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Horizontal directional drilling pilot bore simulation

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Abstract: Various underground communications (water or gas supply systems, sewage, power lines) have been installed in recent years using trenchless technologies, i.e. horizontal directional drilling (HDD). This technique by its nature does not disturb the traffic and is environmentally friendly. One must be aware that, with HDD, pipeline building time is shortened. With respect to the above, HDD is especially effective in highly populated or industrial areas. However, it is necessary to ensure that HDD works, in order to prevent damage to installed underground facilities. The aim of this work is to propose a user-friendly and inexpensive simulation tool with required functionality for HDD trajectoryplanning under field conditions. The optimization of HDD trajectory-planning consisted of analyzing the methods for pilot bore trajectory construction: step-by-step, six-point, and four-point control. The statistical evaluation of such methods revealed their similarity regarding time resources (when using any of the methods mentioned above, the upper limit of 95% confidence interval of HDD trajectory-planning duration does not exceed 1 min). The tests of the simulation tool at work places with Ditch Witch JT922 demonstrated good functionality due to its intuitive interface and simple and fast ability to replan a HDD trajectory. Increased accuracy of bore path planning and simpler pilot bore construction were achieved.

Key words: Horizontal directional drilling, pipeline, pilot bore planning, simulation

1. Introduction

Trenchless technologies, employed in engineering networks and repair areas, have been rapidly developing over the last few decades. They greatly facilitate tiresome ground works, almost not disturbing the traffic (traffic may be restricted in exceptional cases only), and by shortening the time spent for construction of engineering infrastructure. In principle, these technologies are environmentally friendly, causing the least possible impact on ecosystems. One trenchless technology is horizontal directional drilling (HDD). This is a technique used to install an underground infrastructure (pipelines, water supply systems, sewerage, and power lines) by overcoming natural or artificial obstacles such as streets, buildings, railways, rivers, and lakes. Most commonly used is wet HDD. A greater part of the soil is removed from the pilot bore hole with drilling fluid support, but some soil is retained in drilling fluid mixtures and serves as soil stabilization during pipeline building. For this reason, HDD allows performing operations with little impact on the soil and there is no need for trench excavation before drilling [1–3].

As stated in reports presented by PPI and CAPP [2,3], HDD installation commonly consists of four

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Figure 1. Typical HDD equipment and pilot boring process.

It is necessary to perform several small tasks strictly in order when projecting the pilot tunnel path. At first, an identification of work place on a map and its topography evaluation must be performed. Various obstacles and their nature (water, road, viaduct, building, railway, etc.) should be evaluated later, as well as information gathering about the current underground infrastructure at the crossing point. Of great importance to the selection of the required equipment, as well as to the drilling tracks' profile settings, is soil research. With the underground infrastructure and soil data, a pilot borehole trajectory can be set up, i.e. input/output points and section coordinates (distance, depth, direction parameters).

The drill bit operator and position adjuster are directly responsible for drilling the initial pilot bore, usually communicating via a portable receiver (Figure 1). The latter provides information to the operator, according to the incoming data, about the need to perform trajectory corrections; hence, it should be as close to the planned as possible. Most systems that transmit location information directly to the screen of the drilling unit functions are performed automatically. However, bilateral communication is mandatory for coordinating actions and avoiding dangerous situations.

The HDD pilot tunnel must be clearly marked at each observation point. A member of staff finds the drilling location of the head and sends the evidence to the operator for process monitoring and adjustment. Boring corrections are performed when monitoring the drill bit head, and comparisons are made between the current situation and the preplanned path. The position of the drill head should be determined at least every time a new rod is added, in order to accurately follow the planned trajectory.

Drilling process control is very important when drilling the pilot bore. Inaccurate pilot bore drilling can cost a contractor dearly, due to the necessity to rapidly eliminate the consequences of an accident. Thus, public safety and ensuring the protection of the underground infrastructure are the most important HDD goals. Drilling can be done by setting the sloping drill nose into the right direction and pushing drilling rods forward. The readings of the current horizontal and vertical coordinates from the drill bit probe and the initial entry point should be periodically compared; tracking is also available, using a surface monitoring system that determines the probe location by taking measurements from a surface point [3].

