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Analytical modeling of a coaxial cylindrical probe capacitive sensor based on MATLAB/Simulink for conductive liquids level measurements

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Abstract: Measurement of liquid level is an important process variable in the operation of many process industries such as water treatment plants. There are different types of level sensors to measure this parameter, categorized into ultrasonic, capacitive, optical, microwave, magnetostrictive, resistive chain, magnetoresistive, hydrostatic pressure, air bubbler, gamma ray, etc. In this paper, a cylindrical probe capacitive sensor for measuring the level of conductive liquids has been modeled based on a step-by-step mathematical procedure, using MATLAB/Simulink. The steps take the relation between the different capacitances in the model with the height of the conductive liquid into account. The results show a linearity function between the sensor capacitance from 0.1 to 1.1 nF versus the height of liquid from 0 to 90 cm. In order to have a standardized instrumentation output, a signal conditioning circuit was designed to achieve an output current of 4–20 mA, representing the sensor capacitance, and the response of the sensor in terms of current to different heights of liquid was characterized.

Key words: Probe cylindrical capacitive sensor, mathematical modeling, MATLAB/Simulink, conductive liquid, level sensor

1. Introduction

Among all the resistive, inductive, and capacitive sensors, the last type, working on the basis of changes in the electrical charge, are the most precise sensors for their extremely high sensitivity, high resolution ($\leq$0.01 nm), broad bandwidth ($>$100 kHz), robustness, long-term stability and durability, drift-free character, simple structures, low cost, and noncontact detection features [1]. Various application and different measurement techniques of capacitive sensors have been explained in the literature, including capacitive proximity detectors, capacitive switches, capacitive micrometers, etc. [2]. They are also increasingly preferred over other existing ones for measuring the level of liquids due to their remarkable advantages of low cost, high linearity and sensitivity, low energy dissipation, and easy adjustability to the geometry of the application. Some different sensing mechanisms for measuring the height of liquids are given in Table 1 [3].

Many researchers have discussed the principles of capacitive measurement used for instrumentation [4]. Attempts have been made to predict the performance of these sensors by developing a mathematical model and solving the Laplace equation with specified boundary conditions applying analytical and/or numerical techniques [5]. An extracted model of a capacitive-based level sensor with 4 different kinds of dielectric separately simulated in PSpice with good consistency between the simulated and measurement results has been presented by Bande

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Table 1. State of the arts in liquid-level sensors [3].

Sensor	Advantages	Disadvantages
Float liquid-level sensor	Simple structure Low cost	Vulnerability to mechanical damage High maintenance cost
Ultrasonic liquid-level sensor	Simple structure Easy installation and maintenance	Susceptible to interferences Difficult to achieve intrinsically safety performance
Magnetostrictive liquid-level sensor	High precision Large-scale High security	Easy to get stuck in special environments such as turbid liquid
Differential pressure liquid-level sensor	Stable performance High precision Low cost	Easily jammed or blocked
Fiber-optic liquid-level sensor	Anti-electromagnetic interference Robustness toward harsh environments	Unable to measure the turbid liquid level and other liquid with sticky substance
Conventional capacitive liquid-level sensor	Low cost High linearity Low energy dissipation	Complex structure Nonintrinsic safety

et al. [6]. A precise formulation for a cylindrical capacitive sensor has been also reported by Azimi and Golnabi, where they calculated the electrical capacitance through dissimilar theoretical models such as Coulomb's law, Gauss' law, and the Laplace equation. Based on the models, a capacitance geometrical relation has been made and by comparing the results of different methods an evident error difference in short capacitor length range and a trivial difference in long capacitor limits were reported [7]. Furthermore, an electrical technique on the basis of a cylindrical capacitive sensor has been given in [8] to predict the moisture content in the sesame, soybean, and canola seed. Moreover, a semicylindrical capacitive sensor for the purpose of soil moisture measurement was introduced by Das et al., where a numerical-based signal conditioner was used to convert the alterations in the capacitance into voltage variations [8,9]. An active shielding for grounded capacitive sensors, as an approach for decreasing the effects of both external noise/interference and parasitic capacitances of the shielded cable, has been presented by Reverter et al. [10]. Theoretical and experimental analyses of limitations such as measurement circuit instability and inaccuracy due to active shielding and guidelines for its improvement have also been given in detail in their work. A coaxial cylindrical capacitive sensor along with capacitance measuring module for addressing the problems caused by the active shielding technique was designed and characterized by Jin et al. [11]. Furthermore, in [3], an intrinsically safe liquid level sensor using coaxial cable for flammable and explosive environments and a transducer circuit for converting the capacitance variations into voltage have been developed. Moreover, sensor parameters such as resolution, repeatability, and hysteresis were examined and characterized experimentally.

In this paper a liquid-level measurement system based on a probe cylindrical capacitive sensor is modeled and analyzed. In the absence of a simple method to describe the capacitance behavior of a level sensor, the method is well discussed step-by-step here and the analysis and calculation of physical parameters of the model using MATLAB/Simulink can be considered a popular and prevalent method to describe the sensor behavior when immersed in different levels of liquid.

2. Probe cylindrical capacitive sensor: review and analysis

Let us now analyze a solid cylindrical conductor of radius a surrounded by a coaxial cylindrical shell with an inner radius of b, shown in Figure 1, as a basic structure of our desired sensor. For neglecting edge effects, assume that the length of L is much larger than $b-a$ [12]. Due to the cylindrical symmetry of the system, we choose our Gaussian surface to be a coaxial cylinder with length $l < L$ and radius r, where $a < r < b$. Using Gauss' law, the electric field can be written as

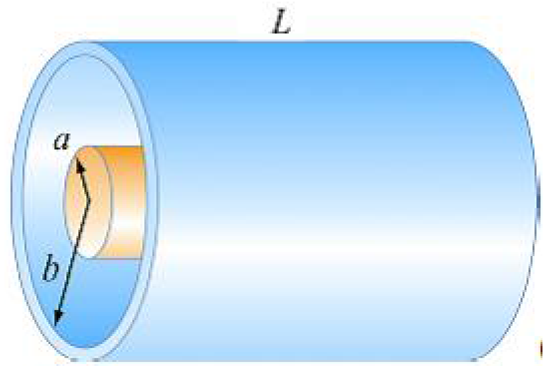


Figure 1. A cylindrical capacitor [12].

