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Multiresponse optimization to improve the torque behavior of an outer-rotor permanent-magnet machine using gray relational analysis based on the Taguchi method

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Abstract: The torque behavior of an outer-rotor surface-mounted permanent-magnet machine is improved by identifying seven pertinent design variables, including rotor height. The optimal design variables are revealed by analyzing 18 experiments determined by the Taguchi method for the minimum torque ripple, minimum total harmonic distortion of the induced voltage, and maximum average torque. In addition, the optimal design variables are obtained very quickly by using gray relational analysis based on the Taguchi method with the single response of the gray relational grade instead of multiple responses. A considerable amount of multiresponse improvement is achieved according to the results of the two optimizations. Performance improvements of 20.6%, 32.0%, and 24.5% are obtained for the average torque, the torque ripple, and the total harmonic distortion of the back-EMF, respectively.

Key words: Outer-rotor permanent-magnet machine, torque ripple, multiresponse, Taguchi method, gray relational analysis

1. Introduction

Recently, considerable interest has arisen in permanent-magnet synchronous machines (PMSMs) that have no rotor windings, resulting in lower copper losses. However, PMSMs suffer from high torque ripple, cogging torque, and large unbalanced magnetic forces. There has been much research on slot and permanent-magnet (PM) designs. These include designing the slot opening to minimize the torque ripple [1]; studying the influence of slot-opening width, pole-arc coefficient, magnet thickness, and air-gap size on electromotive force (EMF) harmonics [2]; varying the width of the magnet poles to minimize the cogging torque [3]; showing the significant effects of pole-arc coefficient, eccentricity, and magnet shape on the harmonic content and cogging torque [4]; increasing the PM offset without significant reduction in back-EMF and motor efficiency [5]; and determining the optimal split ratio for an external-rotor PMSM [6].

Some studies have used the Taguchi method. These include maximizing output power and efficiency (8 design parameters, 18 experiments) [7]; optimizing cogging torque and efficiency (5 design parameters, 16 experiments, including fuzzy logic) [8]; optimizing torque ripple, efficiency, and torque-to-magnet-volume ratio (8 design parameters, 18 experiments, including fuzzy logic) [9]; optimizing efficiency and average torque to torque ripple factor (5 design parameters, 16 experiments, including fuzzy logic) [10]; and optimizing average torque, torque ripple, and the ratio of torque ripple to average torque (4 design parameters, 9 experiments) [11].

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Most studies do not discuss the effect of design parameters on the torque behavior. However, it is important to determine the design variables that will be used in the optimization process. The primary focus of the present study is the effect of design parameters on the torque behavior of a surface-mounted permanent magnet (SPM) machine. The optimal range of design parameters is determined before the optimization process. In addition, the main dimensions of an SPM machine (i.e. the stator and rotor heights) were not considered in previous optimization studies [7–11]. Hence, the present study considers the effects of stator and rotor height on the torque behavior. Seven design variables, including the rotor height, are considered here.

A different set of optimal levels must be obtained for each response (average torque, torque ripple, and total harmonic distortion (THD)) to determine the optimal levels of design parameters based on a single response in a multiresponse problem. Gray relational analysis is used with the Taguchi method to combine the multiple responses into a single response and then obtain the optimal levels very quickly.

2. Torque performance corresponding to design parameters

In this study, an SPM machine with an outer rotor (OR) is selected for study. The geometry of this OR-SPM machine is shown in Figure 1, and its performance parameters are given in Table 1. The important performance values that were obtained by simulating the OR-SPM machine as a two-dimensional finite-element model are given in Table 2 as reference performance values that are used for comparison with the optimization results. ANSYS Maxwell software was used for the finite-element analysis. These values are used as the multiple response/performance characteristics of the optimization process.

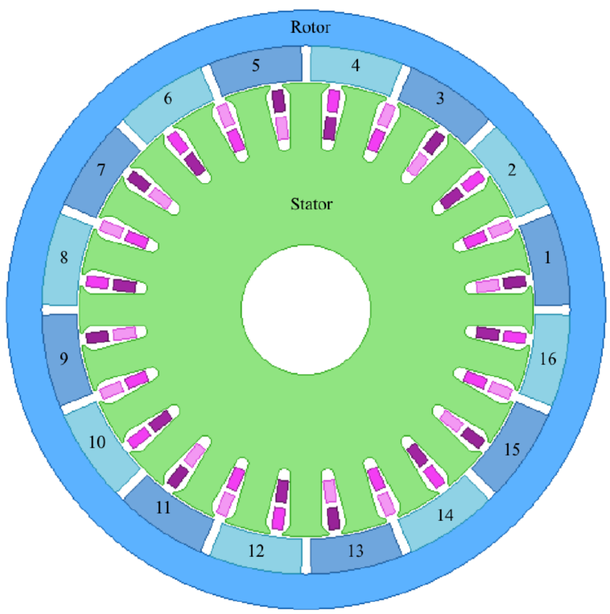

Figure 1. Outer-rotor surface-mounted permanent-magnet (OR-SPM) machine with 24 slots and 16 poles.

