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Performance analysis of the link selection for secure device-to-device communications with an untrusted relay

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Abstract: In this paper, we study the performance of the selection combining and switch and stay combining methods in secure cooperative device-to-device (D2D) communications underlying a cellular network where an untrusted amplify and forward relay helps D2D communications. In our model, we specifically consider the interference from the cellular system over the D2D pair and the interference from the D2D pair over the cellular system. We assume that D2D communication is performed in such a way that the required outage probability of the cellular system is preserved. We obtain an exact closed-form expression for the lower bound of secrecy outage probability of the D2D pair, as well as an asymptotic expression for the outage probability for both the selection combining and switch and stay combining schemes. Using simulations, we study and compare the performance behavior of the selection combining and switch and stay combining schemes as the system parameters change.

Key words: Device-to-device communications, link selection, physical layer security, secrecy outage probability, untrusted amplify and forward relay

1. Introduction

Device-to-device (D2D) communication has been considered as a promising technique for 5G wireless networks [1] in which two devices use the cellular spectrum to directly communicate without the help of a cellular base station (BS). Sharing the same spectrum produces cross-interference between the cellular network (CN) and the D2D pair that makes the D2D communications design a challenging problem. While the broadcast nature of the wireless channel makes information transmission vulnerable against eavesdropping attacks, cooperative communications can be used to improve both security and communications reliability [2].

In [3], the authors considered various selection schemes for decode-and-forward (DF) relay assisted networks. In [4], the authors studied the performance of DF relay selection in the presence of an eavesdropper where they obtained the exact outage probability. This work was further extended to two-way DF relay-assisted communication in [5]. The case of amplify-and-forward (AF) relay selection was considered in [6]. The authors in [7] analyzed the outage probability of the AF relay selection in secondary outages. In their scheme, they considered the cross-interference of both systems. They obtained a closed form outage for their relay selection scheme in case the destination applies the maximal ratio combining (MRC).

The assisting relay may be considered untrusted, which may result in eavesdropping on the information transmission. However, cooperation with untrusted relays can still be beneficial [8–12]. In [8], the authors

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considered an opportunistic transmission scheme for cooperative networks assisted by an untrusted AF relay where the destination opportunistically chooses between the direct link and the relay link based on the achievable secrecy rate of each link. The exact value, as well as a lower bound on the secrecy outage probability (SOP), is obtained. In [9], the authors considered a cooperative network where the relay is untrusted and is equipped with multiple antennas, and the destination performs MRC on the signals received from the direct and the relay links. In addition, they considered the effect of cooperative jamming in their scheme. The authors in [10] considered an artificial noise-forwarding scheme with relay selection for untrusted AF relay networks. This work was further extended in [11] to a case where the relays overhear the signal transmission in the second hop. The authors in [12] studied the secrecy outage performance of cooperative communication where a source wants to communicate with a destination with the help of an untrusted AF relay and obtain the SOP where the destination performs an MRC scheme.

To the authors' knowledge, there is no work that studies the performance of link selection for D2D communications in which physical layer security with an untrusted relay, the effect of cross-interference, and the effect of the direct link are considered in a unified framework. Due to the mathematical relation of these issues with each other, such a unified consideration makes the analysis difficult. In this paper, we propose a link selection for D2D communications that is assisted by an untrusted AF relay. In our scheme, we specifically consider cross-interference. However, cross-interference makes the outage analysis a challenging task. More precisely, the transmit power of the D2D pair is constrained in such a way that the outage probability of the CN remains acceptable. We consider two cases. In the first one, the receiver of the D2D pair (DR) applies a selection combining (SC) scheme [13] to choose between the direct link and the relay link. In the second one, the DR applies the switch and stay combining (SSC) [13] scheme (in which the received signal-to-interference-plus-noise ratio (SINR) of the links is compared to a threshold), selects the link whose SINR is higher than the threshold, and changes the link when the SINR of the selected link becomes lower than the threshold. We obtain the exact expression for the lower bound on the SOP in both cases and study their behavior using simulations. In summary, the contribution of the paper can be stated as follows:

1. The main contribution of the paper is the considered model. In our model, in contrast to previous works, we study the performance of a relay assisted network where the assisting relay is untrusted and hence security should be considered.

2. We consider the effect of security, relaying, cross-interference, and the direct link in a unified framework. Considering all of these issues in a unified problem will make the analysis complicated, involving the computation of the probability density function (PDF) of functions of several related random variables.

3. We obtain the performance of the proposed model based on the outage probability in a closed form for two diversity schemes: SC and SSC.

The organization of the paper is as follows: the system model is presented in Section 2. In Section 3, we provide the outage performance analysis for the proposed system for both the SC and SSC schemes. Asymptotic analysis is provided in Section 4. Simulation results are described in Section 5, and the paper is concluded in Section 6.

2. System model

We consider a communications scenario in which a D2D pair wants to communicate with the help of an untrusted AF relay using the same spectrum shared by the coexisting CN (see Figure 1). This means that the D2D pair and the CN produce interference for each other (cross-interference) that must be considered. We assume the presence of a time-slotted transmission. For the CN, a new message is transmitted from the BS to the cellular receiver (CR) over each slot. However, the transmission of the D2D pair requires two time slots. In the first time slot, the transmitter of the D2D pair (DT) transmits its signal, while the relay and the DR receive. The relay amplifies the received signal and forwards it to the DR in the second time slot. The DR selects either the direct link or the relay link, according to the SC and SSC schemes.

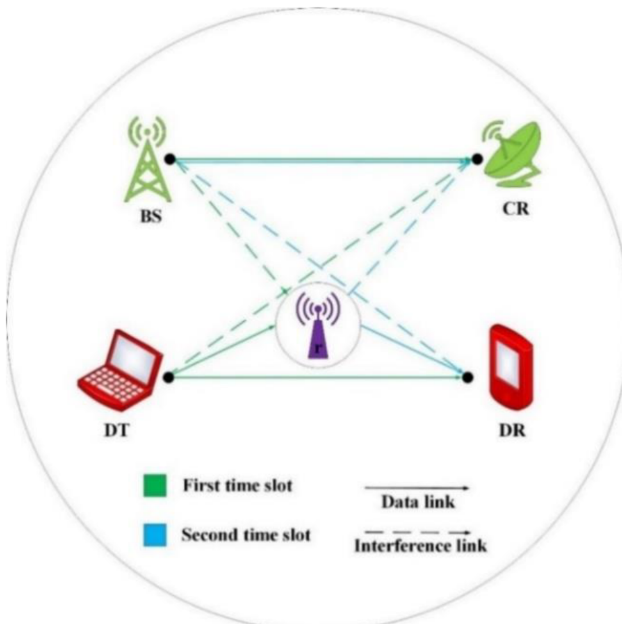

Figure 1. System model.

