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An improved OMTHD technique for an n-level cascaded multilevel inverter with adjustable DC sources

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Abstract: Optimal minimization of total harmonic distortion (OMTHD) and selective harmonic elimination (SHE) switching techniques are usually employed to reduce generated harmonics of multilevel inverters. In the former technique, the THD of waveform is reduced without elimination of any harmonic order and the latter, in contrast, eliminates selected harmonic orders. In this paper, the harmonic elimination ability of the SHE technique is added to OMTHD and an improved OMTHD technique is proposed for an n-level cascaded multilevel inverter with adjustable DC sources. The main novelty of this switching technique is elimination of some harmonic orders, beside THD minimization. Moreover, optimal DC voltages and switching angles can be estimated simpler than conventional OMTHD technique. To show the advantages of the proposed technique, which is formulated for an n-level inverter, a 7-level one is simulated in MATLAB/Simulink with both improved and conventional OMTHD techniques. The obtained results show the elimination of two elective harmonics as well as THD minimization. Moreover, experiments on a 7-level inverter verify the superiority of the proposed technique.

Key words: Optimal minimization of total harmonic distortion, selective harmonic elimination, cascaded multilevel inverter, adjustable DC sources, genetic algorithm

1. Introduction

Multilevel inverters, because of their numerous advantages in comparison with conventional two-level high frequency PWM converters, can be employed in medium and high power applications such as adjustable speed drives (ASDs) [1], flexible AC transmission system (FACTS) devices [2], and renewable energy sources, as interface converters [3–5]. Producing very low distorted waveform, drawing input current with low distortion, and possibility of operating at lower switching frequencies, which result in lower switching losses, are some attractive features of multilevel inverters. However, their greater number of elements results in more complex and expensive inverters, compared with conventional ones [6].

Among different structures, three main categories are cascaded, diode clamped, and flying capacitor multilevel inverters [6,7]. The first group has attracted more attention, because the number of possible output voltage levels is more than twice the number of DC sources. Moreover, its modularized structure makes its manufacturing as well as extending its voltage levels simpler than the two other groups. However, the necessity of an independent DC source for cascaded multilevel inverters makes them suitable for those applications, in which independent DC sources are available.

Different switching techniques, which have been introduced for multilevel inverters, try to regulate the

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fundamental component of output voltage at the desired value and reduce the harmonic content of this voltage. These switching techniques can be categorized into high and low frequency ones [6,8]. Multilevel PWM and space vector PWM [9] belong to the former and SHE [10,11] as well as OMTHD [12] are examples of the latter, in which reducing harmonic content is considered more. In the SHE technique, some selected harmonics are eliminated, but the OMTHD technique only tries to reduce THD of waveform without elimination of any specific harmonic order. It is obvious that elimination of several harmonic orders in the SHE technique reduces THD, but [13] has shown that the OMTHD technique is more successful in THD reduction. It is worth mentioning that lower switching frequency in the latter techniques decreases switching losses significantly.

In cascaded multilevel inverters with constant DC sources, switching angles are estimated by the SHE and OMTHD techniques to eliminate selective harmonics and minimize THD, respectively. By adjustable DC sources, degrees of freedom is increased and consequently elimination of more harmonic orders and lower THD value can be achieved in the SHE and OMTHD techniques, respectively [12,14]. Moreover, a multilevel inverter with adjustable sources has lower losses, because switching losses depend on DC voltage and these sources do not always operate at their nominal voltage. To benefit from these advantages, adjustable DC voltage can be easily obtained by controlled rectifier and DC–DC converters for AC or DC sources, respectively.

Both the SHE and OMTHD techniques result in nonlinear equations with relevant constraints that can be solved by various methods. Newton–Raphson and sequential quadratic programming (SQP) are iterative methods, whereas resultant theory is an example of an analytical method. However, due to the nonlinearity of such problems, heuristic methods, like genetic algorithm (GA) [15] and particle swarm optimization (PSO) [16], have almost been introduced.

As mentioned earlier, the conventional OMTHD technique does not control any specific harmonic order, and so an unregulated fundamental component as well as presence of all harmonic orders would be expected. Therefore, a nonlinear equality constraint should be considered during THD minimization to regulate the fundamental component and be updated for each new given value. This paper, in contrast, proposes an improved OMTHD technique for a cascaded multilevel inverter with adjustable DC sources, in which regulation of the fundamental component becomes independent from THD minimization. Thus, the optimization process is carried out without any nonlinear equality constraint and consequently there is no need to repeat the process for each new given value. Moreover, some harmonic orders also can be eliminated by the proposed technique.

The remaining parts of the paper are organized as follows: the structure of a cascaded multilevel inverter and its conventional OMTHD technique are described and then an improved OMTHD technique for an n-level inverter is proposed in the next two parts. In part 4, genetic algorithm is briefly explained and employed to estimate the optimal switching angles as well as DC voltages of a 7-level inverter by both OMTHD techniques. Finally, the advantages of the proposed switching technique are verified by experiments on a laboratory test setup.

2. Cascaded multilevel inverter

To propose the improved OMTHD switching technique for an n-level cascaded inverter, the typical structure of this inverter beside the conventional OMTHD technique is briefly described in the next two sections.

