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Constrained control allocation for nonlinear systems with actuator failures or faults

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Abstract: In this paper, a combination of dynamic constrained control allocation with terminal sliding mode control is proposed for a general class of overactuated nonlinear systems with actuator faults/failures. First, the terminal sliding mode control is designed to converge the system tracking error to zero in a finite-time. Then a control allocation strategy is developed and will be solved by a Lyapunov method, which leads to a dynamic update law with finite-time convergence. This strategy satisfies input limits and when faults/failures occur in some of the actuators, the control signals are automatically redistributed among the healthy actuators. Simulation results on a near space vehicle show the effectiveness of the proposed approach.

Key words: Overactuated systems, Lyapunov method, Terminal sliding mode control, Dynamic control allocation

1. Introduction

Actuator faults and failures might have serious damaging effects on the performance of engineering systems if they are not well handled. To maintain specified safety performance of these systems when unexpected faults/failures occur, fault-tolerant control (FTC) is a very valuable control technology. Many FTC methods have been introduced in recent years. Liao et al. [1] used a linear matrix inequality method to design a robust FTC and Yang et al. [2] designed an H_∞ controller for linear systems with sensor and actuator failures. Moreover, adaptive feedback control schemes were developed in [3,4] for linear systems with actuator failures. Adaptive fault tolerant controllers were presented in [5,6] for nonlinear systems with actuator failures. However, in [5,6], input limits were not taken into account, whereas if the actuators reached their constraints, every effort to increase the actuators' output would create no variation in the output and would result in system instability.

To reduce the effects of faults/failures and to achieve FTC, an important factor is the availability of redundant actuators. This is because of more freedom regarding controller design. Control allocation (CA) is an approach that can effectively manage overactuated systems, especially when some of the actuators become damaged and CA redistributes the control signals into the healthy actuators. There has been much study of CA problems, and methods like pseudoinverse [7], linear programming [8], direct CA [8], daisy chain [9], constrained quadratic programming [10], and constrained nonlinear programming [11] have been previously introduced. They all, independently of the dynamic control problem, solve the control problem as a static optimization problem. Unlike the above approaches, Johansen [12] designed an optimal CA scheme in the shape

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of a dynamic update law with infinite-time convergence for nonlinear systems, and Benosman [13] designed an optimal CA for a class of nonlinear systems with unstable internal dynamics, but methods in [12,13] do not consider actuator faults/failures.

Sliding mode control (SMC) schemes have an inherent capability to reject matched disturbances and uncertainties. Therefore, their combination with CA seems to have great potential for the development of FTC [14]. However, robustness in conventional SMC schemes can only be achieved after the sliding phase [14]. To eliminate the reaching phase, an integral sliding mode control (ISMC) was proposed in [14]. To deal with fault, combinations of the CA with ISM are used in [15,16] for linear systems. The integral action has deficiencies, like large overshoot and long settling time. Terminal sliding mode control (TSMC) has improved characteristics such as finite-time convergence and higher control accuracy compared with ISMC [17,18].

This paper investigates a combination of dynamic CA and TSMC according to Figure 1 for a general class of nonlinear systems. The main contributions of this paper are as follows:

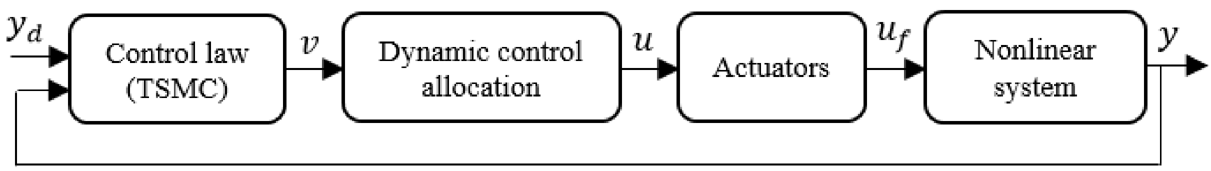

Figure 1. Block diagram of the proposed method.

- The CA problem will be solved by a Lyapunov method, which leads to a dynamic update law with finite-time convergence.
- The proposed CA method can satisfy control constraints in the presence of faults/failures in actuators.
- A TSMC with nonlinear sliding surface will be proposed that can guarantee the finite-time convergence of the system tracking error to zero.

2. Problem formulation

This paper focuses on a class of MIMO nonlinear systems represented by

$$\dot{x} = f(x) + g(x)\, u_f(t)$$

$$y = h(x), \tag{1}$$

where $x \in R^n$ is the state vector; $f(x) \in R^n$ and $g(x) \in R^{n \times n_a}$ are smooth nonlinear functions of x; $u_f(t) = \left[u_{f_1}, \ldots, u_{f_{n_a}}\right]^T \in R^{n_a}$ is the control signal whose components may be failed during operation; and $y \in R^m$ is the output vector.