Let $h_{a\to b}$ denote the channel coefficient of the channel between node a and node b, where $a \in \{BSDTr$ and $b \in \{CR, DR, r\}$, which undergoes independent Rayleigh fading, meaning that the channel power gains are exponentially distributed, i.e. $|h_{a\to b}|^2 \sim \sigma^2_{a\to b} e^{-\sigma^2_{a\to b} t}$ where $\sigma^2_{a\to b} = \frac{1}{E\{|h_{a\to b}|^2\}}$ and $E\{z\}$ is the expectation value of random variable z. Let P_{BS}, P_{DT}, and P_r respectively denote the transmit power of BS, DT, and relay r, which are assumed to be fixed in this paper. The channel capacities from the BS to CR in the first and the second time slots, respectively, are given by $C^1_{BS\to CR}{=}log_2\left(1+\frac{P_{BS}|h_{BS\to CR}|^2}{N_0+P_{DT}|h_{DT\to CR}|^2}\right)$ and $C^2_{BS\to CR}{=}log_2\left(1+\frac{P_{BS}|h_{BS\to CR}|^2}{N_0+P_r|h_{r\to CR}|^2}\right)$. The transmit power of the DT and the relay should be limited to preserve the quality of service (QoS) of the CN transmission. Here we limit the transmission power at the DT and the relay such that the following constraint in time slot i is satisfied:

$$P^{C,i}_{out} = \Pr(C^i_{BS\to CR} \le R_C) \le \bar{P}^C_{out}, \tag{1}$$

where $P^{C,i}_{out}$ is the outage probability of the CN at time slot i, R_C is the transmission rate of CN, and $\bar{P}^C_{out}$ is the required outage probability limit.

Proposition 1 *The transmit powers* P_{DT} *and* P_r *should be respectively constrained as:*

$$P_{DT} \leq \frac{P_{BS}\sigma^2_{DT\to CR}}{\sigma^2_{BS\to CR}\vartheta}[\frac{1}{1-\bar{P}^C_{out}}e^{(-\frac{\sigma^2_{BS\to CR}N_0}{P_{BS}}\vartheta)}-1], \tag{2a}$$

$$P_r \leq \frac{P_{BS}\sigma^2_{r\to CR}}{\sigma^2_{BS\to CR}\vartheta}[\frac{1}{1-\bar{P}^C_{out}}e^{(-\frac{\sigma^2_{BS\to CR}N_0}{P_{BS}}\vartheta)}-1], \tag{2b}$$

where $\vartheta = 2^{R_C}-1$*.*

Proof Please see Appendix A.

As is obvious from Eq. (2), if $\bar{P}^C_{out} < 1 - e^{\left(-\frac{\sigma^2_{BS\to CR}N_0}{P_{BS}}\vartheta\right)}$ happens due to profound fading of the CN channels, the transmit powers of the DT and r should be set to zero and the CN channel is unavailable for the D2D transmitters. Therefore, we assume that the D2D transmitters transmit their data with their maximum allowable power, i.e. Eq. (2) can be given as follows:

$$P_{DT} = \frac{P_{BS}\sigma^2_{DT\to CR}}{\sigma^2_{BS\to CR}\vartheta}\Lambda^+, \tag{3a}$$

$$P_r = \frac{P_{BS}\sigma^2_{r\to CR}}{\sigma^2_{BS\to CR}\vartheta}\Lambda^+, \tag{3b}$$

where $\Lambda^+ = \max(\Lambda, 0)$ and $\Lambda = \frac{1}{1-\bar{P}^C_{out}}e^{\left(-\frac{\sigma^2_{BS\to CR}N_0}{P_{BS}}\vartheta\right)}-1$.

Note that, as can be seen from Eq. (3), there are some values for the system parameters such as $\bar{P}^C_{out}$ that result in zero transmission power for the DT and the relay. This will be observed later in simulations.

In the D2D system, the received signals by the relay r and the DR in the first time slot are respectively given by:

$$y_r = \sqrt{P_{DT}}h_{DT\to r}x_D + \sqrt{P_{BS}}h_{BS\to r}x_C + n_r, \tag{4a}$$

$$y_{DR} = \sqrt{P_{DT}}h_{DT\to DR}x_D + \sqrt{P_{BS}}h_{BS\to DR}x_C + n_{DR}, \tag{4b}$$

where n_r and n_{DR}, respectively, are the additive white Gaussian noise at the relay and DR with zero mean and unit variance, and x_C and x_D are respectively the unit power symbols transmitted by the DT and the BS in the first time slot. The relay scales the received signal by the factor $G = \sqrt{\frac{1}{P_{DT}|h_{DT\to r}|^2+P_{BS}|h_{BS\to r}|^2+1}}$, constructs the symbol $x_r = Gy_r$, and sends x_r to the DR. The received signal by the DR in the second time slot is given by:

$$y'_{DR} = \sqrt{P_r}h_{r\to DR}x_r + \sqrt{P_{BS}}h_{BS\to DR}x'_C + n'_{DR}, \tag{5}$$

where ${x_C}'$ is the transmitted symbol of the BS in the second time slot.

The SINRs at the relay and the DR in the first time slot are respectively given by $\gamma_{DT\to r} = \frac{P_{DT}|h_{DT\to r}|^2}{N_0+P_{BS}|h_{BS\to r}|^2}$ and $\gamma_{DT\to DR} = \frac{P_{DT}|h_{DT\to DR}|^2}{N_0+P_{BS}|h_{BS\to DR}|^2}$. The SINR of the relay link at the DR is given by $\gamma_{DTrDR} = \frac{\gamma_{DT\to r}\gamma_{r\to DR}}{1+\gamma_{DT\to r}+\gamma_{r\to DR}}$, where $\gamma_{r\to DR} = \frac{P_r|h_{r\to DR}|^2}{N_0+P_{BS}|h_{BS\to DR}|^2}$. The cross-interference makes analytical study using the exact SINR for the relaying link, i.e. γ_{DTrDR}, a challenging task, meaning that studying the performance of the proposed scheme is not easy and much more complicated computations are involved. This is mainly due to considering the cross-interference from the cellular system to the D2D pair that changes the SINR expression as the interference appears in the denominator. In such an expression, more than one random variable is involved, and the PDF of such random variables, i.e. the SINRs, will have a complicated form. Therefore, we use the approximation $\gamma_{DTrDR} \approx \hat{\gamma}_{DTrDR} = \min(\gamma_{DT\to r}\,\gamma_{r\to DR})$ [14]. $\square$

It is easy to show that the cumulative distribution function (CDF) of random variable $\gamma_{DT\to DR}$ is given as follows:

$$F_{\gamma_{DT\to DR}}(\gamma) = 1 - \frac{\Psi_1}{\Psi_1+\gamma} e^{-\Psi_{DT\to DR}\gamma}, \tag{6}$$

where $\Psi_1 = \frac{\Psi_{BS\to DR}}{\Psi_{DT\to DR}}$, $\Psi_{a\to b}{=}\frac{N_0\sigma^2_{a\to b}}{P_a}$, $a \in \{BSDTr$, and $b \in \{CR, DR, r$ (please refer to Appendix B). The CDFs of $F_{\gamma_{DT\to r}}(\gamma)$ and $F_{\gamma_{r\to DR}}(\gamma)$ are similar to Eq. (6) with the corresponding parameters.

The secrecy capacity of the proposed scheme with an untrusted relay is given by:

$$C^X_{sec} = \frac{1}{2}\log_2\left(\frac{1+\gamma^X_{e2e}}{1+\gamma_{DT\to r}}\right), \tag{7}$$

where $X = \{SC, SSC\}$. The SOP is defined as the probability that the secrecy capacity is below a positive threshold as [15]:

$$P^X_{out} = \Pr(C^X_{sec} < R_S), \tag{8}$$

where R_S is the secrecy rate of the D2D transmission.

