

Two-group decodable distributed differential space-time code for wireless relay networks based on SAST codes

Obada ABDALLAH^{1,*}, Ammar ABU-HUDROUSS²

¹Ministry of Telecommunications and Information Technology, Gaza Strip, Palestine

²Department of Electrical Engineering, Islamic University of Gaza, Gaza Strip, Palestine

Received: 28.05.2015

Accepted/Published Online: 25.11.2016

Final Version: 30.07.2017

Abstract: Space-time code can be implemented in wireless relay networks when all relays cooperate to generate the code at the receiver. In this case, it is called distributed space-time code. If the channel response changes very quickly, the idea of differential space-time coding is needed to overcome the difficulty of updating the channel state information at the receiver. As a result, the transmitted signal can be demodulated without any knowledge of the channel state information at the relays or the receiver. In this paper, development of new low decoding complexity distributed differential space-time codes is considered. The developed codes are designed using semiorthogonal algebraic space-time codes. They work for networks with an even number of relays and have a two-group decodable maximum likelihood receiver. The performance of the new codes is analyzed via MATLAB simulation which demonstrates that they outperform both cyclic codes and circulant codes.

Key words: Distributed differential space-time codes, cyclic codes, circulant codes, low decoding complexity

1. Introduction

Multiple-input multiple-output (MIMO) system performance can be greatly increased using space-time codes as in [1,2]. Similar to the conventional MIMO system, the idea of space-time codes applied to cooperative systems can improve bandwidth efficiency without the need for feedback [3,4]. Such codes are known as distributed space-time codes (DSTCs), where all relay nodes are allowed to simultaneously transmit over the same channel [2,3,5–8]. We are mainly interested in [5], where a new distributed space-time coding was proposed, which depends on the linear dispersion (LD) space-time codes of [9]. The transmission from the source to the destination is done in two steps. In the first step, the source broadcasts the data. In the second step, the relays encode their received signals into an LD code and then retransmit them to the destination.

In practical implementations, DSTCs share conventional space-time coding used in MIMO systems with the need of channel fading in wireless relay networks to be known at the receiver. This allows for decoding the data at the receiver coherently. Coherent detection needs up-to-date training symbols, which cannot be done in a fast fading environment. Inspired by differential space-time coding, a new technique for wireless relay networks was introduced in [10], which is called distributed differential space-time code (DDSTC). Several authors independently suggested differential encoding/decoding for wireless relay networks in [11–16]. The authors of [11] presented a general approach for DDSTC and they also provided a few code constructions. A partially coherent distributed space-time code was proposed in [12]. It does not require the knowledge of fading

*Correspondence: obada_333@hotmail.com

coefficients between the relay and destination. Cyclic distributed space-time code was presented in [13]. In [14], differential distributed space-time coding with low decoding complexity was considered. The code is four-group decodable and constructed using Clifford’s algebras. The authors of [15] proposed a decode-and-forward cooperative communication scheme that uses differential minimum-decoding complexity quasi-orthogonal space-time block code. The source and destination have multiple antennas while the relays have a single antenna. The first-hop is not coherent while the second-hop is coherent. In [16], the performance of a differential distributed space-time coding scheme was investigated for vehicle-to-vehicle networks employing low-complexity multiple-symbol differential detection for low, medium, and fast fading channels.

In wireless networks, the low decoding complexity becomes an important issue, especially if the number of cooperating terminals is large. Although the four-group decodable DDSTC satisfies the low decoding complexity demand, a limitation of this method is that it can be only implemented with the power of 2 relays. In this paper, semiorthogonal algebraic space-time (SAST) codes are used to overcome this limitation. This allows constructing STCs for even numbers of relay networks. The joint modulation of paper [17] is also extended to a multiple symbol modulation.

