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Research Article

Statistical features-based comparison of analysis and synthesis of normal and epileptic electroencephalograms for various wavelets

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Abstract: An electroencephalogram (EEG) is an electrical signal in microvolts captured noninvasively from the brain, which provides important and unique information about the brain. The frequency of an EEG signal lies between 0 and 100 Hz. Decomposition of an EEG signal into various bands such as alpha, beta, delta, theta, and gamma is essential in seizure-related studies. EEGs play a key role in the diagnosis of epileptic seizures and neurological disorders. In this paper, multiple wavelet families for decomposition and reconstruction are explored and are compared based on risk functions and reconstruction measures. While dealing with the wavelets it is a difficult task to choose the correct/accurate wavelet for the given biosignal analysis. Various statistical properties were studied by the authors to check the suitability of various wavelets for normal and diseased EEG signal decomposition and reconstruction. The methodology was applied to 3 groups (63 subjects) consisting of both sexes and aged between 1 and 80 years: 1) normal healthy subjects, 2) patients with focal seizures, and 3) patients with generalized seizures. Our result shows that the Haar and Bior3.7 wavelets are more suitable for normal as well as diseased EEG signals, as the mean square error, mean approximate error, and percent root mean square difference of these wavelets are much smaller than in other wavelets. The signal-to-error ratio for Haar and Bior3.7 was much higher than in any other wavelet studied.

Key words: Decomposition, wavelet, reconstruction, electroencephalogram, epilepsy, statistical features

1. Introduction

An electroencephalogram (EEG) represents the entire brain dynamics in the form of an electrical signal captured from the surface of the head. It is the superimposition of various processes related to the brain. The amplitude of the EEG signal is in microvolts (200 μ V). The amplitude, frequency, and characteristics of an EEG signal change from one state to the other, and with age. An EEG is decomposed mainly into 5 subbands: delta (0–4 Hz), occurring during deep sleep, during childhood, and in serious organic brain diseases; theta (4–8 Hz), occurring in childhood, during emotional stress; alpha (8–13 Hz), occurring in a normal person in an awake state; beta (13–30 Hz), affected by mental activity; and gamma (above 30 Hz) [1,2]. Instead of studying the complete EEG, EEG subband-related information yields more accurate information about the underlying neuronal activities [3].

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2. Materials and methods

Traditional Fourier transform can be used for the feature extraction of EEG, but it works well for the whole-time series of a stationary signal [4,5]. In this paper a wavelet filter is used for the subband decomposition as it has various advantages such as time-frequency localization, scale-space analysis, flexibility, and multirate filtering. Over the length of a signal, variable window size can be applied in the wavelet transform (WT). Depending on the signal specifications, this allows the wavelet to get stretched or compressed [6,7]. This makes WT a very popularly used feature extraction technique for nonstationary signals, such as EEGs [8,9]. There are two types of WT: continuous wavelet transform (CWT) and discrete wavelet transform (DWT) [10,11].

CWT: Continuous wavelet transform

$$X(a,b) = \frac{1}{\sqrt{b}} \int_{-\infty}^{\infty} X(t) \varphi\left(\frac{t-a}{b}\right) dt$$
(1)

Here ' ψ ' is the mother wavelet, 'a' is a time shift, and 'b' modulates the width.

DWT: Discrete wavelet transform

$$\Psi_{(a,b)}(t) = 2^{a/2} \psi(2^{-a/2} (t-b))$$
⁽²⁾

The DWT means selecting a subset of scales 'a' and positions 'b' of the mother wavelet ' ψ (t)' [12,13]. While dealing with the wavelets the most important concern is how to choose the correct/accurate wavelet for the analysis of a given signal or image. All wavelets can be applied to a given signal/image for its analysis, but the results obtained from each wavelet would differ a bit and might or might not be acceptable depending on the application requirement. The authors have applied various wavelets for analysis and synthesis of an EEG signal to study various risk characteristics.