$$\oiint_S \vec{E}.d\vec{A} = EA = E(2\pi rl) = \frac{\lambda l}{\varepsilon_0} \Rightarrow E = \frac{\lambda}{2\pi\varepsilon_0 r}, \tag{1}$$

where $\lambda = \frac{Q}{L}$ is the charge per unit length. Therefore, the potential difference can be given by

$$\Delta V = V_b - V_a = -\int_a^b E_r dr = -\frac{\lambda}{2\pi\varepsilon_0 r}\int_a^b d_r/r = -\frac{\lambda}{2\pi\varepsilon_0 r}\ln(b/a), \tag{2}$$

in which the integration path was chosen along the direction of the electric field lines. As expected, the outer conductor with negative charge has a lower potential. This gives

$$C = \frac{Q}{|\Delta V|} = \frac{\lambda L}{\lambda \ln(b/a)/2\pi\varepsilon_0} = \frac{2\pi\varepsilon_0 L}{\ln(b/a)} \tag{3}$$

It can be seen that the capacitance C depends on the geometrical factors of L, a and b. We will refer to this equation again in section 3.

A grounded cylindrical capacitive level sensor for liquid measurement is realized by grounding the inner conductor of a shielded cable and using the outer conductor for measuring the liquid level [11]. Figure 2a shows a circuit considered for an active shielding technique. The length of the electrode determines the measuring range of the liquid-level measurement system. However, the material and volume of the electrode limit the application of the system, and the parasitic capacitance C_p cannot be completely offset by the active shielding technique. Figure 2b shows a circuit based on a novel approach of a shielded cable. The difference between the circuit shown in Figure 2b and the one in Figure 2a is that the inner conductor is either connected to the ground or used as an electrode.

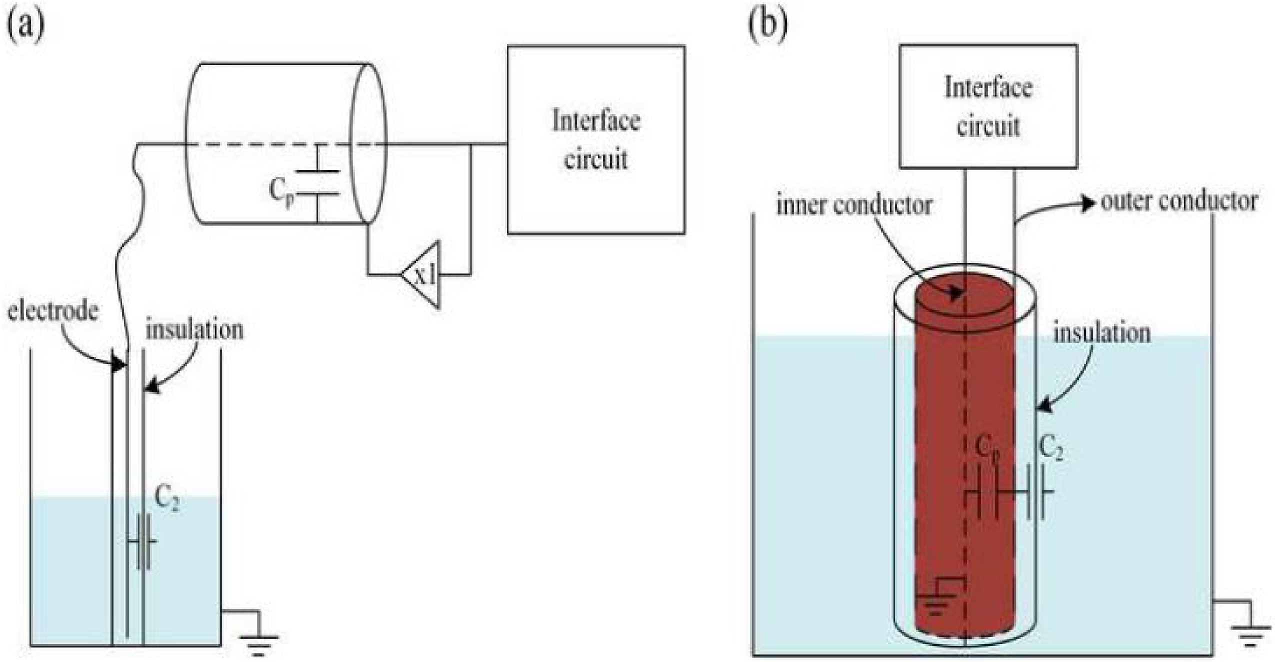


Figure 2. (a) Measurement of a grounded capacitive sensor using active shielding; (b) the novel approach of a shielded cable [11].

A structural model of a cylindrical capacitive sensor composed of four parts: a cylindrical stainless steel column, a hexagon-head bolt, a screw, and a coaxial cable, is illustrated in Figure 3. As shown in Figure 4a, the parasitic capacitance, defined as C_p, consists of C_0 and C_1. The capacitanceC_0 is laid between the inner and outer conductors. C_1 is the capacitance between the outer conductor and the stainless steel column. As shown in Figure 4b, when the grounded cylindrical capacitive sensor is immersed in conductive liquids in a nonmetallic container, the inner conductor and the stainless steel column and conductive liquids are at the same potentials. An additional capacitance, defined as C_2, will appear if the liquid levels rise. C_2 is the capacitance between the outer conductor and conductive liquids. Tables 2 and 3 present the physical parameters of a cylindrical capacitive sensor and the constant parameters used in this research, respectively.

Table 2. Physical parameters of a cylindrical capacitive sensor.

Value	Parameter
$l_{cable} = 3000mm$	coaxial cable length
$l_{column} = 84.5mm$	stainless steel column length
$l_{c_0} = 2990mm$	capacitance C_0 electrode length
$l_{c_1} = 20mm$	capacitance C_1 electrode length
$d_1 = 0.75mm$	External diameter of the inner conductor
$d_2 = 4.8mm$	Internal diameter of the outer conductor
$d_3 = 5.6mm$	External diameter of the outer conductor
$d_4 = 7.2mm$	External diameter of the PVC insulation
$d_5 = 8.2mm$	The through-hole internal diameter of the hexagon-head bolt
$d_6 = 16mm$	Internal diameter of the top threaded groove

