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Research Article

Two-stage optimization method for power loss and voltage profile control in distribution systems with DGs and EVs using stochastic second-order cone programming

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Abstract: This paper introduces a stochastic second-order cone programming (SSCOP) approach to solve the distributed generation coordination problem considering the uncertainty of electric vehicle charging in distribution networks. To minimize the total power loss in the distribution system, the problem is formulated to coordinate the output power of distributed generations (DGs). Two stages are presented to solve the optimization problem: the first stage is to optimize the output power of distributed generators without electric vehicle charging in the distribution system, and the second stage is to optimize the output power of distributed generators according to the stochastic increased load due to the uncertainty of electric vehicle charging. The proposed approach is tested on 69-node and 118-node large-scale distribution systems. The simulated results demonstrate the feasibility and effectiveness of SSCOP.

Key words: Distribution systems, power loss, distributed generation, electric vehicle, stochastic second-order cone programming

1. Introduction

Increasing concerns about greenhouse gas emissions and the energy crisis have motivated the development of transportation electrification. Electric vehicles (EVs) are expected to play a major role in transportation electrification. The cumulative sales of EVs are expected to reach 5 million units by 2020 in China [1]. However, the uncertainty of EV charging has a great impact in the distribution systems, including increasing power loss of distribution systems and nodal voltage offset. It is necessary to develop renewable energy such as wind- and solar-based generation units. The generation units are called distributed generations in the distribution systems to meet the EV random load. It is also necessary to coordinate distributed generations and EVs because of uncertainties of charging power of EVs.

Stochastic second-order cone programming with recourse is a class of optimization problems defined to formulate many applications of DSOCP with uncertain data [2]. Second-order cone programming is a special convex programming model [3], and is one of the important optimization methods in mathematics. The method has been applied in power systems [4,5]. Many scholars have applied cone programming to power flow calculation of power systems. A nonlinear constraint condition is transformed into linear in power flow calculations. The relationship between the cone sets of variables is preserved to improve the calculation speed. For example, a

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power flow calculation model is proposed based on conic programming [6]. A cone programming model for DGs is obtained by minimizing power loss as the objective function [7]. The cone programming model for optimal reconfiguration of the distribution network is solved by taking the switch in the lines as variables [8]. The optimal power flow model based on second-order cone programming is established using an objective function of minimizing active power loss, reactive power loss, and generation cost [9]. In fact, the operating conditions of the power network are always changing, and the above literature fails to consider the impact of various uncertainties on the power grid.

The charging power of the EV is random as the stochastic load of the distribution network. In this paper, the uncertainty of EV charging on the electric network is studied. A stochastic second-order cone programming (SSCOP)-based method is also presented to optimize the output of DGs in order to decrease the power loss and the nodal voltage offset, which will bring great potential risk to the distribution system.

2. Output power of DGs and charging power of EVs

2.1. The stochastic model of solar photovoltaic power generation

Solar irradiation is a random process in a day. That is to say, solar radiation will have different values in different time periods of a day. From the statistical analysis of historical data, solar radiation is subject to beta distribution, and the solar radiation value always corresponds to a probability. In mathematics, the probability density function of solar radiation can be expressed by beta distribution [10]:

$$f(s_i(t)) = \begin{cases} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}[s_i(t)]^{\alpha-1}(1-s_i(t))^{\beta-1} & \text{for} \begin{cases} 0 \leq s_i(t) \leq 1 \\ \alpha \geq 0, \beta \geq 0 \end{cases}, \\ 0 \end{cases} \tag{1}$$

where $f(s_i(t))$ is the beta function of the solar irradiance value $s_i(t)$ at time period t, $s_i(t)$ is solar irradiance value, α and β are the parameters of the beta function, which is determined by historical irradiance data, and $\Gamma()$ is expressed as the beta function.