Assumption 1 *Eq. (1) has a relative degree $r_1, \ldots, r_m$ such that $\sum_{i=1}^{i=m} r_i = n$.*

Assumption 2 *Eq. (1) has more actuators than controlled outputs, i.e. $n_a > m$.*

At the beginning, the system is healthy and devised to track a desirable output y_d. Afterward, faults or failures might occur in some of the actuators in $t_f > 0$. In this case, output of the actuator ρ, $\rho \in \{1, \ldots, n_a$ can be modeled as follows [6]:

$$u_{f_\rho}(t) = (1-\sigma_\rho)\,\delta_\rho u_\rho(t) + \sigma_\rho \tilde{u}_\rho, \tag{2}$$

where $\tilde{u}_\rho$ shows the failure in the ρth actuator, $0 \leq \delta_\rho \leq 1$ shows the amount of the remaining effective section of the actuator ρ, and

$$\sigma_\rho = \begin{cases} 1 & if\, the\, \rho th\, actuator\, is\, failed \\ 0 & otherwise \end{cases} \tag{3}$$

If the actuator ρ is healthy, $u_{f_\rho}(t) = u_\rho(t)$. When a fault or failure happens at the actuator ρ at $t \geq t_f$, multiple usual fault/failure models could exist [5].

I. Loss of effectiveness: in this case, $u_{f_\rho} = \delta_\rho u_\rho$.

II. Lock in place or stuck-type: in this case, $u_{f_\rho} = \tilde{u}_\rho = constant$.

III. Float fault: in this case, $u_{f_\rho} = 0$.

IV. Hard over fault: in this case, $u_{f_\rho} = \bar{u}_\rho$ or $u_{f_\rho} = \underline{u}_\rho$.

Now we can express the input vector $u_f(t)$ in Eq. (1) as follows [6]:

$$u_f(t) = (I-\sigma)\,\delta u(t) + \sigma \tilde{u}, \tag{4}$$

where $u(t)$ is the applied control vector and

$$\begin{cases} \sigma = diag(\sigma_1, \sigma_2, \ldots, \sigma_{n_a}) \\ \delta = diag(\delta_1, \delta_2, \ldots, \delta_{n_a}) \end{cases} \tag{5}$$

Here our aim is to develop a combination of TSMC and CA based on the Lyapunov method for Eq. (1) with the actuator faults from Eq. (4) such that the output $y(t)$ tracks a desired smooth trajectory $y_d(t)$, and at the same time all actuators satisfy control constraints:

$$u(t) \in G = \left\{ u = [u_1 \ldots u_{n_a}]^T - \underline{u}_\rho \leq u_\rho \leq \overline{u}_\rho, \rho = 1, \ldots, n_a \right\}, \tag{6}$$

where $\underline{u} = \left[\underline{u}_1 \ldots \underline{u}_{n_a}\right]^T$ and $\overline{u} = [\overline{u}_1 \ldots \overline{u}_{n_a}]^T$ are lower and upper control constraints.

Assumption 3 *There exists* $B_{di} > 0$ *such that* $\left\| \left(y_{di}, y_{di}^{(1)}, \ldots, y_{di}^{(r_i)} \right) \right\| \leq B_{di}$.

Under assumption 1, there exists a local coordinate transformation:

$$z = T(x) = \left[z_1(x)^T, \ldots, z_m(x)^T \right]^T \tag{7}$$

$$z_i(x) = [z_{i1}(x), \ldots, z_{ir_i}(x)]^T = \left[h_i(x), \ldots, L_f^{r_i-1} h_i(x) \right]^T \tag{8}$$

that converts Eq. (1) to the following form [19]:

$$
\begin{aligned}
&\dot{z}_{i1} = z_{i2} \\
&\quad\vdots \\
&\dot{z}_{i(r_i-1)} = z_{ir_i} \\
&\dot{z}_{ir_i} = v_i(z,t) \\
&y_i = h_i = z_{i1} \qquad (i = 1,\ldots,m),
\end{aligned} \tag{9}
$$

where

$$v_i(z,t) = b_i(z) + \sum_{\rho=1}^{n_a} a_{i\rho}(z)\left[(1-\sigma_\rho)\,\delta_\rho u_\rho(t) + \sigma_\rho \tilde{u}_\rho\right] \tag{10}$$

and where $b_i(z) = L_f^{r_i} h_i$ and $a_{i\rho}(z) = L_{g_\rho} L_f^{r_i-1} h_i$. Eq. (10) can be expressed as follows:

$$v(z,t) = B(z) + A(z)\left[(1-\sigma)\right]\delta u(t) + \sigma\tilde{u} \tag{11}$$

where $B(z) = [b_1 \ldots b_m]^T$ and $A(z) = \begin{bmatrix} a_{11} & \cdots & a_{1n_a} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn_a} \end{bmatrix}$; $u(t) = [u_1 \ldots u_{n_a}]^T \in R^{n_a}$ is the real control input; and $v(z,t) = [v_1 \ldots v_m]^T \in R^m$ is the virtual control effort produced by the TSMC law, which will be designed in the next section. By considering $n_a > m$, we will not have a unique solution (i.e. u) for Eq. (11). In Section 4, a CA problem is formulated to find the best solution for Eq. (11) while satisfying the control constraints in Eq. (6).

