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Accelerating the solving of nonlinear equations using the homotopy method: application on finding the operating point of complex circuits

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Abstract: Analog circuits with nonlinear elements (e.g., diode, BJT, and CMOS) and integrated circuits are very complex systems. As a result, they are very difficult to analyze because of the need to generate a nonlinear equation system solution in order to do so. Solving nonlinear equations is still a challenging problem. Iterative methods, however, are frequently used to solve them. This paper describes a method that can be used to both accelerate the solving of nonlinear equations and find the operating point in various integrated circuits by construction of the global homotopy equation of the analog circuit. This is done by converting the elements in the circuit to their equivalent dependent sources. The proposed method is based on three steps. The first step is the use of the homotopy method to get a continuous function. The second step consists of finding the best direction of the solution by applying prediction and the correction procedures. The third step consists of the control of step size to accelerate the solution search. In order to demonstrate the effectiveness of the proposed method, a new type of software called PyAMS (Python Language for Analog and Mixed Signal) was created by the authors based on the proposed method. A comparison was done between the proposed method and two other methods and, in doing so, many types of integrated circuits were used. Based on the proposed method, an analysis of many universal circuits was carried out to verify the correct function of the circuit.

Key words: Prediction, correction, homotopy, nonlinear equations, analog circuits

1. Introduction

Finding the operating points of nonlinear circuits is an important problem in circuit simulation. Until now, the SPICE circuit simulator has mainly been used to formulate a circuit's equation with the modified nodal analysis method (MNA), and it employs the Newton–Raphson method (NR) to solve it [1,2]. However, the NR method does not always result in a solution, so we can use some techniques developed in SPICE to help with the convergence of the NR method. These techniques are source stepping, temperature sweeping, G_{min} stepping, and voltage limiting [2].

The continuation method, proposed in [3–11], is an effective method to solve nonlinear circuits. The continuation method is based on 3 steps:

1. Construct a continuous function by using the homotopy method.
2. Find the best direction of the solution by using the predictor-corrector method. The predictor delivers a new value of the continuation parameter at each step of the method and predicts an approximate solution of the corresponding new system in the homotopy. To calculate the predictor we can use either the secant

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or Euler methods [12]. The predicted approximate solution is then corrected by applying the corrector, e.g., the NE method.

3. Accelerate the solution search by controlling the step size.

Based on these three steps, this paper investigates the development of an approach to accelerate the solving of nonlinear equations. It also attempts to locate the operating point in various analog and integrated circuits by either modeling the analog devices (e.g., BJT, diode, and resistor) using the homotopy (continuation) parameters or constructing the global homotopy equation of the analog circuit. This is done by converting the elements in the circuit to their equivalent dependent sources.

The paper is organized as follows: Section 2 presents the basis of the homotopy equation. Section 3 discusses the solving of the homotopy equation. Section 4 proposes a new method to control the step size to accelerate the solving. Section 5 examines the modeling of analog elements through the continuous parameter. The simulation results are then given in Section 6. Finally, Section 7 consists of the conclusion and future work.

2. Homotopy function

The homotopy function is a common method used to solve nonlinear algebraic equations systems and can be applied to a large variety of problems [13]. We are most interested in solving the zero-finding problems:

$$F\left(x\right) = 0 \tag{1}$$

where, $x \in R^n$ and $F{:}R^n \longrightarrow R^n$.

The homotopy function $H\left(x, \lambda\right)$ is based on embedding a parameter λ into $F\left(x\right)$. As a result, an equation with a new dimension is created:

$$H\left(x, \lambda\right) = 0 \tag{2}$$

where $\lambda \in R$ and $H{:}\, R^n \times R \longrightarrow R^n$. The parameter λ is the continuation or homotopy parameter.

At present, the most widely used method is the Newton homotopy mapping (NH):

$$H(x, \lambda) \ = \ F(x) \ + \ (1 \ - \lambda) \, F(x_0) \tag{3}$$

$x_0 \in R^n$ is the solution of the homotopy function when $\lambda = 0$.

According to homotopy mapping, the solution of the equation $F\left(x\right) = 0$ can be adopted to solve the homotopy equation $H\left(\lambda, \, x\right) = 0$. For any λ ranging from 0 to 1, if the homotopy equation solution (x, λ) exists, the corresponding curve of (x, λ) starts from $(x_0, 0)$ and ends in the solution $(x^{*}, 1)$.

3. Solving the homotopy equation

The continuation method has widely been used in the literature to solve the homotopy equation (Eq. (2)) [12,14]. First, the homotopy parameter λ should be adjusted by choosing a step size $\Delta\lambda (0 \, < \, \Delta\lambda \, < 1)$; then the predictor-corrector method (illustrated in Figure 1) is used. The predictor computes the approximation of the solution by computing a point along the tangent line to the homotopy path at point (x_0, λ_0) using the following formula:

$$x_1^0 = x_0 + \Delta\lambda \, \dot{x}_0 \tag{4}$$

where

$$\dot{x}_0 = \partial x / \partial \lambda \tag{5}$$

Following this, the solution x_1 of $H(x_1, \lambda_1)$at $\lambda_1 = \lambda_0 + \Delta\lambda$can be computed by the corrector function using NR iteration:

$$\begin{cases} J\left(x_1^i, \lambda_1\right) \Delta x_1^i = -H\left(x_1^i, \lambda_1\right) \\ x_1^{i+1} = x_1^i + \Delta x_1^i \end{cases} \tag{6}$$

where $J{=}\partial H/\partial x$ is the Jacobien function of H, i is the iteration index, and x_1^0 is the initial approximation or the initial of Gauss calculated by the predictor (Eq. (4)).

3.1. Prediction

In this part, how to calculate a good prediction value to obtain a good direction of correction is discussed. There are two methods for calculating prediction: the secant method and the Euler method (Figure 2).

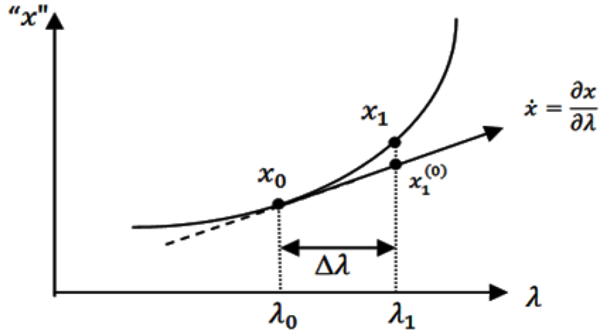

Figure 1. Predictor-corrector method.