Although solar radiation has a different value at each time of day, the solar radiation in a certain period of time can be considered as constant, if the solar irradiance value is supposed to be constant at each time period. The probability of the solar irradiance value may be formulated as [10]

$$p_S(t) = \int_0^t f(s_i(t))s_i(t)\,dt \tag{2}$$

Because of the fluctuation in solar radiation in the beta distribution, the output power of a photovoltaic power generation system is also changed according to the beta distribution in different solar radiation conditions. If the solar radiation is changed in a certain range with the probability of beta distribution, the output power of the photovoltaic power generation system also changes with the probability of beta distribution in a certain range. Therefore, the output power of distributed photovoltaic generation system i at time period t may be formulated by

$$P_{PVi}(t) = \eta_{PVi}P_{Si}(t)p_S(t), \tag{3}$$

where $P_{Si}(t)$ is the solar irradiance power of distributed photovoltaic power generation system i at time period t, and η_{PVi} is the generation efficiency of distributed photovoltaic power generation system i due to the impact of photovoltaic power generation materials on the process and results of photoelectric conversion. Obviously

the output power of distributed photovoltaic generation $P_{PVi}(t)$ is stochastic because the solar irradiance value $s_i(t)$ is stochastic at different time t.

2.2. The stochastic model of distributed wind power generation

It has been demonstrated by a large number of experiments that the stochastic wind speed in most regions approximately follows Weibull distribution, which can be formulated as follows [11]:

$$f(v) = \frac{k}{c^k} v^{(k-1)} \exp(-\frac{v}{c})^k \quad 0 \le v \le \infty, \tag{4}$$

where v is wind speed, and k and c are respectively the shape index and the scale index of the Weibull distribution.

The output power level depends on the wind speed, and the output power is zero when the wind speed is smaller than the cut-in wind speed or not less than the cut-out wind speed. When the wind speed is not less than the cut-in wind speed and not greater than the rated wind speed, the output power increases linearly with the wind speed. When the wind speed is not less than the rated wind speed and less than the cut-out wind speed, the power output is in accordance with the rated power. From the statistical analysis of historical data, it is found that the level of each output power from zero to the rated power is corresponding to a probability value of the Weibull distribution. Therefore, the probability of the wind speed value may be formulated by [12]

$$p_W(t) = \int_0^t f(v_i(t))v_i(t)\, dt, \tag{5}$$

where $f(v_i(t))$ is the Weibull function of the wind speed value $v_i(t)$ at time period t, and $v_i(t)$ is wind speed value.

The output power of distributed wind power generation system i at time period t may be formulated by

$$P_{Wi}(t) = \eta_{Wi} P_{WS}(t) p_W(t), \tag{6}$$

where $P_{WS}(t)$ is wind power under wind speed of $v_i(t)$, and η_{Wi} is generation efficiency of distributed wind power generation system i under wind speed of $v_i(t)$. Similarly, the output power of distributed photovoltaic generation $P_{PVi}(t)$ is stochastic because the wind speed value $v_i(t)$ is stochastic at different time t.

2.3. Stochastic nature of EV load

The main influencing factors of the total EV charging power in the distribution network are determined by state of charge (SOC), the daily mileage, the recharging time of EVs, and charging time length of EVs. Due to differences in people's travel habits and the different characteristics of the EV's battery, there are a lot of uncertainties in the charging of EVs in the distribution network. Generally, the recharging power in a charging station always is uncertain due to uncertainties of SOC in the battery, the expected traveling distance, the recharging time, and so on. It is clear that EV charging is a random event, with a large uncertainty, and its charging power is a random variable. The probability density functions (PDFs) of an EV's daily mileage can be expressed as follows [13]:

$$f(d) = \frac{1}{\sqrt{2\pi}d\sigma_d} \exp[-\frac{(\ln d - \mu_d)^2}{2\sigma_d^2}], \tag{7}$$

where d is the daily distance driven by a vehicle, μ_d is the mean of the distribution, and σ_d is the standard deviation of the probability function.

