A ternary zero-correlation zone sequence sets construction procedure

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Abstract: In the present paper, we propose ternary zero-correlation zone (ZCZ) sequence sets with ideal correlation properties for phase shifts within the ZCZ. The proposed ternary ZCZ sequence sets can be constructed from binary mutually orthogonal complementary sets (MOCS) and ternary perfect sequences.

Key words: Mutually orthogonal complementary sets, perfect sequence, zero correlation zone, ternary zero correlation zone sequence sets

1. Introduction

In CDMA systems, all users simultaneously share the same communication resources; channel separation is provided by the correlation properties of the spreading codes. Therefore, sequences with good autocorrelation and cross-correlation functions play an important role in the design of CDMA systems. Sets of sequences having ideal correlation properties, impulse-like autocorrelation, and zero cross-correlation functions in a specified zone are called zero-correlation zone (ZCZ) sequences [1]. ZCZ codes have been employed in diverse wireless systems, such as sonar sensor network, multiple input multiple output, orthogonal frequency division multiplexing, local positioning systems, and ultrasonic imaging [2–7]. The requirements for the ZCZ sequence set used in a specific system vary with the design and purpose of the system. Therefore, the availability of various types of construction methods makes it easier to select proper ZCZ sets for particular systems [7].

Numerous construction methods of ZCZ sequence sets have been proposed [1,7–17]. Most of the proposed construction methods are based either on complementary sequence sets [18] or perfect sequences. Generally, a ZCZ sequence set is characterized by the sequence period $(\mathbf{N})$, the number of sequences $(\mathbf{M})$, and the ZCZ length $(\mathbf{Z_0})$. The following bound has been set on the parameters of ZCZ sequence set [19,20]:

$$\mathbf{Z_0} \leq \frac{\mathbf{N}}{\mathbf{M}} - \mathbf{1} \tag{1}$$

That is, given the sequence length $(\mathbf{N})$, it is impossible for the set size $(\mathbf{M})$ and the ZCZ length $(\mathbf{Z_0})$ to be large simultaneously. A ZCZ sequence set that satisfies Eq. (1) is called an optimal ZCZ sequence set. Although binary ZCZ sequences facilitate hardware construction, they are not optimal. In fact, the bound is even tighter for binary ZCZ sequences: $\mathbf{Z_0} \leq \mathbf{N / 2M}$ [9]. In order to overcome this difficulty, ternary sequences ZCZ were introduced and various construction methods were proposed. Hayashi proposed many construction

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procedures based on manipulation of perfect sequences in, among others, [10–14]. However, certain constraints are imposed on the construction of these ternary ZCZ sequences. In general, the ZCZ length of a ternary ZCZ sequence based on a perfect sequence is shorter than the length of the perfect sequence. Furthermore, perfect sequences with a small alphabet size do not exist for all lengths [21]. In order to solve this problem, we propose a systematic construction procedure of a new family of ternary ZCZ sequences. Our construction is based on ternary perfect sequences and mutually orthogonal complementary sets (MOCS) [18]. The proposed method can produce ternary ZCZ sequences with flexible parameters depending on the MOCS matrix used for construction. The proposed sequence set is almost optimal, which means that for a given sequence length $(\mathbf{N})$ and ZCZ length $(\mathbf{Z_0})$, it can provide one less than the maximum number of codes. Since the ternary ZCZ sets, whose elements take $\pm\mathbf{1}$ and $\mathbf{0}$, can facilitate hardware construction [22], the proposed ternary ZCZ sets can be applied in various systems such as radar sensor networks, ultrawide band [23,24], or those stated earlier.

After an examination of preliminary considerations in Section 2, a procedure for constructing the proposed sequence sets is presented in Section 3. The proposed sequence sets' properties are discussed in Section 4. Concluding remarks are presented in Section 5.

