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Investigation of the computational speed of Laguerre network-based MPC in the thermal control of energy-efficient buildings

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Abstract: The design of computationally efficient model predictive control (MPC) systems for the thermal control of buildings is a challenging task since long prediction horizons may be needed, which can take a significant computational time, especially when multizone buildings are considered. In this paper, we investigate the computational performance of a potential approach for this purpose, Laguerre network-based MPC (LN-MPC), for thermal control of buildings, where parameterization of control input over the prediction horizon is used to reduce the number of decision variables. The computational performance of the suggested framework with comparison to the classical MPC framework is investigated through a detailed case study. It was observed that although LN-MPC can produce almost the same results as the classical MPC with a considerably smaller number of decision variables, it has no computational advantage. The potential reasons behind the lack of improvement in the computational performance of LN-MPC are also discussed.

Key words: Model predictive control, Laguerre basis functions, computational efficiency, energy-efficient buildings

1. Introduction

Climate change and decreasing fossil fuels are future worries for our world. To alleviate these worries to some extent, legislations are expected to come into existence in the near future to put pressure on greenhouse gas emissions and to motivate the use of renewable energy [1]. A simple solution to address the above problems partly necessitates all energy consuming sectors to be energy efficient, especially the building sector, which is responsible for approximately 40% of total end-use energy consumption [2]. About half of the building energy consumption is through heating, ventilation, and air conditioning (HVAC). Currently, most HVAC systems are either manually controlled or through heuristically designed rule-based controllers, which, most of the time, are not energy efficient, and they can be problematic for satisfaction of the desired thermal comfort level [3,4]. This de facto situation and the importance of the building sector for energy use reduction caused a surge in interest both from control companies (for example, Siemens, Honeywell, Johnson Controls) and the academic community to develop advanced control methods for energy-efficient thermal control of buildings. For example, it was specified in [5,6] that the number of publications on predictive control of buildings (a strain of advanced model-based control methods) in 2015 was 31 times that in 2000.

MPC is the most appropriate and popular control method for thermal control of buildings in terms of energy efficiency [7–11] since (i) they have many measurable or predictable disturbances (ambient temperature, solar, and internal gains), (ii) their thermal response time is not short due to their large thermal mass, which

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can be used to store thermal energy and then use this stored energy effectively later on. The typical control time step for thermal control of a building can vary from one minute to one hour depending on the heating/cooling technology used and the thermal comfort required. Thermal comfort is, in general, characterized by the stay of zone air temperature in specific temperature bands during day and night. In MPC-based thermal control of buildings, the prediction horizon length in terms of the control time step is application dependent, and it can be up to hundreds. For example, consider a single-zone office building heated by a radiator where radiator thermal power can be changed every minute. Assuming that we have accurate weather predictions in the next 6 h, then we can set prediction horizon length $N_p = 360$ in this case.

The main problem for application of MPC for energy-efficient buildings in the case of a long prediction horizon length is the computational demand to solve online MPC optimization problems. In this paper, we investigate the use of a Laguerre network-based MPC (LN-MPC) design framework [12–15] for thermal control of energy-efficient buildings, which can have computational efficiency in some applications compared to the classical MPC framework. In LN-MPC, control input over the prediction horizon is parameterized in terms of a set of orthonormal functions called a Laguerre network. The number of basis functions is typically significantly less than the prediction horizon length, which, in turn, means a much smaller number of decision variables, and hence the possibility of a computationally better performance. The details of the LN-MPC framework will be given in the next section. The computational performance of LN-MPC is compared to that of classical MPC through a detailed case study.

The rest of the paper is organized as follows. In Section 2, the classical MPC framework is briefly given and then LN-MPC framework is introduced in detail. The considered case study for application of the proposed LN-MPC and comparison of the results with those of classical MPC are given in Section 3. The results are discussed in detail in Section 4. Finally, we conclude with the main findings of this work along with some future directions in the Conclusions section.