Because of differences in people's travel habits and pattern of vehicle usage, not all EVs in the distribution system start charging simultaneously, and so the time of switching on an individual charger is a random variable, and the probability density functions of the battery recharging start time t_s can be expressed as follows [14]:

$$f(t_s) = \frac{1}{\sqrt{2\pi}\sigma_{t_s}} \exp[-\frac{(t_s - \mu_{t_s})^2}{2\sigma_{t_s}^2}], \tag{8}$$

where t_s is the recharging time of an EV, μ_{t_s} the mean, indicating the location of maximum probability density, and $\sigma_{t_s}^2$ is the standard deviation of recharging time of an EV in a day.

On the other hand, assuming the SOC of an EV drops linearly with the distance of travel, the SOC at the beginning of recharge cycle can be expressed as follows [15]:

$$E_i = 1 - \frac{d}{L}, \tag{9}$$

where d is the daily distance traveled by a vehicle, E_i represents the initial SOC of an EV battery, and L is the maximum range of the EV.

The probability distribution of initial battery SOC can be expressed as (10), which is derived from (7) and (9):

$$f(E) = \frac{1}{\sqrt{2\pi}L(1-E)\sigma_d} \exp\{-\frac{[\ln(1-E)L - \mu_d]^2}{2\sigma_d^2} \tag{10}$$

The probability density function of recharging power is obtained:

$$p(E) = \frac{1}{\sqrt{2\pi}L(1-E)t_E\sigma_d} \exp\{-\frac{[\ln(1-E)L - \mu_d]^2}{2\sigma_d^2}, \tag{11}$$

where t_E is the charging time length of EVs.

The probability density function of a battery charging load operating at power level P_j at time instant t can be represented as $\xi(P_{j,t})$ [15]:

$$\xi(P_{j,t}) = \sum_{k=1}^{t} f(t_s)f(E_{j-(t-k)}) \quad k \leq t, 1 \leq k \leq 24 \tag{12}$$

Eq. (13) then gives the expression of multiple batteries' charging loads in the distribution system at time instant t at node i.

$$P_{EVi,t} = \sum_{m=1}^{n_i}\sum_{j=1}^{N_T} P_{mj}.\xi(P_{mj,t}), \tag{13}$$

where n_i is the total number of EVs at node i, N_T is the total number of hours of battery to be fully charged from a fully discharged state, and P_{mj} is the discrete power demand value of the m-th EV at charging power level P_j, which can be expressed as follows:

$$P_{mj} = \frac{P_{m,j-1} + P_{m,j}}{2}j = 1, 2, \cdots N_T \tag{14}$$

Based on the above analysis, it is shown that the total EVs' charging loads $P_{EVi,t}$ at node i in a distribution system is a stochastic load, which is influenced by the daily mileages and vehicle traffic patterns (usage).

3. Mathematical model

3.1. Objective functions

It is assumed that a distribution system is installed with the DG of renewable energy systems and charging systems for EVs at different nodes in this paper. Figure 1 is the single radiation line model in the distribution system with DGs and EVs. In the system, it is also assumed that the DG of renewable energy systems includes a DG system and a distributed energy storage system (DS). The DG system is based on solar power, wind power, etc. The output power of the DG of a renewable energy system is dependent on the output power of the DG system, the storage power of the energy storage system, and the load power at the local node. The renewable energy is fully utilized by the local load, and the surplus power will be stored in the DSs when the power of the local load is smaller than that generated from the renewable energy DG system. With this assumption, controlling the output power of the DG means controlling the power output of renewable energy systems, and controlling the power output of renewable energy systems means the controlled power of the renewable energy is used for local storage. Controlling the renewable energy aims at minimizing system loss.
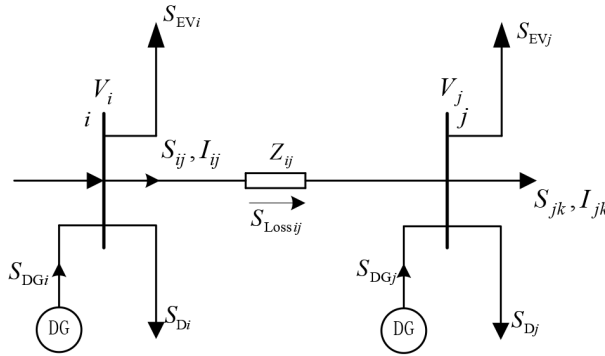
Figure 1. A simplified 2-node distribution system with DGs and EVs.