3. Outage performance analysis

3.1. SC scheme

In the SC scheme, the path with the highest instantaneous SINR is chosen between the direct and the relay link so that the end-to-end SINR can be written as [13]:

$$\gamma^{SC}_{e2e} = \max(\gamma_{DT\to DR}, \hat{\gamma}_{DTrDR}). \tag{9}$$

When the DR adopts the SC scheme, the CDF of the corresponding SINR, i.e. γ^{SC}_{e2e}, is given by:

$$F_{\gamma^{SC}_{e2e}}(\gamma) = \Pr(\gamma^{SC}_{e2e} < \gamma) = F_{\gamma_{DT\to DR}}(\gamma)F_{\hat{\gamma}_{DTrDR}}(\gamma) = F_{\gamma_{DT\to DR}}(\gamma)(1-(1-F_{\gamma_{DT\to r}}(\gamma))(1-F_{\gamma_{r\to DR}}(\gamma))), \tag{10}$$

where the second equality is due to the independency of the involved random variables and the last equality follows from the definition of the CDF for the minimum of two random variables [16].

By substituting Eq. (7) into Eq. (8) and defining $\alpha = 2^{2R_S}$, we have the following:

$$\begin{aligned} P^{SC}_{out} &= \Pr\left(\tfrac{1}{2}\log_2\left(\tfrac{1+\max(\gamma_{DT\to DR}, min(\gamma_{DT\to r}, \gamma_{r\to DR}))}{1+\gamma_{DT\to r}}\right) < R_S\right) \\ &= \Pr\left(\tfrac{1+\max(\gamma_{DT\to DR}, min(\gamma_{DT\to r}, \gamma_{r\to DR}))}{1+\gamma_{DT\to r}} < \alpha\right) = \int_0^\infty F_{\gamma^{SC}_{e2e}(y)}(\alpha+\alpha y-1) f_{\gamma_{DT\to r}}(y)dy, \end{aligned} \tag{11}$$

where $\gamma_{e2e}^{SC}(y) = \max(\gamma_{DT\to DR}, \min(y, \gamma_{r\to DR}))$ and $f_{\gamma_{DT\to r}}(.)$ is the PDF of $\gamma_{DT\to r}$ given by:

$$f_{\gamma_{DT\to r}}(y) = \frac{d}{dy}F_{\gamma_{DT\to r}}(y) = \frac{\Psi_2 e^{-\Psi_{DT\to r}y}}{(y+\Psi_2)^2} + \frac{\Psi_{DT\to r}\Psi_2 e^{-\Psi_{DT\to r}y}}{y+\Psi_2}, \tag{12}$$

where $\Psi_2 = \frac{\Psi_{BS\to r}}{\Psi_{DT\to r}}$.

Proposition 2 *The exact closed-form expression for the lower bound of the SOP of the D2D pair communications for the SC scheme can be obtained by:*

$$P_{out}^{SC} = 1 - I_1 - I_2 - I_3 - I_4 + I_5 + I_6, \tag{13}$$

where:

$$I_1 = \frac{\beta_1}{(\Psi_2-\Psi_1')^2}e^{\Psi_1'\beta_2}E_1(\Psi_1'\beta_2) + \frac{-\beta_1}{(\Psi_1'-\Psi_2)^2}e^{\Psi_2\beta_2}E_1(\Psi_2\beta_2) + \frac{\beta_1}{\Psi_1'-\Psi_2}[\frac{1}{\Psi_2} - \beta_2 e^{\Psi_2\beta_2}E_1(\Psi_2\beta_2)], \tag{14}$$

$$I_2 = \frac{\beta_3}{\Psi_2-\Psi_1'}e^{\Psi_1'\beta_2}E_1(\Psi_1'\beta_2) + \frac{\beta_3}{\Psi_1'-\Psi_2}e^{\Psi_2\beta_2}E_1(\Psi_2\beta_2), \tag{15}$$

$$I_3 = \frac{\beta_4}{(\Psi_2-\Psi_3')^2}e^{\Psi_{3r}'\beta_5}E_1(\Psi_{3r}'\beta_5) + \frac{-\beta_4}{(\Psi_3'-\Psi_2)^2}e^{\Psi_2\beta_5}E_1(\Psi_2\beta_5) + \frac{\beta_4}{\Psi_3'-\Psi_2}[\frac{1}{\Psi_2} - \beta_5 e^{\Psi_2\beta_5}E_1(\Psi_2\beta_5)], \tag{16}$$

$$I_4 = \frac{\beta_6}{\Psi_2-\Psi_3'}e^{\Psi_3'\beta_5}E_1(\Psi_3'\beta_5) + \frac{\beta_6}{\Psi_3'-\Psi_2}e^{\Psi_2\beta_5}E_1(\Psi_2\beta_5), \tag{17}$$

$$\begin{aligned} I_5 \;=\; & \tfrac{\beta_7}{(\Psi_3'-\Psi_1')(\Psi_2-\Psi_1')^2}e^{\Psi_1'\beta_8}E_1(\Psi_1'\beta_8) + \tfrac{\beta_7}{(\Psi_1'-\Psi_3')(\Psi_2-\Psi_3')^2}e^{\Psi_3'\beta_8}E_1(\Psi_3'\beta_8) \\ & + \tfrac{-\beta_7(\Psi_1'+\Psi_3'-2\Psi_2)}{(\Psi_1'-\Psi_2)^2(\Psi_3'-\Psi_2)^2}e^{\Psi_2\beta_8}E_1(\Psi_2\beta_8) + \tfrac{\beta_7}{(\Psi_1'-\Psi_2)(\Psi_3'-\Psi_2)}[\tfrac{1}{\Psi_2} - \beta_8 e^{\Psi_2\beta_8}E_1(\Psi_2\beta_8)], \end{aligned} \tag{18}$$

$$\begin{aligned} I_6 \;=\; & \tfrac{\beta_9}{(\Psi_3'-\Psi_1')(\Psi_2-\Psi_1')}e^{\Psi_1'\beta_8}E_1(\Psi_1'\beta_8) + \tfrac{\beta_9}{(\Psi_1'-\Psi_3')(\Psi_2-\Psi_3')}e^{\Psi_3'\beta_8}E_1(\Psi_3'\beta_8) \\ & + \tfrac{\beta_9}{(\Psi_1'-\Psi_2)(\Psi_3'-\Psi_2)}e^{\Psi_2\beta_8}E_1(\Psi_2\beta_8), \end{aligned} \tag{19}$$