2.1. Wavelet family

Haar is the simplest and oldest of the wavelet family. The most widely used wavelet in signal processing is the Daubechies; it is also called the modified Haar wavelet. Haar, Daubechies, Coiflet, and Symlet are the orthogonal wavelets capable of almost perfect reconstruction of a signal. Generally, a wavelet is selected based on the shape of the mother wavelet and its ability to analyze the signal [14,15]. Specifications of the wavelet family are as shown in Table 1.

| Property | Haar | db | Coiflet | Symlet | Bior |
|------------------------------|--------------|----|--------------|--------------|------|
| Discrete wavelet transform | | | \checkmark | \checkmark | |
| Continuous wavelet transform | \checkmark | | \checkmark | \checkmark | |
| Orthogonal analysis | \checkmark | | \checkmark | \checkmark | |
| Biorthogonal analysis | \checkmark | | \checkmark | \checkmark | |
| Near symmetry | | | \checkmark | \checkmark | |
| Asymmetry | | | | | |

Table 1. Specifications of the wavelet family [16].

2.2. Analysis and synthesis

Decomposition of a signal known as analysis and reconstruction of a signal is called synthesis [16]. Figure 1 shows the diagrammatic representation of decomposition and reconstruction of a signal.



Figure 1. Representation of analysis and synthesis.

2.2.1. Analysis

EEG signal decomposition is carried out by employing multiresolution analysis using complementary low pass (CA) and high pass (CD) filters to obtain progressively finer signal details. Table 2 shows the EEG subband decomposition and corresponding frequency for each subband.

| Level of decomposition | CA/CD | LPF/HPF | Frequency band |
|------------------------|-------|---------|-----------------------------|
| 1st lovel | CA11 | LPF | 0-125 Hz: for further level |
| 1St level | CD12 | HPF | 125–250 Hz: Gamma_3 |
| and lovel | CA21 | LPF | 0–62.5 Hz: Ignore |
| | CD22 | HPF | 125–62.5 Hz: Gamma_2 |
| and lovel | CA31 | LPF | 0–31.25 Hz: Ignore |
| Sid level | CD32 | HPF | 31.25–62.5 Hz: Gamma_1 |
| 4th lovel | CA41 | LPF | 0-15.62 Hz: Ignore |
| 4th level | CD42 | HPF | 15.62–31.25 Hz: Beta wave |
| 5th loval | CA51 | LPF | 0–7.81 Hz: Ignore |
| Jui level | CD52 | HPF | 7.81–15.62 Hz: Alpha wave |
| 6th lovel | CA61 | LPF | 0-3.90 Hz: Delta wave |
| oth level | CD62 | HPF | 3.90–7.81 Hz: Theta wave |

Table 2. EEG subband decomposition and corresponding frequency.

DWT is used for the decomposition. It works as a filter to divide the signal into bands at each specific level known as detailed and approximate components [17,18]. The output CD is known as the detailed component and output CA is known as the approximate component. After the 1st level of decomposition HIGH and LOW level components of the EEG signal are obtained. The LOW level component is further decomposed to obtain HIGH and LOW level components. In this paper a 6-level decomposition is explained for 6 different wavelets. The number of decomposition levels depends on sampling frequency of the signal and bandwidth. Six-level EEG signal decomposition is shown in Figure 2.

2.2.2. Synthesis

Reconstruction of a decomposed signal is known as synthesis. It is the reverse of decomposition, and is achieved by inverse discrete wavelet transform (IDWT). IDWT reconstructs the EEG signal from the given coefficients by performing 6 levels of inverse discrete wavelet transform. Figure 3 shows the reconstructed EEG signal.

3. Database collection

The authors have used the web-based freely available EEG database published by Karunya University, Coimbatore, India. The database consists of real EEG signals of normal subjects and focal and generalized epileptic seizures captured from patients of the age group between 1 and 107 years. It is a 16-channel EEG recording with 250 Hz of sampling frequency [19]. The study was carried out by the authors on 63 subjects aged between 1 and 80 years.

4. Results

Decomposition and reconstruction of an EEG signal (normal, focal epilepsy, and generalized epilepsy) using Haar, Symlet, Coiflet, db18, Bior3.7, and Rbio 6.8 is shown below. Six levels of decomposition are carried out to extract all the required EEG subbands. Figures 4–6 show the EEG signal (normal EEG, EEG with focal seizures, and EEG with generalized seizures) decomposition by Haar and other wavelets. Six levels of reconstruction were carried out to compose the original EEG signal from the decomposition levels. Figure 7 and Figure 8 show the reconstruction of an EEG signal.