2. Preliminary considerations

Given a sequence set $\{a_n^x,\ 1 \leq x \leq M\}$ of set size M, in which each sequence is of length N, where $n = 0, 1, 2 \ldots N-1$, then we can define the following periodic and aperiodic correlation functions, respectively, as follows:

$$R_{a^x,a^y}\left(\tau\right) = \sum_{n=0}^{N-1} a_n^x a_{n+\tau mod N}^{*y} \begin{cases} ACF\, x = y \\ CCF\, x \neq y \end{cases} \tag{2}$$

$$\theta_{a^x,a^y}\left(\tau\right) = \sum_{n=0}^{N-1-\tau} a_n^x a_{n+\tau}^{*y} \begin{cases} ACF\, x = y \\ CCF\, x \neq y \end{cases} \tag{3}$$

where a_n^* denotes the complex conjugate of sequence element a_n.

Let a matrix F of order $M \times M$ be defined such that each element in it is a sequence of length L, arranged as follows:

$$F = \begin{bmatrix} F_{11} & F_{12} & & F_{1M} \\ F_{21} & F_{22} & \cdots & F_{2M} \\ \vdots & & \ddots & \vdots \\ F_{M1} & F_{M2} & \cdots & F_{MM} \end{bmatrix} \tag{4}$$

F is a MOCS if it satisfies the following relations [18]:

$$\sum_{i=1}^{M} \theta_{\mathbf{F_{i,j}F_{i,j}}}(\tau) = 0, \quad for\, \forall j,\, \forall \tau \neq 0 \tag{5}$$

$$\sum_{i=1}^{M} \theta_{\mathbf{F_{i,j}F_{i,k}}}(\tau) = 0,\, for\, \forall j \neq k,\, \forall \tau \tag{6}$$

We can extend the MOCS matrix F to $F_1^{'}$ and $F_2^{'}$ by interleaving or concatenation, as in [18]:

$$F_1^{'} = \begin{bmatrix} F \otimes F & (-F) \otimes F \\ (-F) \otimes F & F \otimes F \end{bmatrix} \tag{7}$$

$$F_2^{'} = \begin{bmatrix} FF & (-F)F \\ (-F)F & FF \end{bmatrix} \tag{8}$$

Note that $F \otimes F$ denotes the matrix whose ijth entry is the interleaved sequence of the ijth entry of F and the ijth entry of F, and that FF denotes the matrix whose ijth entry is the concatenated sequence of the ijth entry of F and the ijth entry of F.

A sequence b of length N_1 with ideal periodic ACF is defined as a perfect sequence [8]:

$$R_{b,b}(\tau) = \begin{cases} E_b, & \tau = 0, \\ 0, & \text{for } \tau \neq 0, \end{cases} \tag{9}$$

where the energy E_b of the sequence b is given by

$$E_b = b_0^2 + , \dots , b_i^2 + , \dots + b_{N_1-1}^2 \tag{10}$$

A set of sequences a_n^x is a ZCZ sequence set denoted by $ZCZ(N, M, Z_0)$, where Z_0 is the ZCZ length [10], if it satisfies

$$\forall x, \forall \tau \ (0 < |\tau| \leq Z_0), \ R_{a^x, a^x}(\tau) = 0 \tag{11}$$

$$\forall xy \ (x \neq y), \forall \tau \ (|\tau| \leq Z_0), \ R_{a^x, a^y}(\tau) = 0 \tag{12}$$

3. Construction of the proposed sequence sets

The construction procedure of the proposed ternary sequence sets is performed as follows.

First, we define a binary starter MOCS matrix denoted by $C^{(0)}$ of M_0 rows, where each row contains M_0 sequence elements each of length L_0. Applying either Eq. (7) or (8) n times ($n \geq 1$) starting from $C^{(0)}$ results in a MOCS matrix $C^{(n)}$ that contains M rows where each row contains M sequences each of length L, where $M = 2^n M_0$ and $L = 2^n L_0$ that is each row of $C^{(n)}$ is of length $N = ML = 2^{2n} M_0 L_0$. The MOCS matrix $C^{(n)}$ can be written as

$$C^{(n)} = \begin{bmatrix} c_0^1 & \cdots & c_i^1 & \cdots & c_{M-1}^1 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ c_0^j & \cdots & c_i^j & \cdots & c_{M-1}^j \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ c_0^M & \cdots & c_i^M & \cdots & c_{M-1}^M \end{bmatrix} \tag{13}$$

where

$$c_i^j = \left(c_{i,0}^j, \dots , c_{i,k}^j, \dots , c_{i,L-1}^j \right) c_{i,k}^j \in \{1, -1\} \tag{14}$$