2. Classical MPC and MPC framework with a discrete-time Laguerre network

A general deterministic convex model predictive control problem can be represented as

$$\text{minimize} \quad \sum_{j=k}^{k+N_p} f(x(j), u(j), w(j)) \tag{1a}$$

subject to

$$x\,(k+j) = Ax\,(k-1+j) + B_u u\,(k-1+j) + B_w w\,(k-1+j) \tag{1b}$$

$$y\,(k+j) = Cx\,(k+j) + D_w w\,(k+j)\,,\ j = 1, \ldots, N_p \tag{1c}$$

$$g\,(x\,(k+j)\,,\ u\,(k+j)\,,\ w\,(k+j)) \le 0,\ \ j = 0, \ldots, N_p - 1, \tag{1d}$$

where $x \,\epsilon\, R^{n_x}$ is the state, $u \,\epsilon\, R^{n_u}$ control input, $w \,\epsilon\, R^{n_w}$ measurable or predictable disturbance, $y \,\epsilon\, R^{n_y}$ measured output, N_p prediction horizon length, and f, g convex functions. In the solution of optimization problem in (1), the initial state at the current time step "k" is necessary for prediction of system response over the prediction horizon, which should be estimated through an observer if it is not measurable. Optimization problem (1) is solved at each time step "k" to determine the control input array over the prediction horizon but the first element of this input array corresponding to the current time step "k" is applied to the system. At the next time step "k + 1", the optimization problem (1) is solved with the new available information. This kind of implementation of model predictive control is called receding horizon control. In this text, we will call the MPC problem in (1) classical model predictive control (C-MPC). Note that in C-MPC the number of decision variables is $N_p n_u$ if we neglect the use of slack variables used in constraint softening. For MIMO systems with a large prediction horizon, $N_p n_u$ may be large, and hence C-MPC may be computationally heavy to be used online. A predictive control method to decrease the number of decision variables, which "may" result in less computational time for solving the online MPC problems, is the Laguerre network-based MPC (LN-MPC). Next, the details of this approach are given.

A Laguerre network (LN) consists of a set of n orthonormal functions $L(k) = [l_1\ (k)\ \ l_2\ (k) \ldots l_n\ (k)]^T$, where $l_i\ (k) \in R$, and they satisfy the recurrence relation

$$L(k+1) = A_L L(k), \tag{2}$$

where $L(0) = \sqrt{\beta}\left[1\ \ -a\ \ a^2\ \ldots\ (-a)^{n-1}\right]^T$,

$$A_L = \begin{bmatrix} a & 0 & 0 & \cdots & 0 & 0 & 0 \\ \beta & a & 0 & \cdots & 0 & 0 & 0 \\ -a\beta & \beta & a & \cdots & 0 & 0 & 0 \\ a^2\beta & -a\beta & \beta & \cdots & 0 & 0 & 0 \\ -a^3\beta & a^2\beta & -a\beta & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots \\ (-a)^{n-2}\beta & (-a)^{n-3}\beta & (-a)^{n-4}\beta & \cdots & \cdots & \beta & a \end{bmatrix}$$

and $\beta = 1 - a^2$ with $0 \le a < 1$. Here, a is called the pole of the LN. A LN is described by two parameters: a and n.

The i-th component of control input vector at time step j over the prediction horizon can be written as

$$u_i\,(k+j) = L_i^T\,(j)\,\eta_i,\ i = 1, 2, \cdots, n_u,\ j = 0, 1, 2, \cdots, N_p - 1, \tag{3}$$

where $L_i\,(j) = \left[l_i^1\,(j)\ \ l_i^2\,(j)\ \cdots\ l_i^{n_i}\right]^T$, $\eta_i = \left[\eta_i^1\ \eta_i^2\ \cdots \eta_i^{n_i}\right]^T$. Here n_i is the number of Laguerre basis functions used in approximating u_i over the prediction horizon window. Moreover, we assume that L_i has pole a_i, which means the possibility of the use of different poles for each input parameterization. Using Eq. (3), we can write

$$u\,(k+j) = L\,(j)\,\eta \tag{4}$$

where $L\,(j) = diag\left(L_1^T\,(j)\,, L_2^T\,(j)\,, \cdots, L_{n_u}^T\,(j)\right)$ and $\eta = [\eta_1\,\eta_2 \cdots \eta_{n_u}]^T$. Using the cost function $f(x\,(j)\,, u\,(j)\,,$ $w\,(j)) = u(j)^T u(j)$, which is input energy and which is what we want to minimize in the thermal control of energy-efficient buildings, and the input and output constraints

$$\Delta u_{\min} \le \Delta u\,(k+j) \le \Delta u_{\max}, \quad j = 0,\ 1, \cdots, N_p - 1,$$