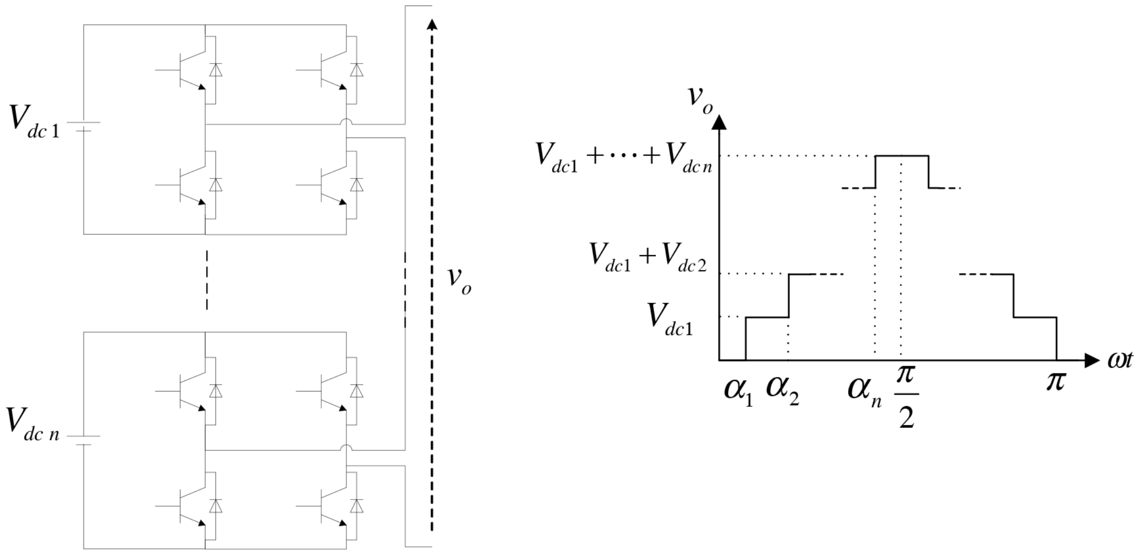
2.1. Structure

The single-phase structure of an n-level cascaded inverter, which is composed of $(n-1)/2$ series H-bridge inverters, beside its half-cycle voltage is presented in Figure 1, in which α_1, α_2, ..., α_n and V_{dc1}, V_{dc2}, ..., V_{dcn}

are switching angles and voltage of DC sources, respectively. It is seen that each H-bridge inverter needs its own independent DC source. Moreover, it is assumed that the maximum voltage of each adjustable DC source is equal to constant value V_{dc} and these can be independently varied between zero and V_{dc}. Hence, the voltage of DC sources can be written as coefficient of V_{dc}.

$$V_{dc1} = a_1\, V_{dc}\,,\; V_{dc2} = a_2\, V_{dc}\,,\; \ldots,\; V_{dc\,n} = a_n\, V_{dc} \tag{1}$$

The Fourier series of produced voltage of an n-level inverter in (2) contains no even-order harmonics, due to symmetry of waveform, and the amplitude of its odd-order harmonics depends on all DC voltage coefficients as well as switching angles [10].



(a) (b)

Figure 1. Single-phase n-level cascaded inverter: (a) structure, (b) its half-cycle voltage.

$$V_o\,(t) = \sum_{k=1}^{\infty} V_k\; \sin\,(k\omega t) \quad , \quad V_k = \begin{cases} \frac{4\,V_{dc}}{k\pi}\,(a_1.\,\cos\,(k\alpha_1) + \cdots + a_n\,.\cos\,(k\alpha_n))\,, & \text{for odd } k \\ 0, & \text{for even } k \end{cases} \tag{2}$$

2.2. Conventional OMTHD technique

THD of any distort waveform can be calculated by (3)

$$THD = \sqrt{\frac{\sum\limits_{k=2,3,\ldots}^{\infty} V_k^2}{V_1^2}}, \tag{3}$$

where V_1 and V_k are amplitude of the fundamental component and harmonic orders of waveform, respectively. Considering all harmonic orders, which is required for precise calculation of THD, is almost impossible for those waveforms like Figure 1b with broad spectral content. Instead of (3), another form of the THD formula, which considers effective value of waveform, can take all harmonic orders into account (see (4)) [12].

$$THD = \sqrt{\left(\frac{V_{rms}^2}{V_{1,rms}^2}\right) - 1}, \tag{4}$$

where $V_{1,rms}$ is the effective value of the fundamental component and V_{rms} represents the effective value of the whole waveform. The amount of V_{rms} for an n-level waveform, with specified parameters in Figure 1b, can be simply calculated by the well-known definition of the effective value (see (5)). The effective value of the fundamental component is also calculated, considering its given amplitude in (2).

$$V_{rms} = V_{dc}\sqrt{\frac{2}{\pi}\left(a_1^2\,(\alpha_2-\alpha_1) + (a_1+a_2)^2\,(\alpha_3-\alpha_2) + \cdots + (a_1+\cdots+a_{n-1})^2\,(\alpha_n-\alpha_{n-1}) + (a_1+\cdots+a_n)^2\,\left(\frac{\pi}{2}-\alpha_n\right)\right)} \tag{5}$$

$$V_{1,rms} = \frac{V_1}{\sqrt{2}} = \frac{4\,V_{dc}}{\sqrt{2}\,\pi}\,(a_1.\,\cos(\alpha_1) + \cdots + a_n\,.\cos(\alpha_n)) \tag{6}$$

Finally, substitution of (5) and (6) in (4) results in a precise formula to calculate THD of a phase voltage of an n-level inverter with adjustable DC sources.

$$THD = \sqrt{\frac{\pi}{4}\left(\frac{a_1^2\,(\alpha_2-\alpha_1) + (a_1+a_2)^2\,(\alpha_3-\alpha_2) + \cdots + (a_1+\cdots+a_{n-1})^2\,(\alpha_n-\alpha_{n-1}) + (a_1+\cdots+a_n)^2\,\left(\frac{\pi}{2}-\alpha_n\right)}{(a_1.\,\cos(\alpha_1) + a_2.\cos(\alpha_2) + \cdots + a_n\,.\cos(\alpha_n))^2}\right) - 1} \tag{7}$$