3. TSMC

In this section, in order to find virtual control $v(z,t) \in R^m$ for Eq. (11), a TSMC is proposed that guarantees the output tracking problem in a finite time. From Eq. (9), the system outputs and its derivatives can be rewritten as

$$
\begin{aligned}
&y_i = h_i = z_{i1} \\
&\dot{y}_i = \dot{z}_{i1} = z_{i2} \\
&\quad\vdots \\
&y_i^{(r_i)} = v_i(z,t) \qquad (i = 1,\ldots,m)
\end{aligned} \tag{12}
$$

By defining the ith element of tracking error as $e_{i1} = y_i - y_{di}$, a nonlinear terminal sliding surface is set to [20]

$$
\begin{aligned}
&e_{i2} = \dot{e}_{i1} + \alpha_{i1} e_{i1} + \beta_{i1} e_{i1}^{\frac{p_{i1}}{q_{i1}}} \\
&\quad\vdots \\
&e_{i(r_i-1)} = \dot{e}_{i(r_i-2)} + \alpha_{i(r_i-2)} e_{i(r_i-2)} + \beta_{i(r_i-2)} e_{i(r_i-2)}^{\frac{p_{i(r_i-2)}}{q_{i(r_i-2)}}} \\
&s_i = \dot{e}_{i(r_i-1)} + \alpha_{i(r_i-1)} e_{i(r_i-1)} + \beta_{i(r_i-1)} e_{i(r_i-1)}^{\frac{p_{i(r_i-1)}}{q_{i(r_i-1)}}},
\end{aligned} \tag{13}
$$

where $i = 1,\ldots,m$ and $j = 1,\ldots,r_i - 1$, $\alpha_{ij} > 0$ and $\beta_{ij} > 0$; $p_{ij} < q_{ij}$ are positive odd constants and s_i is a nonlinear terminal sliding surface for e_{i1}.

The r_ith derivative of e_{i1} becomes

$$e_{i1}^{(r_i)} = v_i(z,t) - y_{di}^{(r_i)} \tag{14}$$

and the lth time derivative of e_{ij} is given by

$$e_{ij}^{(l)} = e_{i(j-1)}^{(l+1)} + \frac{d^{(l)}}{dt^{(l)}} \left(\alpha_{i(j-1)} e_{i(j-1)} + \beta_{i(j-1)} e_{i(j-1)}^{\frac{p_{i(j-1)}}{q_{i(j-1)}}} \right) \tag{15}$$

In accordance with Eqs. (13) – (15), the time derivative of s_i becomes

$$\dot{s}_i = v_i(z,t) - y_{di}^{(r_i)} + \sum_{j=1}^{r_i-1} \left[\alpha_{ij} e_{ij} + \beta_{ij} e_{ij}^{\frac{p_{ij}}{q_{ij}}} \right]^{(r_i-j)} \tag{16}$$

Therefore, the TSMC law is set to

$$v_i\,(z,t) = y_{di}^{(r_i)} - \sum_{j=1}^{r_i-1} \left[\alpha_{ij} e_{ij} + \beta_{ij} e_{ij}^{\frac{p_{ij}}{q_{ij}}} \right]^{(r_i-j)} - \gamma_i s_i - K_i sign\,(s_i) \tag{17}$$

where $\gamma_i > 0$ and $K_i > 0$ $(i = 1,\ldots,m)$.

Lemma 1 *([18,21]): If there are positive definite functions* $V\,(x)$, $\lambda_1 > 0$, $\lambda_2 > 0$ *and* $0 < r < 1$ *such that* $\dot{V}\,(x) \le -\lambda_1 V\,(x) - \lambda_2 V^k\,(x)$, *then the equilibrium point* $x = 0$ *is finite-time stable and the convergence time* t_s *satisfies*

$$t_s \le \frac{1}{\lambda_2\,(1-r)} \ln \frac{\lambda_2 V^{1-r}\,(x_0) + \lambda_1}{\lambda_1} \qquad x_0 = x(0) \tag{18}$$

Theorem 1 *For the nonlinear sliding surface (Eq. (13)) and the control law (Eq. (17)), the overactuated system (Eq. (1)) will be stable and closed-loop signals will converge to the equilibrium point in a finite-time.*

Proof By substituting Eq. (17) into Eq. (16), we have

$$\dot{s}_i = -\gamma_i s_i - K_i sign(s_i) \tag{19}$$

To prove stability of Eq. (1), we choose the Lyapunov function as

$$V_1 = \frac{1}{2} S^T S, \tag{20}$$

where $S = [s_1,\ldots,s_m]^T$. Differentiating V_1 and using Eq. (19) yields

$$\dot{V}_1 = \sum_{i=1}^{m} s_i \dot{s}_i = -\sum_{i=1}^{m} K_i s_i sign\,(s_i) - \sum_{i=1}^{m} \gamma_i s_i^2 \le -\underline{K} \sum_{i=1}^{m} |s_i| - \underline{\gamma} \sum_{i=1}^{m} s_i^2, \tag{21}$$

where $\underline{\gamma} = \{\gamma_i\} > 0$ and $\underline{K} = \{K_i\} > 0$. Eq. (21) can be expressed as

$$\dot{V}_1 \leq -\sqrt{2}\underline{K}V_1^{\frac{1}{2}} - 2\underline{\gamma}V_1 \tag{22}$$

According to Eq. (22) and Lemma 1, we know that V_1 converges to the equilibrium point in the finite-time $t_{s_1} \leq \frac{1}{\underline{\gamma}} \ln \frac{\sqrt{2}\underline{\gamma}V_1^{\frac{1}{2}}(S_0)+\underline{K}}{\underline{K}}$.

