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Sum rate enhancement and interference alignment for MIMO channels

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Abstract: Interference alignment (IA) is a cooperative technique used for multiple-input, multiple-output (MIMO) interference channels to confine the interference to a reduced signal space of the receiver. IA helps all nodes of the network to achieve half of the degree of freedom that can be achieved if there were no interference. In this paper an iterative alternating minimization algorithm is presented, which aims at providing the higher sum rate along with IA for more than three users having complete channel state information. In each iteration of the algorithm precoders and decoders are calculated to align interference in a reduced interference subspace. The precoders are then further improved for sum rate enhancement by scaling with suitable weights to increase the signal power of the desired signal. The simulation results show that the proposed algorithm achieves significant increase in average sum rate and also minimizes the overall interference as compared to conventional IA algorithms at low signal-to-noise ratio.

Key words: MIMO, interference alignment, degree of freedom, signal to interference and noise ratio

1. Introduction

Interference of two or more wireless links on each other is traditionally avoided by orthogonalizing the channel access, e.g., the channels could be accessed by time-multiplexing or assigned different frequency bands or given random access (e.g., carrier sense multiple access). All of these techniques avoid interference by restricting the number of overlapping transmissions [1]. These traditional techniques of interference avoidance do not scale up the capacity of the system linearly with the increasing number of users.

Interference alignment (IA) is a capacity optimal solution where the capacity of the network can grow linearly with the number of users in a network at high signal-to-noise ratio (SNR). It is a cooperative interference management technique in which transmitters and receivers cooperatively design precoders and decoders respectively such that the interference is aligned in a reduced signal subspace of each receiver's total signal space. The number of antennas of a receiver is considered here as multidimensional signal space of the receiver. This total space can further be divided into two subspaces, one for the intended signal reception and the other for the nonintended interference signal. Each transmitter having global channel state information (CSI) encodes its signal such that interference caused by the transmitter to the nonintended receivers must lie in the interference subspace of the receivers. Each receiver will then have all the interference in the same signal subspace. The receiver can now decode its own signal by projecting the received signal to the subspace orthogonal to the interference subspace. The receiver does this by multiplying the received signal with the decoder that has dimensions equal to the signal subspace.

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In an IA scheme, all the transmitters design their precoders to cause less interference to other nonintended receivers by aligning all interfering signals into interference subspace [1, 2]. The extent to which these precoders bring the useful signal vectors closer to the signal subspace is important for sum rate enhancement. The closer the signal vectors get to the signal space, a better rate is achievable. Thus, if maximization of the sum rate is taken as an objective function with the constraints of IA, then precoders also have to be optimized for a direct link between the transmitter and its intended receiver. This is the focus of this paper: to achieve a higher sum rate with the constraints of IA.

A three-users case of a multiple-input, multiple-output (MIMO) interference channel was considered in [3], where the maximum degree of freedom was achieved using an IA closed form solution. However, the closed form solution does not exist for more than three users. In [3], a distributed algorithm was given for an IA solution, which requires channel reciprocity. Peters et al. proposed an IA algorithm that eliminates the requirement of channel reciprocity [2]. They used an alternating minimization procedure that does not require perfect CSI and can be generalized for more than three users. In order to achieve the maximum sum rate along with the interference alignment, the authors of [4] and [5] optimized the precoders and decoders by moving along the direction of the gradient of the maximum sum rate. In [1] the sum rate was maximized per stream basis via a maximum signal to interference and noise ratio (MAX-SINR) algorithm. IA is usually optimal at high SNR and is suboptimal in low to moderate SNR regions in terms of sum rate enhancement [1]. In [6] a hierarchy precoder design was given, which is only practical in a multicell environment. Precoders and decoders can also be calculated at a centralized location [7] or a relay [8].

In this paper we have proposed an iterative algorithm based on an alternating minimization procedure that does not require a centralize location or relay. The algorithm calculates the precoders and decoders for IA and then precoders are optimized for sum rate enhancement. Further optimization of the precoders is done by multiplying the precoders with suitable weight factors to increase the sum rate.