It is obvious that minimization of (7) cannot guarantee obtaining the desired fundamental component and so a nonlinear equality constraint should also be considered.

$$M = \frac{V_1}{n\,V_{dc}} = \frac{4}{n\,\pi}\,(a_1.\,\cos(\alpha_1) + a_2.\cos(\alpha_2) + \cdots + a_n\,.\cos(\alpha_n)) \quad , \quad 0 < M \leq \frac{4}{\pi}, \tag{8}$$

where M is modulation index, which is obtained from division of the amplitude of the fundamental component by summation of nominal values of DC sources. In other words, M can be assumed as the normalized value of the fundamental component. To obtain meaningful results, inequality constraints for DC voltage coefficients as well as switching angles, which are presented in (9), should be also considered beside (8) as constraints of the THD formula.

$$0 \leq a_1, a_2, \ldots, a_n \leq 1 \quad ; \quad 0 \leq \alpha_1 \leq \alpha_2 \leq \cdots \leq \alpha_n \leq \frac{\pi}{2} \tag{9}$$

Finally, it can be concluded that exerting the conventional OMTHD technique in an n-level inverter with adjustable DC sources results in an objective function with $2n$ independent variables beside equality and inequality constraints.

3. Improved OMTHD technique

3.1. Description

As it was mentioned, the conventional OMTHD technique does not eliminate any harmonic order. Moreover, elimination of dominant low-order harmonics by passive filters is more difficult, compared with high-order harmonics. Thus, some switching techniques have been introduced to eliminate low-order harmonics beside THD minimization by merging the conventional SHE and OMTHD techniques [17,18]. In these techniques, the weighted amplitude of some low-order harmonics and THD formula form the objective function, which is minimized by considering its constraints. It is clear that such optimization cannot guarantee elimination of specific harmonic orders for all modulation indices. Moreover, these switching techniques also need to consider a nonlinear equality constraint for regulation of the fundamental component at the desired value.

The improved OMTHD technique proposed in this paper can eliminate $n-1$ harmonic orders and also minimize THD without considering any nonlinear equal constraint to regulate the fundamental component. For a detailed explanation, it is assumed that in an n-level cascaded inverter with adjustable DC sources, we want to eliminate first $n-1$ non-triplen harmonics, minimize THD, and regulate the fundamental component of the produced voltage at a given value. The obtained THD formula includes $2n$ variables, n DC voltage coefficients, and n switching angles, and with given constraints in (9). It is clear that elimination of $n-1$ low-order harmonics means turning their amplitudes to zero, and so the $n-1$ nonlinear equality constraints in (10) are obtained.

$$THD = \sqrt{\frac{\pi}{4}\left(\frac{a_1^2\,(\alpha_2-\alpha_1)+(a_1+a_2)^2\,(\alpha_3-\alpha_2)+\cdots+(a_1+\cdots+a_{n-1})^2\,(\alpha_n-\alpha_{n-1})+(a_1+\cdots+a_n)^2\left(\frac{\pi}{2}-\alpha_n\right)}{(a_1.\,\cos(\alpha_1)+a_2.\cos(\alpha_2)+\cdots+a_n\,.\cos(\alpha_n))^2}\right)-1}$$

$$\begin{cases} \frac{4}{n\,\pi}\,(a_1.\,\cos(\alpha_1)+a_2.\cos(\alpha_2)+\cdots+a_n\,.\cos(\alpha_n))=M \\ \frac{4\,V_{dc}}{5\,\pi}\,(a_1.\,\cos(5\alpha_1)+a_2.\cos(5\alpha_2)+\cdots+a_n\,.\cos(5\alpha_n))=0 \\ \vdots \\ \frac{4\,V_{dc}}{k\,\pi}\,(a_1.\,\cos(k\alpha_1)+a_2.\cos(k\alpha_2)+\cdots+a_n\,.\cos(k\alpha_n))=0 \end{cases} \tag{10}$$

In (10), k is $n-1$ low-order and non-triplen harmonics. It is worth mentioning that any $n-1$ harmonics can be selected and it is not restricted to low-order or non-triplen harmonics. Now, the THD formula and its n nonlinear equality constraints are divided by one of the DC voltage coefficients like a_1 and consequently (11) is derived. Modified equality constraints of $n-1$ low-order harmonics in (11) can be considered as a linear $(n-1)$-variable problem with $n-1$ variables b_2 to b_n, and so these $n-1$ variables, which depend on switching angles, can be simply obtained in (12).

$$THD = \sqrt{\frac{\pi}{4}\left(\frac{(\alpha_2-\alpha_1)+(1+b_2)^2\,(\alpha_3-\alpha_2)+\cdots+(1+b_2+\cdots+b_{n-1})^2\,(\alpha_n-\alpha_{n-1})+(1+b_2+\cdots+b_n)^2\left(\frac{\pi}{2}-\alpha_n\right)}{(\cos(\alpha_1)+b_2.\cos(\alpha_2)+\cdots+b_n\,.\cos(\alpha_n))^2}\right)-1}$$