$$u_{\min} \le u\,(k+j) \le u_{\max}, \qquad j = 0, 1, \cdots, N_p - 1,$$

$$y_{\min} \le y\,(k+j) \le y_{\max}, \qquad j = 1, \cdots, N_p,$$

the LN-MPC equations can be summarized as follows:

$$\text{minimize} \sum_{j=k}^{k+N_p} \eta^T L(j)^T L(j)\eta \tag{5a}$$

subject to

$$x\left(k+1+j\right)=Ax\left(k+j\right)+\ B_u L\left(j\right)\eta+B_w w\left(k-1+j\right)$$

$$j=0,\ 1,\cdots,N_p-1, \tag{5b}$$

$$y\left(k+j\right)=Cx\left(k+j\right)+D_w w\left(k+j\right),\quad j=\ 1,\cdots,N_p, \tag{5c}$$

$$\Delta u_{\min}\leq\Delta u\left(k+j\right)=\ L\left(j\right)\eta-\ \left(\,L\left(j-1\right)\eta\ or\,u\left(k-1\right)\ \ if\,j=0\,\right)\leq\ \Delta u_{\max}\,,$$

$$j=0,\ 1,\cdots,N_p-1, \tag{5d}$$

$$u_{\min}\leq u\left(k+j\right)=L\left(j\right)\eta\ \leq\ u_{\max},\ j=0,\ 1,\cdots,N_p-1, \tag{5e}$$

$$y_{\min}\leq y\left(k+j\right)\leq\ y_{\max},\quad j=\ 1,\cdots,N_p. \tag{5f}$$

The control objective is the satisfaction of the operational constraints in (5d)–(5f) with use of minimum thermal energy given by (5a). The output constraint (5f) typically represents the stay of the zone air temperature in a temperature bound for thermal comfort of occupants.

Remark 1 In MPC problems, in general constraints regrading control input and control input rates are hard constraints meaning that violation of them means infeasibility of the optimization. However, constraints on outputs can be soft constraints, which means that violation of outputs is allowed whenever necessary but this violation is penalized in the cost function to allow output violation when only it is needed. Output constraint softening helps avoid infeasibility during online optimization. The output constraint softened version of the optimization problem in (5) is obtained by replacing (5a) and (5f) with the following equations:

$$\text{minimize} \sum_{j=k}^{k+N_p} \eta^T L(j)^T L(j)\eta+\rho(\epsilon_1\left(j\right)+\epsilon_2\left(j\right)), \tag{5a'}$$

$$y_{\min}-\epsilon_1\left(j\right)\leq y\left(k+j\right)\leq\ y_{\max}+\epsilon_2\left(j\right),\quad j=0,\ 1,\cdots,N_p, \tag{5f'}$$

where $\epsilon_1\left(j\right)>0$, $\epsilon_2\left(j\right)>0$ and $\rho\gg 0$. The parameter ρ is a large positive number to penalize output constraint violations.

Remark 2 It can be shown that C-MPC can be obtained as a special case of LN-MPC by setting $a_i=0$, $n_i=N_p$, $i=1,2,\ldots,n_u$. For details the reader is referred to [13].

The design of MPC using Laguerre orthonormal basis functions is a relatively new idea. This new MPC framework may be computationally very advantageous for some applications. Its computational speed is not tested in MPC of energy-efficient buildings. In this sense, to the best knowledge of the author, this is the first study in the open literature investigating this point. It is important to emphasize that having a smaller number of variables in an optimization problem does not always mean better computational speed. As will be discussed later, the computational speed depends on many factors such as the number of constraints, the number of active constraints during optimization, the number of iterations, and the algorithm used.

3. Case study

In this section, we consider a case study for illustration of the proposed LN-MPC framework. The case study considered is a single-zone office building heated by a radiator. The thermal model of the building shown in Figure 1 was created through the MATLAB-based BRCM toolbox [16]. BRCM building thermal models are based on a resistance/capacitance network approach, and this toolbox was validated by its developers against EnergyPlus [17] (which is a popular building thermal dynamics and energy simulation software program). During validations it was observed that the air zone temperatures in building thermal models in the two simulation environments deviated from each other less than 0.5 °C. Hence, the BRCM toolbox has high accuracy in building thermal modeling.