2. Problem analysis

The need to develop new trenchless methods, such as microtunneling, pipe-jacking, and HDD, was stated several years ago [4]. It was important to accurately identify the needs and priorities of future research. The most urgent

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areas for research were agreed as follows: whole-life costing of trenching and trenchless technology operations; issues associated with connections; mapping of underground infrastructure, multi-utility tunnels, and drilling fluid reuse and disposal; improved modeling of ground movements; development of 'see ahead' technology; and development of economic sensors for pipes and joints. These areas were discussed in several research works in the past years [5–8]. Additionally, attempts were made to summarize trenchless technology applications. Research has been conducted that analyzes a selection of the most suitable construction methods of engineering geology and hydrogeological conditions [9].

HDD is carried out in accordance with a predesigned pilot bore trajectory. However, when arriving at the place of the prospective HDD work, it often turns out that there is not enough space for drilling and pipe assembly. This is because it must often work not only in rural but also in urban areas; the latter is defined as a high-traffic and densely populated area. Furthermore, underground communications do not meet those in the documentation. For these reasons, the predesigned drilling trajectory must often be corrected at the location. Therefore, the pilot boring duration increases, as does the duration of all HDD executions. However, this is not desirable due to the inconvenience caused by possible traffic restrictions and by placing the equipment on sidewalks. Therefore, it is highly recommended to execute the HDD in the daytime.

A change of location of the pilot bore may affect the quality of HDD, i.e. the drilling fluid compound or its overall amount can be incorrectly selected. For example, bore hole walls are weakened when fluid amount is not sufficient; therefore, a pulling of the tube into such a bore hole will result in accumulation of ground water. This can lead to damage to the main road's surface, which may just collapse. Therefore, it is important to reuse the drilling fluid and remove any excess [4].

Requirements to the operator qualification are very high, because, in most cases, there are various engineering communications previously installed at the locations of planned drilling, e.g., drainage, pipeline, power supply, various communication lines, oil, steam lines, and sewage systems. An operator controls a probe of the drilling rig, i.e. the drilling depth and angle, having only the information about the specific drilling place and the depths of the existing communications. Due to the drilling trajectory's improper adjustment, the already installed underground or municipal infrastructures can be damaged by human error (e.g., perforated storm sewer damaged road surfaces) [10]. A life-threatening situation can arise if the main gas pipeline or power supply line become perforated, resulting in a heavy explosion in urban and densely populated areas. A huge economic loss can result from backbone optical communication cable breakage.

Although, due to high demand, several solutions have already been proposed for the automation of various stages of HDD implementation to eliminate, as much as possible, the influence of human factors on the work quality, these solutions are hardly available to small businesses due to their high cost.

As was mentioned earlier, a correction of the HDD trajectory at the work place is time-limited. Therefore, it is highly important to propose user-friendly and inexpensive simulation tools with the required functionality for HDD trajectory planning under field conditions. A question arises about which HDD trajectory planning method to choose. Authors claim that there is no substantial difference among HDD trajectory-planning methods regarding the time resources needed. They are mostly similar and appropriate for use. However, such a hypothesis should be examined.

This paper is organized as follows: Section 2 presents the research objectives by conducting a literature survey of HDD problem analysis. Sections 3–5 explain the pilot bore planning methods used in HDD. Section 6 presents a solution for determining the drill pipes' end points, i.e. their coordinates. Section 7 presents an evaluation of HDD trajectory-planning methods, using the paired sample t-test method. The results of HDD

pilot bore planning were obtained with a pilot bore simulation (Section 8) and testing under field conditions (Section 9). Finally, the results are discussed in Section 10.

3. Pilot bore planning methods

The main problem to be solved during the development stage of the simulating tool is to find a pilot bore construction method that ensures that the bending angle of drill pipes does not exceed the maximum value allowed.

Three pilot bore construction methods were chosen for further analysis:

- 1) Step-by-step method: the pilot bore trajectory is constructed by adding a drill pipe to the end of the previous pipe (Figure 2). The maximum $\Delta \alpha$ value is limited, which ensures that the maximum drill pipe bending radius is not exceeded.
- 2) Six-point control method: drill pipes are lined automatically according to the trajectory, which is constructed by taking into account the six control points (Figure 3). The user can change the position of each control point with the computer mouse, and the drill pipes are aligned automatically in real time. The trajectory is calculated using cubic spline interpolation between 1–3 and 4–6 control points, and line interpolation between 3 and 4 control points.



