

Discrete design optimization of distribution transformers with guaranteed optimum convergence using the cuckoo search algorithm

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Abstract: Transformer design optimization methods presented in the literature rarely yield solutions directly applicable in production; the design engineer usually needs to convert the theoretical solution to a practical one. This problem is addressed in this paper, and a discrete transformer design optimization method is proposed that yields solutions with commercially available or productionally feasible dimensions, thus eliminating the need for further efforts of the design engineer to make the theoretical solution a feasible one. The cuckoo search, a nature-inspired metaheuristic algorithm, is used as the optimization algorithm in this study, and it is shown that the guaranteed global optimum solution is attained in a single run. Furthermore, a simple method is proposed to reduce the number of objective function and constraint calculations. The method is based on skipping calculations for design vectors recurring during the search process by use a caching technique. It is envisaged that the use of the proposed method will make a significant contribution to the streamlining of the quotation and design processes in the transformer industry as well as standardization of production materials.

Key words: Distribution transformer, discrete transformer design optimization, cuckoo search, guaranteed global convergence, caching technique

1. Introduction

Transformers are the most important and costly components of electrical transmission and distribution (T&D) networks. About 7.4% of global power production was lost in T&D networks in 2011, typically shared equally between power lines and transformers, and the estimated value of these losses is 186.3 billion USD. The losses that occurred in distribution transformers in the same year is estimated to 657 TWh of electrical energy per annum, and it is projected that 402 TWh can be saved annually by 2030 with the adoption of higher-efficiency distribution transformers [1].

Due to the significant savings potential, many economies in the world have recently adopted energy-efficiency requirements or incentive programs mandating or promoting the use of energy-efficient transformers to reduce losses in their T&D networks. On the other hand, increases in transformer efficiency are subject to increases in transformer weight and size, sometimes as much as 50% or more. The transformer manufacturing industry is thus faced with the challenge to develop the indisputably best (optimum) designs since, in today's highly competitive market environment, it will be too difficult to expect their customers to fully compensate for the inevitable increase in material costs.

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Transformer design optimization (TDO) is defined in the literature as a mixed-integer nonlinear programming problem having complex and discontinuous objective functions and constraints, with the objective of the detailed calculation of the characteristics of a transformer based on national and/or international standards and transformer user requirements, using available materials and manufacturing processes, to minimize manufacturing cost or total owning cost (TOC), while maximizing operating performance [2,3].

There are several types of objective functions defined in the literature for TDO, but the most commonly used ones are given below. A detailed description of the objective function types can be found in Chapter 2 of [2].

- Minimization of transformer manufacturing cost; that is, total material cost, labor cost, and the manufacturing overhead.
- Minimization of total owning cost (TOC), which can be defined as the life cycle costs associated with purchasing and operating a transformer, calculated as the sum of purchase price and annualized cost of losses in present value terms.

Transformers and transformer design optimization are two areas that have been extensively studied in the literature, for which bibliographical surveys and general backgrounds of research and developments can be found in review papers [4–9].

The remainder of this paper is organized as follows: Section 2 presents the need for discrete TDO (DTDO) and the relevant literature. Section 3 describes the mathematical formulation of the DTDO problem and provides a brief theoretical background for the proposed method comprising the cuckoo search optimization algorithm and the caching technique. Section 4 presents the results of the application of the proposed method to three different distribution transformer ratings. Finally, Section 5 concludes this paper.

2. Discrete transformer design optimization (DTDO)

In many real-world engineering design optimization problems, design variables cannot have arbitrary or continuous values. Instead, some or even all of the design variables must be chosen from a set of discrete or integer values for practical reasons. For example, the diameters of a pipe, thickness of a structural member, or size of a screw are discrete design variables since they may have to be selected from commercially available or standard sizes. Also, many other design variables such as the number of bolts, number of teeth of a gear, or number of coils of a spring must be integers.

This is valid for the TDO problem as well; for instance, winding conductors must be selected from commercially available sizes, core material must be slit to certain widths, etc.

The TDO methods presented in the literature rarely yield solutions directly applicable in production; the design engineer usually needs to spend additional efforts to convert the theoretical solution to a practically feasible one.

One method to tackle this problem, which is still being used by major players in the transformer industry, is to use a two-step optimization process. In the first theoretical design optimization step, feasible design requirement is relaxed and consequently several good theoretical designs are obtained. In the second feasible design optimization step, one good theoretical design is selected as the base to narrow the range of a few design variables to their selected good theoretical design counterparts, and an exhaustive search is made in the space defined by these ranges with standard material dimensions.