$$\begin{cases} \frac{4}{n\,\pi}\,(\cos(\alpha_1)+b_2.\cos(\alpha_2)+\cdots+b_n\,.\cos(\alpha_n))=\frac{M}{a_1} \\ \frac{4\,V_{dc}}{5\,\pi}\,(\cos(5\alpha_1)+b_2.\cos(5\alpha_2)+\cdots+b_n\,.\cos(5\alpha_n))=0 \\ \vdots \\ \frac{4\,V_{dc}}{k\,\pi}\,(\cos(k\alpha_1)+b_2.\cos(k\alpha_2)+\cdots+b_n\,.\cos(k\alpha_n))=0 \end{cases} \quad , \qquad b_2=\frac{a_2}{a_1}\ ,\ \ldots\ ,\ b_n=\frac{a_n}{a_1} \tag{11}$$

$$b_i = \frac{\det\begin{pmatrix} b_2\,.\cos(5\alpha_n) & \cdots & b_{i-1}\,.\cos(5\alpha_{i-1}) & -\cos(5\alpha_1) & b_{i+1}\,.\cos(5\alpha_{i+1}) & \cdots & b_n\,.\cos(5\alpha_n) \\ \vdots & & \vdots & \vdots & \vdots & & \vdots \\ b_2\,.\cos(k\alpha_n) & \cdots & b_{i-1}\,.\cos(k\alpha_{i-1}) & -\cos(k\alpha_1) & b_{i+1}\,.\cos(k\alpha_{i+1}) & \cdots & b_n\,.\cos(k\alpha_n) \end{pmatrix}}{\det\begin{pmatrix} b_2\,.\cos(5\alpha_2) & \cdots & b_n\,.\cos(5\alpha_n) \\ \vdots & & \vdots \\ b_2\,.\cos(k\alpha_n) & \cdots & b_n\,.\cos(k\alpha_n) \end{pmatrix}}$$

$$i = 2, 3, \ldots, n \qquad (12)$$

Substitution of these b_i in the THD formula of (11) results in an objective function with no DC voltage coefficients and any nonlinear equality constraint. The obtained formula consists of only n switching angles and it should only satisfy inequality constraints in (9). In other words, this formula represents THD of a voltage waveform with no $n-1$ low-order harmonic orders. After estimating switching angles by optimizing this THD formula, coefficients b_i are calculated. Now, coefficient a_1 can be simply calculated for each modulation index from the first equality constraint in (11) and finally each coefficient a_i $(i = 2, ..., n)$ is simply computed from $a_i = b_i \times a_1$.

With a glance at the proposed switching technique, THD of produced voltage in the absence of some specific harmonic orders is firstly minimized without any nonlinear equality constraint and optimal switching angles are estimated once. Then DC voltage coefficients are simply calculated for each modulation index. It should be mentioned again that optimal switching angles are estimated only once, due to independence of modulation index regulation from THD minimization and then different modulation indices are obtained only by choosing appropriate DC voltage coefficients.

3.2. Discussion

Compared with the conventional OMTHD technique, the main contributions of the proposed switching technique are as follows:

- Elimination of $n-1$ harmonic orders in n level inverter, beside THD minimization
- Very simpler optimization process, because:
- Derived THD formula only consists of n switching angles that lead to smaller search space.
- The problem has no nonlinear equality constraint to satisfy the modulation index.
- The optimal switching angles are estimated only once for all modulation indices.
- DC voltage coefficients depend linearly on the modulation index and are simply calculated for each M, after estimation of switching angles.

4. Genetic algorithm

Among different optimization algorithms, GA has been found on the natural selection process, which drives biological evolution. In this process, the reproduction act of a set of individuals forms a certain generation, which is called a population. Then those individuals of this population with high environmental adaptation survive and produce the next generation through crossover and mutation.

To imitate this natural process, the GA randomly selects individuals, as parents, from the current population and uses them to produce the children of the next generation. For example, each individual has $2n$ chromosomes (n switching angles and n DC voltage coefficients) for the conventional OMTHD technique and n chromosomes (n switching angles) for improved OMTHD. In successive generations, the population satisfies optimization conditions more and so it evolves toward an optimal solution. In the present problem, those individuals that result in THD values, as cost function, with lower amounts are selected. This reproduction process is repeatedly carried out until the last generation satisfies the end condition. This routine is shown in Figure 2. It should be mentioned that the GA employs three main rule types to produce the next generation from the current population:

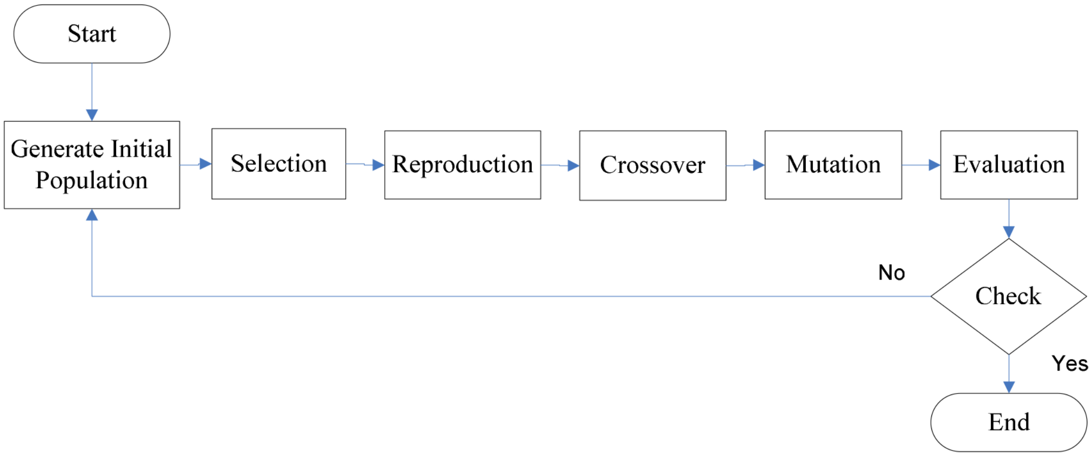


Figure 2. The GA flowchart.