where $\Psi_3 = \frac{\Psi_{BS\to DR}}{\Psi_{r\to DR}}$, $\Psi_1'=\frac{\Psi_1+(\alpha-1)}{\alpha}$, $\Psi_2'=\frac{\Psi_2+(\alpha-1)}{\alpha}$, $\Psi_3'=\frac{\Psi_3+(\alpha-1)}{\alpha}$, $\beta_1 = \frac{\Psi_1\Psi_2}{\alpha}e^{-\Psi_{DT\to DR}(\alpha-1)}$, $\beta_2 = \Psi_{DT\to DR}\alpha + \Psi_{DT\to r}$, $\beta_3 = \frac{\Psi_1\Psi_{BS\to r}}{\alpha}e^{-\Psi_{DT\to DR}(\alpha-1)}$, $\beta_4 = \frac{\Psi_2\Psi_3}{\alpha}e^{-\Psi_{r\to DR}(\alpha-1)}$, $\beta_5 = \Psi_{r\to DR}\alpha + \Psi_{DT\to r}$, $\beta_6 = \frac{\Psi_3\Psi_{BS\to r}}{\alpha}e^{-\Psi_{r\to DR}(\alpha-1)}$, $\beta_7 = \frac{\Psi_1\Psi_2\Psi_3}{\alpha^2}e^{-(\Psi_{DT\to DR}+\Psi_{r\to DR})(\alpha-1)}$, $\beta_8 = (\Psi_{DT\to DR}+\Psi_{r\to DR})\alpha + \Psi_{DT\to r}$, $\beta_9 = \frac{\Psi_1\Psi_3\Psi_{BS\to r}}{\alpha^2}e^{-(\Psi_{DT\to DR}+\Psi_{r\to DR})(\alpha-1)}$, *and* $E_1(.)$ *is the exponential integral function [17].*

Proof Please see Appendix C. □

3.2. SSC scheme

By substituting Eq. (7) into Eq. (8), we get the following:

$$P_{out}^{SSC} = \Pr(\frac{1}{2}\log_2(\frac{1+\gamma_{e2e}^{SSC}}{1+\gamma_{DT\to r}}) < R_S) = \int_0^\infty F_{\gamma_{e2e}^{SSC}(y)}(\alpha+\alpha y-1) f_{\gamma_{DT\to r}}(y)dy. \tag{20}$$

When the DR adopts the SSC scheme, the CDF of the corresponding SINR, i.e. γ_{e2e}^{SSC}, is given by [13]:

$$F_{\gamma^{SSC}_{e2e(y)}}(\gamma) = \begin{cases} F_{\gamma_{DT\to DR}}(\gamma_T) F_{\hat{\gamma}_{DTrDR}(y)}(\gamma), & \text{if } \gamma < \gamma_T, \\ Pr(\gamma_T \le \gamma_{DT\to DR} \le \gamma) & \\ +F_{\gamma_{DT\to DR}}(\gamma_T) F_{\hat{\gamma}_{DTrDR}(y)}(\gamma), & \text{if } \gamma \ge \gamma_T, \end{cases} \tag{21}$$

where $\hat{\gamma}_{DTrDR}(y) = \min(y\gamma_{r\to DR})$.

Proposition 3 *The exact closed-form expression for the lower bound of the SOP of the D2D pair communications for the SSC scheme will be given by:*

$$P_{out}^{SSC} = (1-\frac{\Psi_1}{\Psi_1+\gamma_T}e^{-\Psi_{DT\to DR}\gamma_T})(1-\Theta_1-\Theta_2) + (\frac{\Psi_1}{\Psi_1+\gamma_T}e^{-\Psi_{DT\to DR}\gamma_T})\Theta_3 - \Theta_4 - \Theta_5, \tag{22}$$

where, defining $\tau = \frac{\gamma_T-\alpha+1}{\alpha}$*, we have the following expressions:*

$$\Theta_1 = \frac{\beta_4}{(\Psi_2-\Psi_3')^2}e^{\Psi_3'\beta_5}E_1(\Psi_3'\beta_5) + \frac{-\beta_4}{(\Psi_3'-\Psi_2)^2}e^{\Psi_2\beta_5}E_1(\Psi_2\beta_5) + \frac{\beta_4}{\Psi_3'-\Psi_2}[\frac{1}{\Psi_2} - \beta_5 e^{\Psi_2\beta_5}E_1(\Psi_2\beta_5)], \tag{23}$$

$$\Theta_2 = \frac{\beta_6}{\Psi_2-\Psi_3'}e^{\Psi_3'\beta_5}E_1(\Psi_3'\beta_5) + \frac{\beta_6}{\Psi_3'-\Psi_2}e^{\Psi_2\beta_5}E_1(\Psi_2\beta_5), \tag{24}$$

$$\Theta_3 = \frac{\Psi_2}{\tau+\Psi_2}e^{-\tau\Psi_{DT\to DR}}, \tag{25}$$

$$\begin{aligned}\Theta_4 \quad &= \quad \tfrac{\beta_1}{(\Psi_2-\Psi_1')^2}e^{\Psi_1'\beta_2}E_1((\Psi_1'+\tau)\beta_2) - \tfrac{\beta_1}{(\Psi_1'-\Psi_2)^2}e^{\Psi_2\beta_2}E_1((\Psi_2+\tau)\beta_2) \\ &\quad + \tfrac{\beta_1}{\Psi_1'-\Psi_2}[\tfrac{e^{-\tau\beta_2}}{\tau+\Psi_2} - \beta_2 e^{\Psi_2\beta_2}E_1((\Psi_2+\tau)\beta_2)],\end{aligned} \tag{26}$$

$$\Theta_5 = \frac{\beta_3}{\Psi_2-\Psi_1'}e^{\Psi_1'\beta_2}E_1((\Psi_1'+\tau)\beta_2) + \frac{\beta_3}{\Psi_1'-\Psi_2}e^{\Psi_2\beta_2}E_1((\Psi_2+\tau)\beta_2). \tag{27}$$

Proof Please see Appendix D. □

4. Asymptotic analysis

We can now provide analysis of the asymptotic expression of the SOP of the SC and SSC schemes here. We first define the parameter $\delta_{BS} = \frac{P_{BS}}{N_0}$, which is noise power normalized transmit power of the cellular BS (see [18]). This parameter tends to infinity, i.e. $\delta_{BS} = \frac{P_{BS}}{N_0} \to \infty$. Using this and the power series expansion of the exponential function, i.e. $e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$ [17], along with using the fact that for $x \ll 1$ we can approximate the function as $e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} \approx 1 + x$ and following the same step outlined in Appendix A, we can show that the expressions in Eq. (3) change to $P_{DT,\infty} \approx \frac{P_{BS}\sigma^2_{DT\to CR}}{\sigma^2_{BS\to CR}\vartheta}\left[\frac{\bar{P}^C_{out}}{1-\bar{P}^C_{out}}\right]$ and $P_{r,\infty} \approx \frac{P_{BS}\sigma^2_{r\to CR}}{\sigma^2_{BS\to CR}\vartheta}\left[\frac{\bar{P}^C_{out}}{1-\bar{P}^C_{out}}\right]$.