The above problem is addressed in this paper, and a DTDO method is proposed that yields solutions with commercially available or productionally feasible dimensions, thus eliminating the need for further efforts of the design engineer to make the theoretical solution a feasible one.

A brief overview of the TDO literature in which the use of discrete design variables is explicitly indicated is given in the following:

In most of the literature where discrete design variables are used, this is done in conjunction with the use of the multiple design method, which is a heuristic technique that assigns many alternative values to the design variables so as to generate a large number of alternative designs and finally to select the design that satisfies all the problem constraints with minimum manufacturing cost [2]. Examples of such literature are [2,3,10–12].

Azizian et al. [13] used the genetic algorithm, artificial bee colony, and particle swarm optimization algorithms to optimize cast-resin dry-type transformers using 18 design variables, all of them being discrete.

In their paper, Zhang et al. [14] employed an improved adaptive genetic algorithm to optimize the TOC of a 50-kVA transformer, where binary coding of chromosomes was used and design variables were discrete.

Tamilselvi and Baskar discussed in their paper [15] the application of the covariance matrix adaptation evolution strategy for the TDO of a 400-kVA transformer, minimizing four different objectives. All the design variables used are integer or discrete; however, discrete variables are employed for current densities of LV and HV windings, which would most probably not result in standard conductor dimensions for those windings.

3. Methodology

3.1. Purpose and scope of the study

The purpose of this study is to address the two-step design optimization problem, and in this regard to propose a DTDO method that yields solutions with commercially available or productionally feasible dimensions, thus eliminating the need for the additional efforts of the design engineer to make the theoretical solution a feasible one.

Main technical characteristics of the transformers considered in this study are as follows:

- Three-phase, oil-immersed distribution transformers
- Wound core construction
- Copper foil for low-voltage (LV) and enameled copper round wire for high-voltage (HV) conductors

Minimization of main material cost, manufacturing cost, and TOC are the three objective function options available in the software prepared for this study.

3.2. Objective function, design variables, and constraints

The objective function used in this study is to minimize the transformer main material cost as defined with the following formula:

$$\min Z(\vec{x}) = \min \sum_{j=1}^8 c_j f_j(\vec{x}) \quad (1)$$

where c_j and f_j are the unit cost (USD/kg) and the weight (kg) of each component j of the eight main materials listed in Table 1, and \vec{x} is the vector of the design variables [3].

The eight design variables used in this study, together with their type and unit of measure, are given below:

Table 1. List of main materials with unit costs.

Main material	Unit cost USD/kg
LV winding material	12.01
HV winding material	12.01
Core material	6.01
Insulating paper	7.72
Duct strips	8.58
Insulating liquid	1.72
Tank sheet steel	1.03
Corrugated panel material	1.20

1. Number of turns of the LV winding (integer)
2. Magnetic induction (integer; Gauss)
3. Width of the core leg (discrete; multiples of user selectable steps, 10 mm default)
4. Width of LV conductor (discrete; multiples of user selectable steps, 10 mm default)
5. Thickness of LV conductor (discrete; 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.2, 1.4, 1.6, 1.8, 2.0, 2.24 mm)
6. Nominal diameter of HV conductor (discrete; 0.50, 0.56, 0.63, 0.71, 0.80, 0.90, 1.00, 1.12, 1.25, 1.40, 1.60, 1.80, 2.00, 2.24, 2.50, 2.80, 3.15, 3.55, 4.00, 4.50, 5.00 mm)
7. Number of end cooling ducts in the LV winding (integer)
8. Number of end cooling ducts in the HV winding (integer)

The DTDO problem must satisfy the following constraints:

- Designed no-load losses must be smaller than guaranteed no-load losses plus tolerance
- Designed load losses must be smaller than guaranteed load losses plus tolerance
- Designed total losses must be smaller than guaranteed total losses plus tolerance
- Transformer impedance voltage must be between a minimum and a maximum impedance voltage plus tolerance
- The total heat produced by the total losses of the transformer must be smaller than the total heat that can be dissipated by the combined effects of conduction, convection, and radiation
- Transformer temperature rise must be smaller than maximum temperature rise
- $lb_j \leq x_j \leq ub_j, j = 1, 2, \dots, 8$, where lb and ub are lower and upper boundaries on the design variables, respectively
- $x_j \geq 0, j = 1, 2, \dots, 8$

The method given in Chapter 2 of [2] is used for the design calculation of the transformer, which, in our opinion based on our experience in the transformer industry, is the most realistic and complete conventional design method available in the literature, to the best of our knowledge. The method was coded in MATLAB, extended where necessary to cover other power/voltage ratings based on our experience. Mathematical functions of core loss and heat transfer curves were obtained by curve fitting; these mathematical functions are given in Appendix A. A routine was added to calculate the minimum width of the corrugated panel with which the heat produced in the transformer can be dissipated.