4.1. Statistical features: MSE, MAE, SER, and PRMSD

Six wavelets were applied to the EEG data for analysis and synthesis. When more than one wavelet is applied to the data, a situation arises where it becomes important to decide which one is the best suited. Risk function or mean square error (MSE) is the property of data that measures the average squared difference between the actual signal and the reconstructed signal. Mean absolute error (MAE) is also a measure to know how much deviation is present in the reconstructed signal from the original signal.

$$MSE = \sum_{n=1}^{n} [a (n) - b (n)]^2 / N$$
(3)

a (n): original signalb (n): reconstructed signalN: no. of samples

$$MAE = \sum n = i^{j} |a(n) - b(n)| / (j - i)$$
(4)

Normal EEGs and epileptic EEGs (focal or generalized) are used to evaluate the performance of a wavelet. Table 3 shows the performance analysis of various wavelets. The calculations of mean square error and mean absolute error are shown for normal and epileptic EEGs. The maximum error between the original EEG and reconstructed EEG using Haar wavelet is 1.2294e-013, while error by using Bior3.7 wavelet is 1.5101e-013. It shows that the maximum error introduced during reconstruction by both wavelets is almost the same, which is lower than the rest of the wavelets studied in this paper. The error, MSE, and MAE for various wavelets are shown in Table 3.



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Figure 2. Six-level EEG signal decomposition.



Figure 3. EEG signal reconstruction.



Figure 5. Decomposition of an EEG signal with focal seizures using Coiflet5 wavelet.





Figure 7. Normal EEG signal reconstruction using Haar wavelet.



Figure 8. Reconstruction of an EEG signal with generalized seizures using Symlet18 wavelet.

| EEG Type | Wavelet | MSE (μV^2) | MAE (μV) |
|---------------------------|----------|-----------------|---------------|
| | Haar | 2.4571e-029 | 3.0360e-015 |
| | db18 | 8.8805e-021 | 7.5233e-011 |
| | Symlet18 | 3.9468e-019 | 5.0506e-010 |
| Normal EEG | Coif5 | 2.6378e-016 | 1.2907e-008 |
| | Bior3.7 | 1.2045e-028 | 6.9175e-015 |
| | rBio6.8 | 2.4512e-024 | 1.2484e-012 |
| | Haar | 6.2740e-030 | 1.6898e-015 |
| | db18 | 4.8132e-023 | 5.4889e-012 |
| | Symlet18 | 3.9532e-022 | 1.5620e-011 |
| Focal epileptic EEG | Coif5 | 1.6111e-017 | 3.1912e-009 |
| | Bior3.7 | 1.1062e-029 | 2.0429e-015 |
| | rBio6.8 | 7.0952e-026 | 2.0956e-013 |
| | Haar | 6.0507e-030 | 1.6566e-015 |
| | db18 | 1.9171e-022 | 1.0905e-011 |
| | Symlet18 | 1.1972e-021 | 2.7176e-011 |
| Generalized epileptic EEG | Coif5 | 4.0830e-017 | 5.0639e-009 |
| | Bior3.7 | 9.1290e-030 | 1.9568e-015 |
| | rBio6.8 | 1.9558e-025 | 3.4709e-013 |

Table 3. MSE and MAE of EEG signals for various wavelets.