Next, we choose a ternary perfect sequence $b = (b_0 \ldots, b_i, \ldots, b_{N_1-1})$ of length N_1 having elements from $-1, 0, 1\}$. If N_1 and M are relative primes, a ternary ZCZ sequence set can be constructed by using the following equation:

$$t_i^j = b_{i \bmod N_1} c_{i \bmod M}^j, \tag{15}$$

where $i = 0, 1, \ldots, (N_1 M) - 1$, and t_i^j is the new ternary set's entry sequence, obtained by multiplying ternary perfect sequence element b_i by the MOCS entry sequence c_i^j.

$$C^{(n)} = \begin{bmatrix} t_0^1 & \cdots & t_i^1 & \cdots & t_{M-1}^1 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ t_0^j & \cdots & t_i^j & \cdots & t_{M-1}^j \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ t_0^M & \cdots & t_i^M & \cdots & t_{M-1}^M \end{bmatrix} \tag{16}$$

The obtained ternary set contains M rows and each row is of length $N_t = N_1 N$. As for the ZCZ length, it is equal to $Z_{0i} = ((N_1 - 1)L) + 1$ when interleaving is used to obtain $C^{(n)}$ and equal to $Z_{0c} = (2N_1 - 1)L/2$ when concatenation is used. By replacing M, N, and L with their values, two ternary ZCZ sequence sets of $ZCZ_i(N_1 2^{2n} M_0 L_0 2^n M_0, ((N_1 - 1) 2^n L_0) + 1)$ and $ZCZ_c(N_1 2^{2n} M_0 L_0 2^n M_0, (2N_1 - 1)2^{n-1} L_0)$ can be constructed depending on whether interleaving or concatenation is respectively used. The two sets have the same ZCZ size only when $L = 2$.

Example 1 *We choose a starter MOCS $C^{(0)}$ of set size $M_0 = 2$ and entry sequence length $L_0 = 2$ that satisfies Eqs. (5) and (6) as*

$$C^{(0)} = \begin{bmatrix} ++ & +- \\ +- & ++ \end{bmatrix} \tag{17}$$

where $+$ and $-$ denote 1 and -1, respectively. Two extended sets $C_1^{(1)}$ and $C_2^{(1)}$, of $M = 4$ and $L = 4$, are obtained by using Eqs. (7) and (8), respectively, once $(n = 1)$ as follows:

$$C_1^{(1)} = \begin{bmatrix} ++++ & ++-- & -+-+ & -++- \\ ++-- & ++++ & -++- & -+-+ \\ -+-+ & -++- & ++++ & ++-- \\ -++- & -+-+ & ++-- & ++++ \end{bmatrix}$$

$$C_2^{(1)} = \begin{bmatrix} ++++ & +-+- & --++ & -++- \\ +-+- & ++++ & -++- & --++ \\ --++ & -++- & ++++ & +-+- \\ -++- & --++ & +-+- & ++++ \end{bmatrix}$$

Applying Eq. (15) to MOCS sets $C_1^{(1)}$ and $C_2^{(1)}$ and a ternary perfect sequence such as $b = (1, 1, 0, 1, 0, 0, -1)$ of length $N_1 = 7$, we can obtain two sets: the first set $T_1^{(1)}$ of $ZCZ_c(112 \ 4, 25)$ and the second set $T_2^{(1)}$ of

ZCZ(112, 4, 26). The first and second sequences are denoted by $T_{1,1}^{(1)}$, $T_{1.2}^{(1)}$ from the first set $T_1^{(1)}$ and $T_{2,1}^{(1)}$, $T_{2,2}^{(1)}$ from the second set $T_2^{(1)}$. These sequences are presented in the following and their correlation functions are illustrated in Figures 1 and 2.