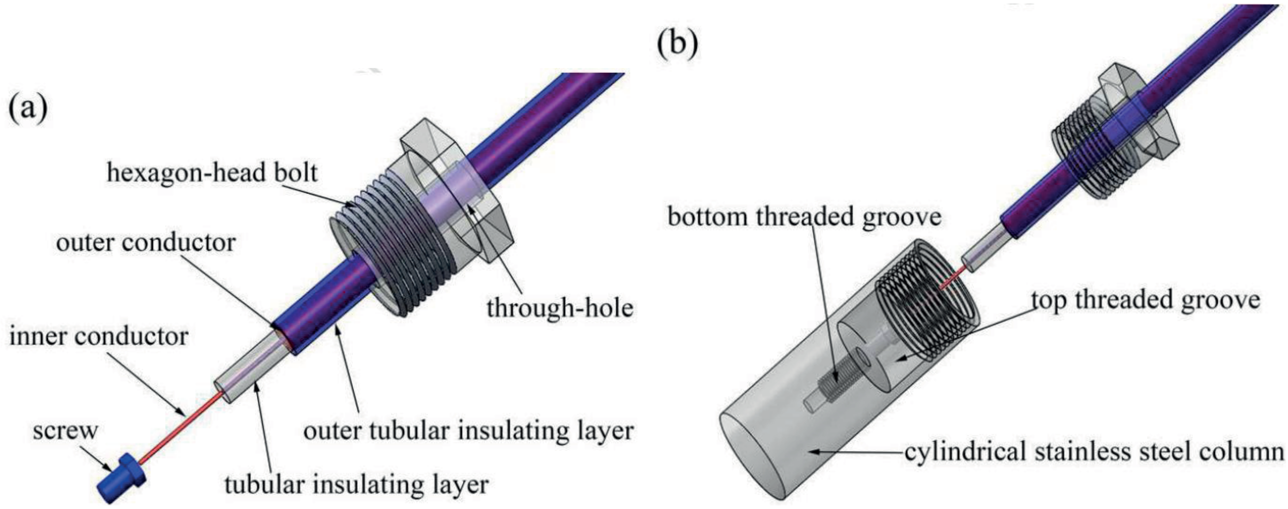


Figure 3. The two-step procedure to obtain a grounded cylindrical capacitive sensor: (a) Step 1; (b) Step 2 [11].

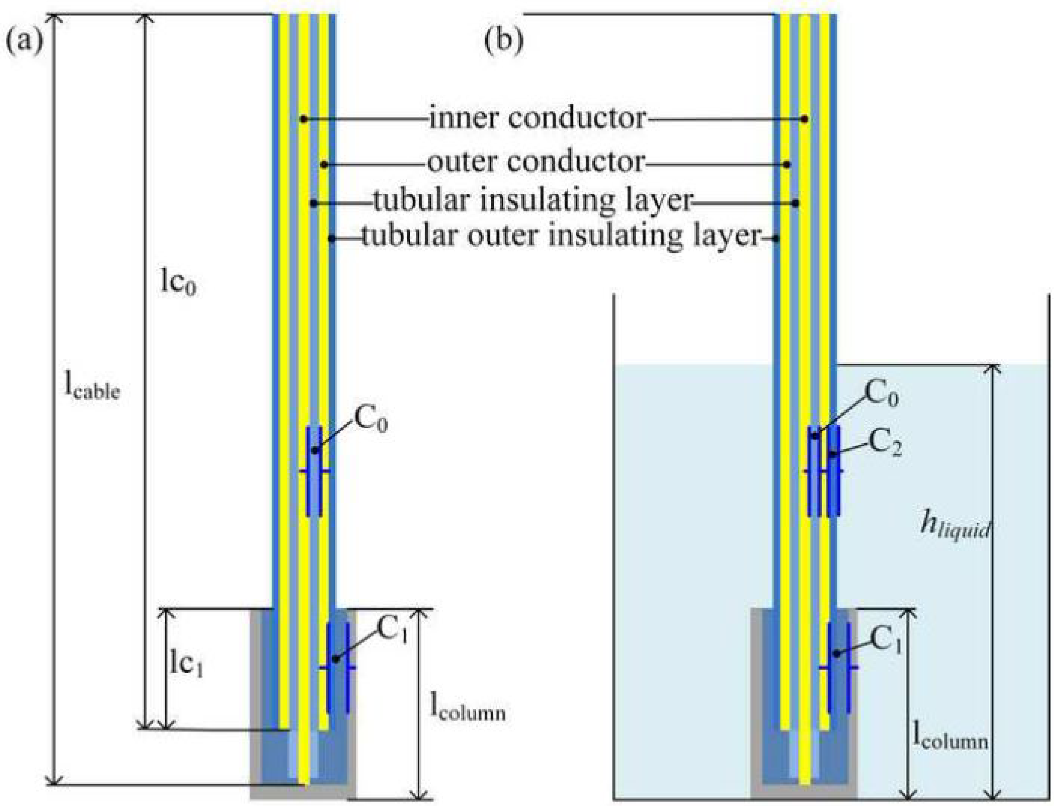


Figure 4. Analysis of the grounded cylindrical capacitive sensor: (a) the sensor in the air; (b) the sensor immersed in conductive liquids in a nonmetallic container [11].

3. Mathematical model

The MATLAB and Simulink environments are integrated in one entity, and thus analyzing, simulating, and revising the models in each environment at any point can be done accordingly. Here, we call Simulink from MATLAB to analyze and simulate the sensor explained in the previous section. A gradual approach has been

Table 3. Constant parameters of sensor.

Value	Constant parameters
$\varepsilon_0 = 8.8542 \times 10^{-12} F/m$	Electric permittivity of vacuum
$\varepsilon_1 = 2.3$	The relative permittivity of the tubular insulating layer (PE)
$\varepsilon_2 = 4.8$	The relative permittivity of the tubular outer insulating layer (PVC)
$\varepsilon_3 = 4.4$	The relative permittivity of the solid mixture composed of epoxy resin and curing agent
h_{liquid}	Height of the conductive liquids

used to model the sensor and to determine the value of sensor capacitance C_{sensor} through 5 steps.

Step 1: At the first step, C_0, the initial value of the capacitance, is configured. It can be calculated based on Eq. (4):

$$C_0 = \frac{Q}{|\Delta V|} = \frac{2\pi\varepsilon_0\varepsilon_1}{\ln(d_2/d_1)}.l_{c_0} \tag{4}$$

The value of $C_0 = 2.068e - 10 \cong 206.1pF$ is obtained using MATLAB/Simulink depicted in Figure 5.