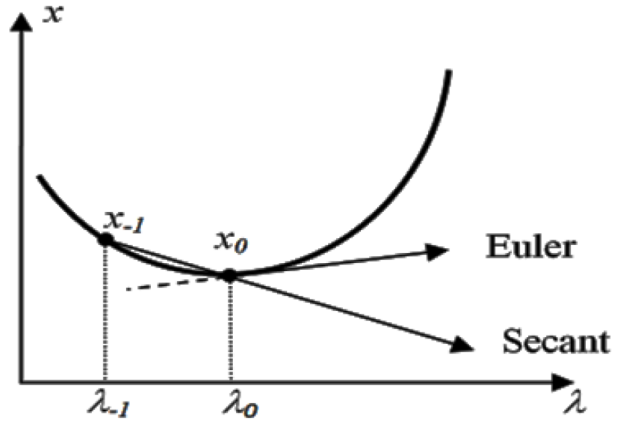

Figure 2. Calculating the tangent with the secant method and the Euler method.

Secant prediction is a very simple method based on two previous points of the vector x. The format of the tangent in the secant prediction is:

$$\dot{x}_0 = \frac{x_0 - x_{-1}}{\lambda_0 - \lambda_{-1}} \tag{7}$$

Euler prediction is a popular method for calculating the prediction tangent. It is based on the Davidenko differential equation [13]:

$$\frac{\partial H}{\partial x}\dot{x} + \frac{\partial H}{\partial \lambda} = 0 \tag{8}$$

Using Eq. (10), we obtain the tangent equation:

$$\dot{x} = -\left(\frac{\partial H}{\partial x}\right)^{-1} \frac{\partial H}{\partial \lambda} \tag{9}$$

Numerical analysis is an effective method to calculate the tangent $\dot{x}$ in Euler prediction:

- Replacing $(\partial H/\partial x)$ by the Jacobian J, which is calculated by the NR algorithm.

- The derivative $(\partial H/\partial\lambda)$ can be presented by the backward difference methods [12].

Therefore, we obtain a new equation close to the numerical derivation of the Euler prediction method:

$$\dot{x} = -J\left(x,\lambda\right)^{-1}\frac{H\left(x,\lambda\right)-H\left(x,\lambda-\partial\lambda\right)}{\partial\lambda} \tag{10}$$

3.2. Correction

To attain an effective calculation in the correction process, we must first find another performance direction in the 5th stage of the algorithm (01):

$$d_i = -H\left(x_1^i,\lambda_1\right)/J\left(x_1^i,\lambda_1\right) \tag{11}$$

For the Newton step, we can consider the convergence using the Armijo rule [1], which is based on the line search method for optimization or decrease in $\|H\|$ along the line segment $[x_1^i, x_1^i{+}d_i]$. It is based on finding the smallest value σ such that:

$$\left\|H\left(x_1^i+\sigma d_i+\lambda_1\right)\right\| < \left(1-\alpha d_i\right)\left\|H\left(x_1^i,\lambda_1\right)\right\| \tag{12}$$

The parameter $\alpha\in[0,1]$ is a small number intended to make Eq. (12) as easy as possible. It results in this new step:

$$x_1^{i+1} = x_1^i+\sigma d_i \tag{13}$$

Notes: 1. The stop iteration of the correction process in analog circuits is the same as the one used in the SPICE simulator [2].

2. A good initial approximation x_0 is essential. This will be discussed below.

Improving the continuation parameter requires either a good predictor value or a good tangent value that has a relationship with the performance step $\Delta\lambda$. If there is a problem in the convergence in the correction process, half of the previous step $(\Delta\lambda/2)$ should be used to find a new prediction. An algorithm proposed in [12] to solve the homotopy equation is given below:

Algorithm 01 (Alg. 01)

```
Choose $\Delta\lambda$ and $x_0$
Start with $\lambda$ = 0 and solve H(x,$\lambda$) = 0
Step $\lambda$ forward: $\lambda = \lambda + \Delta\lambda$
Calculate the predictor: $x_1^0{=}x_0{+}\Delta\lambda\,\dot{x}_0$
Calculate the corrector with the NR algorithm:
    • $x_1^{i+1}{=}x_1^i{-}H\left(x_1^i{,}\lambda_1\right)/J\left(x_1^i{,}\lambda_1\right)$
    • Use the line search method for optimization
If there is no convergence, a half step of $\Delta\lambda$, ($\Delta\lambda{=}\Delta\lambda/2$), then go to stage 3
Choose a good step $\Delta\lambda$
$x_0{=}x_1$
If $\lambda < 1$, go to stage 3
Return the final solution
```

4. New method to control the step size

In Algorithm (01) proposed in [12], the prediction-correction of the homotopy method uses a half-step control strategy. More specifically, the step length is increased by the user if it converges rapidly. If not, it will be decreased by a half step. Other methods such as the asymptotic expansion and the Den Heijer and Reinbolt step length adaptation are discussed for Newton–Gauss iterations in [14]. Unfortunately, although a half step is used, the convergence remains a problem. To overcome this, a new method of choosing a new step size $\Delta\lambda$ based on an adaptive step length is proposed. The subject of step-length adaptation is to eliminate the problem of slow convergence or even divergence in the correction process. This method is mainly based on the performance of the Newton iterations. A simple strategy may be to increase the step length where a few Newton iterations are needed to compute the last point; conversely, we can decrease the step length when the previous point needs many Newton iterations.

To choose the best adaptive step length, we must know the relationship between the step length and the convergence problem in Algorithm (01). To do this, we choose the value of $\Delta\lambda$ that can give the convergence in the process of correction (i.e. the NR iteration $\|\Delta x_i\| > \varepsilon_x$ or $\|H(x_i,\lambda_1)\| > \varepsilon_H$, where ε_x and ε_H are the errors of the stop iteration in the NR method [1]). The correction process results in the following relation:

$$1 \geq 1/\left(1+\frac{\|H\|}{\varepsilon_H}\right) > 0 \tag{14}$$

The value $1/\left(1+\frac{\|H\|}{\varepsilon_H}\right)$ is close to 0 when there is divergence. It takes the value 1 when there is convergence. For this, the adaptive step length must be directly proportional to $\Delta\lambda_0$ and inversely proportional to $1+\frac{\|H\|}{\varepsilon_H}$. It can be given by the following equation:

$$\Delta\lambda_1 = \left(\frac{1}{1+G\,\|H\|}\right)\Delta\lambda_0 \tag{15}$$

where G is a gain in the interval $1 < G < 1/\varepsilon_H$. By using Eq. (17), we can automatically increase or decrease the step length if there is convergence or divergence, respectively. However, there is another problem; when we have slow convergence, Eq. (17) will not increase the step length $(1+G\,\|H\| \approx 1)$. In order to correct this, we must use an n-step length. Eq. (17) can be then improved to:

$$\Delta\lambda_1 = \left(\frac{n}{1+G\,\|H\|}\right)\Delta\lambda_0 \quad \text{for } n > 1 \tag{16}$$

The new proposed algorithm for this new adaptive step is given in Algorithm 02.

The difference between Alg. (01) and Alg. (02) in the new step-length adaptation is seen on the circuit modeling using the homotopy method in the simulation section.