□

4. CA

After finding the virtual control vector $v(z,t) \in R^m$ in the previous section, we intend to find the law control $u(t) \in R^{n_a}$ from Eq. (11) so as to satisfy the control constraints in Eq. (6). However, considering $n_a > m$, we will not have a unique solution for Eq. (11). Therefore, a secondary objective is added to minimize the magnitude of the control vector, or its distance from a preferred control value, u_0. Based on the above-mentioned, the CA problem can be formulated as follows:

$$J(z,u,t) \tag{23}$$
$$subject\,to\;\; \underline{u} \leq u \leq \bar{u}$$

where $J = J_1(z,u,t) + J_2(u)$ and

$$J_1(z,u,t) = \|c_1 (B(z) + A(z)[(I-\sigma)\delta u + \sigma\tilde{u}] - v(z,t))\|_2^2 \tag{24}$$

$$J_2(u) = \|c_2(u - u_0)\|_2^2 \tag{25}$$

where $c_1 \in R^{m \times m}$ and $c_2 \in R^{n_a \times n_a}$ are positive definite diagonal weighting matrices. The cost function J_1 minimizes the allocation error of the virtual control v into u in Eq. (11) and J_2 minimizes the magnitude of the control input u.

Now the CA problem (Eq. (23)) is converted to a quadratic problem (QP). First, let

$$X_0 = u_0 - \underline{u}\;,\;\; X = u - \underline{u}\;,\;\; X_{max} = \overline{u} - \underline{u}$$

$$a_p(z,t) = -[B(z) + A(z)[(I-\sigma)\delta\underline{u} + \sigma\tilde{u}] - v(z,t)] \tag{26}$$

By substituting Eqs. (24) – (26) into Eq. (23), we have

$$J(z,u,t) = \|c_1[A(z)(I-\sigma)\delta X - a_p(z,u,t)]\|_2^2 + \|c_2(X - X_0)\|_2^2 \tag{27}$$
$$subject\,to\;\; 0 \leq X \leq X_{max}$$

By extending Eq. (27), we obtain

$$\begin{aligned} J(z,u,t) \;\; = \;\; & X^T\delta^T (I-\sigma)^T A^T(z) c_1^T c_1 A(z)(I-\sigma)\delta X \\ & - X^T\delta^T (I-\sigma)^T A^T(z) c_1^T c_1 a_p(z,t) - a_p^T(z,t) c_1^T c_1 A(z)(I-\sigma)\delta X \\ & + a_p^T(z,t) c_1^T c_1 a_p(z,t) + X^T c_2^T c_2 X - X^T c_2^T c_2 X_0 \\ & - X_0^T c_2^T c_2 X + X_0^T c_2^T c_2 X_0 \end{aligned} \tag{28}$$

If we define $H_1 = c_1^T c_1, H_2 = c_2^T c_2$, $C^T(z,t) = -2\left[a_p^T(z,t)H_1A(z)\,(I-\sigma)\,\delta + X_0^T H_2\right]$, $R(z,t) = a_p^T(z,t)H_1 a_p(z,t) + X_0^T H_2 X_0$, and $H(z) = 2\left[\delta^T\,(I-\sigma)^T A^T(z)H_1A(z)\,(I-\sigma)\,\delta + H_2\right]$, then Eq. (28) can be rewritten as follows:

$$J(z,u,t) = \frac{1}{2}X^T H(z)X + C^T(z,t)X + R(z,t) \tag{29}$$

$$subject\,to\;\; 0 \leq X \leq X_{max}$$

Remark 1 *$R(z,t)$ is a constant matrix and does not affect the optimal solution (Eq. (29)).*

By defining $E = \begin{bmatrix} E_1 \\ \vdots \\ E_{2n_a} \end{bmatrix} = \begin{bmatrix} I_{n_a} \\ -I_{n_a} \end{bmatrix}$ and $F = \begin{bmatrix} F_1 \\ \vdots \\ F_{2n_a} \end{bmatrix} = \begin{bmatrix} X_{max} \\ 0_{n_a \times 1} \end{bmatrix}$, the optimal problem Eq. (29) becomes a QP problem, as follows:

$$J(z,u,t) = \frac{1}{2}X^T H(z)X + C^T(z,t)X \tag{30}$$

$$subject\,to\;\; EX - F \leq 0$$

For solving the QP problem (Eq. (30)), we consider a Lagrange function:

$$L\,(X,\lambda,z,t) = \frac{1}{2}X^T H\,(z)\,X + C^T\,(z,t)\,X + \lambda^T\,(EX - F) \tag{31}$$

where $\lambda \in R^{2n_a}$ is the Lagrange multiplier. Now we define the limiting optimal set E^* as

$$E^* = \left\{(X,\lambda,z)\,|\,\frac{\partial L}{\partial X} = 0,\; \frac{\partial L}{\partial \lambda} = 0\right\} \tag{32}$$

In theorem 2, a dynamic update law is designed so as to attract (X,λ,z) to E^*. However, before this theorem, an important lemma is presented.