Previous studies have tended to overlook the effect of design parameters such as the stator and rotor heights, slot opening, and magnet dimensions on the torque behavior. Hence, the primary goal here is to improve the torque behavior of the OR-SPM machine under consideration by focusing on the influence of such design parameters. The relevant design parameters are the stator height (H_s), the rotor height (H_r), the air-gap length (g), the magnet-arc to pole-pitch ratio (K_m), the magnet thickness (T_m), the magnet offset (O_m), the

Table 1. Reference parameters of the OR-SPM machine.

Reference parameter name	Value
Inner & outer diameter of rotor	92–120 mm
Inner & outer diameter of stator	26–91 mm
Height of rotor & stator (H_r, H_s)	14–32.5 mm
Stack length	65 mm
Magnet-arc (A_m) & pole-pitch (P_p)	20.25–22.5°
Magnet-arc to pole-pitch ratio (K_m)	0.9
Offset of magnet (O_m)	0 mm
Thickness of magnet (T_m)	7 mm
Slot opening height & width (h_{s0}, b_{s0})	0.5–2.5 mm
Slot body height & bottom width	9.48–2.5 mm
Slot wedge width	5 mm
Slot and pole number	24s–16p
Rated output power	0.55 kW
Rated voltage	220 V
Rated speed	1500 rpm
Material of steel	M19–24G
Material of magnet	XG196/96

Table 2. Reference performance characteristics.

Slot/pole	q	k_{w1}	T_{avg} (Nm)	T_{rip} (%)	THD (%)
24/16	0.5	0.866	12.55	5.15	1.63

slot-opening width (b_{s0}), and the slot-opening height (h_{s0}). All these design parameters are marked on the geometry of the OR-SPM machine shown in Figure 2, and their reference values are given in Table 1. The slot number of an OR-SPM machine is selected as $Q_s = 24$. Additionally, the fundamental winding factor is chosen as $k_{w1} = 0.866$. In this case, the value of slot/pole/phase and the pole number are respectively equal to 0.5 and 16. A double-layered concentrated winding in this study is chosen. The winding layout of the OR-SPM machine is performed to provide the given value of slot/pole/phase. The concentrated winding layout is used as |AC'||CB'||BA'|.

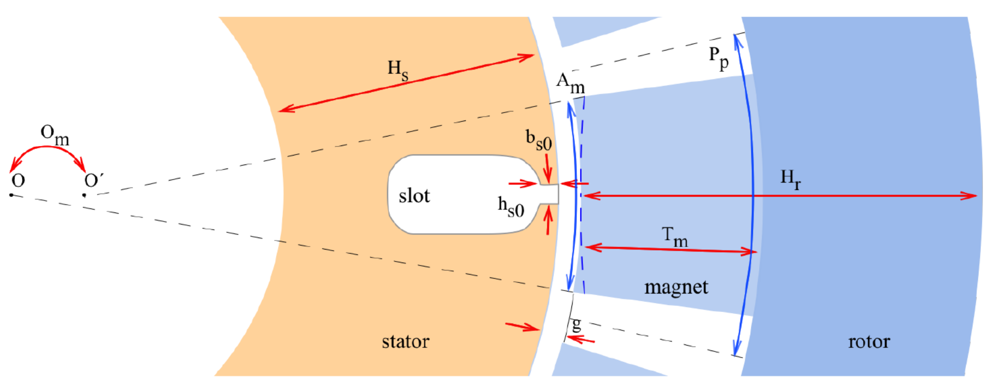

Figure 2. Design parameters for the OR-SPM machine.

The torque ripple (T_{rip}) is defined as:

$$T_{rip} = \frac{T_{rms}}{T_{avg}} \times 100\%, \tag{1}$$

where T_{rms} and T_{avg} are the root-mean-square of the ripple component and average values, respectively, of the instantaneous torque. The variations in the average torque and the torque ripple with each of the design parameters were obtained by analyzing the finite-element model. The outer diameter of the OR-SPM machine was kept constant during this process. These variations are shown in Figure 3. The torque behavior (i.e. the average torque and the torque ripple) is extremely sensitive to each of the design parameters, except for the stator height. Hence, the stator height was excluded as a design variable for the optimization process. Instead, its reference value was fixed at 18 mm as the optimal value. The other design parameters are accepted as variables in the optimization process because of their torque sensitivity.