In the distribution system, S_{ij} is the power flow from node i to j. S_{DGi} or S_{DGj} is a flow-in power injected by the DG of renewable energy systems at node i or j. S_{Di} or S_{Dj} is a flow-out power outpoured by local general load at node i or j, which is uncontrollable at any time; S_{EVi} or S_{EVj} is a flow-out power outpoured by changing load of EVs, which is a stochastic load.

The objective function in this paper is to minimize the power loss, which can be expressed as follows:

$$\min \sum_{t=1}^{T} [\sum_{(ij)\in\Omega_l} P_{Loss}(V_{i,t}; V_{j,t}; \theta_{ij,t})] \tag{15}$$

$$\sum_{(i,j)\in\Omega_L} P_{Loss}(V_{i,t}; V_{j,t}; \theta_{ij,t}) = \sum_{(i,j)\in\Omega_L} G_{ij}(V_{i,t}^2 + V_{j,t}^2 - 2V_{i,t}V_{j,t}\cos\theta_{ij,t}) = \sum_{i=1}^{N} (P_{DGi,t} - P_{Di,t} - P_{EVi,t}), \tag{16}$$

where V_i and V_j are the voltage magnitude of node i and j, θ_{ij} is the voltage angle between node i and j, and Ω_L is a nodal set in the distribution network. T is the study period. N is the number of nodes in the

distribution system and $P_{DGi,t}$ is real power produced by the DG of renewable energy system at node i at time instant $t.P_{Di,t}$ is real power load demand at node i at time instant $t.P_{EVi,t}$ is the overall real power demand for the all EV battery chargers at node i at time instant t, which is modeled as random variables across time from a probability density function. The control task is to minimize the total power loss by properly setting the control variables all day. The constraints associated with the above objective consist of the following.

1) Equal constraint condition

$$\sum_{j \in \Omega_i} (G_{ij}V_{i,t}^2 - V_{i,t}V_{j,t}(G_{ij}\cos\theta_{ij,t} - B_{ij}\sin\theta_{ij,t})) = P_{DGi,t} - P_{Di,t} - P_{EVi,t} \tag{17}$$

$$\sum_{j \in \Omega_i} (B_{ij}V_{i,t}^2 - V_{i,t}V_{j,t}(B_{ij}\cos\theta_{ij,t} + G_{ij}\sin\theta_{ij,t})) = Q_{DGi,t} - Q_{Di,t} - Q_{EVi,t} \tag{18}$$

Power flow as equal constraint condition must satisfy Eqs. (16) and (17). Ω_i is the nodal set connected to node i.

2) Nodal voltage constraint

As for each node in the distribution systems, its voltage must be not greater than its maximal limit and not less than its minimal limit:

$$V_{i,\min} \leq V_{i,t} \leq V_{i,\max}, \tag{19}$$

where V_i, $V_{i,\max}$, and $V_{i,\min}$ are respectively the actual value, the maximal limit, and the minimal limit of nodal voltage at node i.

3) Branch current constraint

Current magnitude of each branch must lie within their permission ranges:

$$I_{ij,t}^2 = (G_{ij}^2 + B_{ij}^2)(V_{i,t}^2 + V_{j,t}^2 - 2V_{i,t}V_{j,t}\cos\theta_{ij,t}) \leq I_{ij,\max}^2, \tag{20}$$

where $I_{ij,\max}$ is the maximal limit of the branch L_{ij}.