$T_{1,1}^{(1)} = \{+ + + + + + - - 0000 - + + -00000000 + - + - - + + - + + + + +0000 - + - +000000\,00 - - + +$

$\qquad - + - + - + + -0000 + + - -00000000 - - - - + + - - - - + - +0000 + + + +00000000 + - - +\}$

$T_{1,2}^{(1)} = \{+ + - - + + + +0000 - + - +00000000 + - - + - + - + + + + - -0000 - + + -000000\,00 - - - -$

$\qquad - + + - - + - +0000 + + + +00000000 - - + + + + + + - + + -0000 + + - -00\,000000 + - + -\}$

$T_{2,1}^{(1)} = \{+ + + + + - + - 0000 - + + -00000000 + + - - - + + - + + + + +0000 - - + +00000000 - + - +$

$\qquad - - + + - + + -0000 + - + -00000000 - - - - + - + - - - + +0000 + + + +00000000 + - - +\}$

$T_{2.2}^{(1)} = \{+ - + - + + + +0000 - - + +00000000 + - - + - - + + + + - + -0000 - + + -000000\,00 - - - -$

$\qquad - + + - - - + +0000 + + + +00000000 - + - + + + + + + - + + -0000 + - + -00000000 + + - -\}$
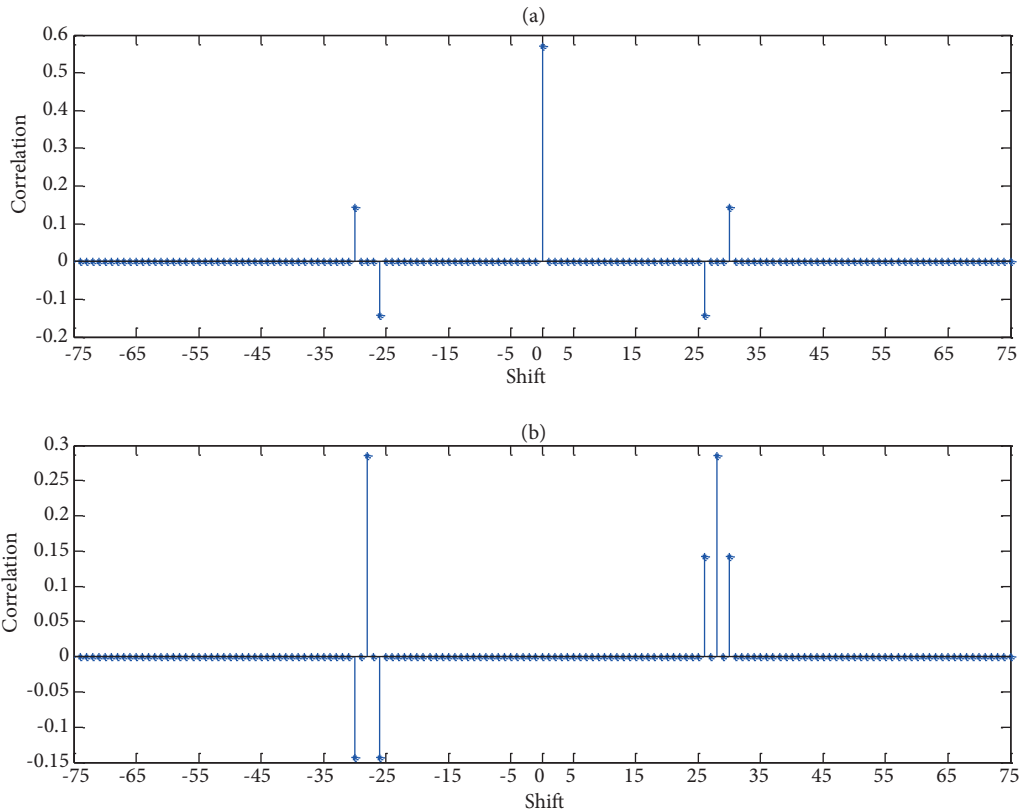