Here we introduce the notations. $\log$ refers to $\log_2$. Uppercase letters such as A denote matrices, bold lowercase letters such as $\mathbf{a}$ denote column vectors, and normal letters such as a denote scalars. A^* is the Hermitian transpose of matrix A and $\|A\|_F$ is the Frobenius norm of the matrix A. $\mathbb{C}^N$ is complex space of dimension N. $\mathcal{V}^R(A)$ is a matrix whose columns are eigenvectors corresponding to the R largest eigenvectors of matrix A. I_N is an $N \times N$ identity matrix. Finally, we use $\{V_k\}$ when referring to the set of precoders and V_k when referring to the precoder at transmitter k and similarly for received decoder filters $\{U_k\}$.

The rest of the paper is organized as follows. In Section 2 the system model for the MIMO interference channel is given. Section 3 is about the necessary conditions and feasibility of the IA. The proposed algorithm for maximum sum rate and IA is presented in Section 4. Simulation results are given in Section 5 and finally the paper is concluded in Section 6.

2. System model

Consider a K-users MIMO interference channel as shown in Figure 1, where each transmitter j with M_j antennas wants to send $d_k \leq M_k$ data streams (or degrees of freedom) to receiver k with $N_k \geq d_k$, where N_K is the number of antennas at the kth receiver node. Each transmitter causes interference to all other nonintended receivers. Most of the time, for the ease of simulation and analysis, a symmetric case is considered where all transmitters and receivers have the same number of antennas, i.e $M_k = M$, $N_k = N$ and $d_k = d$. We denote such a symmetric case of users K having d degrees of freedom with interference channel (M, N,

$d)^K$. Assuming $V_k \in \mathbb{C}^{M\times d}$ is a unitary precoder matrix for the transmitter of user k, then the received signal at receiver k is given by:

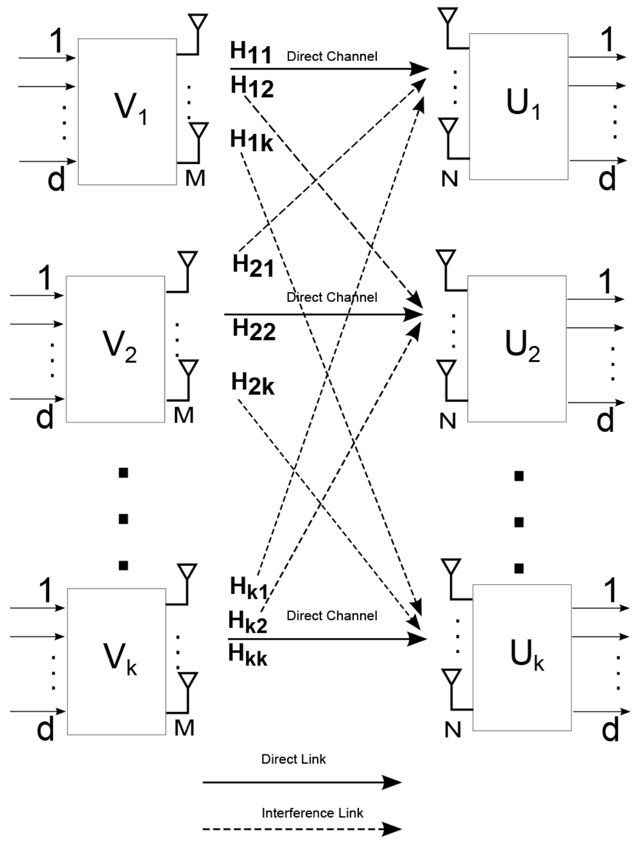

Figure 1. K-users MIMO interference channel.

$$y_k = H_{kk}V_k d_k + \sum_{j\neq k} H_{kj}V_j d_j + n_k, \tag{1}$$

where $H_{kk} \in \mathbb{C}^{N\times M}$ is a MIMO direct channel between two coupled transmitters and receivers, $H_{kj} \in \mathbb{C}^{N\times M}$ is a cross channel or interference channel, and it is assumed that all the channels between transmitter j and receiver k during each transmission interval are static. Each entry of the MIMO channel matrix is taken from an independent and identical distributed (i.i.d.) Rayleigh fading model. n_k is additive white Gaussian noise (AWGN) at receiver k with zero mean and unit covariance.