4) Power output constraint for DG

$$0 \leq \sqrt{(P_{DGi,t})^2 + (Q_{DGi,t})^2} \leq S_{DGi,\max}, \tag{21}$$

where $S_{DGi,\max}$ is the maximal power limit of the DG of the renewable energy system at node i.

3.2. Stochastic second-order cone programming for the problem

In this section we will formulate the power loss and voltage profile control in distribution systems with stochastic charging power of EV problem presented in the previous section as a stochastic second-order cone SSOCP problem. Stochastic second-order cone programming with recourse is a class of optimization problems defined to handle uncertainty in data defining deterministic second-order cone programming [2]. The standard form of stochastic second-order cone programming is the following:

$$minimize \quad \mathbf{c}^T\mathbf{x} + \sum_{k=1}^{\zeta} \mathbf{Q}^{(\mathbf{k})}(\mathbf{x}) \quad subject\,to \quad \mathbf{A}\mathbf{x} - \mathbf{b} \succeq_{r_1} 0, \tag{22}$$

where $r_1 \geq 1$ and $r_2 \geq 1$ are integers $i = 1, 2, ..., r_1$ and $j = 1, 2, ..., r_2$. $\mathbf{A} \in R^{m_1 \times n_1}$, $\mathbf{b} \in R^{m_1}$, and $\mathbf{c} \in R^{n_1}$ are the deterministic data, $\mathbf{x} \in R^{m_1}$ is the first-stage decision variable, and for $k = 1, 2, 3, ...\zeta$, ζ is the number of realizations. $Q^{(k)}(\mathbf{x})$ is the minimum value of the problem

$$minimize \quad \mathbf{d}^{(k)^T}\mathbf{y}^{(k)} \quad subject\,to \quad \mathbf{W}^{(k)}\mathbf{y}^{(k)} + \mathbf{T}^{(k)}\mathbf{x} - \mathbf{h}^{(k)} \succeq_{r_2} 0, \tag{23}$$

where $\mathbf{y}^{(k)} \in R^{n_2}$ is the second-stage variable, and $(\mathbf{T}^{(k)}, \mathbf{W}^{(k)}, \mathbf{h}^{(k)}, \mathbf{d}^{(k)})$ are the set of the possible values of the random variables $(\mathbf{T}(\omega), \mathbf{W}(\omega), \mathbf{h}(\omega), \mathbf{d}(\omega))$, where ω is underlying outcome in a finite event space Γ with a known probability function η_k.

Likewise, the power loss and voltage profile control with stochastic charging power of EV problem given by (15)–(21) is a problem with uncertain data. In this model, the average user load is the deterministic load in the distribution systems, while the total power demand of EVs at every node is a stochastic load due to the randomness of the charging power of EVs. Obviously the load of the distribution system with EVs contains two parts. Hence the problem of power loss and voltage profile control in distribution systems with DGs and EVs can be formulated as a two-stage programming problem. In the first stage, we optimize the output power of the DG of renewable energy systems when there is only the deterministic load in distribution systems for minimizing power loss and controlling voltage profile. In the second stage, we optimize the output power of the DG of renewable energy systems when the stochastic load of charging power of EVs in the distribution system is added.

For predicting the charging load at each time period, the continuous distribution function of the starting charging time and the length charging time can be discretized with its discrete values P_j with 1 hour as the unit in a day. With the notation, our two-stage stochastic programming problem can be written as

$$\min \quad \sum_{t=1}^{T} \sum_{(i,j)\in\Omega_L} P_{Loss}^{DG}(V_{i,t}; V_{j,t}; \theta_{ij,t}) + \sum_{t=1}^{T} \mathbf{Q}^{(t)}(P_{DGi,t}) \tag{24}$$

$$\sum_{(i,j)\in\Omega_L} P_{Loss}^{DG}(V_{i,t}; V_{j,t}; \theta_{ij,t}) = \sum_{(i,j)\in\Omega_L} G_{ij}(V_{i,t}^2 + V_{j,t}^2 - 2V_{i,t}V_{j,t}\cos\theta_{ij,t}) = \sum_{i=1}^{N}(P_{DGi,t} - P_{Di,t}) \tag{25}$$

