag{14}$$

where F_{M_i} and F_{P_i} are the ith values on CDF curves obtained by MCS and the proposed PPF method, respectively, and N is the total number of points chosen in the range between the 1st and the 99th percentiles of the CDF obtained by MCS (here, N is determined by selecting an interval between neighboring points equal to 10^{-4} p.u. for voltages and 10^{-3} p.u. for power).

In this test, base power of 100 MVA is used. For illustration, the PDFs and CDFs of selected output random variables are shown: the PDF and CDF of real power flow through branch 6-11 ($\tilde{P}_{6-11}$) in Figure 3 and Figure 4, respectively; the PDF of reactive power flow through branch 9-10 ($\tilde{Q}_{9-10}$) in Figure 5; the PDF of voltage at bus 11 ($\tilde{V}_{11}$) in Figure 6; and then PDF of real power flow through branch 2-4 ($\tilde{P}_{2-4}$) in Figure 7.

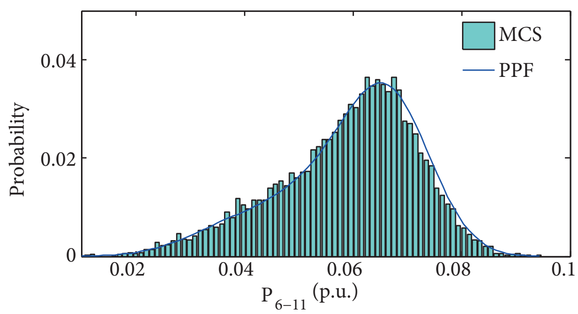

Figure 3. PDFs of $\tilde{P}_{6-11}$.

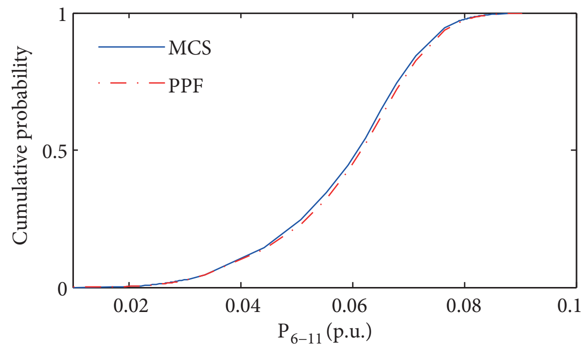

Figure 4. CDFs of $\tilde{P}_{6-11}$.

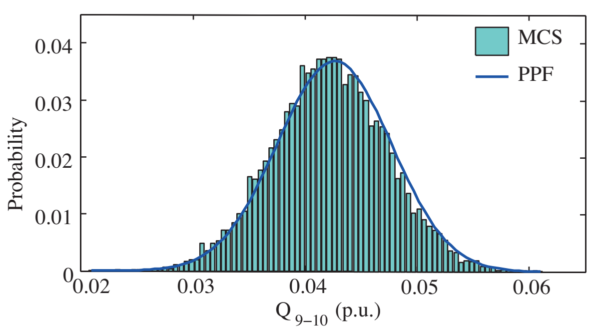

Figure 5. PDFs of $\tilde{Q}_{9-10}$.

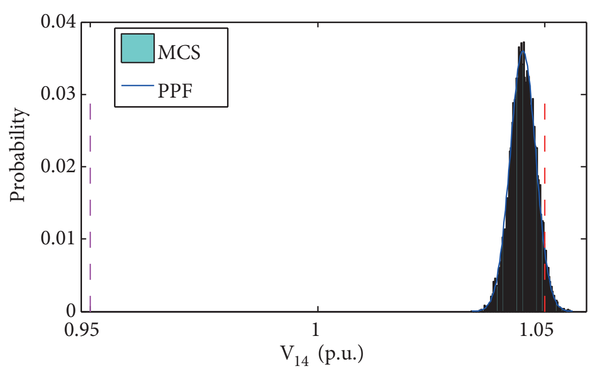

Figure 6. PDFs of $\tilde{V}_{14}$.

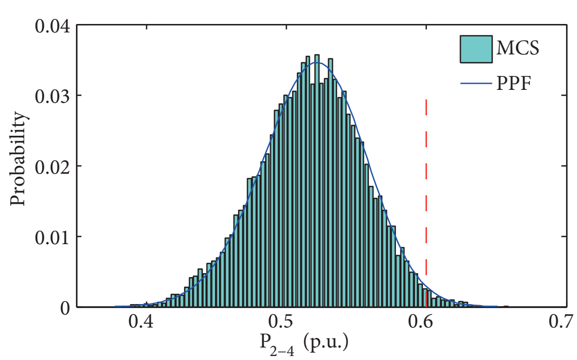

Figure 7. PDFs of $\tilde{P}_{2-4}$.

It can be seen from the figures that the curves obtained by the proposed PPF are very close to the results obtained by MCS. In particular, looking at Figure 3, the distribution of $\tilde{P}_{6-11}$ has non-Gaussian shape, mostly caused by binomial distributions of generating units, discrete distribution of wind power, and beta distribution of PV power generation.

ARMS calculated for $\tilde{P}_{6-11}$, $\tilde{Q}_{9-10}$, $\tilde{V}_{11}$, and $\tilde{P}_{2-4}$ are 0.09%, 0.11%, 0.12%, and 0.08%, respectively, indicating very good accuracy of the proposed PPF method and good performance of the approximation technique presented in Section 3.