2. System model

Consider a wireless network consisting of $R+ 2$ nodes. One node acts as the source, another one acts as the destination node, and the rest act as relay nodes. The wireless channels between the terminals are assumed to be quasistatic flat fading, where f_i and g_i are the channel fading gains from the source to the i th relay and from the i th relay to the destination, respectively (Figure 1). All channel coefficients are assumed to be independent and identically distributed complex Gaussian random variables with zero mean and unit variance. All nodes cannot receive and transmit at the same time. Moreover, the system needs to be synchronized at the symbol level.

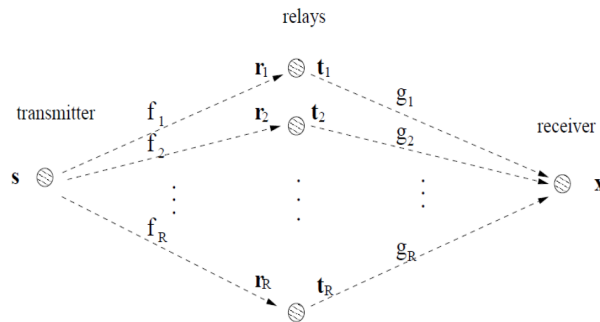


Figure 1. System model for DDSTC.

The transmission from the source to the destination occurs in two steps:

First step: In the first step and at τ th time, a data vector of T symbols ($T = R$) is encoded into a $T \times T$ unitary matrix $U^\tau \in L$, where L is the set of all possible code words. The source sends the differentially encoded signal. This means that the differential scheme allows for overlapping by one block. One block acts as a reference for the next, which is similar to the differential space-time coding in [10]:

$$s^\tau = \sqrt{P_1 T} U^\tau s^{\tau-1}, \tag{1}$$

where $P_1 = \frac{P}{2}$ is the average power transmitted from the source and equal to half of the total power used in

the whole network, and s^τ is normalized such that it satisfies $E\bar{s}s = 1$. The first block, s_o , is the initial vector known to the destination and with a unit-norm, $E\bar{s}_o s_o = 1$. The assumption that U^τ is unitary preserves the transmitted power from vanishing or blowing up [10]. The received vector at the i th relay is given by:

$$r_i = \sqrt{P_1 T} f_i s^\tau + n_i, \tag{2}$$

where f_i is the channel fading coefficient and $n_i = [n_{i,1}, \dots, n_{i,T}]^T$ is the additive white Gaussian noise, which is assumed to be i.i.d. with zero mean and unit variance $CN(0,1)$.

Second step: The second step of transmission starts from time $T + 1$ to $2T$, and the relays amplify and forward the r_i signals to the destination. The idea of LD space-time code is used [9]. This means that all relays perform a linear operation on the r_i vector or on its conjugate to generate t_i :

$$t_i = \sqrt{\frac{P_2}{P_1 + 1}} (A_i r_i + B_i \bar{r}_i), \quad i = 1, 2, \dots, R, \tag{3}$$

where the relay matrices A_i and B_i are $T \times T$ unitary matrices and $P_2 = \frac{P}{R}$, the received vector at the τ th time, can be written as:

$$\begin{aligned} X^\tau &= \sum_{i=1}^R g_i t_i + w \\ X^\tau &= \sqrt{\frac{P_1 P_2 T}{P_1 + 1}} S^\tau H^\tau + W^\tau \\ &= \sqrt{\frac{P_1 P_2 T}{P_1 + 1}} [\hat{A}_1 s^\tau \dots \hat{A}_R s^\tau] H^\tau + W^\tau \\ &= \sqrt{\frac{P_1 P_2 T}{P_1 + 1}} [\hat{A}_1 \hat{U}^\tau \hat{s}_1^{\tau-1} \dots \hat{A}_R \hat{U}^\tau \hat{s}_R^{\tau-1}] H^\tau + W^\tau \end{aligned} \tag{4}$$

where

$$\begin{aligned} H &= [\hat{f}_1 g_1 \dots \hat{f}_1 g_1]^t, \\ W &= \sqrt{\frac{P_2}{P_1 + 1}} \sum_{i=1}^R g_i \hat{A}_i \hat{n}_i + w, \\ \begin{cases} \hat{A}_i = A_i, \hat{f}_i = f_i, \hat{v}_i = v_i, \hat{U}^\tau = U^\tau & \text{if } B_i = 0 \\ \hat{A}_i = B_i, \hat{f}_i = \bar{f}_i, \hat{v}_i = \bar{v}_i, \hat{U}^\tau = \bar{U}^\tau & \text{if } A_i = 0 \end{cases} \end{aligned}$$