Lemma 2 *([18,21]): Suppose there are positive definite functions $V\,(x)$, $\lambda_1 > 0$ and $0 < r < 1$ such that $\dot{V}\,(x) + \lambda_1 V^k\,(x) \leq 0$, then the equilibrium point $x = 0$ is finite-time stable. The convergence time t_s satisfies*

$$t_s \leq \frac{V^{1-r}\,(x_0)}{\lambda_1\,(1-r)} \qquad x_0 = x(0) \tag{33}$$

Theorem 2 *(X,λ,z) tend to E^* in a finite-time if the following dynamic update laws are adopted:*

$$\dot{X} = -\Gamma_1\tau_1 \tag{34}$$

$$\dot{\lambda} = -\Gamma_2\tau_2 - \tau_2\left(\tau_2^T\tau_2\right)^{-1}(\Delta + \omega V_2^a) \tag{35}$$

where $\Gamma_1 \in R^{n_a \times n_a}$, $\Gamma_2 \in R^{2n_a \times 2n_a}$ are positive definite matrices, $\omega > 0$, $0 < a < 1$ and

$$\tau_1 = H\,(z)\left(H\,(z)\,X + C\,(z,t) + E^T\lambda\right) + E^T\,(EX - F) \tag{36}$$

$$\tau_2 = E\left(H\left(z\right)X + C\left(z,t\right) + E^T\lambda\right) \tag{37}$$

$$\Delta = \left(\frac{\partial L}{\partial X}^T \frac{\partial^2 L}{\partial z \partial X}\right)\dot{z} + \frac{\partial L}{\partial X}^T \frac{\partial^2 L}{\partial t \partial X} + \frac{\partial L}{\partial \lambda}^T \frac{\partial^2 L}{\partial t \partial \lambda} \tag{38}$$

Proof Consider the following Lyapunov function:

$$V_2\left(X,\lambda,z,t\right) = \frac{1}{2}\left(\frac{\partial L}{\partial X}^T \frac{\partial L}{\partial X} + \frac{\partial L}{\partial \lambda}^T \frac{\partial L}{\partial \lambda}\right) \tag{39}$$

The time derivative of V_2 will be as follows:

$$\dot{V}_2 = \frac{\partial V_2}{\partial X}\dot{X} + \frac{\partial V_2}{\partial \lambda}\dot{\lambda} + \frac{\partial V_2}{\partial z}\dot{z} + \frac{\partial V_2}{\partial t}$$

$$= \left[\left(H\left(z\right)X + C\left(z,t\right) + E^T\lambda\right)^T H\left(z\right) + \left(EX - F\right)^T E\right]\dot{X}$$

$$+ \left[\left(H\left(z\right)X + C\left(z,t\right) + E^T\lambda\right)^T E^T\right]\dot{\lambda} + \left(\frac{\partial L}{\partial X}^T \frac{\partial^2 L}{\partial z \partial X}\right)\dot{z} + \frac{\partial L}{\partial X}^T \frac{\partial^2 L}{\partial t \partial X} + \frac{\partial L}{\partial \lambda}^T \frac{\partial^2 L}{\partial t \partial \lambda} \tag{40}$$

By substituting Eqs. (36) – (38) into Eq. (40), we have

$$\dot{V}_2\left(X,\lambda,z,t\right) = \tau_1^T\dot{X} + \tau_2^T\dot{\lambda} + \Delta \tag{41}$$

Now by choosing $\dot{X}$ and $\dot{\lambda}$ according to Eqs. (34) and (35), Eq. (41) becomes

$$\dot{V}_2\left(X,\lambda,z,t\right) = -\tau_1^T\Gamma_1\tau_1^T - \tau_2^T\Gamma_2\tau_2^T - \omega V_2^a \leq -\omega V_2^a \tag{42}$$

From Eq. (42) and lemma 2, we know that $(X\lambda z)$ converge to E^* in finite-time as

$$t_{s_2} \leq \frac{V_2^{1-a}(x_0,\lambda_0,z_0,t_0)}{\omega(1-a)} \tag{43}$$

The design procedure of terminal sliding mode control allocation can be summarized in the following algorithm:
□

Algorithm 1

Given the nonlinear overactuated system (Eq. (1)) with the actuator faults/failures (Eq. (4)) and the input limits (Eq. (6)), design FTC by performing the following steps:

Step 1. Given the desired trajectories y_{di}, obtain the virtual control using Eqs. (13) and (17).

Step 2. Given the weighting matrices c_1, c_2, Γ_1 and Γ_2, obtain the CA result $\dot{X}$ using Eq. (34).

Step 3. According to Eq. (26), the derivative of control inputs will be $\dot{u} = \dot{X} + \dot{\underline{u}} = \dot{X}$.