PPF provides not only the information as obtained by DPF calculation but also the overall spectrum of all probable values of output variables, which is useful for probabilistic analysis of power systems and security assessment under uncertainty [12]. From the probability functions of output random variables, we can estimate, for example, the probability so that voltage at a specific bus is out of the operating range or power through a specific line is over its limit [12], and so on.

For example, the probability that voltage at bus 14 within the operating range [0.95; 1.05] p.u. (values between the two vertical dashed lines in Figure 6) can be calculated as:

$$P\left\{0.95 \leq \tilde{V}_{14} \leq 1.05\right\} = 95.4\%. \tag{15}$$

Analogously, assuming that the upper bound of the real power flow (e.g., due to thermal rating) of line 2-4, for instance, is 0.60 p.u. (the vertical dashed line in Figure 7), the probability of it being greater than its upper bound is:

$$P\left\{\tilde{P}_{2-4} > 0.60\right\} = 1\%. \tag{16}$$

6. Conclusions

A cumulant-based probabilistic power flow approach is presented in this paper. It can handle several types of probability distributions, which represent different sources of uncertainty in power systems. Correlations of input power injections can also be taken into consideration. In addition, due to the fact that it is difficult to fit the power output of a wind farm to a common probability distribution, we propose the use of a clustering technique to build a discrete distribution for wind power by using measured data.

The proposed approach has been tested on a modified IEEE-14 bus test system and it gives a good performance in comparison with the result obtained by MCS.

Acknowledgment

We are grateful to TERNA (Italian TSO) for the wind data provided for the testing of the proposed approach.

References

[1] Borkowska B. Probabilistic load flow. IEEE T Power Ap Syst 1974; PAS-93: 752-759.

[2] Carpinelli G, Caramia P, Varilone P. Multi-linear Monte Carlo simulation method for probabilistic load flow of distribution systems with wind and photovoltaic generation systems. Renew Energ 2015; 76: 283-295.

[3] Hajian M, Rosehart WD, Zareipour H. Probabilistic power flow by Monte Carlo simulation with Latin supercube sampling. IEEE T Power Syst 2013; 28: 1550-1559.

[4] Su CL. Probabilistic load-flow computation using point estimate method. IEEE T Power Syst 2005; 20: 1843-1851.

[5] Aien M, Khajeh MG, Rashidinejad M, Fotuhi-Firuzabad M. Probabilistic power flow of correlated hybrid wind-photovoltaic power systems. IET Renew Power Gen 2014; 8: 649-658.

[6] Mohammadi M, Shayegani A, Adaminejad H. A new approach of point estimate method for probabilistic load flow. Int J Elec Power 2013; 51: 54-60.

[7] Allan RN, Al-Shakarchi MRG. Probabilistic techniques in A.C. load-flow analysis. P I Electr Eng 1977; 124: 154-160.

[8] Sanabria LA, Dillon TS. Stochastic power flow using cumulants and Von Mises functions. Int J Elec Power 2004; 8: 676-682.

[9] Zhang P, Lee TS. Probabilistic load flow computation using the method of combined cumulants and Gram-Charlier expansion. IEEE T Power Syst 2004; 19: 676-682.

[10] Yuan Y, Zhou J, Ju P, Feuchtwang J. Probabilistic load flow computation of a power system containing wind farms using the method of combined cumulants and Gram-Charlier expansion. IET Renew Power Gen 2011; 5: 448-454.

[11] Ruiz-Rodriguez FJ, Hernandez JC, Jurado F. Probabilistic load flow for radial distribution networks with photo-voltaic generators. IET Renew Power Gen 2012; 6: 110-121.

[12] Le DD, Berizzi A, Bovo C, Ciapessoni E, Cirio D, Pitto A, Gross G. A probabilistic approach to power system security assessment under uncertainty. In: Bulk Power System Dynamics and Control - IX Optimization, Security and Control of the Emerging Power Grid, 2013 IREP Symposium; 25–30 August 2013; Rethymnon, Greece. New York, NY, USA: IEEE. pp. 1-7.

[13] Usaola L. Probabilistic load flow with wind production uncertainty using cumulants and Cornish Fisher expansion. Int J Elec Power 2009; 31: 474-481.

[14] Fan M, Vittal V, Heydt GT, Ayyanar R. Probabilistic power flow studies for transmission systems with photovoltaic generation using cumulants. IEEE T Power Syst 2012; 27: 2251-2261.

[15] Zheng J, Wen F, Ledwich G, Huang J. Risk control in transmission system expansion planning with wind generators. Int T Electr Energy 2014, 24: 227-245.

[16] Hu Z, Wang X. A probabilistic load flow method considering branch outages. IEEE T Power Syst 2006; 21: 507-514.

[17] Gan G, Ma C, Wu J. Data Clustering: Theory, Algorithms, and Applications. Philadelphia, PA, USA: SIAM, 2007.

[18] Tian WD, Sutanto D, Lee YB, Outhred HR. Cumulant based probabilistic power system simulation using Laguerre polynomials. IEEE T Energy Conver 1989; 4: 567-574.

[19] Fabbri A, Roman TGS, Abbad JR, Quezada VHM. Assessment of the cost associated with wind generation prediction errors in a liberalized electricity market. IEEE T Power Syst 2005; 20: 1440-1446.