5. Modeling of analog elements using the continuous parameter

In this section, we will look at how to change the structure of an analog element by embedding the homotopy parameter to acquire good acceleration in the operating point with use of the continuation solver (Alg. (01) and Alg. (02)). The continuation solver starts from a linear element ($\lambda = 0$) and then reverts to the original problem ($\lambda = 1$). To do so, we need to create a global homotopy equation of the analog circuit. It consists of three steps:

Algorithm 02 (Alg. 02)

```
1: Chose $\Delta\lambda_0$, n, G and x$_o$
2: Start with $\lambda = 0$, $\Delta\lambda_1$=$\Delta\lambda_0$ and solve H(x,$\lambda$) = 0
3: Step $\lambda$ forward $\lambda = \lambda + \Delta\lambda_1$
4: Calculate the predictor using the Euler method: $x_1$=$x_0$+$\Delta\lambda_1 \dot{x}_0$
5: Calculate the corrector with the NR algorithm and find the smallest value $\sigma$ with the line search method:
       $$x_1^{i+1} = x_1^i + d_i$$
6: If there is convergence by NR: x$_0$=x$_1$
7: $\Delta\lambda_1$=$\frac{n}{1+G\|H\|}\Delta\lambda_0$
8: If $\lambda$ <1, go to step 3
9: Return the final solution
```

1. Convert any element in the circuit to its equivalent in either a dependent voltage source or a dependent current source in the circuit.
2. Create the global equation of the circuit so that it has a current source presented by the KCL (Kirchhoff's current law) method and a voltage source presented by the MNA method.
3. Transform the global equation to the homotopy equation.

5.1. Convert any element in the circuit to its equivalent

In this step, any element should be converted to its equivalent of voltage source or current source in the circuit so that their values depend on the voltage in the nodes of the element or the current across the voltage sources.

Example 1 *A resistor can be represented by a current source* I *(Figure 3a) that is dependent on the voltage between the nodes* v_1 *and* v_2 *and expressed by:*

$$I\left(v_1, v_2\right) = \frac{v_1 - v_2}{R} \tag{17}$$

where R *is the resistance value.*

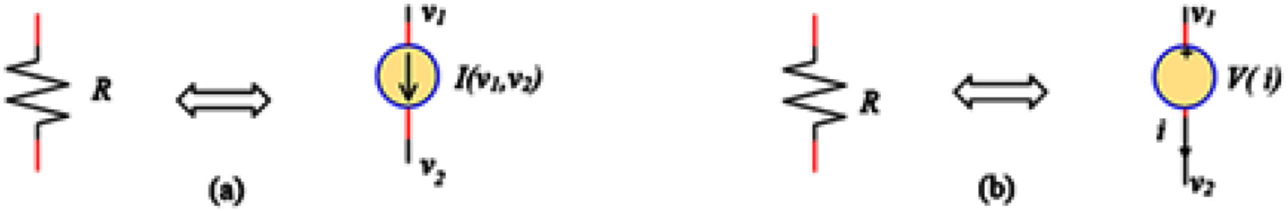

Figure 3. The equivalent of resistor: a) dependent current source, b) dependent voltage source.

The resistor can also be represented by a voltage source V (Figure 3b), dependent on the voltage between the nodes v_1 and v_2 and the current i across source V. It is expressed by:

$$V\left(i\right) = R\, i \tag{18}$$

Example 2 *The diode can be represented by its equivalent nonlinear current source*I, *dependent on the voltage in nodes* v_1 *and*v_2, *shown in Figure 4. It is given as:*

$$I\left(v_1, v_2\right) = Iss(exp\left(\frac{v_1 - v_2}{Vt}\right) - 1) \tag{19}$$

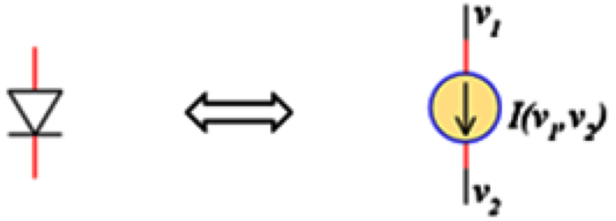

Figure 4. The equivalent of diode by dependent voltage source.

Example 3 *In this example, we deal with the BJT, which is represented by the Ebers–Moll model [2], shown in Figure 5a. For current* I_R *and* I_F, *we have:*

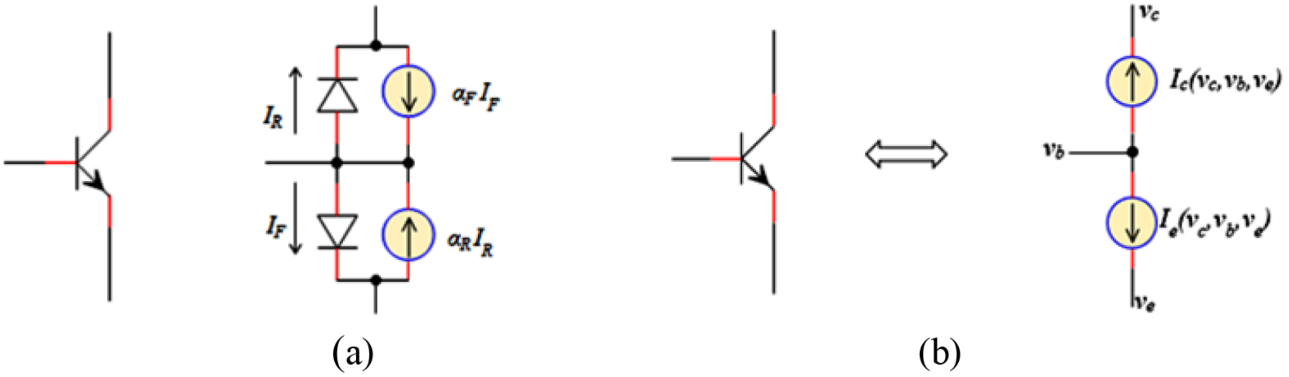

Figure 5. a) The Ebers–Moll model for an NPN BJT, b) the equivalent of BJT by dependent current source.

$$I_R = I_{se}\left(\exp\left(\frac{V_{be}}{V_{te}}\right) - 1\right) \tag{20}$$

$$I_F = I_{sc}\left(\exp\left(\frac{V_{bc}}{V_{tc}}\right) - 1\right) \tag{21}$$

The equivalent of the transistor can be represented by two current sources, namely I_c *and* I_e *(Figure 5b) and is dependent on the voltage in the three nodes* v_c, v_b, *and* v_e:

$$I_c\left(v_c, v_b, v_e\right) = I_{sc}\left(\exp\left(\frac{v_b - v_c}{V_{tc}}\right) - 1\right) - \alpha_R I_{se}\left(\exp\left(\frac{v_b - v_e}{V_{te}}\right) - 1\right) \tag{22}$$

$$I_e\left(v_c, v_b, v_e\right) = -\alpha_F I_{sc}\left(\exp\left(\frac{v_b - v_c}{V_{tc}}\right) - 1\right) - I_{se}\left(\exp\left(\frac{v_b - v_e}{V_{te}}\right) - 1\right) \tag{23}$$