5. Simulation results

The attitude dynamics of a near space vehicle (NSV) is described by [22]

$$\dot{\gamma} = \Xi(\gamma)\,\omega$$

$$\dot{\omega} = J^{-1}\Omega(\omega)\,J\omega + Gu_f(t), \tag{44}$$

where $\gamma = [\mu,\beta,\alpha]^T$ is the attitude angle vector, $\omega = [p,q,r]^T$ is the angular rate vector, and $u_f(t) = [u_{f_1}, u_{f_2}, u_{f_3}, u_{f_4}, u_{f_5}, u_{f_6}]^T$ is the input vector. The matrices $\Xi(\gamma)$, $\Omega(\omega)$, G, and J are given by

$$G = \begin{bmatrix} -0.288 & 0.288 & -0.865 & 0.865 & 0 & 0.002 \\ -0.085 & -0.085 & -0.526 & -0.526 & 0.009 & -0.007 \\ -0.095 & 0.095 & 0.207 & -0.207 & -0.004 & -0.005 \end{bmatrix}, \quad \Omega = \begin{bmatrix} 0 & r & -q \\ -r & 0 & p \\ q & -p & 0 \end{bmatrix}$$

$$J = \begin{bmatrix} 554486 & 0 & -23002 \\ 0 & 1136949 & 0 \\ -23002 & 0 & 1376852 \end{bmatrix}, \quad \Xi(\gamma) = \begin{bmatrix} cos(\alpha) & 0 & sin(\alpha) \\ sin(\alpha) & 0 & -cos(\alpha) \\ 0 & 1 & 0 \end{bmatrix} \tag{45}$$

If we define the state vector of Eq. (44) as $x = \left[\gamma^T, \omega^T\right]^T$, then Eq. (44) converts to Eq. (1), and $f(x) = \begin{bmatrix} \Xi(\gamma)\,\omega \\ J^{-1}\Omega(\omega)\,J\omega \end{bmatrix}$, $g(x) = \begin{bmatrix} 0_{3\times 6} \\ G \end{bmatrix}$ and $y = \gamma = [\mu,\beta,\alpha]^T$.

The initial state and the desired output are chosen as $x_0 = [3.5, 5, 1, 0, 0, 0]$ and $y_d = [3\cos(t) + 0.5, 5, 1 + 4\sin(0.5t)]^T$. The bounds of control input are

$$u_\rho \in \left[-30^\circ, 30^\circ\right]^T \quad (\rho = 1, \ldots, 6) \tag{46}$$

Based on Section 4, the CA parameters are $H_1 = c_1^T c_1 = diag(100, 70, 40)$, $H_2 = c_2^T c_2 = 0.1 I_{6\times 6}$, $\Gamma_1 = 3I_{6\times 6}$, $\Gamma_2 = 10 I_{12\times 12}$, $\omega = 100$, and $a = 0.5$.

To proceed the design of the terminal sliding mode control according to Eq. (17), the design parameters are $\alpha_{ij} = 40$, $\beta_{ij} = 1$, $p_{ij} = 5$, $q_{ij} = 7$, $\gamma_i = 60$, and $K_i = 20$ ($i = 1, 2, 3$ and $j = 1$).

The following cases are simulated to demonstrate the effectiveness of our method:

Case 1: Consider input limits (Eq. (46)) and all actuators fault/failure-free (i.e. $u_{f_\rho} = u_\rho$).

Case 2: Consider all actuators fault-free and without input limits (Eq. (46)).

Case 3: Consider control constraints (Eq. (46)) and

I. For $0 \le t < 5$, all actuators are fault-free.

II. For $5 \le t < 10$, a lock in place fault happens on u_1 ($u_{f_1} = -20$).

III. For $10 \le t < 15$, a hard-over fault occurs on u_6 ($u_{f_6} = -30$) and loss of effectiveness fault occurs on u_3 and u_4 ($u_{f_3} = 0.6u_3$ and $u_{f_4} = 0.7u_4$).

IV. For $15 \le t < 20$, a float fault occurs on u_2 ($u_{f_2} = 0$).

V. For $20 \le t < 25$, a lock in place fault happens on u_5 ($u_{f_5} = 10$).

VI. For $25 \leq t < 35$, float fault occurs on u_1 and u_5 ($u_{f_1} = u_{f_5} = 0$), $u_{f_6} = 10$ and loss of effectiveness fault occurs on u_3 and u_4 ($u_{f_3} = 0.5u_3$ and $u_{f_4} = 0.6u_4$).

VII. For $35 \leq t \leq 40$, all actuators are fault/failure-free.

For case 1, the results of the simulation are illustrated in Figure 2. As seen in Figure 2a, the outputs track all the desired signals precisely. It is obvious from Figure 2b that all controls u_ρ ($\rho = 1, \ldots, 6$) remain within the control constraints (Eq. 46)).