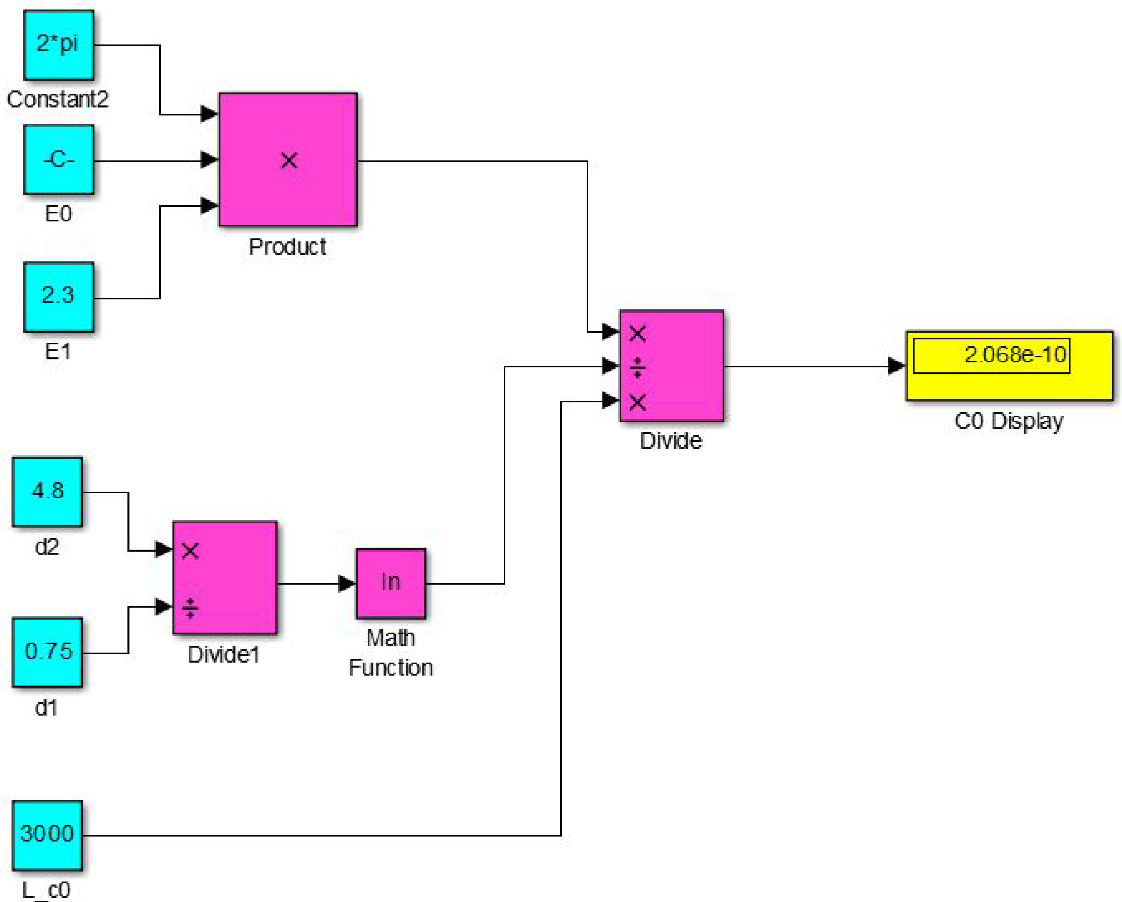


Figure 5. C_0 subsystem, the initial value of the capacitance.

Step 2: The second step is to find the value of C_1. Because of the tubular outer insulating layer (normally is PVC) and solid mixture between two electrodes of capacitance, the value of C_1 cannot be figured out directly from Eq. (3) but can be defined using the following equation, resulted in $C_1 = 1.358e-11 \cong 13.58pF$ shown in Figure 6. Here $h = l_{c_1} = 20mm$.