Example 4 *The capacitor and the inductance are elements of energy storage, and they are dependent on time. The capacitor is equivalent to a current source* I*, which is dependent on the voltage in the nodes* v_1 *and*v_2 *(Eq. (24) and Figure 6a)). The inductance is represented by a source voltage dependent on the current across the source (Eq. (25) and Figure 6b)).*

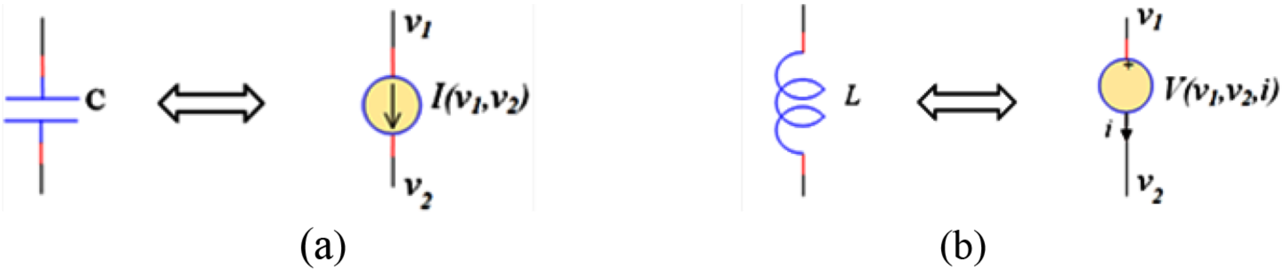

Figure 6. a) The equivalent of capacitor by dependent current source, b) the equivalent of inductance by dependent voltage source.

$$I\left(v_1, v_2\right) = C\,\frac{\partial (v_1 - v_2)}{\partial t} \tag{24}$$

$$V\left(i\right) = L\,\frac{\partial i}{\partial t} \tag{25}$$

Example 5 *The equivalent of the transistor MOSFET (N-channel) can be represented by current sources* I_d *(Figure 7) and is dependent on the voltage in the three nodes* v_g*,* v_s*, and* v_d*:*

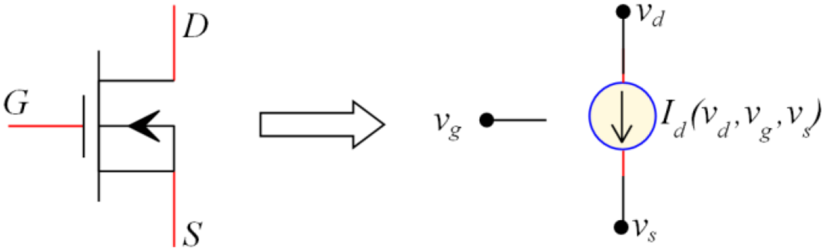

Figure 7. The equivalent of NMOS by dependent current source.

$$I_d\left(v_d, v_s, v_g\right) = \begin{cases} 0 & v_g - v_s \le V_T \\ K_P \frac{W}{L}\left(\left(v_g - v_s - V_T\right)\left(v_d - v_s\right) - \frac{\left(v_d - v_s\right)^2}{2}\right)\left(1 + lambda\ \left(v_d - v_s\right)\right) & v_d \le v_g - V_T \\ K_P \frac{W}{L}\left(v_g - v_s - V_T\right)^2\ \left(1 +\ lambda\ \ \left(v_d - v_s\right)\right) & v_d \ge v_g - V_T \end{cases} \tag{26}$$

5.2. Create the global equation of the circuit

Converting the circuit elements to their dependent sources means simplifying the generation of the global equation of the circuit with the application of two laws: the KCL law for the dependent current source and the

MNA law for the dependent voltage source [2]. The result is the global equation of the circuit described by the following formula:

$$F\left(v,i\right)=\begin{bmatrix} P_i & P_v \end{bmatrix}\begin{bmatrix} i \\ v \end{bmatrix}+\begin{bmatrix} P_I & P_V \end{bmatrix}\begin{bmatrix} I(v,i) \\ V(v,i) \end{bmatrix} \tag{27}$$

where:

- $I(vi)$: the dependent current source.
- $V(vi)$: the dependent voltage source.
- v : the voltage in the node.
- i : the current across the voltage source.
- P_i : position and direction of $i$in the circuit by 0 and ±1.
- P_v : position and direction of v in the circuit by 0 and ±1.
- P_I : position and direction of Iin the circuit by 0 and ±1.
- P_V : position and direction of V in the circuit by 0 and ±1.
- $(v,i)=(v_1v_2,\ldots.v_ni_1,i_2,\ldots,i_m)$.

Note:F is the global function, where F:$R^{n+m}\longrightarrow R^{n+m}$, n represent the number of nodes in the circuit, and m represents the number of voltage sources in the circuit.

Example of a circuit

In the circuit illustrated in Figure 8a, by replacing the equivalent of BJTs with their dependent current sources and the resistor by the dependent voltage source, the equivalent circuit shown in Figure 8b is obtained. The equation system of this circuit uses the KCL and MNA laws in the three nodes.

The expression of the equivalent circuit can be obtained by applying the KCL law in the three nodes and the MNA laws:

$$F\left(v,i\right)=\begin{cases} f_1: i_1+I_{c1}\left(v\right)-I_{c2}\left(v\right)-I_{e2}\left(v\right) \\ f_2: i_2+I_{c2}\left(v\right)-I_{c1}\left(v\right)-I_{e1}\left(v\right) \\ f_3: i_1+i_2-I\left(v\right) \\ f_4: v1-v3+V_{R1}\left(i\right) \\ f_5: v2-v3+V_{R2}(i) \end{cases} \tag{28}$$

where $(vi)=(v_1,v_2,v_3,i_1,i_2)$ because the number of nodes in the circuit shown in Figure 8b is three, and i_1 and i_2 are across the two dependent voltage sources V_{R1} and V_{R2}, respectively. As a result, Eq. (28) can be written as:

$$F\left(v_1,v_2,v_3,i_1,i_2\right)=\left(f_1f_2f_3f_4f_5\right) \tag{29}$$

and can be expressed directly using Eq. (27) as:

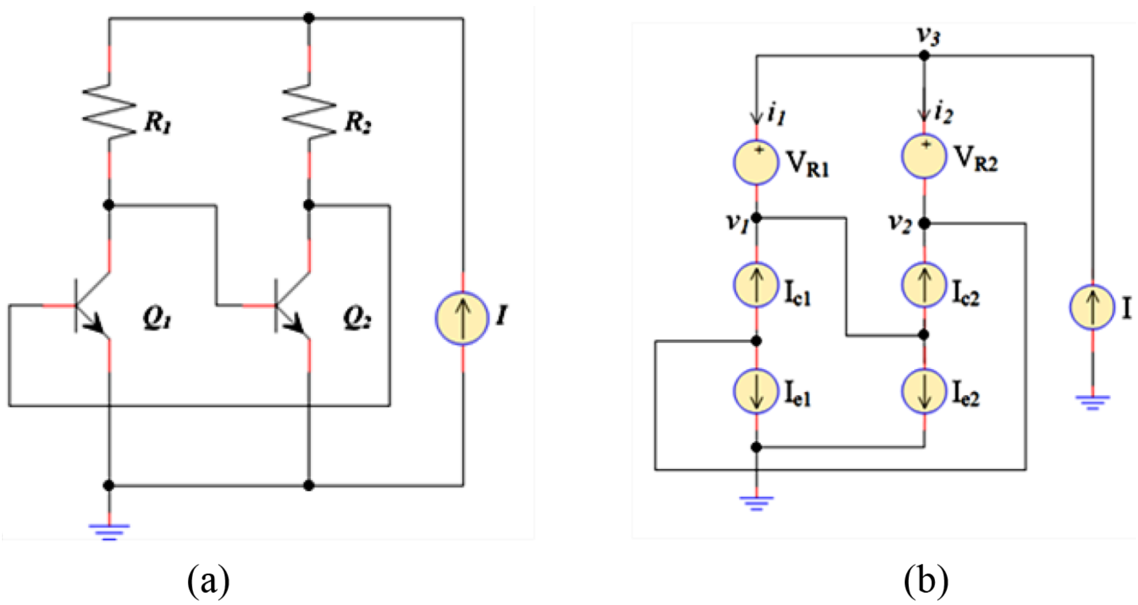

Figure 8. Example of a circuit: a) simple oscillator circuit with BJTs b) the equivalent of the circuit using dependent source.

$$F(v,i) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ v_1 \\ v_2 \\ v_3 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 0 & -1 & 0 & 0 & 0 \\ -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_{c1}(v) \\ I_{c2}(v) \\ I_{e1}(v) \\ I_{e2}(v) \\ I(v) \\ V_{R1}(i) \\ V_{R2}(i) \end{bmatrix} \tag{30}$$

5.3. Transforming the global equation into the homotopy equation

Constructing global equation Eq. (27) simplifies the construction of the homotopy equation that has the general form of Eq. (3); thus, we obtain the global homotopy equation given by the following formula:

$$H(v,i,\lambda) \;=\; F(v,i) \;+\; (1-\lambda)\,F(v_0,i_0) \tag{31}$$

To obtain the same step source used in SPICE, we have to add $(v_0, i_0) = (0,0)$. The final result is:

$$H(v,i,\lambda) = \begin{bmatrix} P_i & P_v \end{bmatrix} \begin{bmatrix} i \\ v \end{bmatrix} + \begin{bmatrix} P_I & P_V \end{bmatrix} \left(\begin{bmatrix} I(v,i) \\ V(v,i) \end{bmatrix} + (1,\lambda) \begin{bmatrix} I(0,0) \\ V(0,0) \end{bmatrix} \right) \tag{32}$$

In this homotopy equation, we obtain the step in the source:

$$\begin{bmatrix} I(v,i) \\ V(v,i) \end{bmatrix} + (1-\lambda) \begin{bmatrix} I(0,0) \\ V(0,0) \end{bmatrix} \tag{33}$$

Eq. (33) eliminates the problem of convergence of the circuit by applying the continuous solver method.

Finally, we can compare Alg. (01) and Alg. (02) to accelerate the solution in the circuit. This is represented by Eq. (32) and will be detailed in the next section.

6. Simulation results

Simulation results were done using a type of new software created by the authors called PyAMS (Python Language for Analog and Mixed Signal). The software is written in Python language version 2.7, and the graphical user interface is programmed with Delphi XE6. The software simplifies the modeling of the elements using their dependent sources and solves the homotopy equation of the circuit using Alg. (2). It can be applied to analyze the circuits. The user interface is illustrated in Figure 9.

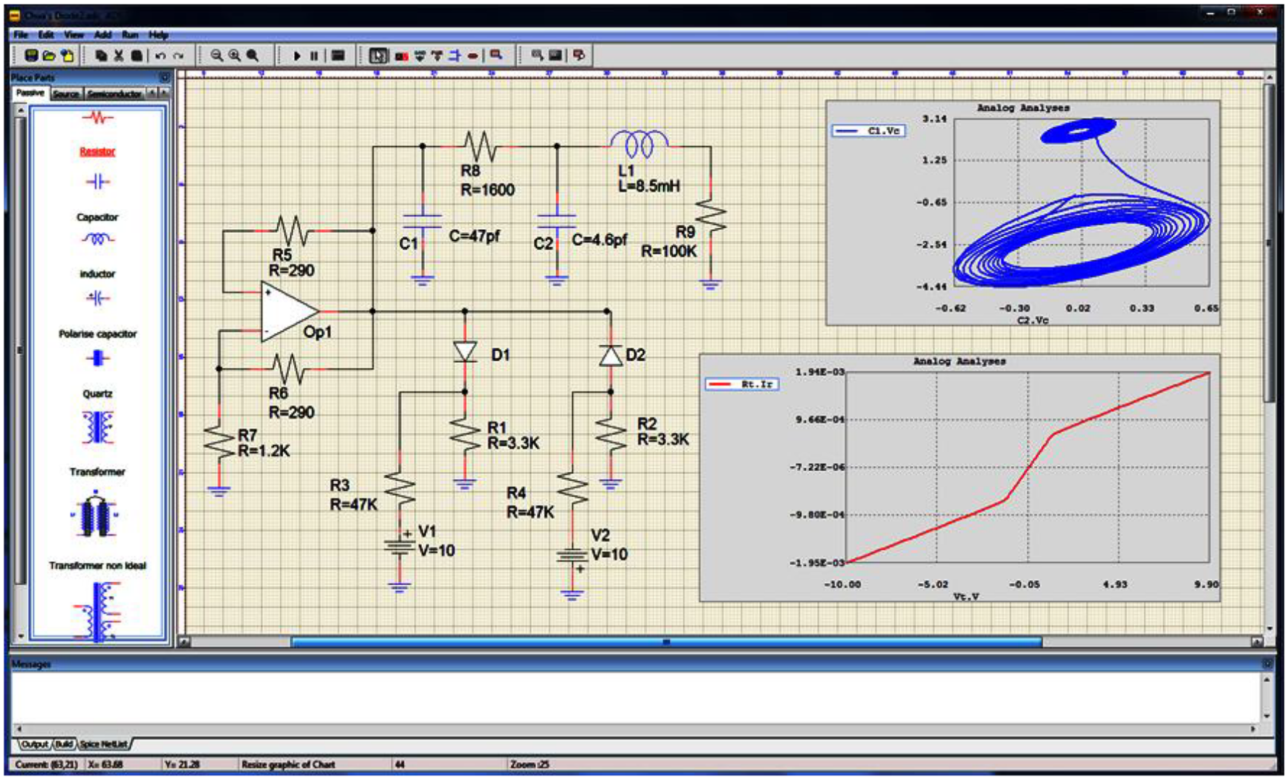

Figure 9. Graphical interface of PyAMS.