- Selection rules choose parents that contribute to the population of the next generation.
- Crossover rules combine two parents to produce children of the next generation.
- Mutation rules apply random changes to parents before child production.

The GA can be employed to solve a variety of constrained and unconstrained optimization problems that are not suitable for standard optimization algorithms. Such cases include problems in which the objective function is nondifferentiable, discontinuous, stochastic, or highly nonlinear.

5. Simulation results

In this part, conventional and proposed OMTHD techniques are applied on a 7-level cascaded inverter and the obtained simulation results are compared. It should be mentioned that GA, which is described in the previous part, is utilized via Genetic Toolbox of MATLAB. It is assumed that we want to eliminate the 5th and 7th harmonic orders from produced voltage of a 7-level inverter and so the proposed generic THD formula and its constraints for an n-level inverter are reduced to (13). Substitution of obtained b_2 and b_3 in the THD formula results in an objective function, which includes only three switching angles.

$$
\begin{gathered}
THD=\sqrt{\frac{\pi}{4}\left(\frac{(\alpha_2-\alpha_1)+(1+b_2)^2\,(\alpha_3-\alpha_2)+(1+b_2+b_3)^2\left(\frac{\pi}{2}-\alpha_3\right)}{(\cos(\alpha_1)+b_2.\cos(\alpha_2)+b_3\,.\cos(\alpha_3))^2}\right)-1}\quad,\ b_2=\frac{a_2}{a_1}\ ,\ b_3=\frac{a_3}{a_1}\\
\frac{4}{3\,\pi}\left(\cos\left(\alpha_1\right)+b_2.\cos\left(\alpha_2\right)+b_3\,.\cos\left(\alpha_3\right)\right)=\frac{M}{a_1}\\
\begin{cases}
\frac{4\,V_{dc}}{5\,\pi}\left(\cos\left(5\alpha_1\right)+b_2.\cos\left(5\alpha_2\right)+b_3\,.\cos\left(5\alpha_3\right)\right)=0\\
\frac{4\,V_{dc}}{7\,\pi}\left(\cos\left(7\alpha_1\right)+b_2.\cos\left(7\alpha_2\right)+b_3\,.\cos\left(7\alpha_3\right)\right)=0
\end{cases}
\Rightarrow
\begin{cases}
b_2=\frac{\cos(7\alpha_3)\,.\,\cos(5\alpha_1)-\cos(5\alpha_3)\,.\,\cos(7\alpha_1)}{\cos(7\alpha_2)\,.\,\cos(5\alpha_3)-\cos(5\alpha_2)\,.\,\cos(7\alpha_3)}\\
b_3=\frac{\cos(7\alpha_1)\,.\,\cos(5\alpha_2)-\cos(5\alpha_1)\,.\,\cos(7\alpha_2)}{\cos(7\alpha_2)\,.\,\cos(5\alpha_3)-\cos(5\alpha_2)\,.\,\cos(7\alpha_3)}
\end{cases}
\end{gathered}
\tag{13}
$$

Figure 3 shows THD of phase voltage versus modulation index for conventional and improved OMTHD techniques. It is seen that two similar constant THD values (11.47% for the conventional and 11.88% for the proposed switching techniques) are obtained for a broad range of modulation indices. In $M = 0.95$, both techniques result in equal THD values that increase steadily for higher modulation indices. The reason for this equality as well as the steady increase in THD value will be explained.

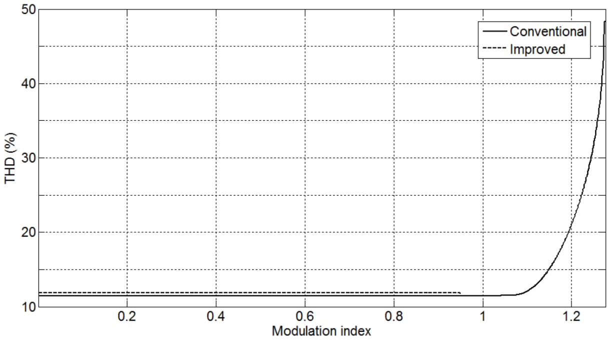
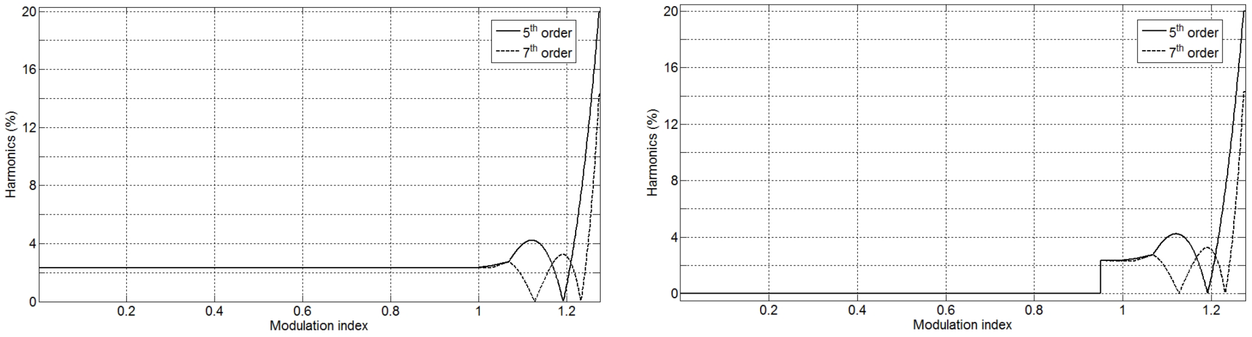


Figure 3. THD of phase voltage vs. modulation index for conventional and improved OMTHD techniques 'simulation result'.