Signal-to-error ratio (SER) is estimated as the power of the original signal divided by the power of the error signal and then it is converted into dB by applying a logarithmic scale. Percent root mean square difference (PRMSD) without considering the mean is estimated for normal and epileptic EEGs using 6 wavelets. It is

one of the most commonly used distortion measures. PRMSD can also be calculated by considering the mean value of the signal. Reconstruction fidelity by pointwise comparison of original data and the reconstructed data is given by the PRMSD. Numerical measure of the residual root mean square error (RMS) is provided by the PRMSD.

$$SER = 10log_{10} \left(EEG_{signal} / ERROR_{signal} \right)^2 \tag{5}$$

 EEG_{signal} : Root mean square amplitude of original EEG signal. $ERROR_{signal}$: Root mean square amplitude of error signal.

$$PRMSD = \sqrt{\frac{\sum_{n=1}^{N} [a(n) - b(n)]^2}{\sum_{n=1}^{N} [a(n)]^2}} \times 100$$
(6)

SER and PRMSD by 6 wavelets is shown in Table 4. MSE, MAE, SER, and PRMSD for each level of EEG reconstruction using Haar wavelet and Bior3.7 wavelet are shown in Table 5 and Table 6 respectively. Good SER is achieved using Haar and Bior3.7 wavelets, and PRMSD is also lower for these two wavelets.

| EEG Type | Wavelet | SER (dB) | PRMSD |
|---------------------------|----------|----------|-------------|
| | Haar | 314.7563 | 1.8289e-014 |
| | db18 | 242.3277 | 7.6492e-011 |
| N LDDO | Symlet18 | 233.8044 | 2.0407e-010 |
| Normal EEG | Coif5 | 190.8965 | 2.8522e-008 |
| | Bior3.7 | 313.6632 | 2.0742e-014 |
| | rBio6.8 | 270.9358 | 2.8393e-012 |
| | Haar | 315.5031 | 1.6782e-014 |
| | db18 | 246.5277 | 4.7165e-011 |
| | Symlet18 | 237.3258 | 1.3605e-010 |
| Focal epileptic EEG | Coif5 | 191.4642 | 2.6717e-008 |
| | Bior3.7 | 313.1457 | 2.2015e-014 |
| | rBio6.8 | 274.9361 | 1.7914e-012 |
| | Haar | 315.2575 | 1.7263e-014 |
| ~ | db18 | 240.2491 | 9.7173e-011 |
| | Symlet18 | 232.2939 | 2.4283e-010 |
| Generalized epileptic EEG | Coif5 | 186.9657 | 4.4845e-008 |
| | Bior3.7 | 313.4713 | 2.1205e-014 |
| | rBio6.8 | 270.1624 | 3.1037e-012 |

Table 4. SER and PRMSD of EEG signals using various wavelets.

5. Discussion

An EEG signal can be categorized into symmetric, orthogonal, nonstationary, and compact [20]. Many advantages of wavelet transform over the other transforms such as Fourier and autoregressive models made us apply the wavelet to an EEG signal for analysis and synthesis [21]. Wavelet transform provides a multiresolution description of a nonstationary EEG signal. At low frequencies it represents good frequency resolution and at high frequencies it gives better time resolution. Selection of the correct mother wavelet plays a very important

| EEG signal with foc | EEG signal with focal epilepsy | | | | | |
|---------------------|--------------------------------|---------------|----------|-------------|--|--|
| EEG subbands | MSE (μV^2) | MAE (μV) | SER (dB) | PRMSD | | |
| EEG signal 250 Hz | 8.8678e-030 | 2.0021e-015 | 315.3630 | 1.7055e-014 | | |
| 64–128 Hz | 1.5722e-029 | 2.5164e-015 | 315.8465 | 1.6125e-014 | | |
| 32–64 Hz | 3.1111e-029 | 3.6472e-015 | 315.7569 | 1.6299e-014 | | |
| 16–32 Hz | 5.8405e-029 | 5.0382e-015 | 315.5090 | 1.6771e-014 | | |
| 8–16 Hz | 5.1260e-029 | 4.7209e-015 | 316.9843 | 1.4309e-014 | | |
| 0–8 Hz | 8.6847e-029 | 6.7963e-015 | 314.5719 | 1.8681e-014 | | |
| EEG signal with ger | eralized epilep | sy | | | | |
| EEG subbands | MSE (μV^2) | MAE (μV) | SER (dB) | PRMSD | | |
| EEG signal 250 Hz | 1.2929e-030 | 8.2916e-016 | 314.7764 | 1.8246e-014 | | |
| 64–128 Hz | 2.4209e-030 | 1.0325e-015 | 314.9730 | 1.7831e-014 | | |
| 32–64 Hz | 3.8787e-030 | 1.3474e-015 | 315.6312 | 1.6536e-014 | | |
| 16–32 Hz | 6.4807e-030 | 1.7725e-015 | 315.3259 | 1.7128e-014 | | |
| 8–16 Hz | 6.7606e-030 | 1.6491e-015 | 315.1442 | 1.7494e-014 | | |
| 0–8 Hz | 2.5182e-030 | 1.0540e-015 | 314.0301 | 1.9884e-014 | | |
| Normal EEG signal | | | | | | |
| EEG subbands | MSE (μV^2) | MAE (μV) | SER (dB) | PRMSD | | |
| EEG signal 250 Hz | 1.0579e-028 | 4.8218e-015 | 315.6833 | 1.6437e-014 | | |
| 64–128 Hz | 1.8285e-028 | 6.3807e-015 | 316.2935 | 1.5316e-014 | | |
| 32–64 Hz | 4.2896e-028 | 9.2733e-015 | 315.5289 | 1.6732e-014 | | |
| 16–32 Hz | 7.3487e-028 | 1.3686e-014 | 315.9641 | 1.5914e-014 | | |
| 8–16 Hz | 2.1723e-027 | 1.9178e-014 | 313.3515 | 2.1499e-014 | | |
| 0–8 Hz | 4.2945e-027 | 3.0153e-014 | 313.0514 | 2.2255e-014 | | |