3. Interference alignment conditions and feasibility

Allocation of d data streams by IA is feasible and possible if we find a set of precoders matrix $\{V_k\}$ and receiver interference suppression or decoders matrix set $\{U_k\}$ where the dimensions of V_k are $V_k \in \mathbb{C}^{M\times d}$ and the dimensions of U_k are $U_k \in \mathbb{C}^{N\times d}$ such that

$$U_k^* H_{kj} V_j = 0, \qquad \forall j \neq k, \tag{2}$$

$$rank(U_k^* H_{kk} V_k) = d_k, \qquad \forall k \in \{1, 2, ..K\}. \tag{3}$$

In the above equations it is clear that for a given receiver k if all the interfering vectors $\sum_{j \neq k} H_{kj} V_j$ are aligned in the subspace of dimensions (N_k - d_k) of the N_k-dimensional space of the receiver k, then choosing the decoder U_k matrix such that it is orthogonal to the interface subspace of receiver will cancel all the interference. This is because the inner product of orthogonal subspaces is zero as given in Eq. (2). Consequently, the receiver can detect d data streams by projecting its received signal to the space orthogonal to the interference subspace as given in Eq. (3). The interference subspace and signal subspace of receivers are orthogonal if all the interfering signal vectors are confined in the interference subspace and all the useful signal vectors are inside the signal subspace. This is an ideal system in terms of sum rate enhancement and interference leakage minimization, but if some part of the signal lies inside the interference subspace the energy of that signal part is lost when the receiver multiplies the received signal vector with the decoder. This useful signal energy reduction will decrease the rate of information transmitted over the link even after interference is canceled out due to Eq. (3). However, if degree of freedom analysis is the only requirement then this power loss is irrelevant [9]. The energy loss is vital in terms of the sum rate point of view because this loss of useful signal energy at the cost of interference cancellation is suboptimal in terms of the sum rate of the network. Thus, in order to achieve both the targets of maximum sum rate and IA, it is necessary to calculate the optimal precoders and decoders that will align all the interference to a reduced signal subspace and move the signal vectors as close as possible to the signal subspace. In order to achieve maximum sum rate along with the IA, the authors of [4] optimized the precoders or decoders by moving them along the direction of the gradient of the maximum sum rate and this optimization was done over the Grassmann manifold. Similarly, [1] enhanced the sum rate on a per stream basis by the MAX-SINR algorithm.

The feasibility of the IA solution depends on network variables such as K, N, M, and d and it is still an open problem to find $\{V_k\}$ and $\{U_k\}$ that will satisfy the IA and degree of freedom achievable as given in Eqs. (2) and (3). The closed form solution for this is only known for certain specific combinations of the above variables such as that for $K = 3$. The algorithmic approach to this open problem, such as iterative the alternating minimization algorithm [2] and the distributed algorithm of [3, 10], attempts to find iteratively exact or very close solutions to this problem for $K \geq 3$. Nevertheless, the iterative algorithm provides numerical insights into the feasibility of IA for the K-users $M \times N$ MIMO interference channel with any IA multiplexing gains [9].

3.1. IA via iterative algorithms

These algorithms of IA normally find optimal precoders $\{V_k\}$ and decoders $\{U_k\}$ iteratively to minimize the interference leakage objective function given by:

$$IL = \min_{\substack{V_j^* V_j = I, \forall j \\ U_k^* U_k = I, \forall k}} \sum_{k=1}^{K} \sum_{j \neq k}^{K} \left\| U_k^* H_{kj} V_j V_j^* H_{kj}^* U_k \right\|_F^2 . \tag{4}$$

The minimization of the above equation is done by first choosing randomly $\{V_k\}$ and $\{U_k\}$, and then one of the variables set, either $\{V_k\}$ or $\{U_k\}$, is kept fixed and the other is optimized so that the overall objective function of minimizing the interference is achieved. In [1], the same interference leakage objective function is minimized by assuming channel reciprocity and shifting the function of the precoder as decoder and the decoder as precoder for reciprocal and straight channels, respectively. However, [2] achieved the objective function by considering a more general and practical approach and without considering the channel reciprocity.