At high transmit power, we can approximate the SINRs, e.g., $\gamma_{DT\to DR} = \frac{X}{1+Y} \simeq \frac{X}{Y}$. Hence, following the same steps as in Appendix B, the corresponding CDFs change, e.g., Eq. (4a) changes to $F_{\gamma_{DT\to DR}}(\gamma) = 1 - \frac{\Psi_1}{\Psi_1+\gamma}$. This is similar to the exact analysis part in Appendix C and with the use of the following [17]:

$$\int_0^{\infty} \frac{\alpha_1}{(y+\alpha_2)(y+\alpha_3)^2} dy = \frac{\alpha_1}{(\alpha_2-\alpha_3)^2}[\frac{\alpha_2-\alpha_3}{\alpha_3} - ln(\frac{\alpha_2}{\alpha_3})]. \tag{28}$$

The asymptotic expression for the SOP of the SC scheme, i.e. Eq. (13), can be written as:

$$\begin{aligned} P^{SC}_{out,\infty} \quad = \quad & 1 - \frac{\frac{\Psi_1\Psi_2}{\alpha}}{(\Psi'_1-\Psi_2)^2}[\frac{\Psi'_1-\Psi_2}{\Psi_2} - ln(\frac{\Psi'_1}{\Psi_2})] - \frac{\frac{\Psi_2\Psi_3}{\alpha}}{(\Psi'_3-\Psi_2)^2}[\frac{\Psi'_3-\Psi_2}{\Psi_2} - ln(\frac{\Psi'_3}{\Psi_2})] \\ & + \frac{\frac{\Psi_1\Psi_2\Psi_3}{\alpha^2(\Psi'_3-\Psi'_1)}}{(\Psi'_1-\Psi_2)^2}[\frac{\Psi'_1-\Psi_2}{\Psi_2} - ln(\frac{\Psi'_1}{\Psi_2})] + \frac{-\frac{\Psi_1\Psi_2\Psi_3}{\alpha^2(\Psi'_3-\Psi'_1)}}{(\Psi'_3-\Psi_2)^2}[\frac{\Psi'_3-\Psi_2}{\Psi_2} - ln(\frac{\Psi'_3}{\Psi_2})]. \end{aligned} \tag{29}$$

Note that (as will be seen in the simulations), in Eq. (13), as $\delta_{BS} \to \infty$, the SOP does not tend to zero. Indeed, the SOP tends to a nonzero constant called the SOP floor, and this SOP floor is Eq. (29).

Following similar steps as for computing the asymptotic expression for the SOP of the SC scheme and the same steps outlined in Appendix D, we can obtain the asymptotic expression for the SOP of the SSC scheme, i.e. Eq. (22), as given in the following:

$$\begin{aligned} P^{SSC}_{out,\infty} \quad = \quad & (1 - \frac{\Psi_1}{\Psi_1+\gamma_T}) + (\frac{\Psi_1}{\Psi_1+\gamma_T})(\frac{\Psi_2}{\Psi_2+\tau}) + \frac{\frac{\Psi_1\Psi_2}{\alpha}}{(\Psi'_1-\Psi_2)}[\frac{1}{\Psi_2} - \frac{1}{\Psi_2+\tau}] \\ & - \frac{(1-\frac{\Psi_1}{\Psi_1+\gamma_T})(\frac{\Psi_2\Psi_3}{\alpha})}{(\Psi'_3-\Psi_2)^2}[\frac{\Psi'_3-\Psi_2}{\Psi_2} - ln(\frac{\Psi'_3}{\Psi_2})] - \frac{\frac{\Psi_1\Psi_2}{\alpha}}{(\Psi'_1-\Psi_2)^2}[\frac{\Psi'_1-\Psi_2}{\Psi_2} - ln(\frac{\Psi'_1}{\Psi_2})] \\ & + \frac{\frac{\Psi_1\Psi_2}{\alpha}}{(\Psi_2-\Psi'_1)^2}[ln(\Psi'_1+\tau) - ln(\Psi'_1)] - \frac{\frac{\Psi_1\Psi_2}{\alpha}}{(\Psi'_1-\Psi_2)^2}[ln(\Psi_2+\tau) - ln(\Psi_2)]. \end{aligned} \tag{30}$$

5. Simulation results

In this section, we provide numerical results for the performance of the proposed link selection scheme for independent Rayleigh distributed fading channels. To study the behavior of the SOP of the D2D pair, we change the noise power normalized transmit power of the cellular BS, i.e. $\delta_{BS} = \frac{P_{BS}}{N_0}$, to obtain the corresponding lower bound of the SOP for the SC and SSC schemes and plot the results in Figure 2. The SOP of both schemes decreases when the transmit SNR δ_{BS} increases, and these SOPs tend to a nonzero constant, i.e. the outage floor. As seen in the figure, this outage floor is indeed the one obtained from the asymptotic analysis. In addition, it can be seen that the SOP of the SC scheme is lower than that of the SSC scheme. This is because in the SC scheme, at all times, we use the link with the highest SINR, which means the SINR of the SC scheme is always higher than or equal to that of the SSC scheme. This results in a lower SOP. Increasing the value of R_S will increase the SOP, as can be seen in Figure 3.

Next, we perform the previous simulation for different values of the outage requirement of the CN, i.e. $\bar{P}^C_{out}$, to study the effect of the outage requirement of the CN on the performance of the proposed link selection

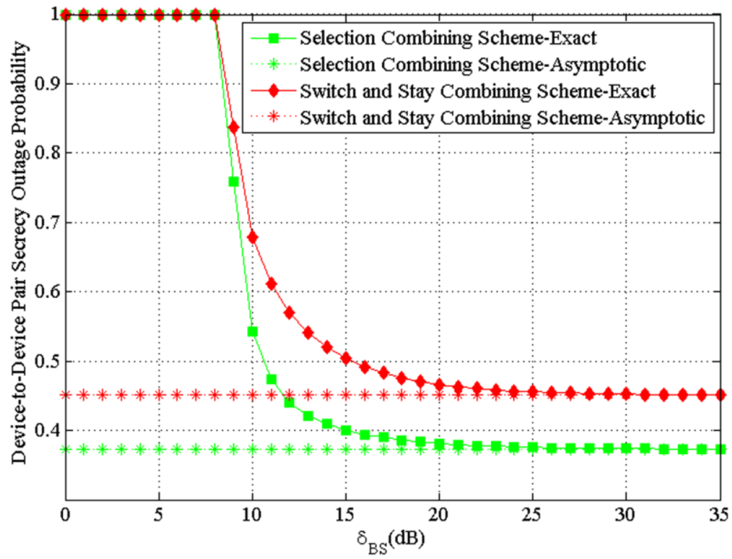

Figure 2. SOP of the D2D pair for the SC and SSC cases versus transmit SNR δ_{BS}. System parameters are: $\sigma^2_{BS\to CR} = \sigma^2_{DT\to DR} = 1$, $\sigma^2_{BS\to DR} = \sigma^2_{DT\to CR} = \sigma^2_{r\to CR} = 2$, $\sigma^2_{r\to DR} = \sigma^2_{DT\to r} = 0.7$, $\sigma^2_{BS\to r} = 1.5$, $\bar{P}^C_{out} = 0.1$, $\gamma_T = 2.5$, $R_C = 0.8\, bit/s/Hz$, $R_S = 0.1\, bit/s/Hz$.