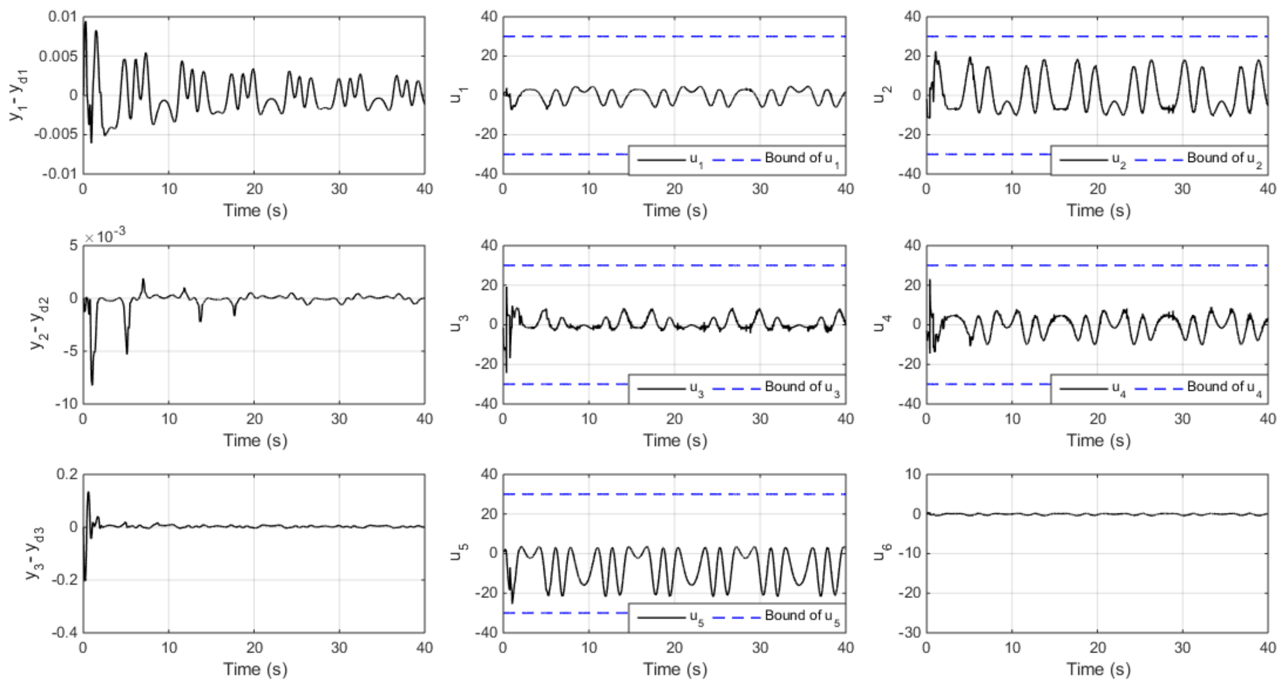

Figure 2. NSV response for case 1. (a) Tracking performance. (b) CA outputs (u).

For case 2, the results of the simulation are illustrated in Figure 3. By increasing the input limits (Eq. (46)), it is obvious from Figure 3b that the inputs u_2 and u_5 indeed violate their bounds. This shows that the proposed CA effectively forces the input signals to stay within their bounds. Of course, according to Figure 3a, tracking is achieved perfectly.

Finally, the simulation results for case 3 are shown in Figure 4. In the presence of the actuator faults/failures (I–VII), the input saturation constraints are still satisfied, as seen in Figure 4b. According to Figure 4b, when an actuator is damaged, control allocation cuts the related control signal of this actuator and redistributes control signals among the healthy actuators. In addition, good tracking is obtained when our system is under actuator faults/failures (Figure 4a).

6. Conclusion

In this paper, a new TSMC allocation algorithm has been successfully designed for overactuated nonlinear systems. The TSMC has a nonlinear sliding surface, which ensures the output tracking performance at finite-time in the presence of actuator faults or failures. The CA problem has been converted to a QP optimization problem and solved by a Lyapunov design approach. The derived CA law is in the form of a finite-time convergent