To demonstrate the effectiveness of the proposed method, the simulation was divided into two parts: the comparison of the proposed algorithm (Alg. 02) to other methods and the circuits' analysis.

6.1. Comparison of the proposed algorithm and other methods

Five types of practical circuits widely used in analog LSIs are considered in this comparison. These circuits consist of an Ebers–Moll BJT and have convergence issues in the SPICE software but are frequently used as a test circuits [7,8]:

- The hybrid voltage reference circuit (HVRef).

- Six-stage limiting amplifier (6sLA).
- Operational amplifier (μA741).
- Wideband amplifier (RCA3040).
- Basic two-stage operational amplifier (2sOA).

The equations of these circuits are realized with the global homotopy equation (Eq. (32)).

Figures 10–13 compare the proposed algorithm, Alg. (02), and Alg. (01) proposed in [12]. The figure on the left gives the schematic diagram of the circuit realized in PyAMS while the figure on the right compares both algorithms. Alg. (02) is represented with a black line and Alg. (01) with a red one. We can count the number of steps λ in the right figure using the points on the lines, and it can be observed that the number of iterations used in Alg. (02) is lower than the number used in Alg. (01); therefore, Alg. (02), the proposed algorithm, is faster than Alg. (01).

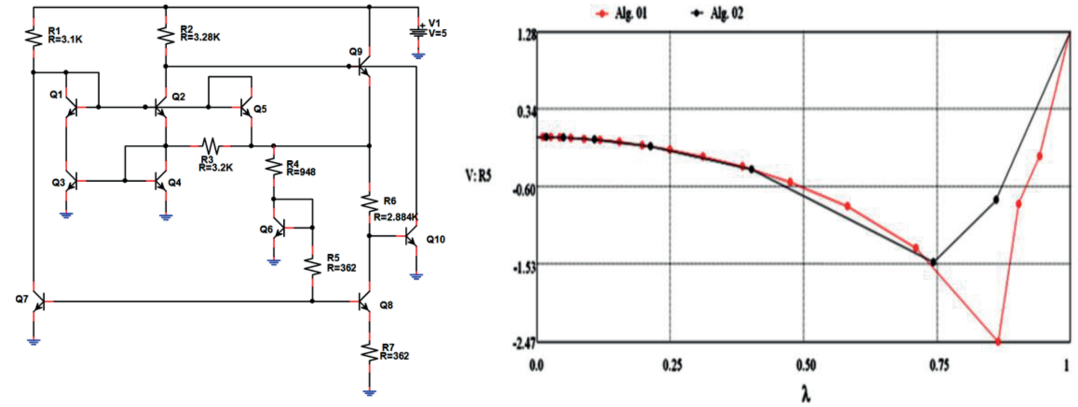

Figure 10. The HVRef circuit: left) the schematic diagram of the circuit, right) the comparison between Alg. (01) and Alg. (02).

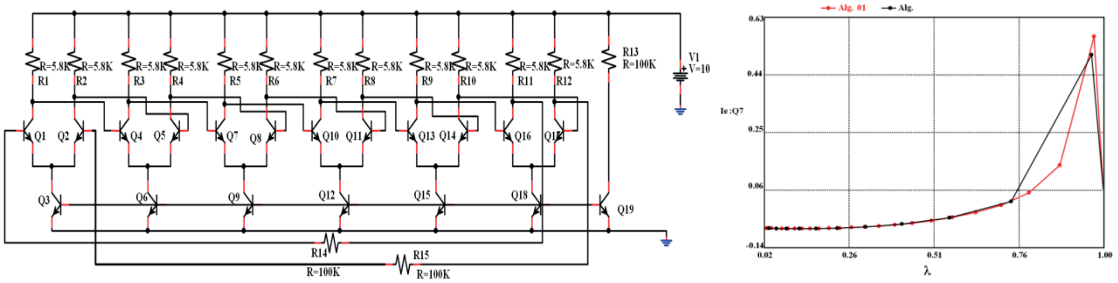

Figure 11. The 6sLA circuit: the schematic diagram of the circuit (left). the comparison (right) between Alg. (01) and Alg. (02).

It can also be noted that the curves in the figure on the right can be obtained using any point in the circuit. In the first circuit (Figure 10), we used the voltage between the terminals of resistor R5. In second circuit of Figure 11, we used the emitter current of transistor Q7. In the circuit shown in Figure 12, the voltage between the terminals of resistor R4 is used. Finally, in the fourth circuit of Figure 13, the emitter current of transistor Q2 is used.

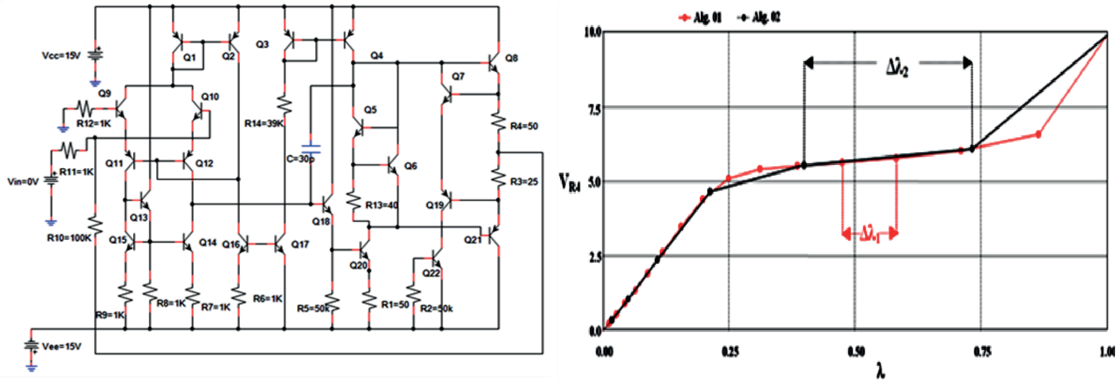

Figure 12. The μA741 circuit: the schematic diagram of the circuit (left) and the comparison (right) between Alg. (01) and Alg. (02).