It should be noted that the THD value for the proposed technique, presented in Figure 3, is derived by minimization of an objective function with three independent variables and without any equality constraint for once. However, in the conventional technique, the THD formula with six variables should be optimized with new nonlinear constraints for each modulation index. Moreover, this optimization has been carried out several times for each modulation index to obtain better estimations.

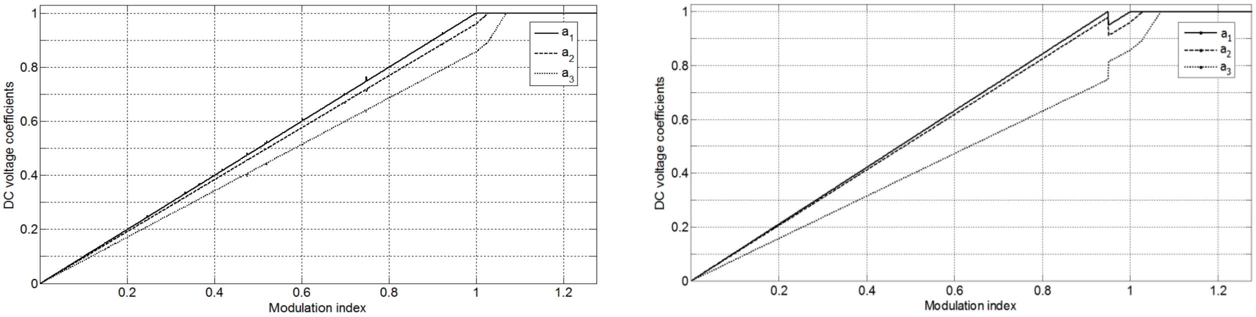
As expected, Figure 4 shows that 5th and 7th harmonic orders are eliminated by the proposed technique for modulation indices below 0.95. It is clear that elimination of low-order harmonics in the proposed technique makes the low-pass filter design easier than the conventional technique, in which there are all harmonic orders, even with reduced amplitude (see Figure 4a). Emerging these two harmonic orders and their variations for higher modulation indices than 0.95 in Figure 4b will be explained.

DC voltage coefficients versus modulation index for both switching techniques are presented in Figure 5. It should be noted that in the conventional technique these coefficients are estimated again for each modulation index, but in the proposed technique and after estimation of switching angles once, these coefficients depend linearly on the modulation index (see (14)).



(a) (b)

Figure 4. Amplitude of 5th and 7th harmonic orders vs. modulation index for (a) conventional, (b) improved OMTHD techniques 'simulation results'.



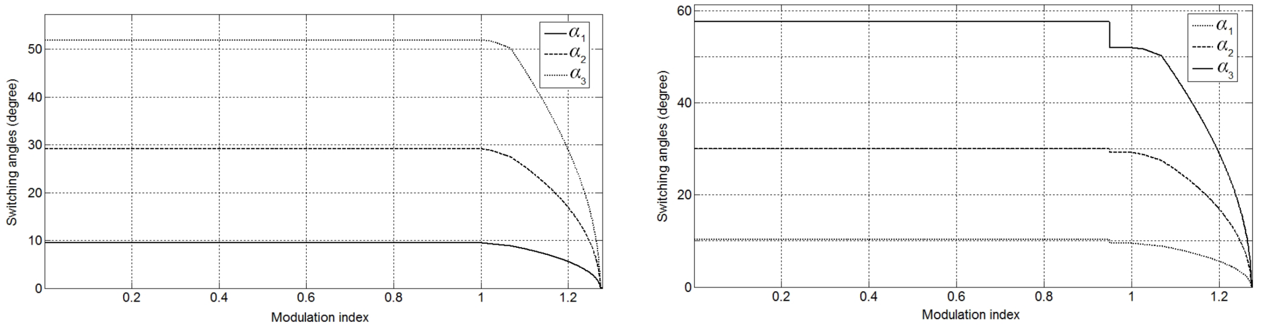
(a) (b)

Figure 5. DC voltage coefficients vs. modulation index for (a) conventional, (b) improved OMTHD techniques 'simulation results'.

$$\begin{cases} \frac{4}{3\,\pi}\left(\cos\left(\alpha_1\right)+b_2.\cos\left(\alpha_2\right)+b_3\,.\cos\left(\alpha_3\right)\right)=\frac{M}{a_1} \\ \frac{4}{3\,\pi}\left(\cos\left(\alpha_1\right)+b_2.\cos\left(\alpha_2\right)+b_3\,.\cos\left(\alpha_3\right)\right)=A \end{cases} \Rightarrow\ a_1=\frac{M}{A}\ \Rightarrow \begin{cases} \frac{a_2}{a_1}=b_2\Rightarrow a_2=\frac{b_2}{A}.M \\ \frac{a_3}{a_1}=b_3\Rightarrow a_3=\frac{b_3}{A}.M \end{cases} \tag{14}$$

Figure 5 can explain the behavior of the THD value as well as the amplitude of two harmonic orders for modulation indices higher than 0.95 in Figures 3 and 4, respectively. It is seen from Figure 5b that coefficient a_1 reaches its maximum value at $M = 0.95$ and for higher modulation indices, coefficients larger than one are required to minimize THD value as well as eliminate 5th and 7th harmonic orders. In other words, reaching DC voltage coefficients to their maximum value means a 7-level inverter with constant DC sources that has only three variable switching angles. In this condition, both conventional and proposed techniques result in the same answers. Moreover, Figure 6 shows that switching angles for modulation indices higher than 0.95 tend to zero to satisfy the desired modulation index. These switching angles mean quasi two-level waveform with high spectral content that can explain high amounts for THD as well as two harmonic orders, in Figures 3 and 4. It worth mentioning that the proposed OMTHD technique despite its limitation for M > 0.95 can regulate the fundamental component at a given value for all modulation indices (see Figure 7).