Figure 1. Correlation properties of the first set (a) ACF of $T_{1,1}^{(1)}$ and (b) CCF of $T_{1,1}^{(1)}$ and $T_{1,2}^{(1)}$.
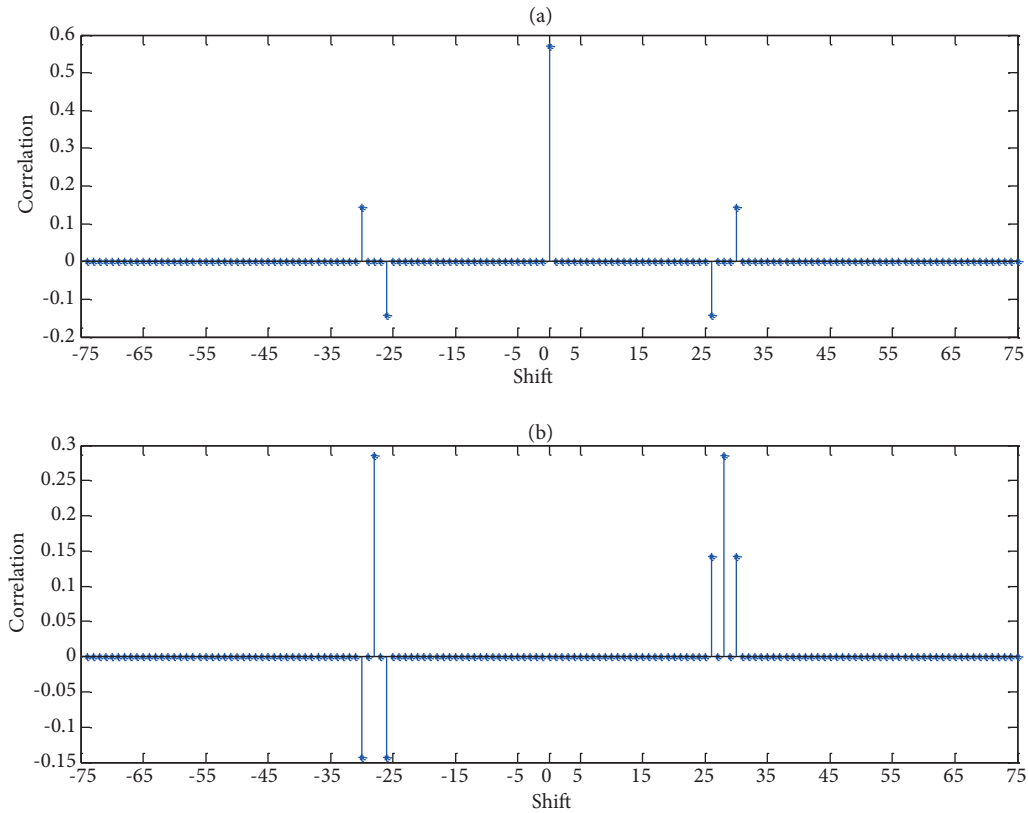
Figure 2. Correlation properties of the second set, (a) ACF of $T_{2,1}^{(1)}$ and (b) CCF of $T_{2,1}^{(1)}$ and $T_{2,2}^{(1)}$.

4. Properties of the constructed sequence set

4.1. Optimality of the constructed sequence set

The proposed sequence set is almost optimal, which means that for a given sequence length (N) and set size M, it can provide one less than the maximum ZCZ length $(Z_{\max})$. The upper bound for a ZCZ length is equal to $Z_{\max} = \frac{\text{sequence length}}{\text{set size}} - 1$ (see Eq. (1)); hence the upper bound of the proposed construction is equal to $Z_{\max} = N_1 L - 1$. Table 1 compares the upper bound $Z_{\max}$ and ZCZ lengths Z_{0i} and Z_{0i} for $M = L = 4$ and some values of N_1.

Table 1. Values of Z_{0i} and Z_{0c} compared to the upper bound.

N_1	Z_{0i}	Z_{0c}	$Z_{\max}$(bound)
7	25	26	27
13	49	50	51
21	81	82	83
31	121	122	123
57	225	226	227

Note that the proposed construction is almost optimal when MOCS matrix C obtained recursively by concatenation is used. The difference between ZCZ of the ternary ZCZ sequence set obtained by concatenation of Z_{0c} and the one obtained by interleaving Z_{0i} is computed by the following formula:

$$d = Z_{0c} - Z_{0i} = \frac{L}{2} - 1 \tag{18}$$

Table 2 shows some values of d for different sets when $L_0 = 2$ and $N_1 = 7$.