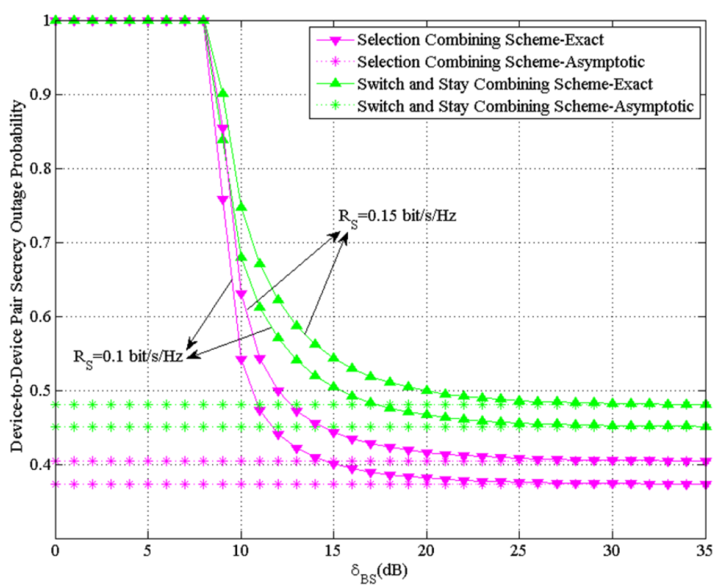

Figure 3. SOP of the D2D pair for the SC and SSC cases versus transmit SNR δ_{BS} for $R_S = 0.1, 0.15\, bit/s/Hz$. System parameters are: $\sigma^2_{BS\to CR} = \sigma^2_{DT\to DR} = 1$, $\sigma^2_{BS\to DR} = \sigma^2_{DT\to CR} = \sigma^2_{r\to CR} = 2$, $\sigma^2_{r\to DR} = \sigma^2_{DT\to r} = 0.7$, $\sigma^2_{BS\to r} = 1.5$, $\bar{P}^C_{out} = 0.1$, $\gamma_T = 2.5$, $R_C = 0.8\, bit/s/Hz$.

scheme. We change the outage probability requirement of the CN, obtain the SOP of the D2D pair, and plot the results for different values of R_S in Figure 4. It can be seen that increasing the outage requirement of the CN decreases the SOP of our scheme. This is mostly because the lower outage requirement for the CN forces the D2D pair to transmit with lower transmit power, which leads to high SOP (see Eq. (3)).

To explicitly show the effect of R_S on outage behavior, we change the value of R_S, compute the outage probabilities of the SC and SSC schemes, and plot the results in Figure 5. It can be seen that the SOP of both

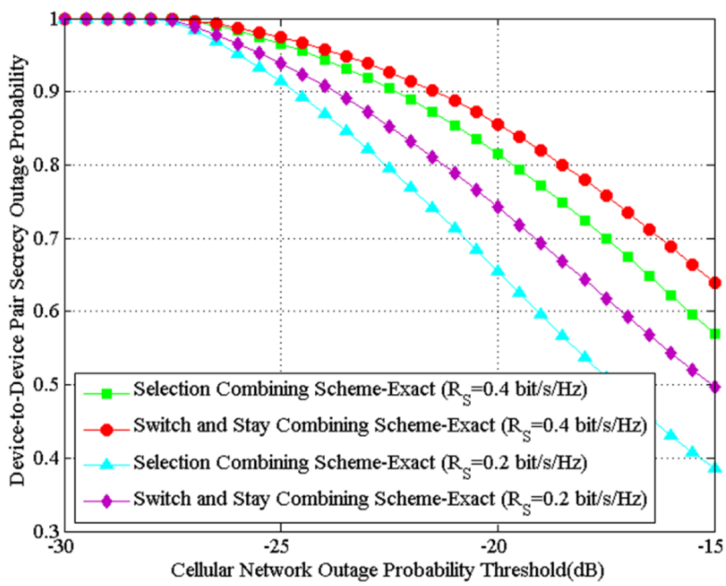

Figure 4. SOP of the D2D pair for the SC and SSC cases versus the outage requirement of the CN. System parameters are: $\sigma^2_{BS\to CR} = \sigma^2_{DT\to DR} = 1$, $\sigma^2_{BS\to DR} = \sigma^2_{DT\to CR} = \sigma^2_{r\to CR} = 2$, $\sigma^2_{r\to DR} = \sigma^2_{DT\to r} = 0.5$, $\sigma^2_{BS\to r} = 0.4$, $\gamma_T = 2.5$, $\gamma_{BS} = 25\,dB$, $R_C = 0.6\,bit/s/Hz$.

schemes tends to one as the secrecy rate requirement increases, and the SOP of the SC scheme is lower than that of the SSC scheme.

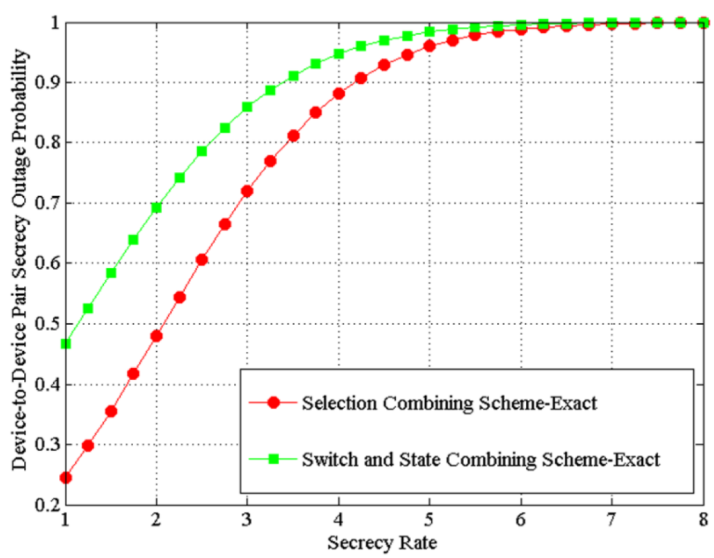

Figure 5. SOP of the D2D pair for the SC and SSC cases versus the secrecy rate. System parameters are: $\sigma^2_{BS\to CR} = \sigma^2_{DT\to DR} = 1$, $\sigma^2_{BS\to DR} = \sigma^2_{DT\to CR} = \sigma^2_{r\to CR} = 4$, $\sigma^2_{r\to DR} = \sigma^2_{DT\to r} = 0.5$, $\sigma^2_{BS\to r} = 0.4$, $\gamma_T = 0.5$, $\gamma_{BS} = 25\,dB$, $\bar{P}^C_{out} = 0.99$, $R_C = 0.1\,bit/s/Hz$.

6. Conclusion

In this paper, we proposed a new communications scheme in which an untrusted relay assists the communication of the D2D link underlying the CN. Because of the shared spectrum, there is cross-interference from the cellular system over the D2D link that was considered in our model. We obtained an exact closed-form expression for the lower bound of the SOP and further studied the asymptotic behavior of the outage probability. We studied the behavior of the SOP when the system parameters changed and it was observed that the SOP of the SC scheme was always lower than that of the SSC scheme. This is because the SINR of the SC scheme is always higher than that of the SSC scheme.

References

[1] Tehrani MN, Uysal M, Yanikomeroglu H. Device-to-device communication in 5G cellular networks: challenges, solutions, and future directions. IEEE Commun Mag 2014; 52: 86-92.

[2] Al-Qahtani FS, Zhong C, Alnuweiri H. Opportunistic relay selection for secrecy enhancement in cooperative networks. IEEE T Commun 2015; 63: 1756-1770.