Here we take an example by considering a system with $N = M = K = 3$ where each transmitter is transmitting $d_k = 1$ stream of data and each receiver k has space $\mathcal{L}$ of dimension N. A subspace $\mathcal{S}$ of dimensions $N_k - d_k$ can be considered as an interference subspace. The subspace orthogonal to this subspace $\mathcal{S}$ of dimensions d_k is the signal subspace for the receiver K. The decoder U_k matrix columns will be the orthogonal basis for this signal subspace or in other words $U_k = I_N - S_k S_k^*$, where S_k is the orthonormal basis for interference subspace $\mathcal{S}$.

The interference and signal subspaces for receiver $k = 1$ are shown in Figure 2, where the interference signals $H_{12}V_2$ and $H_{13}V_3$ at receiver 1 have Euclidean distances $e1$ and $e2$, respectively, from the interference subspace $\mathcal{S}$. The objective is to reduce these Euclidean distances to zero. If precoders are kept fixed, these Euclidean distances can be reduced by optimizing the interference subspace. This is done by moving the subspace close to these interfering signals. Thus, in a single iteration of the algorithm, interference subspace optimization is done by reducing the Euclidean distances of all interfering signals from interference subspace to be as close as possible to zero. Mathematically, this can be represented by:

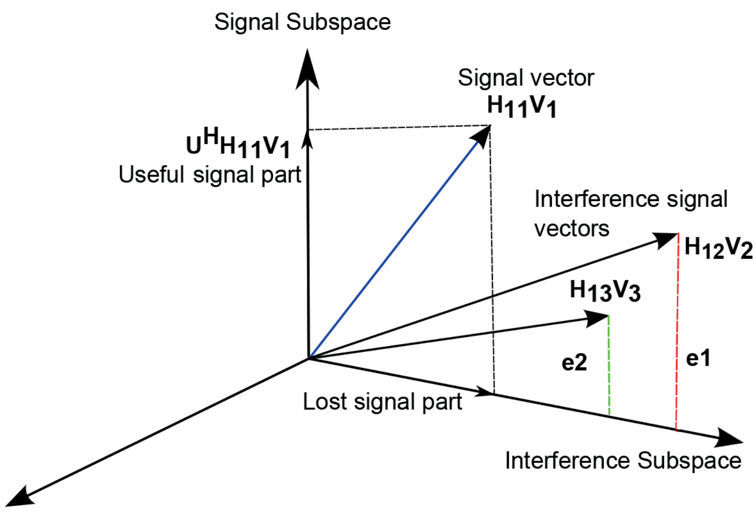

Figure 2. Interference and signal subspaces at receiver 1.

$$\min_{S_k^* S_k} \sum_{j \neq k}^{K} \| H_{kj} V_j - S_k S_k^* H_{kj} V_k \|_F^2 . \tag{5}$$

By using simple properties of linear algebra the solution to Eq. (5) is given by choosing the columns of S from the $N_k - d_k$ dominant eigenvectors of the overall interference matrix $\sum_{j \neq k}^{K} H_{kj} V_j V_j^* H_{kj}^*$. Mathematically, it can be written as:

$$S_k = \mathcal{V}^{N_k - d_k} (\sum_{j \neq k}^{K} H_{kj} V_j V_j^* H_{kj}^*). \tag{6}$$

This is the optimized interference subspace of receiver k and all interference signal vectors have minimum distances to it. Now the second variable to be optimized is the K precoder vectors set $\{V_k\}$. The optimization of the precoder is done such that the transmitter should cause less interference to all the other nodes in the network. In other words, the transmitter should produce the least energy in the signal space of other nonintended receivers. In optimizing the precoder the decoders set $\{U_k\}$ is kept fixed. The signal subspaces of all the other nonintended receivers seen by a transmitter j is given by:

$$\sum_{k\neq j}^{K-1} H_{kj}^*(I_N - S_k S_k^*)H_{kj}^*. \tag{7}$$

The precoder V_j is chosen such that its columns are the d_k least dominant eigenvectors of the above equation; thus:

$$V_j = \mathcal{V}^{d_j}(\sum_{k\neq j}^{K-1} H_{kj}^*(I_N - S_k S_k^*)H_{kj}^*). \tag{8}$$