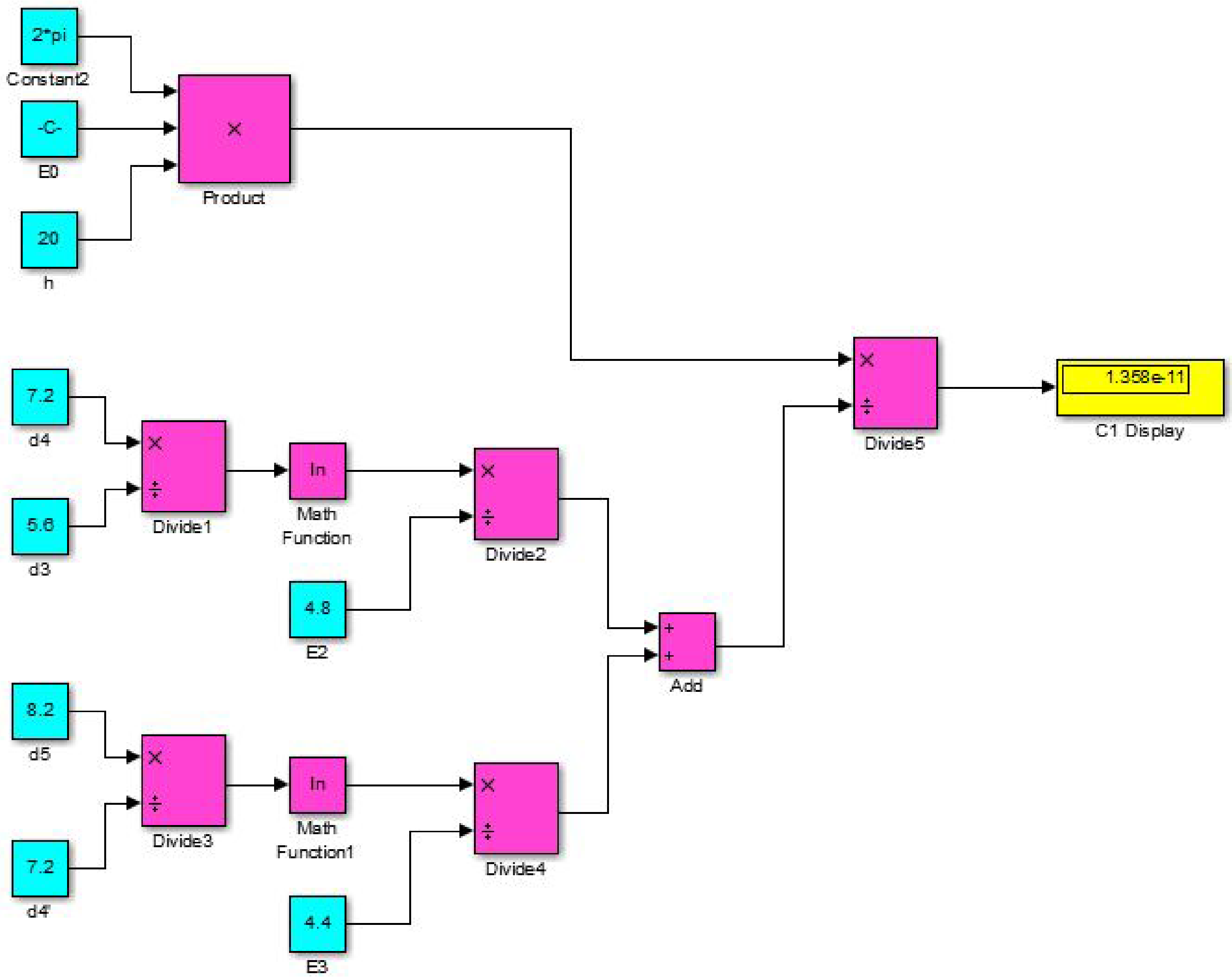


Figure 6. C_1 subsystem.

$$\begin{aligned} C_1 &= \frac{Q}{V_{C1}} = \frac{Q}{-\left[\int_{d_4}^{d_3} Q/2\pi r\varepsilon_0\varepsilon_2 h.dr + \int_{d5}^{d_4} Q/2\pi r\varepsilon_0\varepsilon_3 h.dr\right]} \\ &= \frac{Q}{Q/2\pi r\varepsilon_0\varepsilon_2 h.\ln(d_4/d_3) + Q/2\pi r\varepsilon_0\varepsilon_3 h.\ln(d_5/d_4)} = \frac{2\pi\varepsilon_0 h}{\ln(d_4/d_3)/\varepsilon_2 + \ln(d_5/d_4)/\varepsilon_3} \end{aligned} \tag{5}$$

Step 3: The third step creates the parasitic capacitance, C_p. The model, shown in Figure 7, takes C_0 and C_1 as the inputs and calculates the C_p as the output, so that $C_p = C_1 + C_0 = 219.68pF$

Step 4: C_2 is presented in the fourth step. As shown in Figure 4b, the height of conductive liquids is considered as two separate parts: length of stainless steel column l_{column} and length of the cable immersed in conductive liquids $h_{liquid} - l_{column}$. From Eq. (3), the capacitance C_2 can be defined as

$$C_2 = \frac{2\pi\varepsilon_0\varepsilon_2}{\ln(d_4/d_5)}.(h_{liquid} - l_{column}) \tag{6}$$

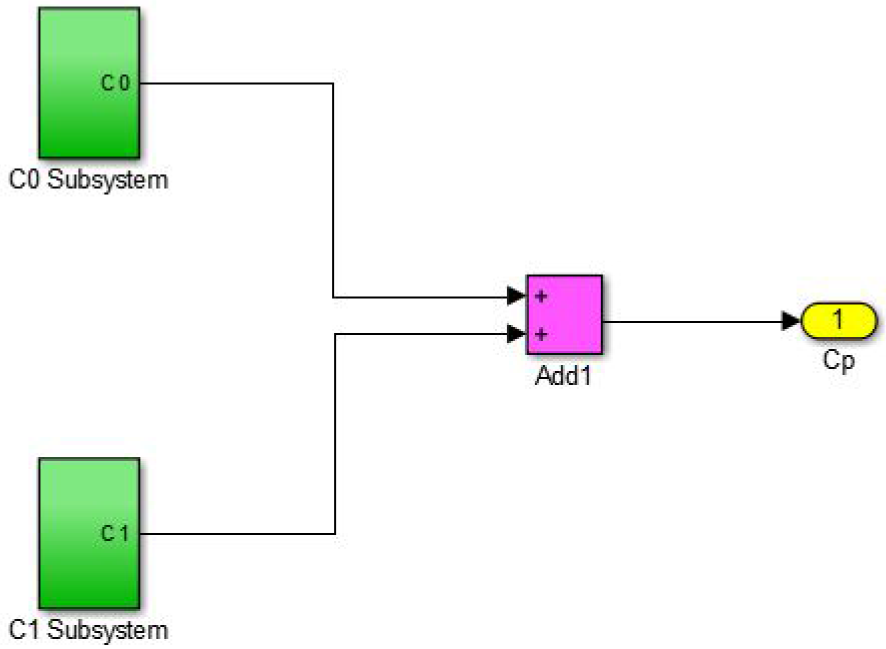


Figure 7. C_p subsystem.

Figure 8 represents the C_2 subsystem. Here h_{liquid} is taken as the input and its value is determined by the liquid level that is intended for measuring.

Step 5: By accomplishing the fifth step, as the final step, C_{sensor} is calculated. Figure 9a and 9b show the overall block diagram of C_{sensor} and its related subsystems, respectively. The capacitance value of the sensor C_{sensor} can be expressed as

$$C_{sensor} = C_p + C_2 = 219.68 + 1.0625(h_{liquid} - l_{column}) \quad (7)$$

Eq. (7) indicates a linear relationship between C_{sensor} and h_{liquid}.

4. Level measurement

Different values of conductive liquids in the range of 0∼900 mm were applied to the sensor and are illustrated in Figure 10 as the constant values in Simulink environment at the h_liquid input. The corresponding values of sensor capacitances were obtained and depicted in terms of farad in the Display block. For the minimum level of the tank, the Display block shows 1.299e-10 which is equal to 129.9 pF, i.e. when the level is zero (empty tank), the C_{sensor} equals 129.9 pF. Moreover, the maximum level of tank, $h_{liquid} = 900$ mm, leads to $C_{sensor} = 1086$ pF. A comparison between the results of Simulink and simulation in the MATLAB environment is illustrated in Figure 11. It can be seen that there is a linear relationship between the sensor capacitance and liquid level as mentioned earlier in section 3.5 by Eq. (7).