[3] Gao Z, Chen D, Chen B, Lu Z, Yao N. Outage probability equivalency of three typical relay selection schemes for selective DF relay networks with selection combining. Wireless Pers Commun 2015; 85: 1205-1215.

[4] Son PN, Kong HY. Exact outage probability of a decode-and-forward scheme with best relay selection under physical layer security. Wireless Pers Commun 2014; 74: 325-342.

[5] Son PN, Kong HY. Exact outage probability of two-way decode-and-forward scheme with opportunistic relay selection under physical layer security. Wireless Pers Commun 2014; 77: 2889-2917.

[6] Poursajadi S, Madani MH. Outage performance analysis of incremental relay selection for STBC AF cooperative networks. Wireless Pers Commun 2015; 83: 2317-2331.

[7] Majhi S, Kalamkar SS, Banerjee A. Secondary outage analysis of amplify-and-forward cognitive relays with direct link and primary interference. In: 21st National Conference on Communications; 27 February1 March 2015; Mumbai, India. pp. 1-6.

[8] Ju M, Kim DH, Hwang KS. Opportunistic transmission of nonregenerative network with untrusted relay. IEEE T Veh Technol 2015; 64: 2703-2709. v Huang J, Mukherjee A, Swindlehurst AL. Secure communication via an untrusted non-regenerative relay in fading channels. IEEE T Signal Proces 2013; 61: 2536-2550.

[9] Sun L, Zhang T, Li Y, Niu H. Performance study of two-hop amplify-and-forward systems with untrustworthy relay nodes. IEEE T Veh Technol 2012; 61: 3801-3807.

[10] Sun L, Ren P, Du Q, Wang Y, Gao Z. Security-aware relaying scheme for cooperative networks with untrusted relay nodes. IEEE Commun Lett 2015; 19: 463-466.

[11] Ju M, Kim DH. Performance of untrusted amplify-and-forward relaying. In: 4th International Conference on Intelligent Systems Modelling & Simulation; 2931 January 2013; Bangkok, Thailand. pp. 575-577.

[12] Goldsmith A. Wireless Communications. 1st ed. New York, NY, USA: Cambridge University Press, 2005.

[13] Anghel PA, Kaveh M. Exact symbol error probability of a cooperative network in a Rayleigh-fading environment. IEEE T Wirel Commun 2004; 3: 1416-1421.

[14] Krikidis I. Opportunistic relay selection for cooperative networks with secrecy constraints. IET Commun 2010; 4: 1787-1791.

[15] Papoulis A. Probability, Random Variables, and Stochastic Processes. 4th ed. New York, NY, USA: McGraw-Hill, 2002.

[16] Gradshteyn IS, Ryzhik M. Table of Integrals, Series, and Products. 7th ed. San Diego, CA, USA: Academic Press, 2007.

[17] Zou Y, Zhu J, Zheng B, Yao YD. An adaptive cooperation diversity scheme with best-relay selection in cognitive radio networks. IEEE T Signal Proces 2010; 58: 5438-5445.

Appendices

A. Appendix Proof of Proposition 1

We obtain the primary outage in Eq. (1) for the first hop as the outage of the second hop can be obtained in a similar way. The outage probability of the primary link in the first hop can be written as follows:

$$\begin{aligned} P_{out}^{C,1} &= \Pr(C_{BS\to CR}^{1} \le R_C) = \Pr(\log_2(1 + \tfrac{P_{BS}|h_{BS\to CR}|^2}{N_0 + P_{DT}|h_{DT\to CR}|^2}) \le R_C) \\ &= \Pr(\tfrac{\delta_{BS}|h_{BS\to CR}|^2}{1+\delta_{DT}|h_{DT\to CR}|^2} \le \vartheta) = \int_0^\infty F_{|h_{BS\to CR}|^2}(\tfrac{\vartheta}{\delta_{BS}} + \tfrac{\delta_{DT}\vartheta}{\delta_{BS}}x) f_{|h_{DT\to CR}|^2}(x)dx, \end{aligned} \tag{A.1}$$

where $F_{|h_{BS\to CR}|^2}(z) = 1 - e^{-\sigma_{BS\to CR}^2 z}$ and $f_{|h_{BS\to CR}|^2}(z) = \sigma_{BS\to CR}^2 e^{-\sigma_{BS\to CR}^2 z}$.

The integral in Eq. A.1 can now be easily computed to obtain the following:

$$P_{out}^{C,1} = 1 - \frac{\frac{\sigma_{DT\to CR}^2}{\delta_{DT}}}{\frac{\sigma_{BS\to CR}^2}{\delta_{BS}}\vartheta + \frac{\sigma_{DT\to CR}^2}{\delta_{DT}}} e^{(-\frac{\sigma_{BS\to CR}^2}{\delta_{BS}}\vartheta)}, \tag{A.2}$$

where $\delta_{BS} = \frac{P_{BS}}{N_0}$, $\delta_{DT} = \frac{P_{DT}}{N_0}$, $\delta_r = \frac{P_r}{N_0}$, and $\vartheta = 2^{R_C} - 1$. Since we must have $P_{out}^{C,1} \le \bar{P}_{out}^{C}$, Eq. (2a) will be easily obtained.

B. Appendix Obtaining the CDF in Eq. (6)

We can write the received SINR at the DR in the first time slot as $\gamma_{DT\to DR} = \frac{P_{DT}|h_{DT\to DR}|^2}{N_0 + P_{BS}|h_{BS\to DR}|^2} = \frac{X}{1+Y}$ where $X = \frac{P_{DT}|h_{DT\to DR}|^2}{N_0}$ and $Y = \frac{P_{BS}|h_{BS\to DR}|^2}{N_0}$. We now have:

$$F_{\gamma_{DT\to DR}}(\gamma) = \Pr(\gamma_{DT\to DR} < \gamma) = \Pr(\frac{X}{1+Y} < \gamma) = \int_0^\infty F_X(\gamma + \gamma y) f_Y(\gamma) d\gamma. \tag{B.1}$$

Since $F_X(z) = 1 - e^{-\Psi_{DT\to DR} z}$ and $f_Y(z) = \Psi_{BS\to DR} e^{-\Psi_{BS\to DR} z}$ from Eq. B.1, the CDF $F_{\gamma_{DT\to DR}}(.)$ can be obtained by $F_{\gamma_{DT\to DR}}(\gamma) = 1 - \frac{\Psi_1}{\Psi_1 + \gamma} e^{-\Psi_{DT\to DR}\gamma}$ where $\Psi_1 = \frac{\Psi_{BS\to DR}}{\Psi_{DT\to DR}}$.