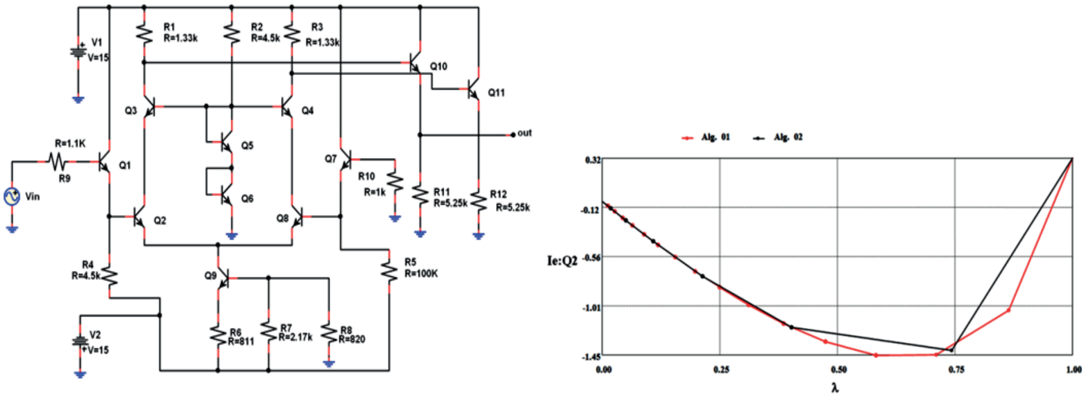

Figure 13. The RCA3040 circuit: the circuit (left) and the comparison (right) between Alg. (01) and Alg. (02).

The comparison of the computational efficiency is summarized in the Table below. It compares the proposed method (Alg. (02)) to two other methods; the first method is the VGNH (variable gain Newton homotopy) implemented with the SPICE software [7,8] while the second is the method (Alg. (01)) proposed in [12]. The parameters used in Alg. (02) are: $G = 10^6, n = 1.6$, and $\Delta\lambda_0 = 0.08$.

Table. Comparison of computational efficiency.

Circuit	Number of steps λ in:		
	VGNH	Alg. (01)	Alg. (02)
HVRef	21	15	07
6sLA	20	26	12
μA741	28	16	07
RCA3040	21	15	08
2sOA	20	15	05
Average	22	17.4	7.8

It can be noted that for the 5th circuit 2sOA in the Table, a textual description of the circuit was used rather than the schematic diagram.

We can also see in the Table that Alg. (02) is faster than the two other methods, namely the method used in SPICE and the one used in [12]. From the average line, Alg. (02) is about three times faster than the VGNH method in SPICE.

6.2. Circuit analysis using the proposed method

In this subsection, the proposed method is used to analyze some universal circuits such as the LM741 noninverting amplifier (Figure 14), the astable multivibrator (Figure 15), the Peltz oscillator (Figure 16), the three-stage ring oscillator (Figure 17), the pulse generator (Figure 18), Chua's circuit (Figure 19), and the CMOS inverter (Figure 20). The figures on the left depict the schematic diagram of the circuit realized by PyAMS while the figure on the right illustrates the circuit analysis using Alg. (02). Using these figures, we can verify the correct functioning of the circuit.

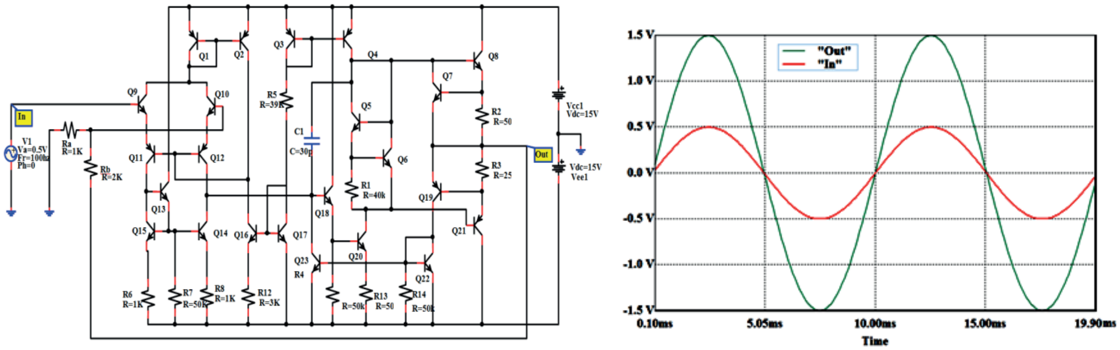

Figure 14. The LM741 noninverting amplifier circuit: the circuit (left) and the waveforms for LM741 input and output (right).

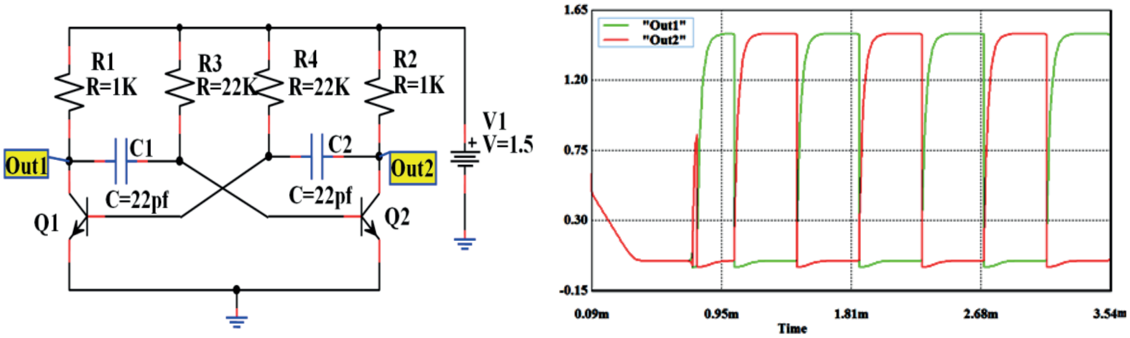

Figure 15. The astable multivibrator circuit: the circuit (left) and simulation results of the astable multivibrator circuit (right).

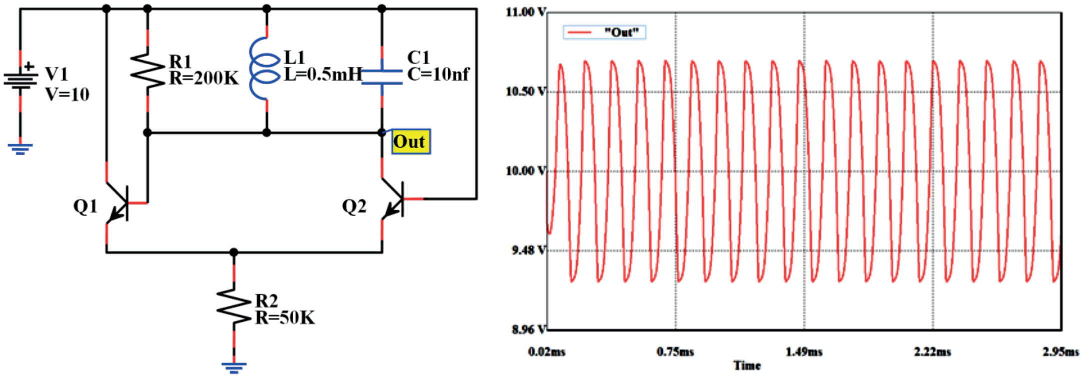

Figure 16. Peltz oscillator circuit: the circuit (left) and simulation results (right).