3.3. Cuckoo search algorithm

The cuckoo search (CS), developed by Yang and Deb [16], is a swarm intelligence-based optimization algorithm built on brood parasitism of some cuckoo species. The cuckoo reproduction process is described with the following three idealized rules:

- Each cuckoo lays only one egg (solution) at a time and drops it in a nest randomly chosen.
- The best nests with high quality of eggs that are more similar to the host bird's eggs have the opportunity to develop and carry over to the next generation.
- The number of host nests (population) is fixed and a host bird can discover an alien egg (low quality solution) with a probability $p_a \in (0, 1)$. In such a case, the host bird can either throw the egg away or abandon the nest and build a completely new one in a new location.

It was mathematically proved in [16,17] that the CS algorithm can satisfy the global convergence requirements and thus has guaranteed global convergence properties.

The pseudocode of the overall optimization algorithm used in this study, comprising CS and the caching method described in Section 3.6, is given in Figure 1.

There is basically a single key parameter for CS, discovery rate p_a , which was suggested as $p_a = 0.25$ in [16].

The MATLAB code for the constrained optimization version given in [18] is used in this study with enhancements made to handle discrete design variables. It should be noted that the objective function is calculated twice for each iteration in this version of CS.

3.4. Parameter tuning of the CS algorithm

The impact of its single key parameter p_a on the performance of the CS algorithm was analyzed by conducting parameter tuning studies, and it was observed in general that the algorithm is insensitive to its key parameter for the TDO problem. Therefore, $p_a = 0.25$ is used in this study as suggested in [16].

3.5. Penalty calculation method

A static penalty method is used in this study for no-load and load loss related constraints, and a death penalty is used for impedance voltage and temperature rise constraints. Details of penalty calculation are given in Appendix B.

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Define objective function as min or max  $f(\vec{x})$ ,  $\vec{x} = (x_1, x_2, \dots, x_d)$  and constraints
Define the discovery rate  $p_a \in [0, 1]$  and MaxGeneration
Initialize a population of  $n$  host nests with random solutions
Create empty hash table
Update hash table
while ( $t < \mathbf{MaxGeneration}$ )
    Get a cuckoo (say  $\vec{x}_i$ ) randomly by Lévy flights
    Evaluate its quality/fitness  $F_i$  as follows:
    if  $\vec{x}_i$  exists in hash table
        Get objective function value  $F_i$  from hash table
    else
        Calculate objective function value  $F_i$ 
        Update hash table
    end if
    Choose a nest among  $n$  (say  $j$ ) randomly
    if ( $F_i < F_j$ )
        Replace  $j$  by the new solution
    end if
    Abandon a fraction  $p_a$  of worse nests and build new ones
    at new locations via Lévy flights
    Keep the best solutions (or nests with quality solutions)
    Rank the solutions and find the current best solution
end while

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Figure 1. Pseudocode for the overall optimization algorithm.

3.6. Reduction of the number of objective function calculations

Stochastic search is used as the core mechanism in many metaheuristic algorithms, in which random choices are made in the search direction as the algorithm iterates toward a solution. Consequently, it is possible that in an iteration the algorithm may come up with the same set of design variable values used in a previous iteration, therefore for which an objective function calculation has already been made.

In the proposed method, referred to as the caching technique, the set of design variable values as well as the corresponding calculated objective function values are stored in a table. At each iteration, where a new set of design variable values are generated, they are first compared with the table to see if the same set has been used before. If so, the previously calculated objective function value is taken from the table, and hence recalculation of the objective function value is eliminated. Otherwise, calculation of the objective function value is performed.