C. Appendix Proof of Proposition 2

To compute the expression given in Eq. (11), we first need to compute the CDF $F_{\gamma_{e2e}^{SC}(y)}(.)$ where $\gamma_{e2e}^{SC}(y) = \max(\gamma_{DT\to DR}, \min(y, \gamma_{r\to DR}))$. Defining $z(y) = \min(y\gamma_{r\to DR})$, we note that the random variables $z(y)$ and $\gamma_{DT\to DR}$ are statistically independent. Therefore, we can write the CDF $F_{\gamma_{e2e}^{SC}(y)}(.)$ as follows [16]:

$$F_{\gamma_{e2e}^{SC}(y)}(\gamma) = F_{\gamma_{DT\to DR}}(\gamma) F_{z(y)}(\gamma), \tag{C.1}$$

where the CDF $F_{\gamma_{DT\to DR}}(.)$ is given in Eq. (6). Using the following result [16]:

$$Pr(min(a, X) < \gamma) = \begin{cases} F_X(\gamma), & \text{if } a \ge \gamma, \\ 1, & \text{if } a < \gamma, \end{cases} \tag{C.2}$$

the CDF $F_{z(y)}(.)$ can be obtained as follows:

$$F_{z(y)}(\gamma) = Pr(min(y, \gamma_{r\to DR}) < \gamma) = \begin{cases} 1 - \frac{\Psi_3}{\Psi_3 + \gamma} e^{-\Psi_{r\to DR}\gamma}, & \text{if } y \ge \gamma, \\ 1, & \text{if } y < \gamma, \end{cases} \tag{C.3}$$

where $\Psi_3 = \frac{\Psi_{BS\to DR}}{\Psi_{r\to DR}}$. Substituting Eq. C.3 and Eq. (4a) into Eq. C.1, we now have the following:

$$F_{\gamma_{e2e}^{SC}(y)}(\gamma) = F_{\gamma_{DT\to DR}}(\gamma)F_{z(y)}(\gamma) = \begin{cases} (1-\frac{\Psi_1}{\Psi_1+\gamma}e^{-\Psi_{DT\to DR}\gamma})(1-\frac{\Psi_3}{\Psi_3+\gamma}e^{-\Psi_{r\to DR}\gamma}), & \text{if } y \geq \gamma, \\ 1-\frac{\Psi_1}{\Psi_1+\gamma}e^{-\Psi_{DT\to DR}\gamma}, & \text{if } y < \gamma, \end{cases} \tag{C.4}$$

and using Eq. (12), the following expression could be obtained for the outage probability:

$$\begin{aligned} P_{out}^{SC} &= \int_0^\infty F_{\gamma_{e2e}^{SC}(y)}(\alpha+\alpha y-1)f_{\gamma_{DT\to r}}(y)dy \\ &= \int_0^\infty \left[\begin{array}{l} (1-\frac{\Psi_1}{\Psi_1+(\alpha+\alpha y-1)}e^{-\Psi_{DT\to DR}(\alpha+\alpha y-1)})(1-\frac{\Psi_3}{\Psi_3+(\alpha+\alpha y-1)}e^{-\Psi_{r\to DR}(\alpha+\alpha y-1)}) \\ (\frac{\Psi_2 e^{-\Psi_{DT\to r}y}}{(y+\Psi_2)^2}+\frac{\Psi_{DT\to r}\Psi_2 e^{-\Psi_{DT\to r}y}}{y+\Psi_2})dy \end{array} \right]. \end{aligned} \tag{C.5}$$

With the help of $\int\limits_0^\infty \frac{e^{-\mu x}}{x+B}dx = -e^{B\mu}Ei(-\mu B)$ and $\int\limits_0^\infty \frac{e^{-px}}{(A+x)^2}dx = pe^{Ap}Ei\left(-Ap\right)+\frac{1}{A}$ [19, Eqs. (3.352.4) and (3.353.3)] where $E_1\left(z\right) = -Ei\left(-z\right) = \int_z^\infty \frac{e^{-x}}{x}dx$ [17], Eq. C.5 can be computed and Eq. (13) follows.

D. Appendix Proof of Proposition 3

To compute the outage expression in Eq. (22), we must compute the integral in Eq. (20). Substituting Eqs. (21) and (12) into Eq. (20), we have the following:

$$\begin{aligned} P_{out}^{SSC} &= \int_0^\tau F_{\gamma_{DT\to DR}}(\gamma_T)F_{\hat{\gamma}_{DTrDR}}(\alpha+\alpha y-1)f_{\gamma_{DT\to r}}(y)dy \\ &\quad +\int_\tau^\infty [Pr(\gamma_T \leq \gamma_{DT\to DR} \leq (\alpha+\alpha y-1)) + F_{\gamma_{DT\to DR}}(\gamma_T)F_{\hat{\gamma}_{DTrDR}}(\alpha+\alpha y-1)]f_{\gamma_{DT\to r}}(y)dy. \end{aligned} \tag{D.1}$$

Similar to Eq. C.3, we obtain the required CDF in Eq. D.1 by $F_{\hat{\gamma}_{DTrDR}}\left(\gamma\right) = \begin{cases} 1-\frac{\Psi_3}{\Psi_3+\gamma}e^{-\Psi_{r\to DR}\gamma}, & \text{if } y \geq \gamma \\ 1, & \text{if } y < \gamma \end{cases}$.

By substituting the corresponding parameters in Eq. D.1, the final integral will be as follows:

$$\begin{aligned} P_{out}^{SSC} &= \int_0^\tau (1-\frac{\Psi_1}{\Psi_1+\gamma_T}e^{-\Psi_{DT\to DR}\gamma_T})(1-\frac{\Psi_3}{\Psi_3+(\alpha+\alpha y-1)}e^{-\Psi_{r\to DR}(\alpha+\alpha y-1)})(\frac{\Psi_2 e^{-\Psi_{DT\to r}y}}{(y+\Psi_2)^2}+\frac{\Psi_{DT\to r}\Psi_2 e^{-\Psi_{DT\to r}y}}{y+\Psi_2})dy \\ &+\int_\tau^\infty \left[\begin{array}{l} (\frac{\Psi_1}{\Psi_1+\gamma_T}e^{-\Psi_{DT\to DR}\gamma_T}-\frac{\Psi_1}{\Psi_1+(\alpha+\alpha y-1)}e^{-\Psi_{DT\to DR}(\alpha+\alpha y-1)}) \\ +(1-\frac{\Psi_1}{\Psi_1+\gamma_T}e^{-\Psi_{DT\to DR}\gamma_T})(1-\frac{\Psi_3}{\Psi_3+(\alpha+\alpha y-1)}e^{-\Psi_{r\to DR}(\alpha+\alpha y-1)}) \end{array} \right](\frac{\Psi_2 e^{-\Psi_{DT\to r}y}}{(y+\Psi_2)^2}+\frac{\Psi_{DT\to r}\Psi_2 e^{-\Psi_{DT\to r}y}}{y+\Psi_2})dy. \end{aligned} \tag{D.2}$$

Using the following results [17, Eqs. (3.352.3) and (3.353.1)]:

$$\int_u^v \frac{e^{-\mu x}}{x+A}dx = e^{A\mu}\{Ei[-(A+v)\mu]-Ei[-(A+u)\mu]\},$$

$$\int_u^\infty \frac{e^{-\mu x}}{(x+B)^n}dx = e^{-u\mu}\sum_{k=1}^{n-1}\frac{(k-1)!(-\mu)^{n-k-1}}{(n-1)!(u+B)^k} - \frac{(-\mu)^{n-1}}{(n-1)!}e^{B\mu}Ei[-(u+B)\mu],$$

the integral in Eq. D.2 can be computed and Eq. (22) follows.