The back-EMF of the OR-SPM was obtained by finite-element analysis. All harmonics of the back-EMF were obtained by fast Fourier transform. The THD of the back-EMF value was then calculated for the 22 harmonics by the following equation:

$$THD = \frac{1}{E_1}\sqrt{\sum_{n=2}^{22} E_n^2} \times 100\%. \tag{2}$$

The optimal range of each design parameter was determined for the reference boundary of the magnetic flux density. The highest magnetic flux density value for the reference model was obtained as 1.78 T. The limit value used in determining the optimum parameters was accepted as B_{limit} = 1.85 T. This limit was exceeded in some values of the slot opening height ($h_{s0} < 0.4$). Although smaller torque ripple was obtained for larger average torque for this range, the values less than 4 mm were not included in optimization. Thus, the first step in the process was the determination of the design parameters and their optimal ranges for use in the optimization. The optimal ranges of the design parameters are H_r = 12–14 mm, g = 0.45–0.55 mm, K_m = 0.88–0.90, T_m = 8–10 mm, O_m = 12–16 mm, b_{s0} = 2.6–2.8 mm, and h_{s0} = 0.4–0.6 mm.

3. Optimization with the Taguchi method

The multivariable Taguchi method provides the designer with an efficient approach for doing experiments to determine near-optimal variable values. The process diagram of the system to be optimized is shown in Figure 4. This process transforms some input/signal factors (M) into an output that has one or more observable response variables (Y). The control factors (X) are controllable, whereas the noise factors (Z) are uncontrollable. The function of the system can be shown in terms of its process diagram, which reflects the output (Y) as a result of the input (M) and other influencing factors (X, Z) on the system [12].

The relevant factors are the variables that have a direct influence on the performance of the process. The levels are the values that define the conditions of the factors that are held during the experiments. An orthogonal array (OA) is used to design the experiments and describe the trial conditions [12]. An experiment to study these factors is accomplished by using an L18 array as shown below:

$$L_n\left(i^x\right) = L_{18}\left(2^1 3^6\right) = L18, \tag{3}$$

where n, i, and x are the number of experiments, the number of levels, and the number of control factors, respectively. The L18 design is called a ‘mixed design’, in which factors have both two and three levels. The L18