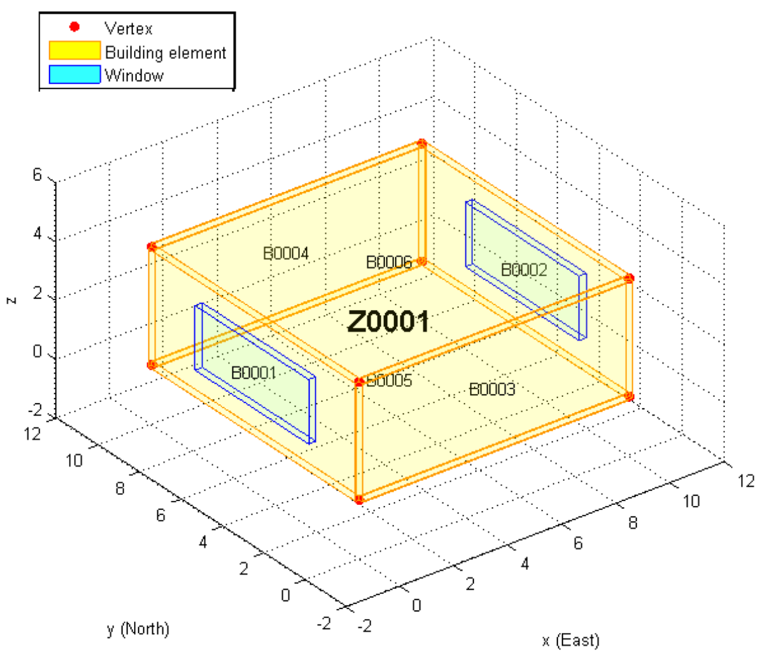

Figure 1. A single-zone office building created using the BRCM toolbox. The building has dimensions 10 m × 10 m × 4 m. The glass area is $10\,m^2$ and the frame area for windows is $2\,m^2$ on the associated walls.

The bilinear building thermal model obtained from the BRCM toolbox has the form

$$x(k+1) = Ax(k) + B_u u(k) + B_w w(k) + B_{wu} w(k)\, u(k) + B_{xu} x(k)\, u(k)\,, \tag{6a}$$

$$y = Cx, \tag{6b}$$

where $x \in R^{n_x}$ denotes the temperatures of building elements and zone air, $u \in R$ the radiator thermal power, $w \in R^{n_w}$ the solar radiations (east, west, south, and north), internal gains, and ambient temperature (which are assumed to be predictable), and $y \in R$ the zone air temperature. This model is a large-scale (with $n_x = 25$) and nonlinear model, and hence cannot be used in a convex MPC. To obtain a linear time-invariant

(LTI) control model to be used in a convex MPC, we use system identification, where the model in (6) is used to create identification and validation input/output data sets. The corresponding signals used for identification and validation purposes are shown in Figures 2–4, where central Europe (Brussels) climate is considered and the data are obtained from the TRNSYS building energy simulation software [18].

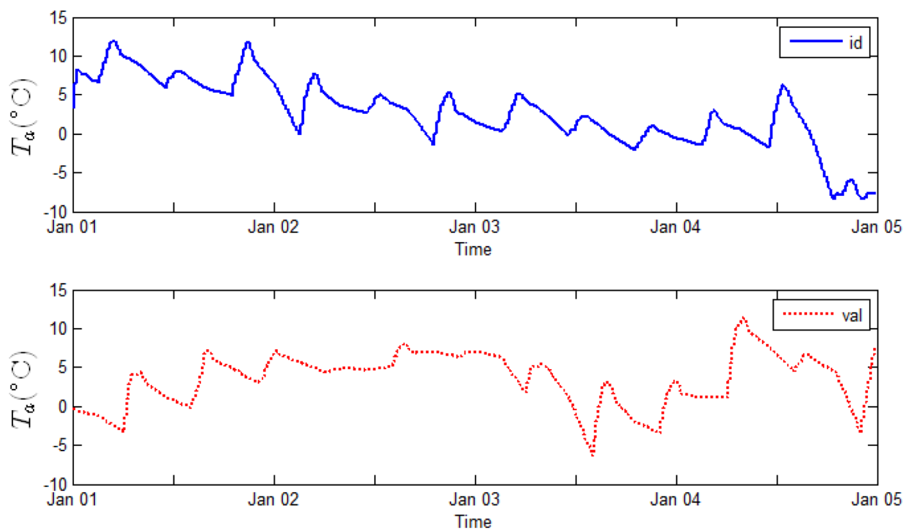

Figure 2. Ambient temperature used for identification (id) and validation (val). Temperature data collected over 4 days were used, which were obtained for the Brussels climate.