If f_i and g_i are kept constant for 2 consecutive blocks, i.e. $H^\tau = H^{\tau-1}$ and $U^\tau \hat{A}_i = \hat{A}_i \hat{U}^\tau$, Eq. (4) becomes:

$$\begin{aligned} X^\tau &= \sqrt{\frac{P_1 P_2 T}{P_1 + 1}} U^\tau [\hat{A}_1 \hat{s}_1^{\tau-1} \dots \hat{A}_R \hat{s}_R^{\tau-1}] H^{\tau-1} + W^\tau \\ &= \sqrt{\frac{P_1 P_2 T}{P_1 + 1}} U^\tau S^{\tau-1} H^{\tau-1} + W^\tau \\ &= U^\tau X^{\tau-1} + W^{\tau'} \end{aligned} \tag{5}$$

where

$$W^{\tau'} = W^\tau - U^\tau W^{\tau-1}.$$

Now to decode the code word U^τ , a maximum likelihood detector can be applied as follows:

$$\arg \max_U \|X^\tau - UX^{\tau-1}\|. \tag{6}$$

The channel information is not needed at f_i or g_i during the detection process. It is obvious that the construction of the DDSTC suggests that the relays cooperate to encode a unitary space-time code. The design problem of the DDSTC can be summarized as:

- Design a family of unitary code words (U^τ) with full diversity.
- Design relay unitary matrices (A_i).
- Make every U^τ commute with every A_i .

3. Code construction

Our coding scheme is based on SAST codes [18]. The SAST codes have rate one and allow decoding the transmitted symbols into two groups. In this section, we will discuss how to implement these codes and apply them to a cooperative network to get DDSTC.

3.1. Code design

The SAST code matrix is constructed using two circulant space-time codes, each of length L , to be employed in a network of $2L$ relays. The circulant space-time codes are written as follows [18]:

$$A = \begin{bmatrix} x_1 & x_2 & x_3 & \cdots & x_L \\ x_L & x_1 & x_2 & \cdots & x_{L-1} \\ x_{L-1} & x_L & x_1 & \cdots & x_{L-2} \\ \vdots & \vdots & \dots & \ddots & \vdots \\ x_2 & x_3 & x_4 & \cdots & x_1 \end{bmatrix}. \tag{7}$$

The SAST code is constructed as:

$$S = \begin{bmatrix} A(x_1, x_2, \dots, x_L) & -B^H(x_{L+1}, x_{L+2}, \dots, x_{2L}) \\ B(x_{L+1}, x_{L+2}, \dots, x_{2L}) & A^H(x_1, x_2, \dots, x_L) \end{bmatrix}. \tag{8}$$

Two examples of SAST codes are presented. The SAST codes for the four-relay network and the relay matrices are given as [18]:

$$S = \begin{bmatrix} x_1 & x_2 & -x_3^* & -x_4^* \\ x_2 & x_1 & -x_4^* & -x_3^* \\ x_3 & x_4 & x_1^* & x_2^* \\ x_4 & x_3 & x_2^* & x_1^* \end{bmatrix}, \tag{9}$$

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$B_1 = B_2 = A_3 = A_4 = 0,$$

$$B_3 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, B_4 = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}.$$