It should be noted that switching angles in the conventional technique, shown in Figure 6a, should be estimated again for each modulation index, but in the proposed technique these angles are estimated once for all modulation indices smaller than 0.95. To give a more comprehensive view, the amplitudes of nontriplen



(a)

(b)

Figure 6. Switching angles vs. modulation index for (a) conventional, (b) improved OMTHD techniques 'simulation results'.

harmonic orders from 11 to 49 at the base of the fundamental component for both switching techniques at $M = 0.8$ are presented in Figure 8. Considering harmonic orders up to 49 has been recommended in most standards, like IEEE 519-2014 [19]. Moreover, triplen harmonics can be simply eliminated by using delta connection, and so they are not shown in this figure.

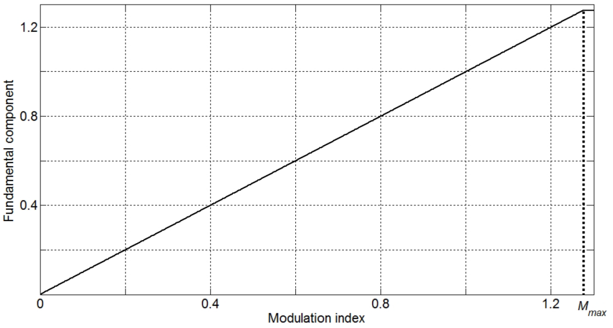


Figure 7. Fundamental component vs. modulation index for the proposed OMTHD techniques 'simulation result'.

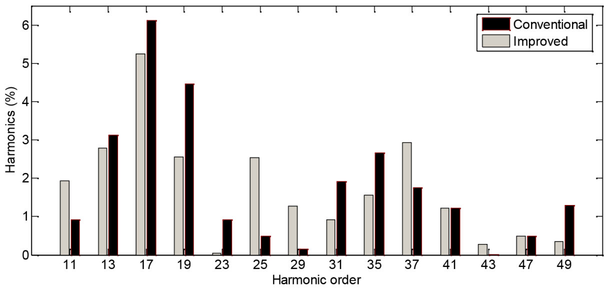


Figure 8. Harmonic spectrum of phase voltage for $M = 0.8$ in the conventional and improved OMTHD techniques 'simulation results'.

Figure 8 shows that none of switching techniques can control the amplitude of harmonics greater than 7th order and consequently no superiority can be mentioned for one of these two techniques. It is worth mentioning that the presented amplitudes in Figure 8 are constant for all modulation indices smaller than 0.95.

6. Experimental results

In this part, a single-phase 7-level cascaded inverter with adjustable DC sources is experimentally tested in the laboratory. Each H-bridge uses SKP10N60A (600V/10A), TLP250, and IR2104 as switching device, isolator, and gate driver, respectively. An AVR microcontroller ATmega32, as an inexpensive and simple controller, is utilized to control the multilevel inverter, due to the simplicity of the proposed technique. The isolator and gate driver of each H-bridge are supplied by its own onboard power supply and an isolated 15 V DC power supply is dedicated to each H-bridge as its adjustable DC source. These parts are indicated in the circuit diagram in Figure 9.

For each modulation index in Table 1, three switching angles and three DC voltage coefficients, which are obtained by both switching techniques, are brought. It is worth mentioning that $M = 0.2$, 0.8 are smaller than boundary M (i.e. 0.95) and $M = 1.2$ belongs to the high modulation indices zone. Applying the obtained

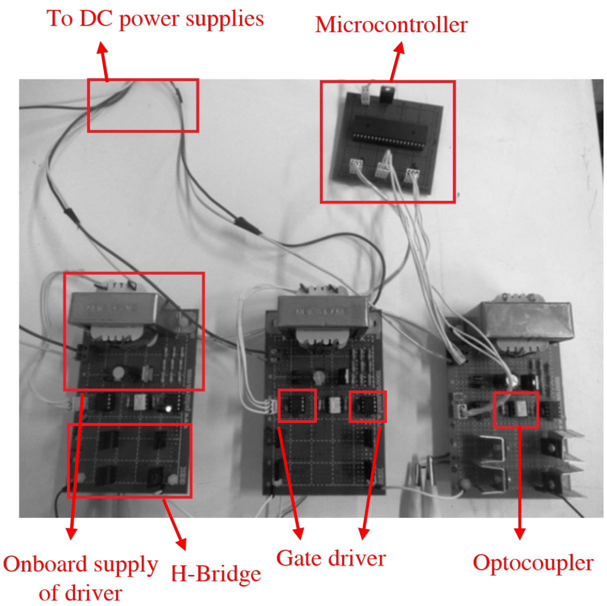


Figure 9. Implemented test circuit of a 7-level cascaded inverter with three isolated adjustable DC sources.