Table 2. Some values of d for different sets.

n	L	Z_{0c}	Z_{0i}	d
0 (starter)	2	13	13	0
1	4	26	25	1
2	8	52	49	3
3	16	104	97	7
4	32	208	193	15
5	64	416	385	31
6	128	832	769	63

4.2. Comparison between the proposed construction and some known ternary ZCZ sequence sets

In this section, we will compare the proposed construction method with well-known ternary ZCZ sequence set constructions [10–14]. The first family of ternary ZCZ sequences, presented in [10], is obtained through several steps:

Step 1: Selection of binary orthogonal matrices of order $h \times h$ satisfying constraints given in Table 3.

Table 3. Comparison between the proposed set and some known ternary ZCZ sequence sets.

	Parameters	Constraints	Flexible	Complex
ZCZ-[10]	$(h+1)2^{m+2}, 2h, 2^{m+1} - 1$	$\theta_{h_r^{(a)} h_s^{(a)}}(\tau) = \theta_{h_r^{(b)} h_s^{(b)}}(\tau), |\tau| = 1$	Yes	Yes
ZCZ-[11]	$4L_p, 2(2l+1), 4k+1$	Perfect sequence	No	Yes
ZCZ-[12]	$2^{m+1}k(2n+h), 2n2^{m+1}k - 1$	Perfect sequence	No	Yes
ZCZ-[13]	$4L_1 L_2, 2L_2, 2L_1 - 1$	Perfect sequence	No	Yes
ZCZ-[14]	$2Lk(2n+h), 2n, 2kL - 1$	Perfect sequence	No	Yes
New	$N_1 M L M (2N_1 - 1)L/2$	$\gcd(N_1, M) = 1$	Yes	No

$\theta_{h_r^{(a)} h_s^{(a)}}(\tau)$ is the aperiodic correlation function of two different rows r and s in orthogonal matrix h and (τ) is the delay. $k \geq 1, l \geq 0, n \geq 2, h = 1, 2$, and $L_p = (2l+1)(2k+1)$.

Step 2: Negation, concatenation, zero padding of binary orthogonal matrices, and interleaving to obtain the ZCZ starter matrix. An interleaving operation is used m times to extend the ZCZ set. Note that several operations are performed in order to obtain the basic starter. In communication systems design, complexity is avoided unless it is absolutely necessary.

The second family, presented in [11–13], is obtained by manipulating pairs of perfect sequences. In each construction method, the constructed ternary ZCZ set's parameters are all dependent on the length of perfect sequences used. As mentioned in the introduction, ternary perfect sequence lengths are severely restricted [21]. Consequently, there is no practical application for this family of ternary ZCZ sequences. An attempt was made in [14] to design sequences with flexible parameters by using Golay complementary sequences [18]. However, in this family, the objective was achieved only in terms of sequence length and ZCZ length. The number of sequences remains dependent on the perfect sequence used. In CDMA systems, each user is assigned a unique code; a larger set size means more simultaneous users admitted to the system.

To alleviate the problem, we proposed a new approach to the design of ternary ZCZ sequence set in Section 3. Our construction is based on MOCS matrices and perfect sequences. MOCS matrices were extensively studied

and can be easily obtained by a recursive method [18]. The ternary perfect sequence used for construction can be of any lengths as long as it satisfies the relative prime with the MOCS set size M. The obtained sequence sets have flexible parameters: sequence length and ZCZ length are dependent on both MOCS and perfect sequence. Furthermore, the number of codes (users) is independent of the perfect sequence. Therefore, the proposed ternary ZCZ sequences construction procedure is more flexible, less complex, and easier to implement in CDMA multiple access systems. Table 3 summarizes the above discussion.

5. Conclusion

A new construction method of ternary sequence sets that have a ZCZ for periodic correlation functions, which closely approaches the theoretical bound, was presented. The proposed construction is based on perfect sequence, the length of which is a relative prime with the binary MOCS size. Note that a ternary ZCZ sequence set based on a binary MOCS matrix obtained recursively by concatenation can provide a longer ZCZ than those obtained by interleaving. The proposed ternary ZCZ sequences construction method is flexible and practical.

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