A hash table, which is a data structure that associates keys with values, is used for implementing the above. The primary operation it supports efficiently is a lookup: given a key (e.g., a person's name), find the corresponding value (e.g., that person's telephone number). It works by transforming the key using a hash function into a hash, a number that the hash table uses to locate the desired value. This hash maps directly to a bucket in the array of key/value pairs, hence the name 'hash map'. The mapping method lets the user directly access the storage location for any key/value pair.

In this study, MATLAB's Map Containers are used to implement hash tables. The key for each iteration is obtained by concatenating design variable values converted to a string by using the num2str function of MATLAB.

Using the caching technique for improving the performance of optimization has been addressed in the literature for evolutionary algorithms, specifically for genetic algorithms and genetic programming.

3.7. Impact of using narrow boundary conditions

In almost all constrained engineering problems, boundary conditions, i.e. upper and lower limits of the design variables are defined, which is the case for the TDO problem as well. It is assumed in this study that an experienced engineer would be able to estimate optimum values of the design variables within $\pm 25\%$ limits. In addition to using the caching technique to reduce the number of objective function calculations, the impact of narrowing boundary conditions has also been investigated in this study.

4. Computational results and discussion

4.1. Technical characteristics and constraints of the transformers studied

Design optimizations were performed for three transformers with the following technical characteristics, in addition to those given in Section 3.1:

- 160, 400, and 630 kVA power ratings
- Primary/secondary voltage 20/0.4 kV
- Loss and short-circuit impedance values in accordance with designs given in [2] (Table 2)
- Maximum temperature rise limits as given below:
 - Average winding temperature rise 60 K
 - Top oil temperature rise 55 K
- Other characteristics in accordance with the standard IEC 60076-1

Table 2. Constraints and tolerances of the transformers studied.

Power rating	Constraints		
	P_0 (W)	P_k (W)	U_k (%)
160 kVA	425	2350	4
400 kVA	750	4600	4
630 kVA	1100	8900	6
Tolerances	+15%	+15%	$\pm 10\%$
	+10%		

P_0 = No-load losses, P_k = load losses, U_k = short-circuit impedance.

For calculation of the main material costs, unit costs given in Table 1 were used.

4.2. Performance tests

The main program, objective function calculation routine, and optimization algorithm subroutine were coded in MATLAB 2014a, including the implementation of the caching technique. The user interface of the TDO software is shown in Figure 2. The software has the feature of exporting all the feasible solutions found during the optimization process to Excel.

Performance tests have been made using a mainstream notebook computer with 2.60 GHz Intel Core i5-3320M CPU and 4 GB RAM,