The optimization for the precoder and decoder is done iteratively as given by Eqs. (8) and (6) and after a number of iterations the objective function of interference alignment is achieved. As we may note, these algorithms are trying to align interference only and are thus serving the objective of interference minimization. Each transmitter is trying to be harmless to the other nodes by considering cross channels that are causing interference. However, the direct channel between a transmitter and its intended receiver that is responsible for the rate of the link is not taken into consideration while designing these precoders. The sum rate of the network is given by:

$$R = \sum_{k=1}^{K} \log \left| I_N + \frac{U_k^* H_{kk} V_k V_k^* H_{kk}^* U_k}{\sigma^2 I_N + \sum_{j\neq k}^{K} U_k^* H_{kj} V_j V_j^* H_{kj}^* U_k} \right|. \tag{9}$$

In this paper, we have considered the objective of sum rate enhancement with the constraint of interference minimization through IA. It is achieved by further optimizing the linear precoders that are achieving IA. We will explore it in detail in the following section.

4. Optimized maximum sum rate and IA algorithm

In this section we present the optimization of the precoders that are achieving IA to maximize the sum rate. The problem is formulated mathematically as:

$$R = \max \sum_{k=1}^{K} \log \left| I_N + \frac{U_k^* H_{kk} V_k V_k^* H_{kk}^* U_k}{\sigma^2 I_N + \sum_{j\neq k}^{K} U_k^* H_{kj} V_j V_j^* H_{kj}^* U_k} \right|, \tag{10}$$

subject to the constraint of interference minimization:

$$\sum_{k=1}^{K} \sum_{j\neq k}^{K} \left\| U_k^H H_{kj} V_j V_j^* H_{kj}^* U_k \right\|_F^2 = 0, \tag{11}$$

where $V_k^* V_k = I_d$, $U_k^* U_k = I_d$.

The above objective along with the constraint was also achieved by [4] but in a highly computationally dense way. The authors of [1] and [11] also achieved the IA objective but did not take the sum rate enhancement into consideration. Here we present an algorithmic solution that achieves the objective of sum rate enhancement along with the constraint of IA without adding an additional computational burden.

Let us consider a receiver k where the interference from all other transmitters to the receiver is given by:

$$\sum_{j \neq k} H_{kj} V_j V_j^* H_{kj}^*. \tag{12}$$

The receiver will optimize its interference subspace by moving it close to the interfering vectors to achieve IA. This is done by choosing its columns vectors from the $N_k - d_k$ dominant eigenvectors of the interference matrix as given in Eq. (12). This will reduce the Euclidean distances of all the interfering vectors $H_{kj}V_j$ from the orthogonal bases of the interference subspace and hence IA is achieved. The decoder for receiver k is then taken as $U_k = I_N - S_k S_k^*$.

Now the precoder matrix V_k for the transmitter k is chosen such that it should cause the least interference to other nonintended receivers. This is achieved by choosing columns of the precoder matrix V_k from the d_k least dominant eigenvectors of the following matrix:

$$\sum_{k \neq j}^{K-1} H_{kj}^* (I_N - S_k S_k^*) H_{kj}^*. \tag{13}$$

The precoder will also help in achieving IA. In order to achieve the objective of maximum sum rate along with IA the precoder matrix is weighted by weight factor w such that $V_k w$ should give the maximum sum rate along with IA. In each iteration of the algorithm the precoder is first calculated to achieve IA and then weighted to improve performance in terms of sum rate. Different ranges of w weights are considered for sum rate enhancement. Empirically it is found that the best performance in terms of rate enhancement and IA is achieved with $1 \leq w \leq 10$. Thus, the algorithm optimizes precoder V and decoder U for IA and precoder V is then again optimized by multiplying it with the weight factor for sum rate enhancement given by:

$$R_{max} = \max \sum_{k=1}^{K} \log \left| I_N + \frac{U_k^* H_{kk} (V_k w)(V_k w)^* H_{kk}^* U_k}{\sigma^2 I_N + \sum_{j \neq k}^{K} U_k^* H_{kj} V_j V_j^* H_{kj}^* U_k} \right|. \tag{14}$$