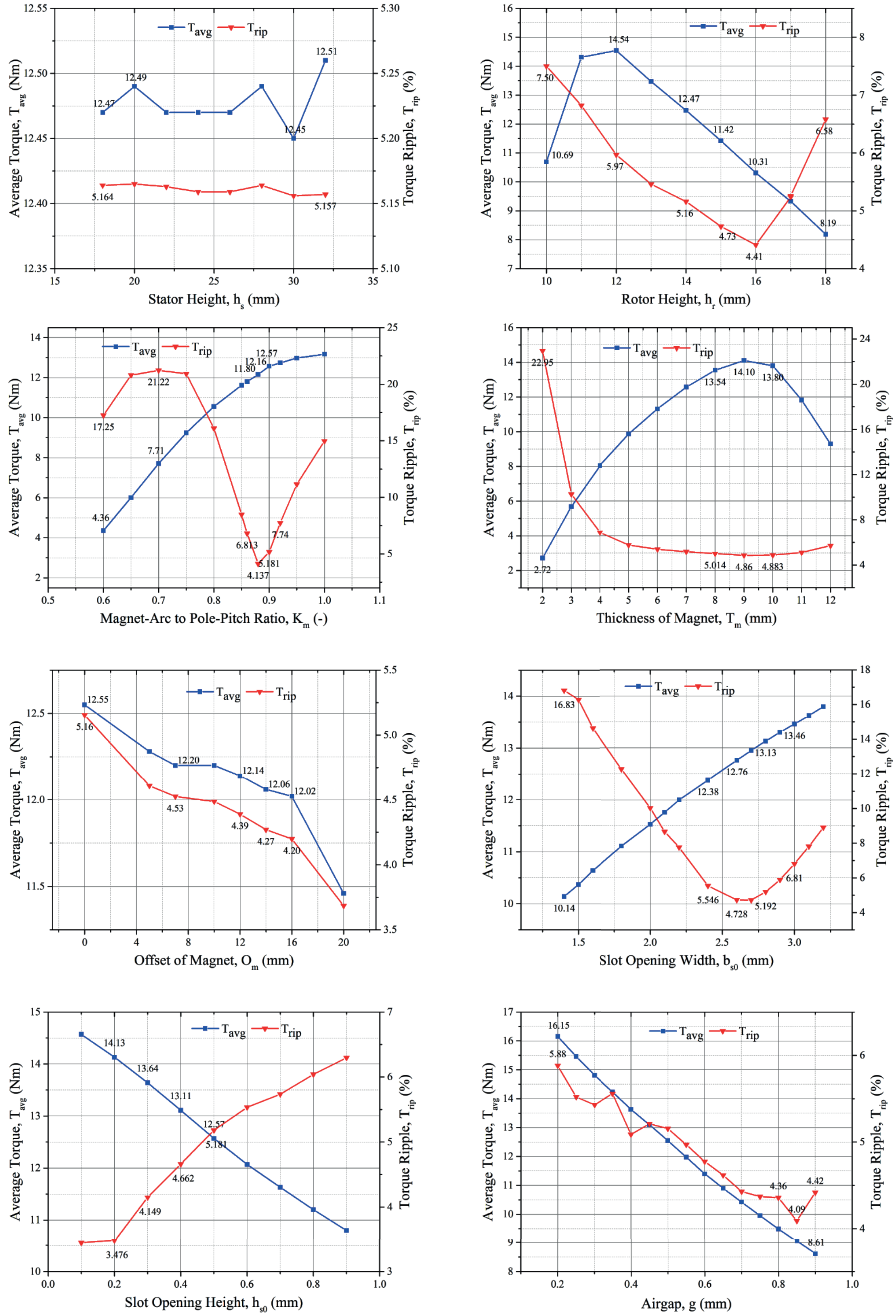

Figure 3. Variations in average torque (blue) and torque ripple (red) with each of the design parameters.

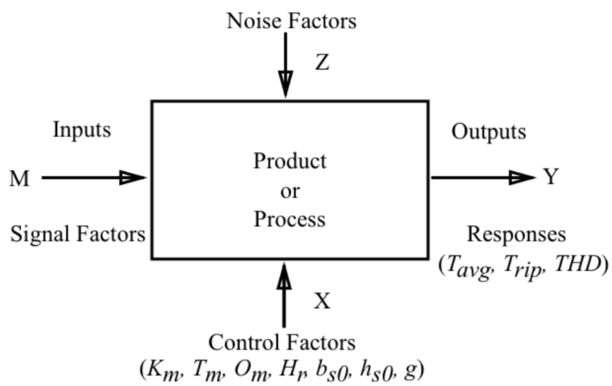

Figure 4. Process diagram of system.

design consists of one factor at two levels and six factors at three levels each (Table 3). There are seven control factors (A–G), one of which has two levels and the others have three levels in this optimization problem. These control factors are also known as the design variables. The L18 array is tabulated with the response values in Table 4. Eighteen experiments with mixed level design are sufficient instead of 27 experiments of single level design.

Table 3. Design variables and levels.

Design variables		Level 1	Level 2	Level 3
K_m	A	0.88	0.9	-
T_m	B	8	9	10
O_m	C	12	14	16
H_r	D	12	13	14
b_{s0}	E	2.6	2.7	2.8
h_{s0}	F	0.4	0.5	0.6
g	G	0.45	0.5	0.55

The signal-to-noise (S/N) ratio measures how the response varies relative to the target. To achieve an optimally designed product, two different S/N ratios are calculated using Eqs. (4) and (5). If the goal is to maximize the response, the S/N ratio is taken as "larger is better" (LIB) and is defined as

$$S/N = -10\log_{10}\left[\frac{1}{n}\sum_{i=1}^{n}\frac{1}{y_i^2}\right]. \tag{4}$$

If the goal is to minimize the response, the S/N ratio is taken as "smaller is better" (SIB) and is defined as

$$S/N = -10\log_{10}\left[\frac{1}{n}\sum_{i=1}^{n}y_i^2\right], \tag{5}$$

where n is the number of experiments for each object function and y_i is the value of the experiment's response.

The LIB case is related to the objective function of the average torque. The SIB case is related to the objective function of the torque ripple and to the THD of the back-EMF; minimizing the THD is the second main goal of this study. The quality characteristics of the average torque, torque ripple, and back-EMF THD as

Table 4. Results/responses of L18 orthogonal array.

n	K_m	T_m	O_m	H_r	b_{s0}	h_{s0}	g	T_{avg}	T_{rip}	THD
	A	B	C	D	E	F	G	(Nm)	(%)	(%)
1	0.88	8	12	12	2.6	0.4	0.45	15.27	3.38	1.25
2	0.88	8	14	13	2.7	0.5	0.5	14.11	4.18	1.19
3	0.88	8	16	14	2.8	0.6	0.55	12.21	5.98	1.15
4	0.88	9	12	12	2.7	0.5	0.55	11.21	5.35	1.28
5	0.88	9	14	13	2.8	0.6	0.45	14.35	5.06	1.16
6	0.88	9	16	14	2.6	0.4	0.5	14.05	3.57	1.19
7	0.88	10	12	13	2.6	0.6	0.5	11.14	4.97	1.27
8	0.88	10	14	14	2.7	0.4	0.55	13.61	4.39	1.19
9	0.88	10	16	12	2.8	0.5	0.45	8.72	9.59	1.32
10	0.9	8	12	14	2.8	0.5	0.5	13.64	4.35	1.41
11	0.9	8	14	12	2.6	0.6	0.55	13.19	4.53	1.47
12	0.9	8	16	13	2.7	0.4	0.45	15.56	3.47	1.39
13	0.9	9	12	13	2.8	0.4	0.55	14.47	3.85	1.40
14	0.9	9	14	14	2.6	0.5	0.45	14.46	3.54	1.44
15	0.9	9	16	12	2.7	0.6	0.5	11.22	4.93	1.45
16	0.9	10	12	14	2.7	0.6	0.45	13.90	3.97	1.50
17	0.9	10	14	12	2.8	0.4	0.5	8.89	7.23	1.56
18	0.9	10	16	13	2.6	0.5	0.55	11.22	4.08	1.45

Table 5. Average torque for the design variables.