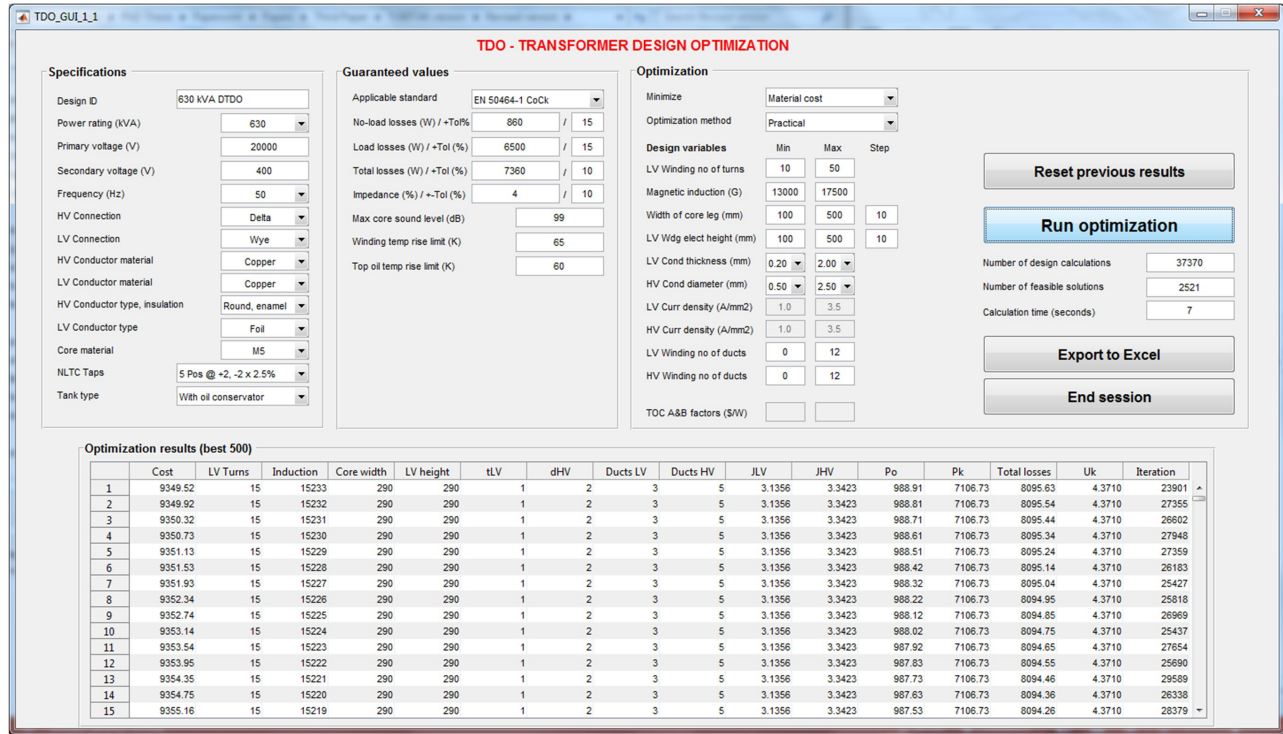


Figure 2. TDO user interface.

- For the three types of transformers with specifications as given in Section 4.1
- Using the same boundary conditions for all three types of transformers, defined widely apart to handle even a wider range of distribution transformer ratings than those tested
- Using the eight design variables as given in Section 3.2, and alternatively, using the first six design variables only, keeping the last two design variables corresponding to the number of end cooling ducts in the LV and HV windings fixed, since these two variables are very rarely used in the literature
- Without using the caching technique
- With the following generation and population values
 - Number of generations: 500, 1000, 1500, 2000
 - Population size: 20, 30, 40, 50
- Key parameter value used is as given in Section 3.4
- No stopping criterion was used
- Tests were repeated 20 times

Two measures are defined to evaluate the performance as follows: ε_{mean} is the mean deviation, and ε_{max} is the maximum deviation of multiple consecutive test runs (minimum 10) as compared to the global optimum. Results of the performance tests evaluated in accordance with these criteria are given in Table 3.

Only the results for the eight design variables case are given in Table 3 since ε_{mean} and ε_{max} values for the six design variables case were all zero, corresponding to global optimum solution for all generation/population size combinations.

Table 3. Performance test results for eight design variables case.

Generation	Population	160 kVA		400 kVA		630 kVA		Overall	
		$\varepsilon_{mean}\%$	$\varepsilon_{max}\%$	$\varepsilon_{mean}\%$	$\varepsilon_{max}\%$	$\varepsilon_{mean}\%$	$\varepsilon_{max}\%$	$\varepsilon_{mean}\%$	$\varepsilon_{max}\%$
500	20	0.28	0.34	0.03	0.07	0.69	1.51	0.33	1.51
500	30	0.22	0.34	0.01	0.05	0.13	0.58	0.12	0.58
500	40	0.11	0.34	0.02	0.05	0.01	0.08	0.05	0.34
500	50	0.08	0.34	0.01	0.05	0.12	0.43	0.07	0.43
1000	20	0.15	0.34	0.01	0.05	0.18	1.47	0.11	1.47
1000	30	0.03	0.34	0.00	0.03	0.00	0.00	0.01	0.34
1000	40	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1000	50	0.03	0.34	0.00	0.00	0.00	0.00	0.01	0.34
1500	20	0.09	0.34	0.00	0.00	0.15	1.47	0.08	1.47
1500	30	0.06	0.34	0.00	0.00	0.00	0.00	0.02	0.34
1500	40	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1500	50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2000	20	0.14	0.34	0.01	0.05	0.08	0.77	0.07	0.77
2000	30	0.03	0.34	0.00	0.00	0.00	0.00	0.01	0.34
2000	40	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2000	50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Following these tests, performance tests were performed for the three types of transformers with both mixed-integer and discrete variable versions of the CS algorithm (for the mixed-integer version, only the number of turns of the LV winding is an integer; all other design variables are continuous) using the first six design variables and with maximum number of generations and population size vales as 1000 and 40, respectively; the tests were repeated 100 times. Statistical results and design variable values for the best solutions are given in Table 4.

After the above tests, another set of performance tests was conducted in which the caching technique was used, with normal and alternatively narrow boundary conditions set at $\pm 25\%$ limits of optimum values of the design variables as described in Section 3.7. These tests were performed for 500/30, 1000/40, and 1500/50 maximum number of generations and population size combinations, repeated 20 times for each case, and the results are given in Table 5.