Table 5. MSE, MAE, SER, and PRMSD for subbands of EEGs using Haar wavelet.

Table 6. MSE, MAE, SER, and PRMSD for subbands of EEG using Bior3.7 wavelet.

| EEG signal with focal epilepsy | | | | | | |
|--------------------------------|------------------|---------------|----------|-------------|--|--|
| EEG subbands | MSE (μV^2) | MAE (μV) | SER (dB) | PRMSD | | |
| EEG signal 250 Hz | 1.3369e-029 | 2.2645e-015 | 313.5802 | 2.0941e-014 | | |
| 64–128 Hz | 2.6094e-029 | 3.3785e-015 | 313.7972 | 2.0424e-014 | | |
| 32–64 Hz | 7.7740e-029 | 5.8480e-015 | 312.5531 | 2.3569e-014 | | |
| 16–32 Hz | 2.6728e-028 | 1.1050e-014 | 311.7838 | 2.5752e-014 | | |
| 8–16 Hz | 7.2196e-028 | 1.6606e-014 | 312.3462 | 2.4137e-014 | | |
| 0-8 Hz | 4.9710e-028 | 1.4351e-014 | 312.9830 | 2.2431e-014 | | |
| EEG signal with ger | neralized epilep | osy | I | 1 | | |
| EEG subbands | MSE (μV^2) | MAE (μV) | SER (dB) | PRMSD | | |
| EEG signal 250 Hz | 1.8331e-030 | 8.9285e-016 | 313.2601 | 2.1727e-014 | | |
| 64–128 Hz | 3.6012e-030 | 1.2898e-015 | 313.5803 | 2.0940e-014 | | |
| 32–64 Hz | 1.1370e-029 | 2.3142e-015 | 312.4807 | 2.3767e-014 | | |
| 16–32 Hz | 3.3370e-029 | 4.2886e-015 | 312.4783 | 2.3773e-014 | | |
| 8–16 Hz | 6.5750e-029 | 5.3862e-015 | 312.4098 | 2.3961e-014 | | |
| 0-8 Hz | 9.5534e-030 | 2.2806e-015 | 312.9774 | 2.2446e-014 | | |
| Normal EEG signal | | | | | | |
| EEG subbands | MSE (μV^2) | MAE (μV) | SER (dB) | PRMSD | | |
| EEG signal 250 Hz | 1.4019e-028 | 5.6864e-015 | 314.4605 | 1.8922e-014 | | |
| 64–128 Hz | 2.1632e-028 | 7.5470e-015 | 315.6355 | 1.6528e-014 | | |
| 32–64 Hz | 8.1694e-028 | 1.4575e-014 | 313.1093 | 2.2107e-014 | | |
| 16–32 Hz | 1.0803e-027 | 2.0905e-014 | 315.3927 | 1.6997e-014 | | |
| 8–16 Hz | 2.7908e-027 | 2.8785e-014 | 315.8641 | 1.6099e-014 | | |
| 0-8 Hz | 1.4737e-026 | 4.9627e-014 | 311.5963 | 2.6314e-014 | | |