Figure 2. Step-by-step HDD pilot bore path construction.



3) Four-point control method: the pilot bore is designed automatically according to the trajectory, which is constructed using four control points (Figure 4). The trajectory is calculated using equations of circle segments between 1–2 and 3–4 control points, and line interpolation between 2 and 3 control points. To reveal the advantages of these point control methods, it is necessary to analyze them in more detail.

4. Six-point control method

To construct the pilot bore, it is necessary to calculate the coordinates of each drill pipe's starting and ending points.

The trajectory of the pilot bore is calculated according to the control point coordinates $((x_i, y_i)i = 1, 2, 3, 4, 5, 6)$:

$$y(x) = \left\{ \begin{array}{ll} F_{S1}(x) & x = x_1 \le x < x_3 \\ F_L(x) & x = x_3 \le x < x_4 \\ F_{S2}(x) & x = x_4 \le x < x_6 \end{array} \right\}.$$
 (1)



Figure 4. HDD pilot bore path construction using four control points.

Here $F_{S1}(x)$ is the equation of the cubic spline segment between 1 and 3, $F_{S2}(x)$ is the equation of the cubic spline segment between the 4–6 control points, and $F_L(x)$ is the equation of linear interpolation between 3 and 4 control points.

The segments of cubic spline interpolation $F_{S1}(x)$ and $F_{S2}(x)$ can be calculated with the common formula:

$$F_{S}(x) = \left\{ \begin{array}{ll} Y_{S1}(x) & x = x_{a} \le x < x_{b} \\ Y_{S2}(x) & x = x_{b} \le x < x_{c} \end{array} \right\},$$
(2)

where indexes a = 1, b = 2, c = 3 for $F_{S1}(x)$, and indexes a = 4, b = 5, c = 6 for $F_{S2}(x)$.

For three points cubic spline interpolation, the values for k_1 , k_2 , k_3 are found by solving the tridiagonal linear equation system:

$$\begin{vmatrix} \frac{2}{x_b - x_a} & \frac{1}{x_b - x_a} & 0 \\ \frac{1}{x_b - x_a} & 2 \cdot \left(\frac{1}{x_b - x_a} + \frac{1}{x_c - x_b}\right) & \frac{1}{x_c - x_b} \\ 0 & \frac{1}{x_c - x_b} & \frac{2}{x_c - x_b} \end{vmatrix} \begin{vmatrix} k_1 \\ k_2 \\ k_3 \end{vmatrix} = \begin{vmatrix} 3 \cdot \left(\frac{y_b - y_a}{(x_a - x_b)^2}\right) \\ 3 \cdot \left(\frac{y_b - y_a}{(x_a - x_b)^2} + \frac{y_c - y_b}{(x_c - x_b)^2}\right) \\ 3 \cdot \left(\frac{y_c - y_b}{(x_c - x_b)^2}\right) \end{vmatrix}.$$
 (3)

The a_1, b_1, a_2, b_2 values are calculated with

$$a_1 = k_1 \cdot (x_b - x_a) - (y_b - y_a); \tag{4}$$

$$b_1 = -k_2 \cdot (x_b - x_a) + (y_b - y_a); \tag{5}$$

$$a_2 = k_2 \cdot (x_c - x_b) - (y_c - y_b); \tag{6}$$

$$b_2 = -k_3 \cdot (x_c - x_b) + (y_c - y_b).$$
(7)

Then the $Y_{S1}(x)$ function in Eq. (2) is calculated with

$$Y_{S1}(x) = (1 - q_1(x)) \cdot y_a + q_1(x) \cdot y_b + q_1(x) \cdot (1 - q_1(x)) \cdot (a_1 \cdot (1 - q_1(x)) + b_1 \cdot q_1(x));$$
(8)

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$$q_1(x) = \frac{x - x_a}{x_b - x_a}.$$
(9)

The $Y_{S2}(x)$ function in Eq. (2) is calculated with

$$Y_{S2}(x) = (1 - q_2(x)) \cdot y_b + q_2(x) \cdot y_c + q_2(x) \cdot (1 - q_2(x)) \cdot (a_2 \cdot (1 - q_2(x)) + b_2 \cdot q_2(x));$$
(10)

$$q_2(x) = \frac{x - x_b}{x_c - x_b};$$
(11)

The linear interpolation between 3 and 4 control points is given by

$$F_L(x) = y_4 + (x - x_3) \cdot \frac{y_4 - y_3}{x_4 - x_3}.$$
(12)

Figure 5 explains how the cubic spline interpolation equation is calculated between A (1 or 4 control point), B (2 or 5 control point), and C (3 or 6 control point) points.