System identification was used to identify a reduced-order LTI state-space model to be used for controller designs. The controller model has the form

$$x^r(k+1) = A^r x(k) + B_u^r u(k) + B_w^r w(k) + K^r e(k), \tag{7a}$$

$$y = C^r x^r(k), \tag{7b}$$

where e represents the white noise and the term $K^r e(k)$ the unmodeled dynamics. The state order of the reduced-order model in (7) is 4, which is significantly less than the state order of the large-scale model, which is 25.

Remark 3 The model in (7) is in a special stochastic form known as the "innovation form". Its multistep ahead prediction is given as follows [19]:

$$x_{pr}^r(k+1) = (A^r - K^r C^r) x_{pr}^r(k) + B_u^r u(k) + B_w^r w(k) + K^r y(k), \tag{8a}$$

$$x_{pr}^r(k+j) = A^{j-1} x_{pr}^r(k+1) + \sum_{i=0}^{j-2} A^{j-2-i} \left[B_u^r \; B_w^r\right] \begin{pmatrix} u(k+i+1) \\ w(k+i+1) \end{pmatrix},$$

$$j = 2,\ 1, \cdots, N_p \tag{8b}$$

$$y_{pr}(k+j) = C^r x_{pr}^r(k+j) \tag{8c}$$

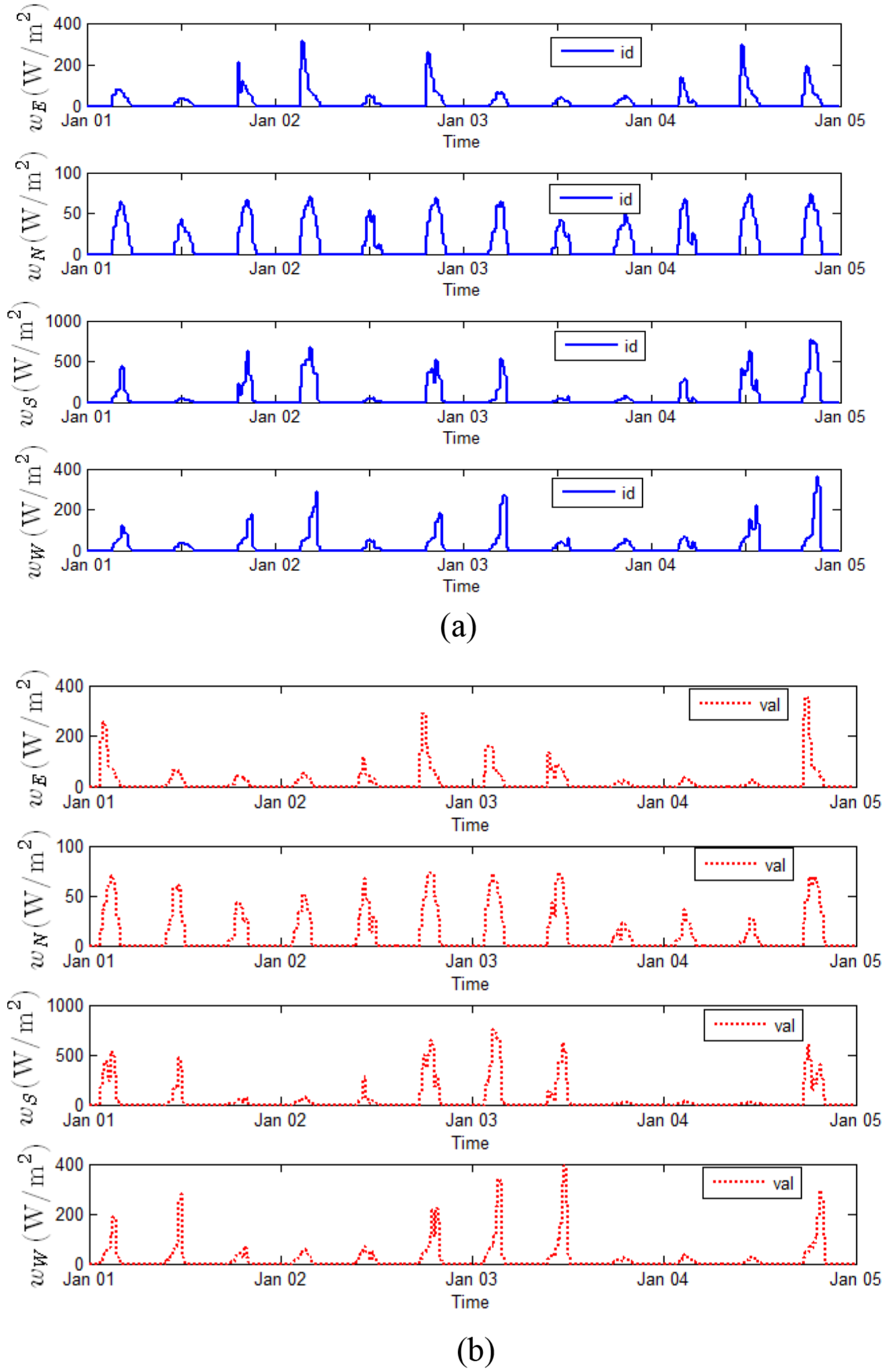