Level	A	B	C	D	E	F	G
i = 1	12.74	14.00	13.27	11.42	13.22	13.64	13.71
i = 2	12.95	13.29	13.10	13.48	13.27	12.23	12.17
i = 3	-	11.25	12.16	13.65	12.05	12.67	12.65
Rank	7	1	6	2	5	4	3
LIB	A2	B1	C1	D3	E2	F1	G1

Table 6. Torque ripple for the design variables.

Level	A	B	C	D	E	F	G
i = 1	5.16	4.32	4.31	5.84	4.01	4.32	4.84
i = 2	4.44	4.38	4.82	4.27	4.38	5.18	4.87
i = 3	-	5.71	5.27	4.30	6.01	4.91	4.70
Rank	6	3	4	2	1	5	7
SIB	A2	B1	C1	D2	E1	F1	G3

Table 7. Total harmonic distortion of back-EMF for the design variables.

Level	A	B	C	D	E	F	G
i = 1	1.22	1.31	1.35	1.39	1.35	1.33	1.34
i = 2	1.45	1.32	1.34	1.31	1.33	1.35	1.35
i = 3	-	1.38	1.33	1.31	1.33	1.33	1.32
Rank	1	3	4	2	7	6	5
SIB	A1	B1	C3	D23	E23	F13	G3

calculated by the S/N ratio are given in Tables 5–7, respectively. In addition, Figures 5–7 show the main-effect plots for the S/N ratio according to each design variable.

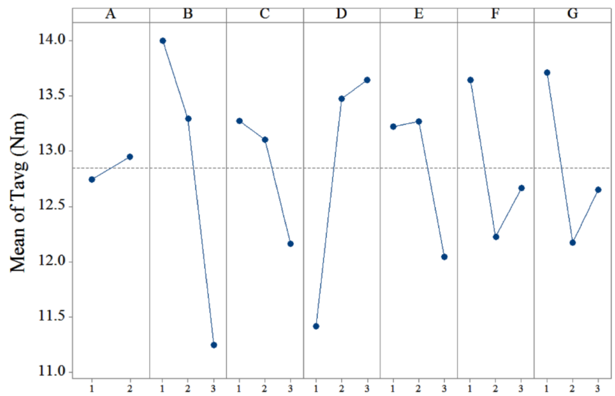

Figure 5. Effect of main design variables on average torque T_{avg}.

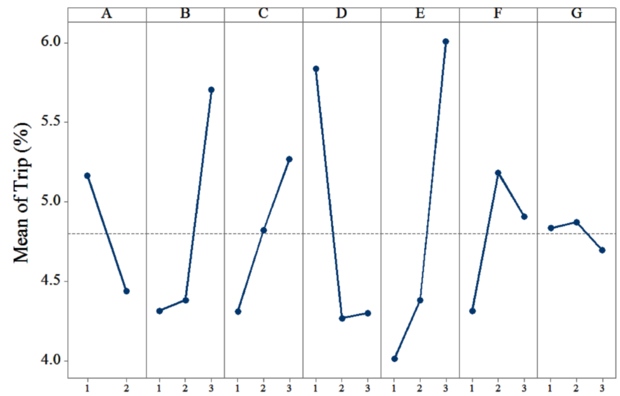

Figure 6. Effect of main design variables on torque ripple T_{rip}.

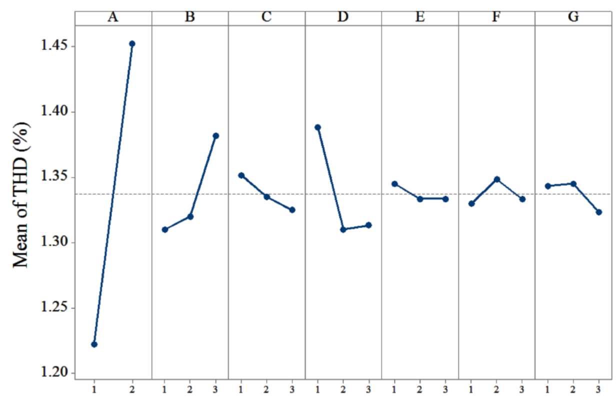

Figure 7. Effect of main design variables on back-EMF THD.