5. Four-point control method

As in the previous case (see Section 4), it is necessary to calculate the starting and ending point coordinates of each drill pipe. The trajectory of the pilot bore is calculated according to the control point coordinates $((x_i, y_i)i = 1, 2, 3, 4)$, as follows:

$$y(x) = \left\{ \begin{array}{ll} F_{C1}(x) & x_1 \le x < x_2 \\ F_L(x) & x_2 \le x < x_3 \\ F_{C2}(x) & x_3 \le x \le x_4 \end{array} \right\}.$$
 (13)

Here $F_{C1}(x)$ is the equation of the circle segment between the 1 and 2 control points, $F_{C1}(x)$ is the equation of the circle segment between 3 and 4 control points, and $F_L(x)$ is the equation of linear interpolation between



Figure 5. Cubic spline interpolation details.



Figure 6. Circle segment details.

2 and 3 control points. The circle segments for $F_{C1}(x)$ and $F_{C2}(x)$ can be calculated with the same formulas with indexes a = 1, b = 2 for $F_{C1}(x)$ and a = 3, b = 4 for $F_{C2}(x)$.

The C point is the center of the circle. H is the midpoint of the chord $c = \sqrt{(x_b - x_a)^2 + (y_b - y_a)^2}$, and $d = \sqrt{R^2 + c^2/4}$ – is the height of the triangular portion. The value of the circle radius R must be equal to or higher than the allowed drill pipe bend radius (e.g., JT922 drill pipes have the tested minimum bend radius of 32 m).

The C point coordinates are given by

$$\begin{aligned} x_c &= \frac{x_a + x_b}{2} + d \cdot \sin(\beta + \pi); \\ y_c &= \frac{y_a + y_b}{2} + d \cdot \cos(\beta); \end{aligned}$$
(14)

here $\beta = \arcsin((y_b - y_a)/c)$ and

$$F_C(x) = y_c - R \cdot \sin\left(\arccos\left(\frac{x - x_c}{R}\right)\right).$$
(15)

The linear interpolation between 2 and 3 control points is given by

$$F_L(x) = y_3 + (x - x_2) \cdot \frac{y_3 - y_2}{x_3 - x_2}.$$
(16)

Figure 6 helps to explain how the circle segment equation is calculated between A (1 and 3 control points) and B (2 and 4 control points) points.

6. Drill pipe's end point coordinates

If L is the length of the drill pipe (in meters) and x_s is the small step used to increase the x value ($x_s \ll L$), then the (xp_i, yp_i) coordinates of *i*-th drill pipe's end point can be calculated with the algorithm presented in Figure 7.

If n is the number of drill pipes in the planned pilot bore, then the angle (in degrees) of *i*-th pipe is

$$\alpha_{i} = \frac{180}{\pi} \cdot \arcsin\left(\frac{yp_{i+1} - yp_{i}}{L}\right), \quad i = 1, 2..n - 1.$$
(17)

The angle differences between adjacent drill pipes are

$$\Delta \alpha_i = \alpha_{i+1} - \alpha_i, \quad i = 1, 2...n - 1.$$
(18)

To ensure that the bending radius is not less than the minimal $\Delta \alpha_i$, the values must be less than $\Delta \alpha_{\max}$ (e.g., $\Delta \alpha_{\max} = 3.43^{\circ}$ for the JT922 drill pipes).

7. Evaluation of pilot bore trajectory planning methods

The paired sample t-test method was used to evaluate the difference among HDD trajectory planning methods. For this purpose, twenty (N = 20) HDD sites (different locations) were selected. Regarding the hypothesis stated above, we only need to evaluate the duration of the pilot HDD trajectory planning, using step-by-step (M1), six-point (M2), and four-point control (M3) methods. The elevation profiles with obstacles and with



Figure 7. Calculation of *i*-th drill pipe's end point coordinates.

entry/exit points for each HDD site were created and saved. Profile preparing time was not evaluated, because we are only interested in the time it takes to plan HDD pilot bore trajectory, using different planning methods. The results of the statistical evaluation are given in Table 1.