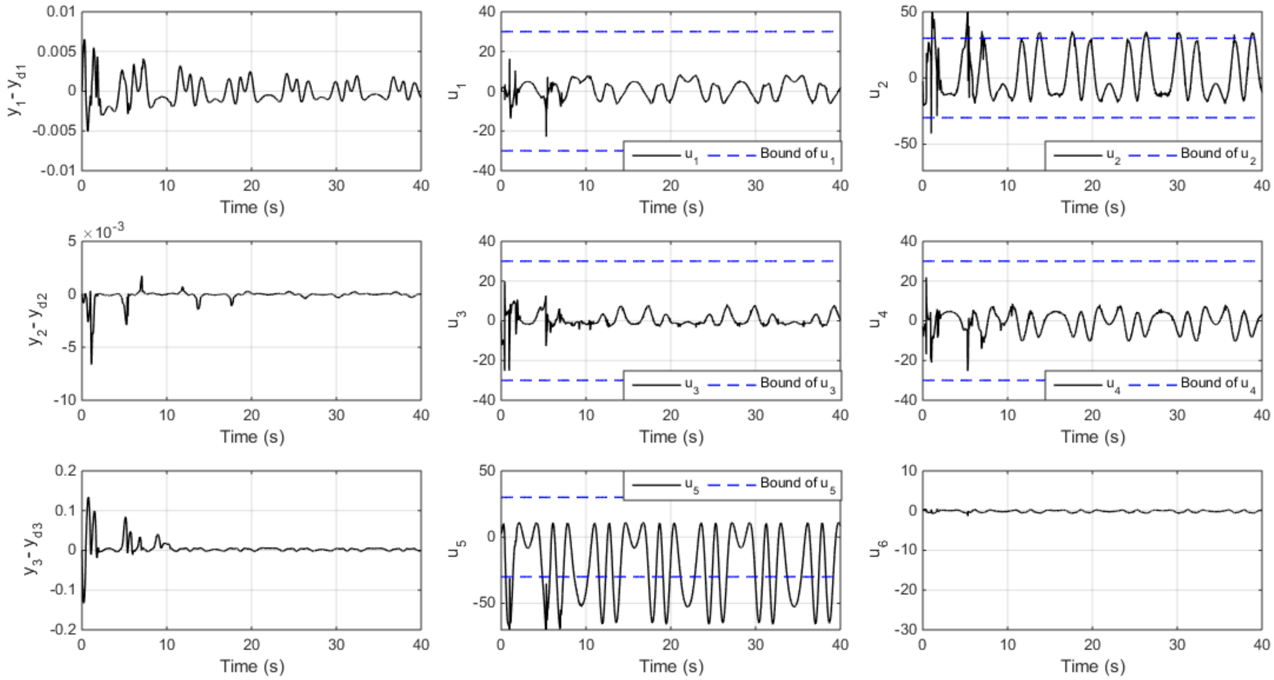

Figure 3. NSV response for case 2. (a) Tracking performance. (b) CA outputs (u).

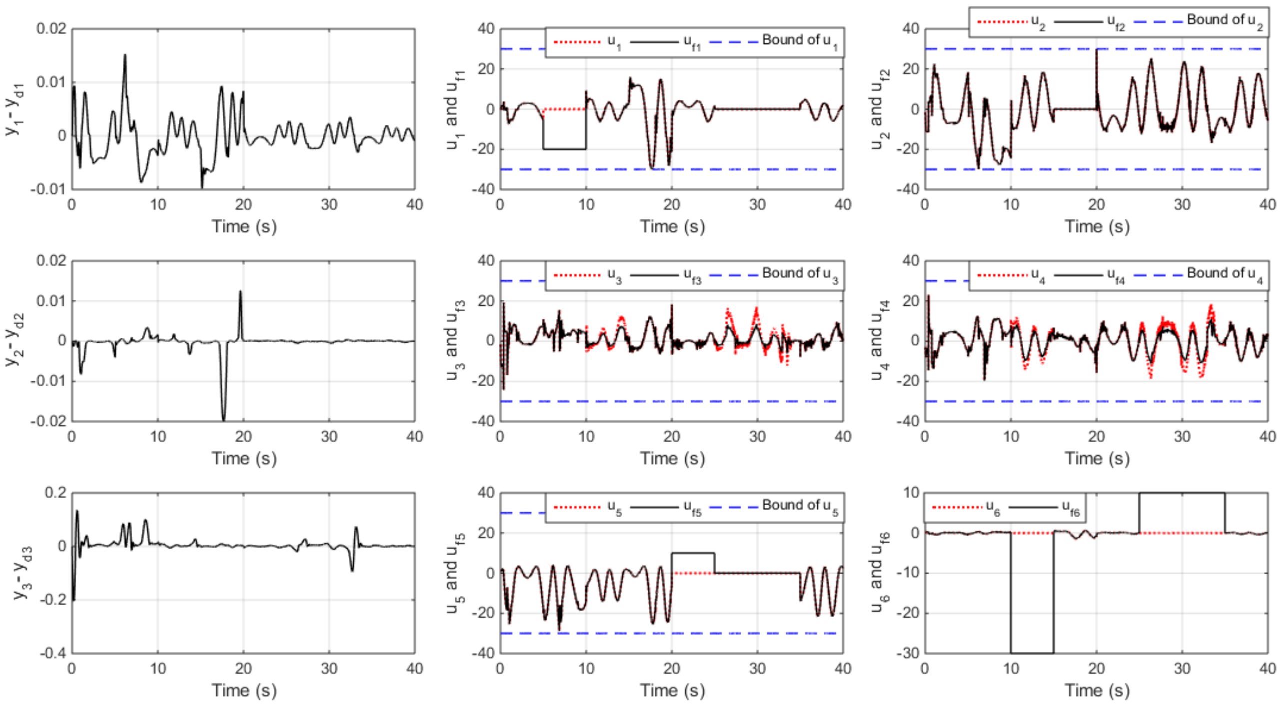

Figure 4. NSV response for case 3. (a) Tracking performance. (b) Plant inputs (u_f) and CA outputs (u).

dynamic update law, which satisfies control constraints and manages the actuators when faults/failures occur. The effectiveness of the proposed method has been demonstrated by applying it to an NSV example. The simulation results show that in the presence of actuator faults/failures, good tracking performance is obtained and CA redistributes control signals among the healthy actuators.

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