5. Transducer circuit

Not many level transducer circuits have been presented in the literature. Mathews et al. [13] presented a design of a capacitance transducer using a 555-timer. The circuit measures the capacitance in terms of frequency and uses a microcontroller to obtain the accurate level. Capacitive transducer circuits for measurement of liquid level

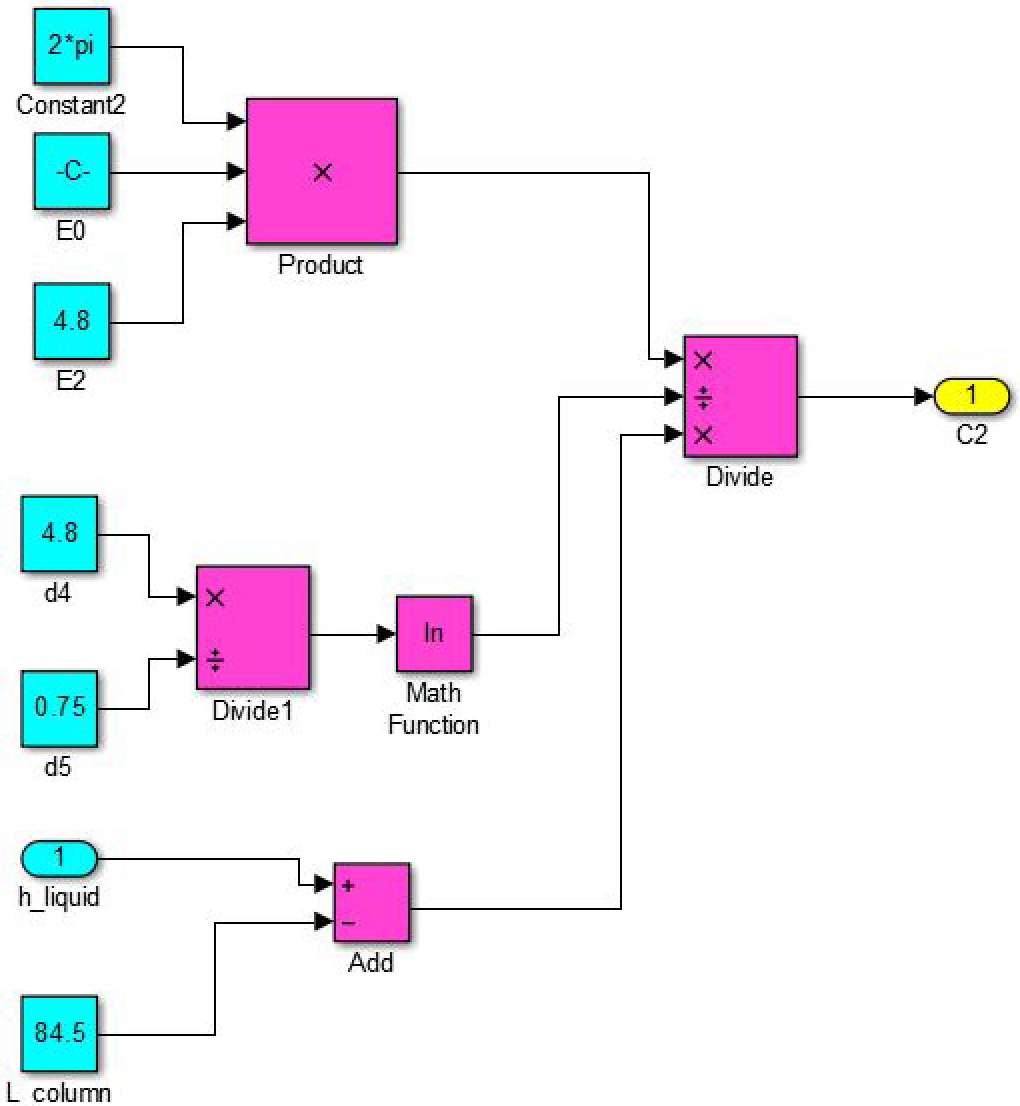
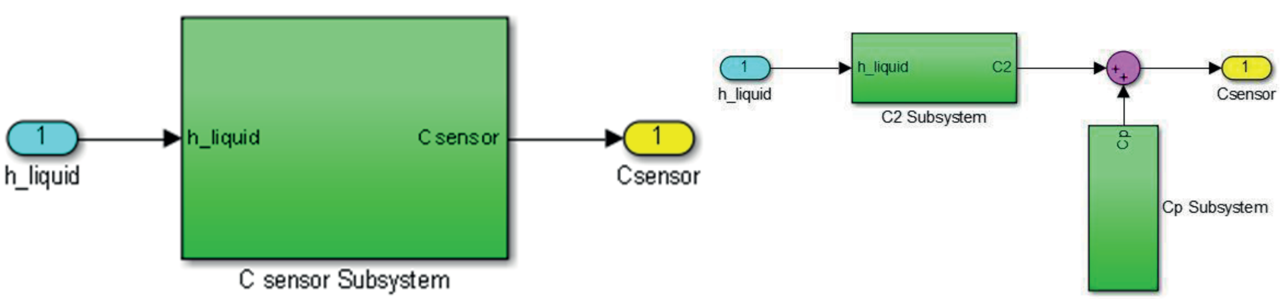


Figure 8. C_2 subsystem.



(a) (b)

Figure 9. C_{sensor} system: (a) overall; (b) subsystems.