The SAST code for a six-relay network is given as [18]:

$$S = \begin{bmatrix} x_1 & x_2 & x_3 & -x_4^* & -x_6^* & -x_5^* \\ x_3 & x_1 & x_2 & -x_5^* & -x_4^* & -x_6^* \\ x_2 & x_3 & x_1 & -x_6^* & -x_5^* & -x_4^* \\ x_4 & x_5 & x_6 & x_1^* & x_3^* & x_2^* \\ x_6 & x_4 & x_5 & x_2^* & x_1^* & x_3^* \\ x_5 & x_6 & x_4 & x_3^* & x_2^* & x_1^* \end{bmatrix}.$$

These codes have unitary relay matrices, and by direct multiplication, we can see that any code matrix commutates with all relay matrices. Therefore, two out of the three required conditions to construct the DDSTC are satisfied. The next subsection demonstrates how these codes can be made unitary.

3.2. Signal set design

In this subsection, signal set design conditions will be derived to make the SAST codes unitary. The idea of joint modulation mentioned in [17] will be used. The authors of [17] suggested that several information symbols can be modulated jointly, where pairwise constellation sets were proposed. The extension to more than two symbols is easy as we will see. Two examples of wireless network are introduced: wireless networks with four and with six relays.

Case 1: Four-relay networks

We can start by computing $S_4^H S_4$ as follows:

$$S^H S = \begin{bmatrix} a & b & 0 & 0 \\ b & a & 0 & 0 \\ 0 & 0 & a & b \\ 0 & 0 & b & a \end{bmatrix}, \tag{10}$$

where

$$a = \sum_{i=1}^4 |x_i|^2 \text{ and } b = x_1^* x_2 + x_2^* x_1 + x_3^* x_4 + x_4^* x_3.$$

Eq. (10) is unitary if and only if $a = 1$ and $b = 0$, so some constraints are required between the elements of first group symbols X_1 and X_2 , and between elements of the second group, symbols X_3 and X_4 . No joint constraint between these two groups is required in order to obtain two-group decodability. In [17], a joint constellation set M consists of the L complex-valued constellation pair $\{a_k, b_k\}$ where $1 \leq k \leq L$. Every constellation pair is mapped to one group. The proposed constellation M has the following form:

$$\begin{cases} a_k = \exp[j(2k\pi/M)]/\sqrt{2} \\ b_k = 0 \end{cases} \quad \text{for } 1 \leq k \leq \frac{L}{2} ,$$

$$\begin{cases} a_k = 0 \\ b_k = \exp[j(2(k - L/2)\pi/M + \theta)]/\sqrt{2} \end{cases} \quad \text{for } \frac{L}{2} \leq k \leq L ,$$
(11)

where $M = L/2$ is an integer and θ is a constellation rotation angle between 0 and $2\pi/M$. To achieve the full diversity and the maximum coding gain, the rotation angle θ is equal to π/M for even M and θ is equal to $\pi/2M$ or $3\pi/2M$ for odd M . The proof can be found in [17]. Applying this constellation set to the SAST code will satisfy the unitary condition of DDSTC. For rate 1 bit per channel use (bpcu), the constellation set has sixteen pairs.

Case 2: Six-relay networks

Similar to the previous case, $S_6^H S_6$ is computed as follows:

$$S_6^H S_6 = \begin{bmatrix} a & b & c & 0 & 0 & 0 \\ c & a & b & 0 & 0 & 0 \\ b & c & a & 0 & 0 & 0 \\ 0 & 0 & 0 & a & b & c \\ 0 & 0 & 0 & c & a & b \\ 0 & 0 & 0 & b & c & a \end{bmatrix} ,$$
(12)

where

$$a = \sum_{i=1}^6 |x_i|^2 ,$$

$$b = x_1^* x_2 + x_3^* x_1 + x_2^* x_3 + x_4^* x_5 + x_6^* x_4 + x_5^* x_6 .$$

$$c = x_2^* x_1 + x_1^* x_3 + x_3^* x_2 + x_5^* x_4 + x_4^* x_6 + x_6^* x_5$$

Similar to the four-relay network case, let the first group contain symbols X_1, X_2 , and X_3 and the second group contain symbols X_4, X_5 , and X_6 . We have the M joint constellation set consisting of the L complex-valued constellation group $\{a_k, b_k, c_k\}$ where $1 \leq k \leq L$. Every constellation group is mapped into one symbol group. For rate 0.5 bpcu, the constellation set has eight pairs. The proposed constellation has the following form:

$$\begin{cases} a_k = \exp[j(2k\pi/3)]/\sqrt{2} \\ b_k = c_k = 0 \end{cases} \quad \text{for } 4 \leq k \leq 6$$

$$\begin{cases} b_k = \exp[j(2k\pi/3 + \theta_1)]/\sqrt{2} \\ a_k = c_k = 0 \end{cases} \quad \text{for } 1 \leq k \leq 3 ,$$
(13)

$$\begin{cases} c_k = \exp[j(2k\pi/2 + \theta_2)]/\sqrt{2} \\ a_k = b_k = 0 \end{cases} \quad \text{for } 7 \leq k \leq 8$$

where θ_1 and θ_2 are constellation rotation angles and have values of $2\pi/3$ and $\pi/2$, respectively. Applying this constellation set to the SAST code satisfies the unitary condition of the DDSTC.

4. Low decoding complexity

In addition to the full rate and full diversity, the decoding complexity of maximum likelihood plays an important role in the space-time code design, which should be minimized as much as possible. Orthogonal space-time code is characterized by full diversity and linear decoding complexity, but its rate is not more than $3/4$ for more than two transmit antennas. On the other hand, quasiorthogonal space-time code has full rate with exponential decoding complexity.

This paper presents low decoding complexity based on group structure, where the transmitted code word can be divided into g groups for maximum likelihood detection, where g is greater than one. The decoding metric of the maximum likelihood detection splits, which means that every group can be decoded separately, so instead of searching in the whole space we can do it in a reduced one.

The search spaces of the proposed DDSTC, the four-group decodable DDSTC in [14], and the cyclic codes in [13] increase exponentially while the transmission rates increase, which means they need more time to recover transmitted data. The cyclic DDSTC has only one maximum likelihood decoder working at a time, but the other two codes have four and two parallel maximum likelihood decoders working at the same time. These parallel decoders reduce the search space of each maximum likelihood decoder and so reduce the overall complexity. For four-relay networks, if the transmission rates are 1 bpcu, the decoding search space for each decoder is 16 for the proposed code, whereas it is 256 for cyclic codes and 4 for four-group decodable DDSTC. For six-relay networks, if the transmission rates are 0.5 bpcu, the decoding search space for each decoder is 8 for the proposed code, whereas it is 64 for cyclic codes.

5. Simulation results

In this section, the performance of the proposed codes is studied through simulations. In all simulations, the channels and noises are assumed to be independent and identically distributed complex Gaussian random variables with zero mean and unit variance. The error event is block error rate (BLER). The fading coefficients are assumed to be constant over each block and vary independently from one block to another. The total power is distributed among the source and the relays equally.

We give two cases of simulated performance for our new codes, which are introduced in Section 3. The first simulation follows case 1, where the relay's number is 4 and the signal set chosen for the proposed code is as given by Eq. (11). We compare the performance of the proposed DDSTC with that of the four-group decodable DDSTC in [14] and the cyclic codes in [13] for transmission rates of 1 bpcu.

In the 2nd case, the relay's number is 6 and the signal set chosen for the proposed code is as given by Eq. (13). We compare the performance of the proposed DDSTC with that of the cyclic codes in [13]. It is not possible to implement four-group decodability for 6 relays as it can only be done for a power of two relays. The adopted transmission rate is 0.5 bpcu.

The performances of our distributed differential space-time codes for case 1 and case 2 are shown in Figures 2 and 3, respectively. It is obvious from Figure 2 that the performances of the proposed DDSTCs outperform the cyclic code with 2 dB at BLER of 10^{-2} . However, the four-group decodable DDSTC is about 2 dB better than our proposed code. For the 6-relay case, Figure 3 shows that the proposed code has better BLER performance than the cyclic codes as there is about 3 dB gain at BLER of 10^{-3} .