4.3. Evaluation of results

The first set of performance tests showed that 1000/40 maximum number of generations and population size combination is the one with the least number of objective function calculations that yields zero ε_{mean} and ε_{max} performance measures for the eight design variables case. The algorithm yielded global optimum results for all maximum number of generations and population size combinations for the six design variables case.

The performance tests regarding comparison of the mixed-integer and discrete variable versions using 1000/40 maximum number of generations and population size combination clearly showed that the CS algorithm yields very robust results for the mixed-integer TDO problem with an average standard deviation of 0.3; it excels

Table 4. Statistical results and design variable values for the best solutions.

		160 kVA		400 kVA		630 kVA	
Statistics	Unit	MI	DV	MI	DV	MI	DV
Best solution	USD	3531	3569	6354	6428	7402	7442
Mean solution	USD	3532	3569	6354	6428	7402	7442
Worst solution	USD	3533	3569	6357	6428	7403	7442
Standard deviation		0.313	0.000	0.565	0.000	0.060	0.000
Average processing time	s	4.7	7.7	4.6	6.0	5.3	5.1
Best MI & DV cost difference	%		1.076		1.165		0.540
Design variables							
LV number of turns		28	27	18	18	15	14
Magnetic induction	Gauss	16,781	16,855	16,597	16,610	17,500	17,453
Width of the core leg	mm	196.1	220.0	248.9	260.0	268.4	280.0
Width of LV conductor	mm	197.9	180.0	239.0	240.0	229.9	210.0
Thickness of LV conductor	mm	0.383	0.400	0.772	0.700	0.907	1.000
Diameter of HV conductor	mm	0.968	1.000	1.534	1.600	1.649	1.600
Calculated constraints							
No-load losses	W	489	489	863	862	1,214	1,247
Load losses	W	2564	2530	5023	5021	9786	9753
Total losses	W	3053	3019	5885	5884	11,000	11,000
Short-circuit impedance	%	4.068	4.397	4.266	4.318	5.526	5,468

MI = Mixed-integer, DV = discrete variable.

Table 5. Comparison of the number of objective function calculations.

Description	Number of function calculations (FC)			Average	
	160 kVA	400 kVA	630 kVA	FC	% of DV
No. of generations/pop. size 500/30					
Discrete variables (DV)	30,000	30,000	30,000	30,000	
DV with caching	16,502	15,435	14,087	15,341	51.1
DV with caching & narrow bounds	11,985	11,009	8,993	10,662	35.5
No. of generations/pop. size 1000/40					
Discrete variables (DV)	80,000	80,000	80,000	80,000	
DV with caching	26,933	25,336	21,525	24,598	30.7
DV with caching & narrow bounds	19,470	18,112	13,576	17,052	21.3
No. of generations/pop. size 1500/50					
Discrete variables (DV)	150,000	150,000	150,000	150,000	
DV with caching	36,117	34,046	27,816	32,660	21.8
DV with caching & narrow bounds	24,553	23,021	17,242	21,605	14.4

for the DTDO problem with zero standard deviation for all three types of transformers tested, thus verifying that the CS algorithm possesses the guaranteed global convergence property as pointed out in Section 3.3.

The average cost increase of the best DTDO solutions as compared to the best TDO solutions is less than 1%, which is very reasonable.

It should be noted that, for 160- and 630-kVA transformers, the number of turns of the LV winding, an important parameter in transformer design, was decreased for the best DTDO solutions as compared to the

best TDO solutions. The impacts of this, such as increase in no-load losses and decrease in impedance voltage, have been offset by decreasing the height of the core window through decreasing the width of the LV conductor. This can be considered as a justification of the well-functioning of the optimization algorithm used.

The performance tests also showed that, with the use of the caching technique, the number of objective function calculations would be reduced by 69% on the average; another 10% reduction is obtained when the caching technique is used together with narrow boundary conditions, without sacrificing the guaranteed global convergence property in both cases. As expected, the rate of reduction in the number of objective function calculations decreases for smaller maximum number of generations and population size combinations, and increases for higher combinations.

5. Conclusions

In this study, a DTDO method is proposed that yields solutions with commercially available or productionally feasible dimensions, so that they are directly applicable in production. CS is used as the optimization algorithm for the proposed method, which has been mathematically proved that it can satisfy the global convergence requirements.