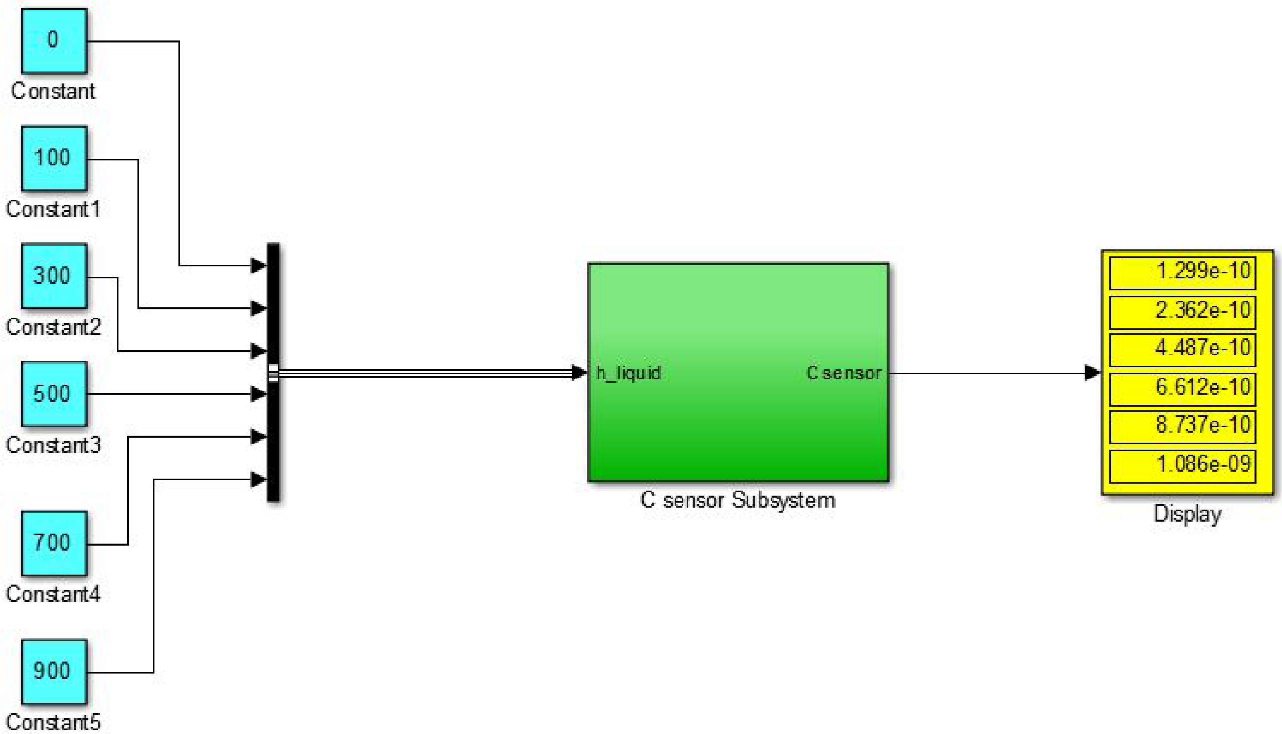


Figure 10. C_{sensor} response versus h_{liquid} stimulus (Simulink environment).

have been discussed by Khan et al. [14]. Bhardwaj and Singh [15] have used an integrated circuit of CAV24 together with an ADC and a microcontroller to determine the percentage of carbon in oil. The changes in capacitance with the variation in soil moisture by using a proper signal conditioning circuit have been presented by Das et al. [9].

In our work, using a level transducer, shown in Figure 12, the sensor capacitance is converted to the standard instrumentation currents of 4$\sim$20 mA based on a signal amplitude variation method. The level transducer block consists of four main parts: a derivative circuit, a half-wave rectifier, a low-pass filter, and a zero-span circuit. In this method, the coaxial cylindrical capacitor (C_{sensor}) plays a major role to change the input signal in a different shape by varying the amplitude, whereas it has been established by a simple differentiator circuit. An input signal is fed to a differential circuit through a measured capacitor and produces an output signal in a square waveform with the same input frequency but dissimilar amplitude. When the coaxial cylindrical capacitive sensor probe (C_{sensor}) is covered by the liquid whose level has to be measured, the triangular wave signal is differentiated by R_1 and C_{sensor}, producing a square wave output. The amplitude of the output signal depends on the differentiating gain factor of R_1 and C_{sensor}, where the value of R_1 is constant but C_{sensor} varies according to the level of liquid (h_{liquid}). Eventually, for sensing the amplitude of the output signal, a half wave precision rectifier with filter is used together with a zero-span adjustment circuit for proper calibration and acquisition of the liquid level in a recordable current form. The internal circuit of each block is given in Figure 13. Using the parametric analysis capability of PSPICE software and by varying the liquid level to change the sensor capacitance from 130 pF to 1086 pF, from empty to full tank, respectively, the current through R_L is changed from 4 to 20 mA. The simulation result for parametric analysis of C_{sensor} is represented in Figure 14. We have assumed that the load is floated; otherwise, the zero-span circuit should have been modified for a grounded load. The calibration results for the transducer are given in Table 4.

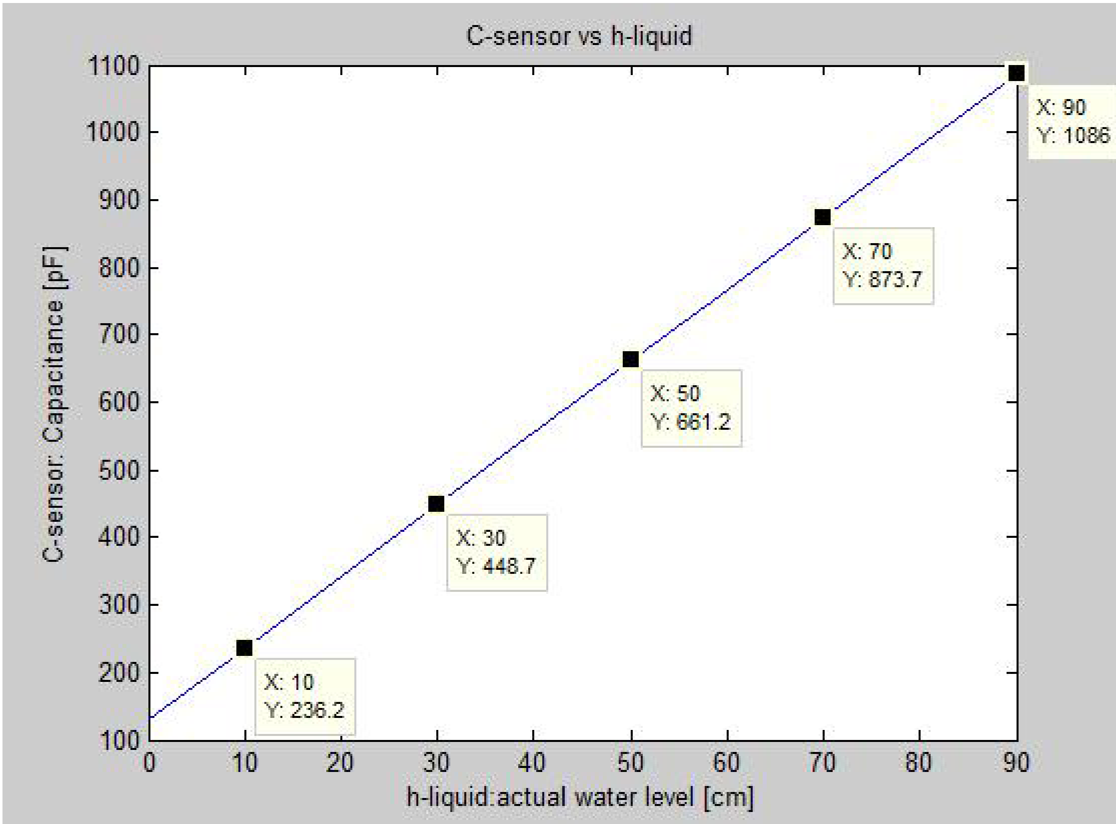


Figure 11. C_{sensor} response versus h_{liquid} stimulus (MATLAB environment).