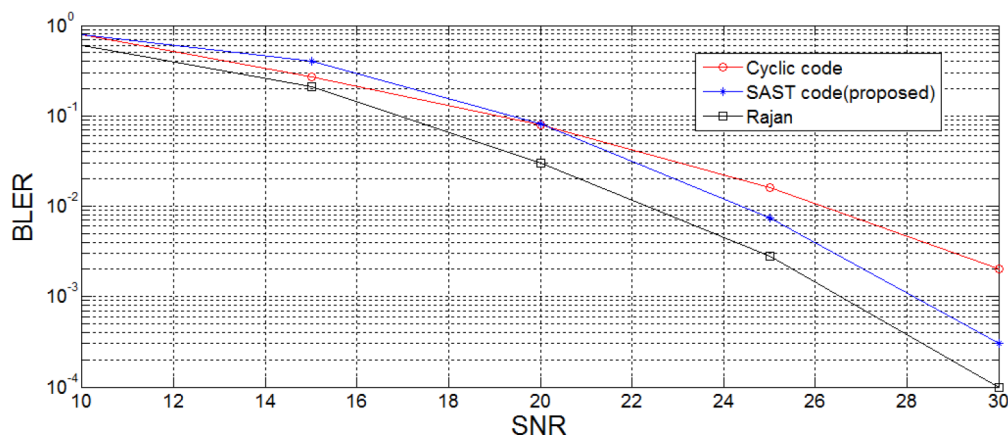


Figure 2. Block error versus SNR for DDSTC with 4 relay networks.

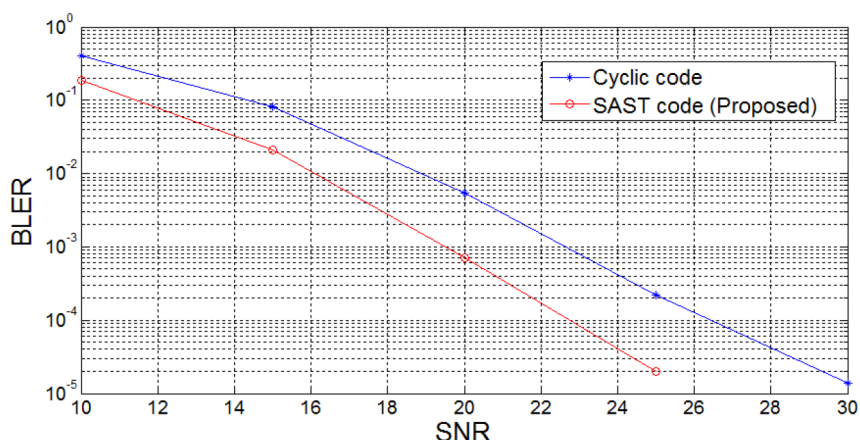


Figure 3. Block error rate versus SNR for DDSTC with 6 relay networks.

6. Conclusion

Since channel state information in cooperative networks is difficult to obtain, developing a differential transmission scheme is necessary. In this paper, the designing problem of a distributed differential space-time code with low decoding complexity was investigated. It is more challenging than differential space-time code for MIMO systems, where no channel information is needed at the transmitter, relays, or receiver. Based on circulant space-time codes, a new class of distributed differential space-time codes was approached, namely SAST codes. The new code is characterized by low decoding complexity, is two-group decodable, and is valid for even numbers of relay networks. The signal set of [17] was used, where the joint modulation is extended to more than two symbols. The performances of these codes were compared with two already existing DDSTCs, cyclic codes and four-group DDSTC. The new codes outperformed the cyclic codes. The proposed code is worse than the four-group DDSTC with a maximum of 2 dB loss. However, the proposed code based on SAST overcomes the basic limitation of the four-group DDSTC. The four-group DDSTC is valid only for the power of two relays while the proposed code is valid for any number of even relays.