Figure 3. Solar radiation for east (w_E), north (w_N), south (w_S), and west (w_W) facades for the Brussels climate: (a) identification, (b) validation.

where "*pr*" means prediction. As we observe, the multistep ahead prediction of an LTI state-space model is obtained by using the one-step ahead prediction as initial condition and iterating the deterministic part with this knowledge.

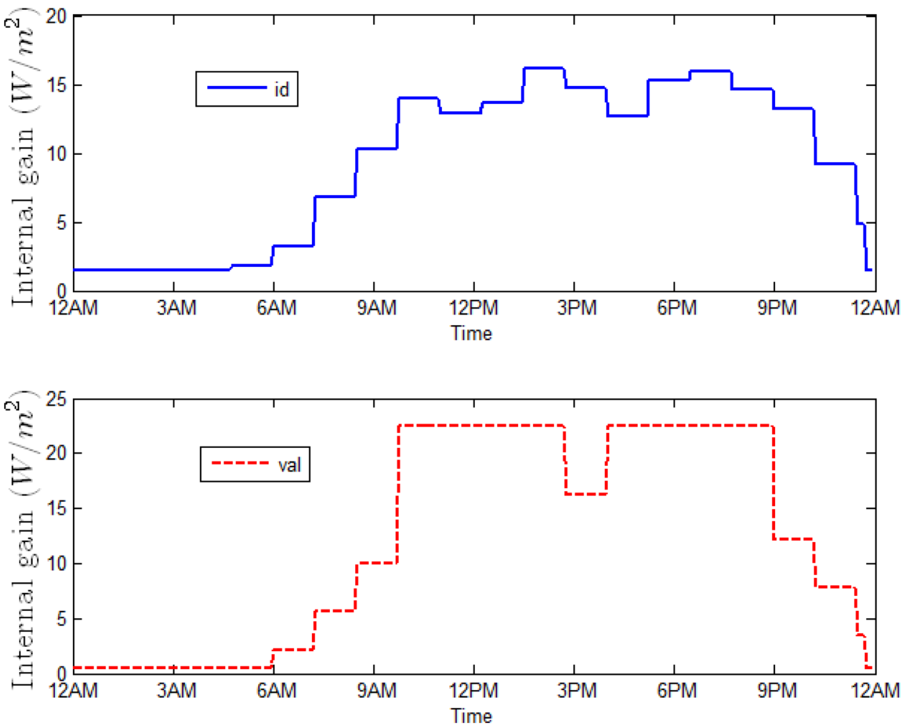

Figure 4. Internal gains: (top) identification, (bottom) validation. This daily variation is assumed to be the same for all other days.

The normalized root mean-squared error (NRMSE) multistep ahead prediction response performance of the reduced-order model with respect to the large-scale model is 98.63 and 98.57 for identification and validation cases, respectively, where NRMSE is defined as

$$NRMSE = 100\,x\left(1 - \frac{\|y - \hat{y}\|}{\|y - mean(y)\|}\right),$$

where $\hat{y}$ is the $N_p = 48$-step ahead zone air temperature prediction response of the reduced-order model, and y is the corresponding value from the large-scale model, which is used as reference.