5. Performance simulation and results

In this section we make a simulation comparison of the proposed algorithm with conventional IA via alternating minimization and MAX-SINR [1] with orthogonal and nonorthogonal precoders. The capacity (sum rate) objective along with interference minimization are compared using MATLAB simulations. The network parameters chosen are $K = 4$, the transmitter and receivers both have the same number of antennas $M = N = 5$, and each transmitter is transmitting $d = 2$ number of streams. All the channels from transmitter to receiver are independent and identically distributed (i.i.d.) Rayleigh faded. Noise is considered to be AWGN with zero mean and unit variance. We can denote this system by $(5 \times 5, 2)^4$. In the simulation we have considered a fixed channel realization of the $(5 \times 5, 2)^4$ interference MIMO channels.

The average sum rate is evaluated for a range of SNR for the proposed algorithm and also the results are compared with the existing schemes. We ran each of the algorithms for 1000 iterations for the range of the SNR. It can be observed that the sum-rate of the proposed scheme outperforms the others schemes MAXSINR and MAX-SINR with orthogonal precoders and conventional IA with the alternating minimization method at both low and high SNR regions as shown in Figure 3. The difference is more pronounced at the lower SNR region than at the higher SNR region. This is because at low SNR the noise term in the denominator of Eq. (14) is the dominant factor and hence an increase in the signal component (numerator of Eq. (14)) would not be compromised by a similar increase in the interference term. The percentage improvement of the sum-rate of the proposed scheme over MAX-SINR and conventional IA with alternating minimization scheme is shown in Figure 4, where it is also evident that improvement is higher at low SNR values. The proposed algorithm achieves interference minimization reasonably well as compared to conventional IA with the alternating minimization method but requires more iterations for convergence, as shown in Figure 5. However, the difference is not significant and is within acceptable limits.

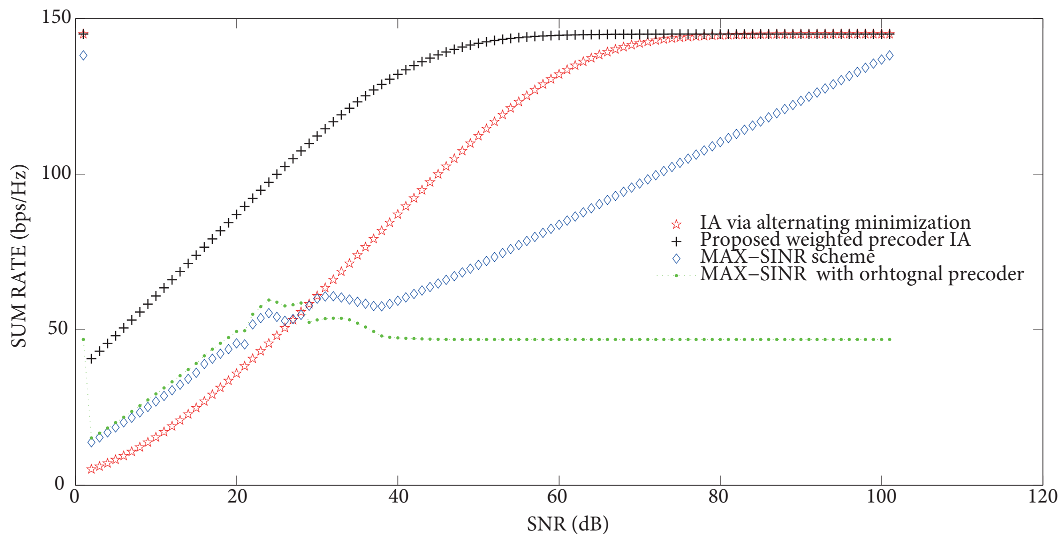

Figure 3. Sum rate of the proposed algorithm for $(5 \times 5, 2)^4$ system.