Analysis of variance does not analyze the optimization problem directly, but rather it extracts the relative importance of the design variables. To evaluate the significant effect of the design variables on the responses, the sum of squares (SS) due to various design variables can be calculated by

$$SS_A = 3\sum_{i=1}^{3}(m_{A_i} - m)^2. \tag{6}$$

The SS for the other seven variables (SS_B, SS_C, and so on) can be obtained in the same way. These results are summarized in Table 8. The magnet thickness (B) and rotor height (D) are revealed as the dominant design variables for the average torque. The slot-opening width (E), rotor height (D), and magnet thickness (B) stand out for the torque ripple. The magnet-arc to pole-pitch ratio (A) is revealed as the only dominant design variable for the THD.

If the dominant variables of each response are changed with maximum and minimum values, while the other variables are kept constant at the maximum value for the average torque and the minimum value for the torque ripple and the THD, two combinations for each response are obtained. It is seen that each response varies considerably for the limit values of the dominant variables (Table 9).

Table 8. Significant effects of the design variables.

Factors		T_{avg} (Nm)		T_{rip} (%)		THD (%)	
		SS	Effect ratio (%) ratio (%)	SS SS	Effect Effect ratio (%)	SS (e-3)	Effect ratio (%)
K_m	A	0.20	0.3	2.36	5.8	238.05	82.4
T_m	B	24.49	34.9	7.37	18.2	18.08	6.3
O_m	C	4.28	6.1	2.76	6.8	2.18	0.8
H_r	D	18.46	26.3	9.62	23.8	23.54	8.1
b_{s0}	E	5.75	8.2	13.56	33.5	0.54	0.2
h_{s0}	F	6.29	9.0	2.35	5.8	1.14	0.4
g	G	7.41	10.6	0.10	0.3	1.74	0.6

Table 9. Effect of dominant variables on each response.

Model	T_{avg} (Nm)	T_{rip} (%)	THD (%)
T_{avg}: A2-B1-C1-D3-E2-F1-G1	14.54	3.77	1.44
T_{avg}: — B3 — D1 — — —	9.11	6.11	1.64
T_{rip}: A2-B1-C1-D2-E1-F1-G3	14.28	3.64	1.37
T_{rip}: — B3 — D1-E3 — —	8.48	7.36	1.53
THD: A1-B1-C3-D2-E3-F1-G3	14.19	4.63	1.16
THD: A2 — — — — — —	14.52	3.74	1.34

4. Optimal design variables for Taguchi analysis

From Tables 5–7, it is clear that the design-variable/level combination of A2-B1-C1-D3-E2-F1-G1 maximizes the average torque, A2-B1-C1-D2-E1-F1-G3 minimizes the torque ripple, and A1-B1-C3-D23-E23-F13-G3 minimizes the THD. From Table 8, the most influential design variables in relation to average torque, torque ripple, and THD are B-D, B-D-E, and A, respectively. It is primarily these variables that are evaluated.

First, evaluation is performed in relation to average torque and torque ripple. The average torque is largest for B, D, and E, whereas the torque ripple is lowest. In addition, variables A, C, and F have the same effects on the average torque and the torque ripple. Variable G has a larger effect on the average torque than on the torque ripple. Therefore, the design variables A2, B1, C1, D23, E1, F1, and G1 are selected to constitute the maximum average torque and the minimum torque ripple.

In addition, design-variable A has a larger effect on the THD than on the average torque or the torque ripple. Thus, design-variable A2 can be replaced with A1. In this case, the THD decreases while the two torque values change. The optimal combination of design variables was determined as A12-B1-C1-D23-E1-F1-G1 for the maximum average torque, the minimum torque ripple, and the minimum THD.

The three optimal responses corresponding to the optimal design variables are given in Table 10. Four trials are available for two different values of A and D. When design-variable D varies from D2 to D3, the average torque decreases. Similarly, when design-variable A varies from A2 to A1, the THD decreases because A has a larger effect on the THD than on either of the two torque values. The improvements in the average torque, torque ripple, and THD are 23.3%, 33.0%, and 24.5%, respectively. Response #3 is more successful than #1 in terms of the torque ripple and the THD.

Our optimization problem has a multiresponse property that also requires evaluation in order to reveal the optimal design variables. As can be observed, the optimization period is extended because of the abovementioned evaluation process.

Table 10. Responses for the Taguchi analysis.