The results of application of LN-MPC and its comparison with C-MPC are given in the following Table and Figure 5 for different LN parameters. Two solvers were used: "quadprog" and "cplex". The results are the same except for the mean computational time for the control input calculation, which is specified separately in the Table for the two solvers. The designed controllers are tested on the large-scale nonlinear model for one day period. In the Table, different aspects are compared: number of decision variables, total control input over one day, mean thermal comfort violation, mean number of iterations during solution of online MPC optimization problems, and mean computational time during solution of online MPC optimization problems. These results were obtained using a personal computer with Intel Core and i7-3740 QM CPU @ 2.70 GHz. The MPC problems were formulated using YALMIP with "optimizer", which creates an object with a low-level numerical format that can be used to efficiently solve a series of similar problems (reduces YALMP analysis and compilation overhead) [20]. As the thermal comfort specification, the zone air temperature bound was set to 20–21 $^\circ$C for the day time and 19–20 $^\circ$C for the night time.

In the Table, the minimum number of Laguerre basis functions for a given pole to produce results close

Table. Application of LN-MPC and its comparison with C-MPC for different LN parameters.

	Classical MPC	LN-MPC-1 (a = 0.2)	LN-MPC-2 (a = 0.6)	LN-MPC-3 (a = 0.8)
Number of decision variables	48	33	14	10
Total control input (=total used energy)	3.4026e+03 J	3.4558e+03 J	3.4459e+03 J	3.4282e+03 J
Mean thermal comfort violation	0.0233 °C	0.0231 °C	0.0232 °C	0.0236 °C
Mean number of iterations	17.9094	18.3380	17.6237	18.7979
Mean input computation time (for one control time step)	quadprog: 0.090 s cplex: 0.018 s	quadprog: 0.3649 s cplex: 0.0458 s	quadprog: 0.2565 s cplex: 0.0359 s	quadprog: 0.1623 s cplex: 0.0311 s

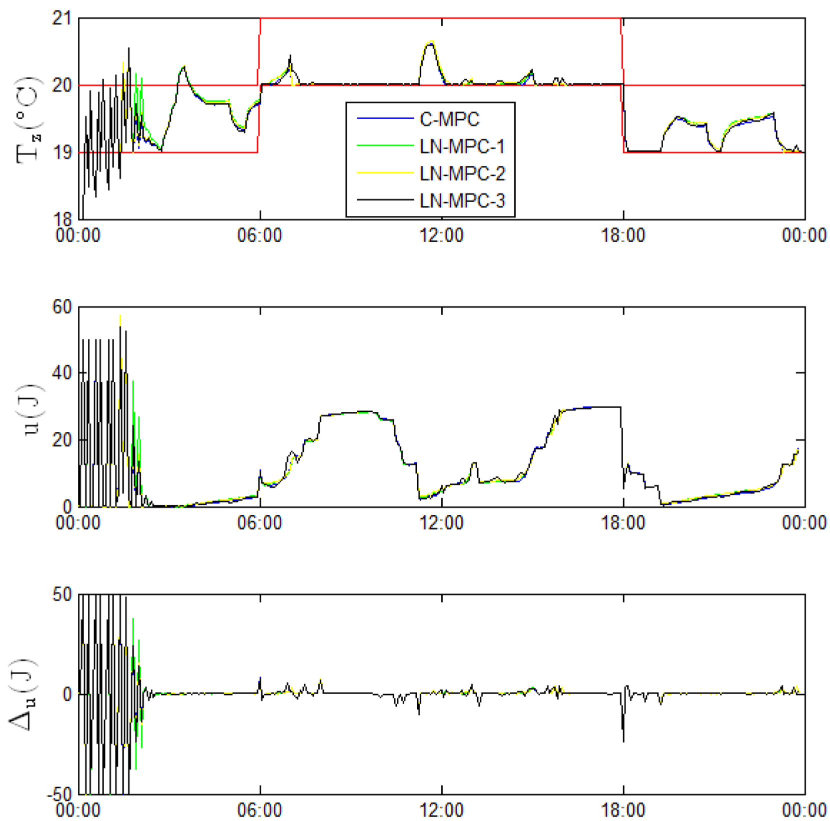

Figure 5. Comparison of classical MPC and Laguerre-based MPC (with different poles) over one day: zone air temperature (top), control input (middle), and rate of change of control input (bottom). The red lines in the top figure are lower and upper thermal comfort bounds for day and night.

to C-MPC results was found by trial and error since there is no analytical method to determine their optimal values.