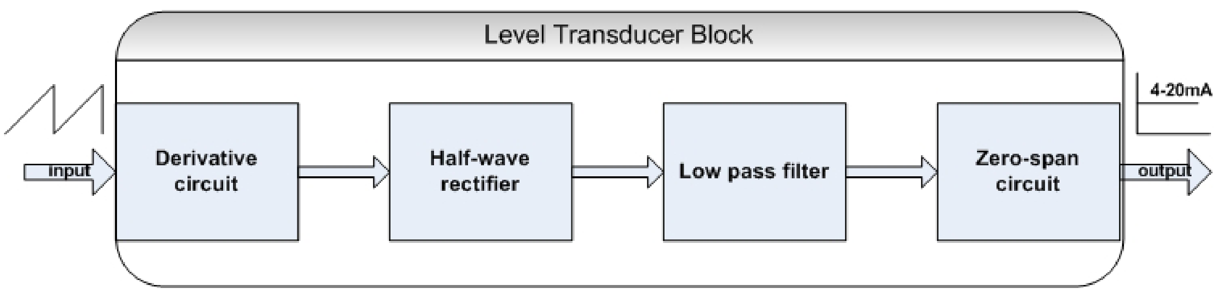


Figure 12. Level transducer block diagram.

Table 4. Calibration results of sensor according to various level values.

Level percent (%)	I (4–20 mA)	Capacitance (pF)
0	4	130
25	8	369
50	12	608
75	16	847
100	20	1086

6. Conclusion

A coaxial cylinder structure liquid-level sensor system was successfully modeled and simulated using MATLAB/Simulink. The simulation results indicated a linear relationship between C_{sensor} and h_{liquid}. Consequently, this model can be used as a level sensor in the library of the Sim-scape/SimElectronics/Sensor in MATLAB/Simulink toolbox in the field of sensors. Finally, a low cost opamp-based transducer circuit was designed to convert variations in the sensor capacitance to a standard instrumentation current (4–20 mA).

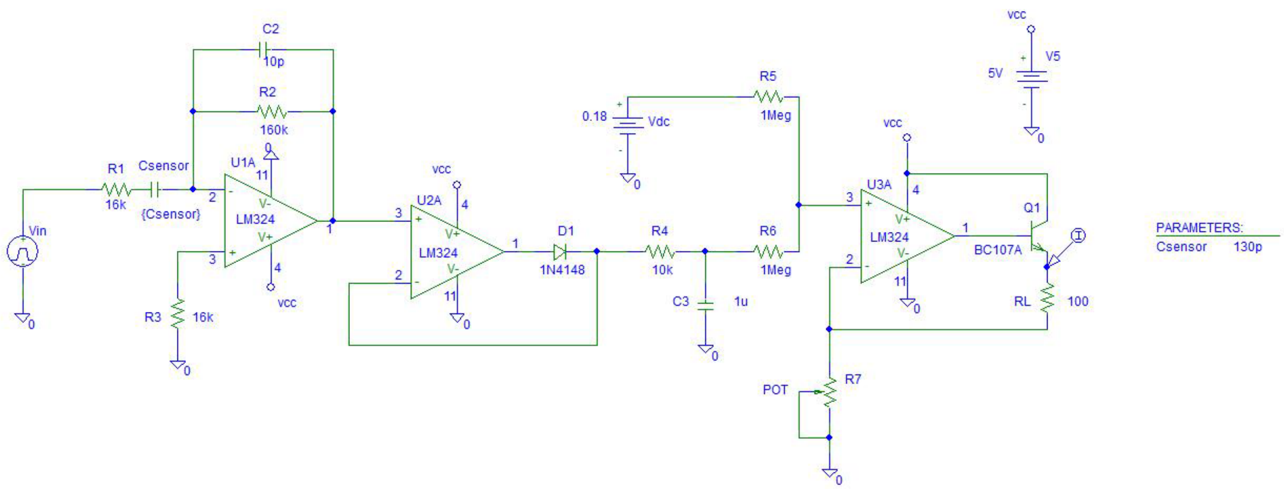


Figure 13. Level transducer electrical circuit.

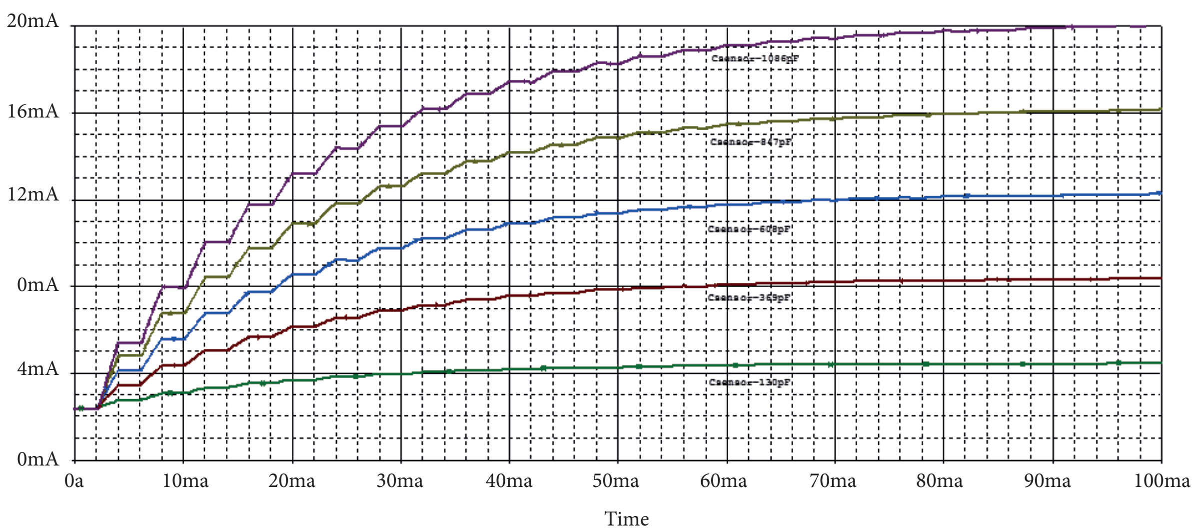


Figure 14. Transducer circuit output (4∼20 mA) versus parametric variation in C_{sensor}.

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