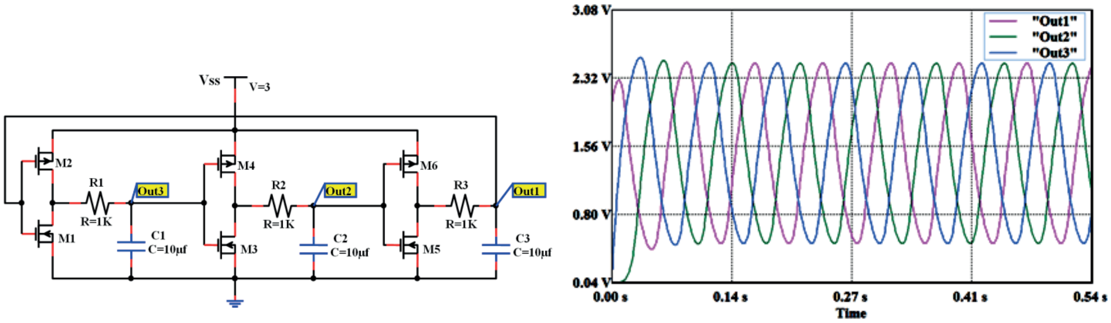

Figure 17. Three-stage ring oscillator circuit: the circuit (left) and simulation results (right).

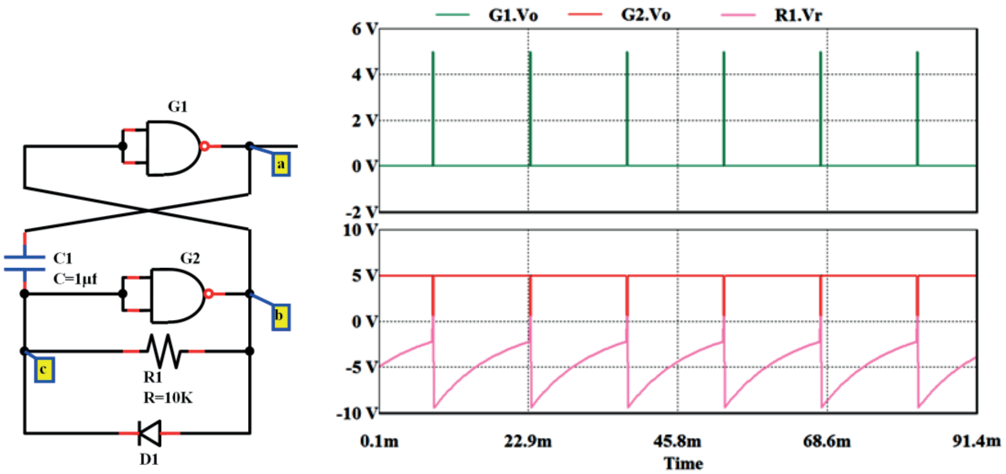

Figure 18. Pulse generator circuit: the circuit (left) and simulation results (right).

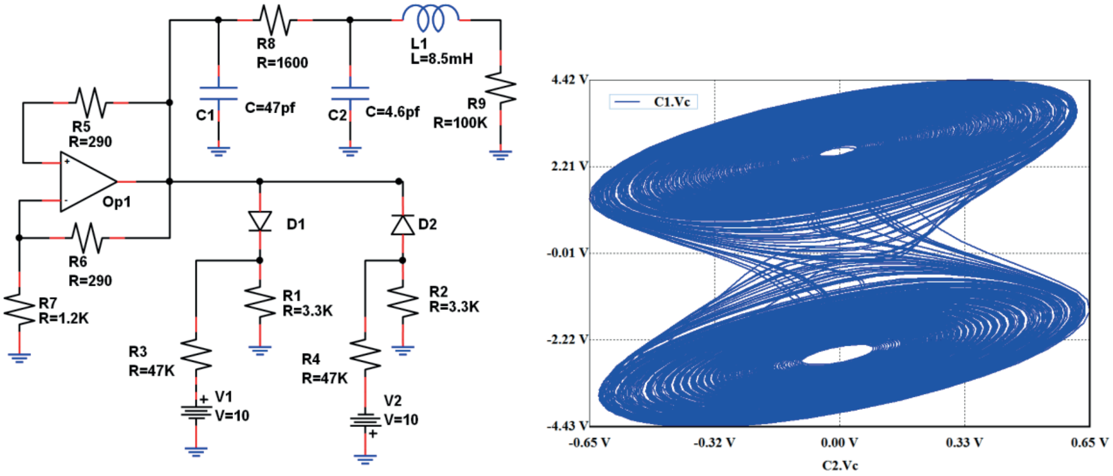

Figure 19. Chua's circuit: the circuit (left) and simulation results (right).

7. Conclusion

A method to accelerate the solving of nonlinear equations has been presented. The proposed method is based on three steps. The first step is the use of the homotopy method to get a continuous function. The second

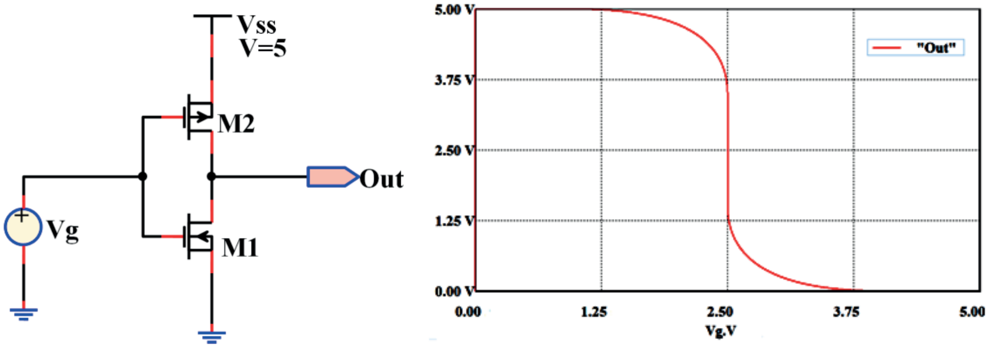

Figure 20. CMOS inverter circuit: the circuit (left) and simulation results (right).

step is applying the continuation parameter solver with a modification in the predictor (initial approximation) by using the Euler method. The predicted approximate solution is then corrected by applying the corrector. The third step is the control of the new step size to accelerate the solution search. In order to demonstrate the effectiveness of the proposed method, the authors created new software based on the proposed method called PyAMS (Python Language for Analog and Mixed Signal). A comparison was done between the proposed method and two other methods, and many types of integrated circuits were used in the process. Circuit analysis of many universal circuits was carried out to verify the correct functioning of the circuit based on the proposed method. Simulation results showed the effectiveness of the proposed method. It is approximately three times faster than the VGNH method implemented in the SPICE software. As future work, the proposed approach can be generalized to include time functions.

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