Model	T_{avg} (Nm)	T_{rip} (%)	THD (%)
Reference	12.55	5.15	1.63
1: A2-B1-C1-D2-E1-F1-G1	15.48	3.78	1.43
2: A2-B1-C1-D3-E1-F1-G1	14.36	3.65	1.44
3: A1-B1-C1-D2-E1-F1-G1	15.14	3.50	1.23
4: A1-B1-C1-D3-E1-F1-G1	14.02	3.94	1.24
Improvement in # 1	23.3%	–26.6%	–12.3%
Improvement in # 3	20.6%	–32.0%	–24.5%

5. Optimization with gray relational analysis

Gray relational analysis is used for solving interrelationships among multiple response/performance characteristics. In this approach, a gray relational grade is obtained for analyzing the relational degree of the multiple performance characteristics [12]. The normalized S/N ratio z_{ij} for the ith experiment and the jth performance quantity with LIB and SIB can be written respectively as

$$z_{ij} = \frac{y_{ij} - \min y_{ij}}{\max y_{ij} - \min y_{ij}};$$

$$z_{ij} = \frac{\max y_{ij} - y_{ij}}{\max y_{ij} - \min y_{ij}}, \tag{7}$$

where y_{ij} is the ith experiment and the jth performance quantity, and max y_{ij} and min y_{ij} are the largest and smallest values of y_{ij}, respectively.

The gray relational coefficient $\gamma_{Ij} = (x_{oj}(k) - x_{ij}(k))$ is calculated for the normalized S/N ratios as

$$\gamma_{ij}(k) = \frac{\Delta_{\min} + \xi \cdot \Delta_{\max}}{\Delta_{ij}(k) + \xi \cdot \Delta_{\max}}, \tag{8}$$

where $\Delta_{ij}(k) = |x_{oj}(k) - x_{ij}(k)|$ is an absolute value, $x_{oj}(k)$ is the ideal normalized value of the jth performance quantity, $x_{ij}(k)$ is the normalized comparability sequence,
$\Delta_{\min} = \min_i \min_k = |x_{oj}(k) - x_{ij}(k)|$ and $\Delta_{max} = \max_i \max_k = |x_{oj}(k) - x_{ij}(k)|$ are the smallest and the largest value of $\Delta_{ij}(k)$, and ξ is the distinguishing coefficient defined in the range $0 < \xi < 1$. Finally, the gray relational grade (G_i) is computed as

$$G_i = \frac{1}{m} \sum \gamma_{ij}(k), \tag{9}$$

where m is the number of responses.

The average torque is investigated for the LIB condition, whereas the torque ripple and THD are investigated for the SIB condition. The S/N ratios, the normalized S/N ratios, the gray relational grade (GRG), and the gray relational grades are obtained using Eqs. (4), (5), and (7)–(9), respectively. The various calculated values of these quantities are given in Table 11.

When gray relational analysis based on the Taguchi method is used, the optimal design variables are determined very quickly and easily because of the multiple responses. The multiresponse (T_{avg}, T_{rip}, THD) is reduced to a single response (GRG), in which case the optimal design variables are obtained very quickly. The values and the main effects on the GRG are shown in Table 12 and Figure 8, respectively.

Table 11. Responses for the gray relational analysis.

	S/N			Normalized S/N			Gray relational coefficient			Grade
n	T_{avg}	T_{rip}	THD	T_{avg}	T_{rip}	THD	T_{avg}	T_{rip}	THD	-
1	23.6768	–10.5783	–1.9382	0.9675	1.0000	0.7266	0.9685	1.0000	0.7853	0.9179
2	22.9905	–12.4235	–1.5109	0.8311	0.7963	0.8879	0.8555	0.8308	0.8992	0.8618
3	21.7343	–15.5340	–1.2140	0.5813	0.4529	1.0000	0.7049	0.6464	1.0000	0.7838
4	20.9921	–14.5671	–2.1442	0.4338	0.5596	0.6488	0.6385	0.6943	0.7401	0.6909
5	23.1370	–14.0830	–1.2892	0.8602	0.6131	0.9716	0.8773	0.7210	0.9724	0.8569
6	22.9535	–11.0534	–1.5109	0.8237	0.9475	0.8879	0.8501	0.9502	0.8992	0.8998
7	20.9377	–13.9271	–2.0761	0.4230	0.6303	0.6745	0.6341	0.7301	0.7544	0.7062
8	22.6772	–12.8493	–1.5109	0.7688	0.7493	0.8879	0.8122	0.7995	0.8992	0.8370
9	18.8103	–19.6364	–2.4115	0.0000	0.0000	0.5479	0.5000	0.5000	0.6886	0.5629
10	22.6963	–12.7698	–2.9844	0.7726	0.7581	0.3315	0.8147	0.8052	0.5994	0.7398
11	22.4049	–13.1220	–3.3464	0.7146	0.7192	0.1949	0.7780	0.7807	0.5540	0.7042
12	23.8402	–10.8066	–2.8603	1.0000	0.9748	0.3784	1.0000	0.9754	0.6167	0.8640
13	23.2094	–11.7092	–2.9226	0.8746	0.8752	0.3549	0.8886	0.8890	0.6079	0.7951
14	23.2034	–10.9801	–3.1673	0.8734	0.9556	0.2625	0.8876	0.9575	0.5755	0.8069
15	20.9999	–13.8569	–3.2274	0.4353	0.6380	0.2398	0.6391	0.7342	0.5681	0.6472
16	22.8603	–11.9758	–3.5218	0.8052	0.8457	0.1286	0.8369	0.8663	0.5344	0.7459
17	18.9780	–17.1828	–3.8625	0.0333	0.2709	0.0000	0.5085	0.5783	0.5000	0.5289
18	20.9999	–12.2132	–3.2274	0.4353	0.8195	0.2398	0.6391	0.8471	0.5681	0.6848

Table 12. GRG for the design variables.