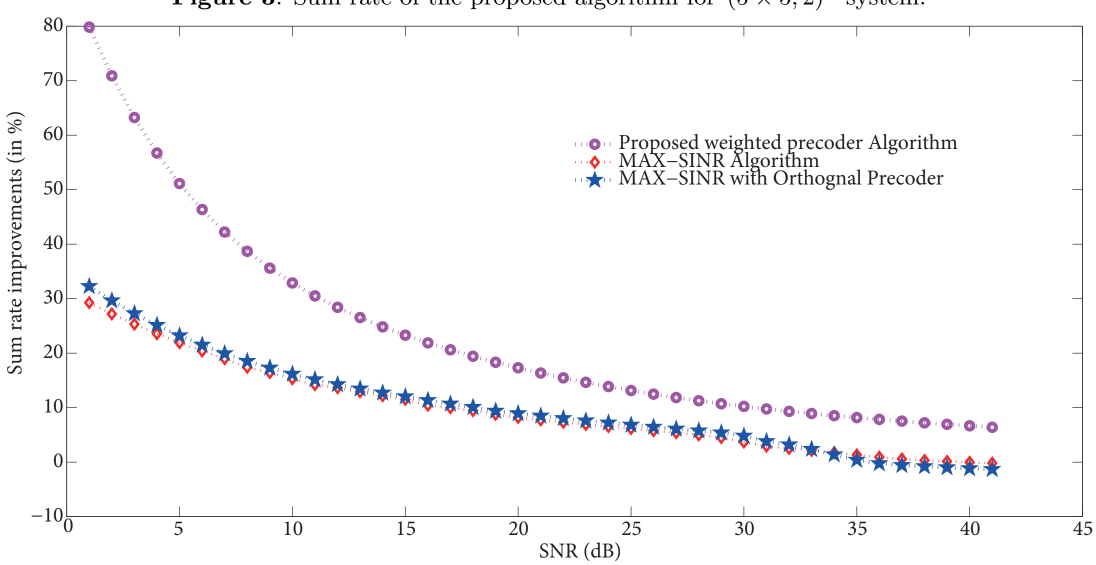

Figure 4. Percentage sum rate improvement of the proposed scheme over conventional IA algorithms for $(5 \times 5, 2)^4$ system.

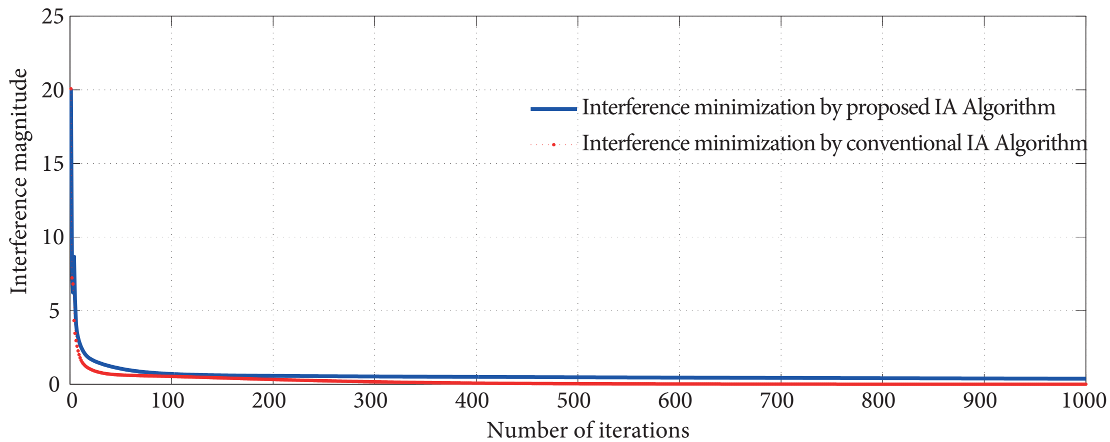

Figure 5. Interference leakage minimization comparison.

6. Conclusions

In this paper we have proposed an iterative algorithm based on an alternating minimization procedure. The algorithm calculates the precoders and decoders for IA. The precoders are then optimized for sum rate enhancement. Further optimization of the precoders is done by multiplying the precoders with suitable weight factors that results in increased sum rate as evident from the simulation results.

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