Table 1. Paired samples statistics of HDD trajectory planning duration when using step-by-step method (M1), six-point control method (M2), and four-point control method (M3).

Sample	Ν	Mean, s	St. err., s	St. dev., s	[95% conf. interval], s
M1	20	44.300	5.800	25.939	[32.932, 55.668]
M2	20	42.800	6.931	30.998	[29.215, 56.385]
M3	20	40.800	3.110	13.909	[34.704, 46.896]

The paired sample t-test has two competing hypotheses:

- The null hypothesis (H_0) assumes that the true mean difference (μ_d) between the mean HDD trajectory planning durations of two different methods is equal to zero $(\mu_d = 0)$.
- The two-tailed alternative hypothesis (H_1) assumes that μ_d is not equal to zero $(\mu_d \neq 0)$.

Paired diff.	Mean, s	St. err., s	St. dev., s	[95% conf. interval] of the difference, s	t-value	df	P value (two-tailed)
M1–M2	1.500	5.160	23.077	[-9.300, 12.300]	0.291	19	0.774
M1–M3	3.500	4.766	21.313	[-6.475, 11.475]	0.734	19	0.472
M2–M3	2.000	5.529	24.729	[-9.573, 13.573]	0.362	19	0.722

The results of the statistical evaluation of paired sample differences are given in Table 2.

Table 2. Paired samples t-test of: M1–M2, M1–M3, M2–M3.

Statistical significance between the methods is determined by looking at the P value. Because the P value in all cases is greater than 0.05, we can assume that the null hypothesis (H_0) is statistically significant, ensuring approximately 95% confidence in the results. Therefore, the paired samples t-test shows that the difference between the evaluated HDD planning methods is statistically insignificant.

8. Simulation tool for HDD pilot bore planning

It is possible to use all three methods described above for HDD pilot bore planning by means of the created simulation tool (Figure 8).

The HDD pilot bore simulation tool has a control panel (input parameters: bore diameter, drill pipe length, soil type), and five tabs are used:

- 1. Profile tab: to create and edit the pilot bore path;
- 2. Maps tab: to integrate a web browser with Google Maps API script, which is used to provide the Earth elevation details between the desired bore entry and exit points;
- 3. Document tab: to create the bore documentation;
- 4. Materials tab: to calculate the amount of drilling fluid in accordance with the selected soil type;
- 5. Details tab: to view the position, angle, and depth of each drill pipe.

To simulate a new HDD pilot bore, it is necessary to know the exact earth elevation profile between our starting and ending points. The ground height profile can be calculated and drawn using Google Maps API, postponing only two points on the map (Figure 8).

Having the latter data, it is possible to assess a depth position for every drill pipe regarding the earth surface. The developed software allows editing ground elevation profiles in the Profile tab. User-friendly interface allows the operator to add/remove drilling pipes using the step-by-step method on planning trajectory, which meets the bending angle requirements, using one mouse click only. The six-point and four-point control methods place drill pipes automatically. The trajectory can be properly adjusted by moving the control points with a computer mouse.

This tool also allows us to add various obstacles, which must be evaluated when planning and carrying out the drilling work. Such obstacles could be the underground power cables, water or gas lines, etc.

Figure 9 demonstrates the three examples of bore path fragment and reveals the ability of the simulation tool to calculate the bending angle and coordinates of every drill pipe. Each drill pipe coordinate is calculated



Figure 8. Screenshot of simulation tool.

according to the algorithm given in Figure 7. The bending angles are calculated with Eqs. (10) and (11). Additionally, the angles are expressed in percentages, and are given in the Details tab and on the bore picture in the Profile tab. The program also calculates the recommended amount of HDD fluids according to the selected soil type in the Materials tab. The exact fluid volume is estimated according to the length and diameter of the simulated HDD bore. The actual pilot bore path calculation does not depend on the selected soil type. The type of fluid and the amount of its constituents are calculated using HDD equipment manufacturer's reference formulas. Therefore, it is not discussed in detail here.

The HDD pilot bore picture and the data of each drill pipe position and bending angle are saved in PDF and Microsoft Excel files. These files can be used for documentation and as a reference during the practical drilling works.