4. Discussion of results

The first observation from the Table and Figure 5 is that the control input results and the zone air temperatures are almost the same for both C-MPC and LN-MPC with different pole values where in LN-MPC a smaller number of variables is used, especially a significantly smaller number of variables in the LN-MPC-3 case.

This, expectedly, shows that with the LN-MPC framework the number of decision variables can be reduced significantly. Since the total control input energy levels used are almost the same, C-MPC and LN-MPC frameworks have energy saving performances close to each other under almost the same thermal comfort violation degrees. However, unexpectedly, we have a LN-MPC framework with a worse computational speed performance (for all considered LN-MPC cases) compared to C-MPC. At first glance, this may be surprising, for example for the LN-MPC-3 case since we have 10 decision variables in this case compared to 48 decision variables in C-MPC. Next, we discuss this unexpected computational performance of the LN-MPC framework.

Here, in comparing the LN-MPC with C-MPC, we use the same solvers ("quadprog" or "cplex") and the same algorithm (C-MPC results are obtained from LN-MPC by setting n= $N_p = 48$, $a = 0$). Although "cplex" is faster than "quadprog" by an order of magnitude, LN-MPC is slower than C-MPC independent of which solver is used. As a result, under the same solver and algorithm, if the computational speed of the LN-MPC framework with a much smaller number of variables is worse than C-MPC, one may think that the number of iterations may be higher in LN-MPC. However, from the Table, we see that the average numbers of iterations in both frameworks are close to each other, around 18 iterations per each time step, and hence this is not the cause.

Another cause may be active constraints encountered during optimization. The thermal comfort of the building is specified by lower and upper zone air temperature bounds. The designed controllers try to keep the zone air temperature within these bounds with minimal energy. Most of the time, minimal energy use corresponds to bringing zone air temperature to the lower bound as seen clearly in the top subfigure in Figure 5. As a result, active input and especially output constraints are unavoidable in thermal control of energy-efficient buildings. However, these active constraints equally exist for both LN-MPC and C-MPC since the numbers of constraints in both frameworks are the same. LN-MPC's being worse implies that there should be another reason for its worse computational performance.

Our guess about the cause of the worse computational speed of the LN-MPC framework is that when a basis is used for the control input, the optimization problem becomes less sparse, and this deteriorated sparsity deteriorates the structure in the optimization, which, in turn, leads to poor linear algebra during optimization. However, theoretical analysis is necessary to confirm this.

As a final comment, output temperature violations, especially in the initial period in the top subfigure in Figure 5, may come from model mismatches (since MPC controllers are designed based on an LTI state-space model but tested on the large-scale nonlinear BRCM building model), or from Kalman filter state estimation error.

5. Conclusions

Thermal control of buildings may need large prediction horizons, and when this fact was combined with large-scale multizone building cases, use of a computationally fast MPC framework is a must. In this paper, we investigated through simulations of a detailed case study whether or not the Laguerre network-based MPC, which uses basis functions to parametrize control input over the prediction horizon, provides a computational speed advantage in the context of thermal control of energy-efficient buildings. In fact, to the best of the author's knowledge this is the first study in the literature including other fields that investigates and compares with classical MPC the computational speed of the LN-MPC framework. The building thermal model was created through the high-fidelity BRCM software, and the control model was obtained using system identification. The performance of the designed controllers all was tested on the detailed nonlinear large-scale model.

From the studied case study, it was seen that although the LN-MPC framework uses a significantly smaller number of decision variables, it does not provide a computational advantage over the classical MPC. The main culprit is guessed to be the destroyed sparsity when a basis function formulation is adopted, and hence as a future work, a theoretical analysis and justification are planned. The unexpectedly slow performance of LN-MPC on the studied case opens up a new research area for the community working on MPC to study theoretically computational speed analysis of the LN-MPC framework. Such studies may help predetermine whether the LN-MPC framework will provide a computational speed advantage for a given application or not. Alternatively, for a given application modification of the LN-MPC framework in terms of better computational speed in the case the original LN-MPC framework fails can be another research direction.

The main finding or message of this study is that in an MPC optimization problem, even if the same solver and algorithms are used, a smaller number of decision variables may not always lead to better computational performance.

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