References

- [1] Yang S, Belfiore JC. Optimal space-time codes for the MIMO amplify-and-forward cooperative channel. *IEEE T Inform Theory* 2007; 53: 647-663.
- [2] Oggier F, Hassibi B. An algebraic family of distributed space-time codes for wireless relay networks. In: *IEEE Information Symposium of Information Theory*; July 2006; Seattle, WA, USA. New York, NY, USA: IEEE. pp. 538-541.
- [3] Laneman JN, Wornell GW. Distributed space-time-coded protocols for exploiting cooperative diversity in wireless networks. *IEEE T Inform Theory* 2003; 49: 2415-2425.
- [4] Nabar RU, Bölcskei H, Kneubühler FW. Fading relay channels: performance limits and space-time signal designs. *IEEE J Sel Area Comm* 2004; 22:1099-1109.
- [5] Anghel PA, Leus G, Kaveh M. Multi-user space-time coding in cooperative networks. In: *IEEE International Conference on Acoustics, Speech and Signal Processing* ; 6–10 April 2003. New York, NY, USA: IEEE. pp. 73-76.
- [6] Jing Y, Hassibi B. Distributed space-time coding in wireless relay networks. *IEEE T Wirel Commun* 2006; 5: 3524-3536.
- [7] Hua Y, Mei Y, Chang Y. Wireless antennas-making wireless communications perform like wireline communications. In: *IEEE AP-S Topical Conference on Wireless Communications Technology*; 15–17 October 2003. New York, NY, USA: IEEE. pp. 47-73.
- [8] Jing Y, Jafarkhani H. Using orthogonal and quasi-orthogonal designs in wireless relay networks. In: *IEEE GlobeCom*; 27 November–1 December 2006; San Francisco, CA, USA. New York, NY, USA: IEEE. pp. 1-5.
- [9] Hassibi B, Hochwald B. High-rate codes that are linear in space and time. *IEEE T Inform Theory* 2002; 48: 1804-1824.
- [10] Hochwald BM, Swelden W. Differential unitary space-time modulation. *IEEE T Commun* 2000; 48: 2041-2052.
- [11] Jing Y, Jafarkhani H. Distributed differential space-time coding for wireless relay networks. *IEEE T Commun* 2008; 56: 1092-1100.
- [12] Kiran T, Rajan BS. Partially-coherent distributed space-time codes with differential encoder and decoder. In: *IEEE International Symposium on Information Theory*; July 2006; Seattle, WA, USA. New York, NY, USA: IEEE. pp. 547-551.
- [13] Oggier F, Hassibi B. Cyclic distributed space-time codes for wireless relay networks with no channel information. *IEEE T Inform Theory* 2007; 56: 250-265.
- [14] Rajan GS, Rajan BS. Algebraic distributed differential space-time codes with low decoding complexity. *IEEE T Wirel Commun* 2008; 7: 3962-3971.
- [15] El-Mahdy A, Desouky A. Differential space-time coding for partially-coherent decode and forward cooperative communication. In: *IEEE 2014 International Conference on Engineering and Technology*; 19–20 April 2014; Cairo, Egypt. New York, NY, USA: IEEE. pp. 1-6.
- [16] Maeeni SA, Kapinas VM, Muhaidat S, Karagiannidis GK, Al-Araji SR. Differential distributed space-time coding for vehicle-to-vehicle networks. In: *2015 International Conference on Information and Communication Technology Research*; 17–19 May 2015; Abu Dhabi, UAE. New York, NY, USA: IEEE. pp. 124-127.
- [17] Yuen C, Guan YL, Tjhung TT. Unitary space time modulation with joint modulation. *IEEE T Veh Technol* 2007; 56: 3937-3944.
- [18] Đào DN, Tellambura C. Capacity-approaching semi-orthogonal space-time block codes. In: *IEEE GlobeCom*; 2005; St. Louis, MO, USA. New York, NY, USA: IEEE. pp. 3304-3309.