9. Simulation tool testing in field conditions

As was stated earlier, underground communications often do not meet the documentation. Therefore, an HDD operator needs to correct the trajectory at the work place. Due to time limitations, such a process should be performed as operatively as possible. HDD bore trajectory construction, using a simulation tool under field conditions, can be illustrated with the practical scenario operating with Ditch Witch JT922. An HDD pilot bore trajectory was presented to the operator, including the location of the water pipes beneath the road as an



Figure 9. Fragment of simulated pilot bore: a) initial entry angle is too big (the bore line turns red), b) bending angles in bore sections do not meet the bending requirements (the color of the bore line turns to blue, and the problem sections are shown in red), c) entry and bending angles meet the requirements (the bore sections are shown in alternating red and green.)

obstacle (Figure 10a). It is not possible to follow the preplanned trajectory when such an unforeseen obstacle occurs. The simulation tool allows us to replan the drilling trajectory considering any new obstacle by moving a control point only. This case is illustrated in Figure 10b. The results analysis of the real case revealed that operating without such a simulation tool can lead to the situation described in Figure 10c, if changing the trajectory does not meet the requirements of pipe-bending ($\Delta \alpha < 3.43^{\circ}$ for the JT922 drill pipes). The position of the bending angle violation is presented in Figure 11, where the angle difference curve crosses the critical level of 3.43° .

10. Conclusion

In this study, we discussed the optimization of HDD trajectory-planning, which mainly consisted of an analysis of the methods for pilot bore trajectory construction and the creation of the simulation tool.

We proposed and evaluated three methods for pilot bore construction: adding a drill pipe according to the step-by-step method, arranging a drill pipe using six control points, and another method that only allows the use of four control points. Statistical analysis was used to estimate the differences among these methods regarding the time resources needed to plan the HDD trajectory.

The paired samples t-test method showed that there were no statistically significant differences between methods regarding HDD trajectory planning duration. This can be explained with the tool functionality, which allowed the HDD trajectory to change easily, using all the methods described above. The upper limit of a 95% confidence interval of HDD trajectory planning duration did not exceed 1 min.



Figure 10. Pilot HDD profile with: a) initially planned HDD trajectory, b) correct HDD trajectory to avoid unexpected obstacles, c) improper HDD trajectory to avoid unexpected obstacles—maximum allowed bending angle is exceeded.



Figure 11. Angle differences between drill pipe end points in HDD trajectories.

The six-point control method would be better suited than the four-point control method, if the HDD bore entry and exit curve radius were different. In the other case, the four-point control method was superior. Nevertheless, the method based on four control points proved to be the fastest and the most user friendly due to the least amount of control over the user interface.

The simulation tool allowed the automatic evaluation of the proper initial angle of the bore hole and the bending radius of every drill pipe by using only a few control points. The drill pipe entry and bending angles were recalculated in percentage (%) to match the readings of the walk-over receiver. Therefore, bilateral communication remained mandatory, even when the simulation tool was used.

By means of the developed simulation tool the detailed plan of the HDD bore hole under field conditions was performed more operatively. Furthermore, the operator could apply any corrections to the HDD trajectory plan if s/he encountered a previously unforeseen obstacle. Due to a faster evaluation of HDD trajectory adjustments, it was possible to economize on total time for the entire HDD project.

Finally, the developed simulation tool is applicable to a variety of HDD drilling equipment.

Nomenclature and units

x	value on x (horizontal) axis, m
y	value on y (vertical) axis, m
(x_i, y_i)	coordinate of i -th point
(xp_i, yp_i)	coordinate of i -th drill pipe end point
A	variable to find the end point of the drill pipe along trajectory $y(x)$, m
R	drill pipe bend radius, m
L	drill pipe length, m
$F_S(x)$	equation of cubic spline segment
$F_L(x)$	equation of linear interpolation
$F_C(x)$	equation of circle segment
c	chord between control points, m
d	height of triangular portion, m
β	angle between c and x axis, rad
α_i	angle of <i>i</i> -th pipe, $^{\circ}$
$\Delta \alpha_i$	angle difference $(\alpha_{i+1} - \alpha_i)$ between adjacent drill pipes, °
n	number of drill pipes
Ν	number of items in a sample
df	degrees of freedom $(df = N - 1)$
H_0	null hypothesis
H_1	alternative to null hypothesis

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