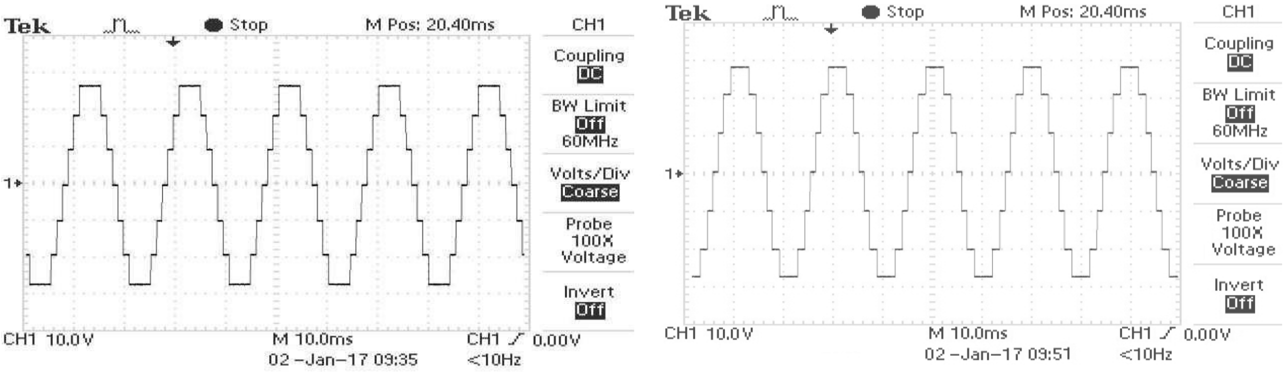
values of $M = 0.8$ on the test setup results in output voltages with $f = 50$ Hz in Figure 10, which are captured by a digital storage oscilloscope, Tektronix TDS1002B. Spectral analysis of the produced voltages in Figure 10 are carried out by MATLAB and harmonic orders up to 49, which also includes triplen harmonics, are presented in Figure 11. Moreover, the results of spectral analysis for two other modulation indices also are brought in Table 2. It should be noted that these modulation indices are chosen like those of Table 1. These results, which are similar to the simulation ones, show elimination of 5th and 7th harmonic orders beside THD minimization. Furthermore, it is seen that two modulation indices smaller than 0.95 (i.e. 0.2 and 0.8) for each switching technique result in the same spectrum, which differ from another technique. However, for $M = 1.2$, both techniques lead to similar answers, because this M is outside the valid range of the proposed OMTHD technique.

Table 1. Switching angles and DC voltage coefficients, obtained by conventional and improved OMTHD techniques.

	M	Switching angles			Voltage coefficients		
		α_1	α_2	α_3	a_1	a_2	a_3
Conventional technique	0.2	9.48	29.20	51.88	0.2	0.19	0.17
	0.8	9.48	29.20	51.88	0.80	0.77	0.69
	1.2	5.55	16.87	28.93	1	1	1
Proposed technique	0.2	10.36	29.97	57.53	0.21	0.21	0.16
	0.8	10.36	29.97	57.53	0.84	0.83	0.63
	1.2	5.55	16.87	28.93	1	1	1

7. Conclusion

In this paper, an improved OMTHD technique was proposed for an n-level cascaded inverter with adjustable DC sources. This switching technique can eliminate $n-1$ selective harmonic orders beside THD minimization, in contrast with the conventional OMTHD technique. As one of the features of the proposed technique, the THD formula only consists of n switching angles that led to shrinking search space, reduced required calculation,



(a)

(b)

Figure 10. Output phase-voltage of 7-level inverter with estimated values by (a) proposed, (b) conventional OMTHD techniques 'experimental results'.

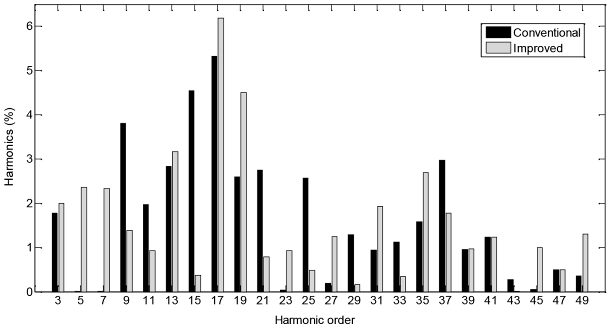


Figure 11. Harmonic spectrum of phase voltage for $M = 0.8$ in the conventional and improved OMTHD techniques 'experimental results'.

Table 2. Spectral analysis of output voltage of 7-level inverter with estimated values by both switching techniques.

	M	THD		5th		7th	
		M*	C**	M*	C**	M*	C**
Conventional technique	0.2	11.41	11.47	2.32	2.34	2.30	2.31
	0.8	11.35	11.47	2.31	2.34	2.29	2.31
	1.2	21.03	21.1	1.08	1.18	3.09	3.12
Proposed technique	0.2	11.81	11.88	≈0	0	≈0	0
	0.8	11.7	11.88	≈0	0	≈0	0
	1.2	21.23	21.1	0.97	1.18	3.05	3.12

* Measured, ** Calculated

and more precise estimation of optimal answers. Moreover, the obtained THD formula is optimized once and without any nonlinear equality constraint.

It was explained that optimal switching angles are estimated only once and then required DC voltages to obtain any desired fundamental component are calculated by simple linear equations. This is in contrast with the conventional technique, in which optimization should be repeated with new nonlinear equality constraints for each modulation index. Constant switching angles beside simple calculation of DC voltages for each modulation index give the opportunity of online implementation of the proposed technique on the simple microcontroller. Finally, the improved and conventional OMTHD techniques for a 7-level cascaded inverter were simulated in

MATLAB and also implemented on a laboratory test setup. Both simulation and experimental results showed the mentioned advantages of the proposed switching technique.

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