Level	A	B	C	D	E	F	G
i = 1	0.7908	0.8119	0.766	0.6753	0.7866	0.8071	0.7924
i = 2	0.7241	0.7828	0.766	0.7948	0.7745	0.7245	0.7306
i=3		0.6776	0.7404	0.8022	0.7112	0.7407	0.7493
Rank	5	1	7	2	4	3	6
LIB	A1	B1	C1	D3	E1	F1	G1

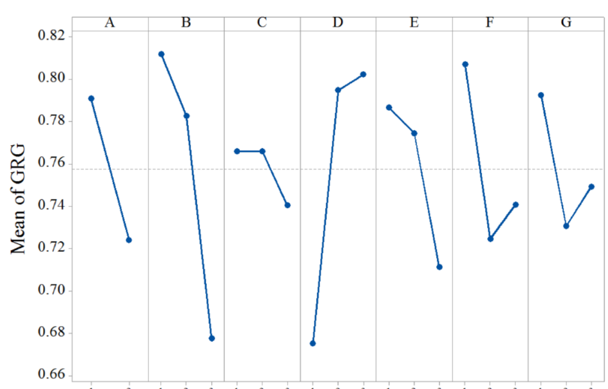

Figure 8. Effect of main design variables on gray relational grade.

As a result of the gray relational analysis, the design variables can be divided into three groups in terms of their variations. The first group consists of magnet thickness (B) and rotor height (D), and it has the most effective multiresponse. The second group consists of magnet-arc to pole-pitch ratio (A), slot-opening width (E), slot-opening height (F), and air gap (G), and it has a very effective multiresponse.

According to the gray relational analysis based on the Taguchi method, the optimal combination of design variables is A1-B1-C1-D23-E1-F1-G1. Both D2 and D3 were tried for obtaining optimal variable values because design-variable D has a larger effect on the average torque and the torque ripple and has a similar value for the two cases (D2, D3).

The responses that correspond to the optimal design variables are given with two combinations in Table 13. These two combinations are the same as the third and fourth solutions of the Taguchi analysis.

Table 13. Responses for the gray analysis based the Taguchi method.

Model	T_{avg} (Nm)	T_{rip} (%)	THD (%)
Reference	12.55	5.15	1.63
1: A1-B1-C1-D2-E1-F1-G1	15.14	3.50	1.23
2: A1-B1-C1-D3-E1-F1-G1	14.02	3.94	1.24
Improvement in # 1	20.6%	–32.0%	–24.5%
Improvement in # 2	11.7%	–23.5%	–23.9%
$K_m = 0.88$ (A), $T_m = 8$ mm (B), $O_m = 12$ mm (C), $H_r = 13$ mm (D), $b_{s0} = 2.6$ mm (E), $h_{s0} = 0.4$ mm (F), $g = 0.45$ mm (G)			

The fuzzy-based Taguchi analysis resulted with five variables in an improvement of 50.8% in the torque ripple [8], and with eight variables in an improvement of 49.2% in the torque ripple and 11.6% in torque-to-magnet-volume ratio [9]. The improvements in average torque, torque ripple, and THD are 20.6%, 32.0%, and 24.5%, respectively. The torque ripple can be reduced by ensuring that both the THD value and the average torque are remarkably improved in this study.

6. Conclusion

Eight design parameters, including the stator and rotor heights, were investigated in relation to how they affected the torque behavior of an OR-SPM machine. Excluding the stator height, seven design parameters were defined as design variables with optimal ranges. The Taguchi method was used to realize 18 experiments that used these optimal ranges. The magnet thickness (B) and rotor height (D) were identified as common high-impact design variables for the two responses of average torque and torque ripple. In addition, the slot-opening width (E) was found to be the most effective for minimizing the torque ripple.

The multiresponse that consisted of the average torque, the torque ripple, and the THD was reduced to a single response (GRG). Following this, the optimal design variables were obtained very quickly by using gray relational analysis based on the Taguchi method. The optimal combination of design variables was obtained as A1-B1-C1-D2-E1-F1-G1 according to both the Taguchi method and the gray relational analysis based on the Taguchi method.

A considerable amount of multiresponse improvement was achieved according to the results of the two optimizations. The performance improvements in relation to average torque, torque ripple, and THD were 20.6%, 32.0%, and 24.5%, respectively. In a future study, experimental research can be carried out to support these optimization results.

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