and crucial role [22]. Six wavelets were selected based on various properties such as symmetry, near symmetry, orthogonality, biorthogonality, and possibility of CWT/DWT. The importance of maintaining the similarity between the biosignal and the mother wavelet is verified as the work is carried out. Daubechies is a modified Haar wavelet, which is highly asymmetric. The Symlet and Coiflet family is also designed by the Daubechies to be a symmetrical wavelet with compact support. Bior and Rbio are symmetric wavelets with very compact support [23]. The nature of the Haar wavelet resembles a step function, which does not match with the nature of EEG signals, while the nature of Bior3.7 matches the EEG signal's nature. Mean and standard deviation in the performance parameters like MSE, MAE, PRMSD, and SER are shown in Table 7.

Table 7. Mean and standard deviation in MSE, MAE, PRMSD, and SER for 3 categories of EEG.

| Mean \pm standard deviation using Haar wavelet | | | | | | |
|---|-----------------------------|--------------------------|--------------------|--------------------------|--|--|
| EEG type | MSE (μV^2) | MAE (μV) | SER (dB) | PRMSD | | |
| Normal | $4.636e-028 \pm 1.1e-028$ | $7.79e-15 \pm 3.854-015$ | 315.6 ± 0.4081 | $8.43e-014 \pm 2.44e-13$ | | |
| Focal epilepsy | $2.427e-030 \pm 2.31e-030$ | $9.47e-30 \pm 4.28e-030$ | 315 ± 0.2459 | $1.77e-014 \pm 4.98e-16$ | | |
| Generalized epilepsy | $2.837e-030 \pm 2.77e-030$ | $1.06e-15 \pm 4.61e-016$ | 315.2 ± 0.2802 | $1.75e-014 \pm 5.69e-16$ | | |
| Mean \pm standard deviation using Bior3.7 wavelet | | | | | | |
| EEG type | MSE (μV^2) | MAE (μV) | SER (dB) | PRMSD | | |
| Normal | $8.89e-028 \pm 1.2e-027$ | $9.66e-15 \pm 5.05e-015$ | 313.2 ± 0.8516 | $2.21e-014 \pm 2.17e-15$ | | |
| Focal epilepsy | $3.467e-030 \pm 3.58e-030$ | $1.05e-15 \pm 4.95e-016$ | 313.2 ± 0.2695 | $2.19e-014 \pm 6.84e-16$ | | |
| Generalized epilepsy | $4.57e-0.30 \pm 4.718e0.30$ | $1.21e-15 \pm 5.77e-016$ | 313.1 ± 0.3599 | $2.19e-14 \pm 7.99e-16$ | | |

6. Conclusion

To check the suitability of various mother wavelets based on statistical properties, they were applied to a total of 63 normal and epileptic real EEG signals for analysis and synthesis. The EEG signal contains sharp spikes, and db and Haar wavelets cannot completely reveal the characteristics of impulsive components. The result shows that Haar and Bior3.7 wavelets show better reconstruction of an EEG signal (normal as well as diseased) as compared to any other wavelet. This shows that near symmetric or symmetric wavelets are more suitable for EEG signal analysis. Statistical features like MSE and MAE improvements or SER and PRMSD are measured and compared during analysis and synthesis of normal EEGs and EEGs with generalized and focal seizures. Better results of analysis and synthesis for normal and diseased EEGs were obtained using Haar and Bior3.7 wavelets. This shows that the similarity between the nature of the mother wavelet and the biosignal should not always be the only important criterion for signal processing, but that optimization of various statistical parameters and the knowledge about the features need to be studied, and will help the user in selecting the suitable wavelet. It is always recommended to check the suitability of more than one wavelet for the given data. Other statistical features like PSD and SNR can also be applied to subbands of EEG signals to classify normal and epileptic EEGs.

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