The results of the performance tests conducted by using three different distribution transformer ratings proved that the CS algorithm yields results with zero standard deviation in a single run for the DTDO problem, in less than 10 s, thus verifying the validity of its guaranteed global convergence property, achieved in a very reasonable time for an optimization run.

The results also showed that feasible solutions are obtained with acceptable cost deviation from theoretically the best solutions found by using the conventional design method.

Use of the caching technique, especially together with narrow boundary conditions, is recommended since it better offsets the disadvantage of the CS algorithm, in which the objective function is calculated twice for each iteration, while still preserving its guaranteed global convergence property.

We are of the opinion that this paper will make an important contribution to the streamlining of the quotation and design processes in the distribution transformer industry as well as standardization of production materials.

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Appendix A: Curve fitting

There are four curves used for conventional design of distribution transformers in Chapter 2 of [2]. Mathematical functions obtained for these curves by using curve fitting methods are given below:

- No-load loss curve (Figure 2.8 in [2])

$$\begin{aligned} SNLL_{TF} = & 59.352025131083FD_{\max}^6 - 490.061843662741FD_{\max}^5 \\ & + 1682.589027202759FD_{\max}^4 + 3072.639488094771FD_{\max}^3 \\ & + 3146.803358051449FD_{\max}^2 - 1712.517232171855FD_{\max} \\ & + 387.025142780563 \end{aligned}$$

where $SNLL_{TF}$ is no-load loss (W/kg) and FD_{\max} is magnetic induction.

- Tank heat transfer by convection coefficient curve (Figure 2.13 in [2])

$$\begin{aligned} TCC = & -0.0000545744472AOR^4 + 0.0092765754930AOR^3 \\ & - 0.5503041809674AOR^2 + 19.5703665004558AOR \\ & - 163.3915803824412 \end{aligned}$$

where TCC is tank convection coefficient (W/m²) and AOR is average oil temperature rise (°C).

- Tank heat transfer by radiation coefficient curve (Figure 2.13 in [2])

$$\begin{aligned} TRC = & 0.88733882883505677AOR^{1.5} \\ & + 20.258956145521502 \log(AOR) \\ & + 2.6813522474580846 \sin(AOR) \\ & - 0.80512690158999911 \cos(AOR) \end{aligned}$$

where TRC is tank radiation coefficient (W/m²) and AOR is average oil temperature rise (°C).

- Corrugated panel heat transfer coefficient curves (Figure 2.14 in [2])

$$\begin{aligned} CPC = & e^{(2.1390993595909102 - 0.20995987044882686 \log(D_{Panel}) + 1.2033475993994058 \log(AOR))} \\ & - 8.6930169541921476 \end{aligned}$$

where CPC is corrugated panel coefficient (W/m²), D_{Panel} is width of corrugated panel (mm), and AOR is average oil temperature rise (°C).

Appendix B: Penalty calculation method

The basic version of the total owning cost (TOC) formula is given by the following:

$$TOC = PC + A \times P_0 + B \times P_k \quad (A.2)$$

where A (USD/W) is the cost of no-load losses, B (USD/W) is the cost of load losses, P_0 (W) is the no-load loss, P_k (W) is the load loss, and PC (USD) is the transformer purchase cost.

A and B coefficients are given as 13.39 USD/W and 2.09 USD/W in Chapter 2 of [2], respectively. Calculations of these coefficients are given in Chapter 8 of [2].

The approach employed to calculate penalty is given below:

- The no-load and load loss factors A and B in Eq. (A.2), which are given as 13.39 USD/W and 2.09 USD/W in Chapter 2 of [2], are used for penalizing. However, since these are given at bid price level, they are normalized to main material cost level.
- For total loss and heat transfer constraints, total loss factor is used, calculated as the weighted average of guaranteed no-load and load losses.
- No-load losses exceeding the guaranteed no-load losses plus the tolerance are penalized by the no-load loss factor.
- Load losses exceeding the guaranteed load losses plus the tolerance are penalized by the load loss factor
- Total losses exceeding the guaranteed total losses plus the tolerance are penalized by the total loss factor.
- The heat that cannot be dissipated by the tank and corrugated panels is penalized by the total loss factor.

The resulting penalty is the sum of all penalties calculated as described above.

The “death penalty” is used for the impedance voltage and temperature rise constraints.