subject to

$$\sum_{j\in\Omega_i}(G_{ij}V_{i,t}^2 - V_{i,t}V_{j,t}(G_{ij}\cos\theta_{ij,t} - B_{ij}\sin\theta_{ij,t})) = P_{DGi,t} - P_{Di,t} \tag{26}$$

$$\sum_{j\in\Omega_i}(B_{ij}V_{i,t}^2 - V_{i,t}V_{j,t}(B_{ij}\cos\theta_{ij,t} + G_{ij}\sin\theta_{ij,t})) = Q_{DGi,t} - Q_{Di,t} \tag{27}$$

$$V_{i,\min} \leq V_{i,t} \leq V_{i,\max} \tag{28}$$

$$I_{ij,t}^2 = (G_{ij}^2 + B_{ij}^2)(V_{i,t}^2 + V_{j,t}^2 - 2V_{i,t}V_{j,t}\cos\theta_{ij,t}) \leq I_{ij,\max}^2 \tag{29}$$

$$0 \leq \sqrt{(P_{DGi,t})^2 + (Q_{DGi,t})^2} \leq S_{DGi,\max}, \tag{30}$$

where $T = 24$ represents there are 24 hours in a day. $P_{DGi,t} \in R^{n_1}$ is the first-stage decision variable when there is only the deterministic load in the distribution systems. $Q^{(t)}(P_{DGi,t})$ is the minimum value of the problem

$$\min \quad \sum_{t=1}^{T} \sum_{(i,j) \in \Omega_L} P_{Loss}^{DGEV}(V_{i,t}^*; V_{j,t}^*; \theta_{ij,t}^*) \tag{31}$$

$$\sum_{(i,j) \in \Omega_L} P_{Loss}^{DGEV}(V_{i,t}^*; V_{j,t}^*; \theta_{ij,t}^*) = \sum_{(i,j) \in \Omega_L} G_{ij}((V_{i,t}^*)^2 + (V_{j,t}^*)^2 - 2V_{i,t}^* V_{j,t}^* \cos \theta_{ij,t}^*)$$

$$= \sum_{i=1}^{N} (P_{DGi,t}^* - P_{Di,t} - P_{EVi,t}) \tag{32}$$

subject to

$$\sum_{j \in \Omega_i} (G_{ij}(V_{i,t}^*)^2 - V_{i,t}^* V_{j,t}^* (G_{ij} \cos \theta_{ij,t}^* - B_{ij} \sin \theta_{ij,t}^*)) = P_{DGi,t}^* - P_{Di,t} - P_{EVi,t} \tag{33}$$

$$\sum_{j \in \Omega_i} (B_{ij}(V_{i,t}^*)^2 - V_{i,t}^* V_{j,t}^* (B_{ij} \cos \theta_{ij,t}^* + G_{ij} \sin \theta_{ij,t}^*)) = Q_{DGi,t}^* - Q_{Di,t} - Q_{EVi,t} \tag{34}$$

$$V_{i,\min} \leq V_{i,t}^* \leq V_{i,\max} \tag{35}$$

$$(I_{ij,t}^*)^2 = (G_{ij}^2 + B_{ij}^2)((V_{i,t}^*)^2 + (V_{j,t}^*)^2 - 2V_{i,t}^* V_{j,t}^* \cos \theta_{ij,t}^*) \leq I_{ij,\max}^2 \tag{36}$$

$$0 \leq \sqrt{(P_{DGi,t}^*)^2 + (Q_{DGi,t}^*)^2} \leq S_{DGi,\max}^*, \tag{37}$$

where $P_{DGi,t}^* \in R^{n_2}$ is the second-stage decision variable different for $P_{DGi,t}$ due to the uncertain load growth of charging power of EVs with its random charging schedule. Variables with the superscript "*" are second-stage variables.

Because the above optimization model is not the standard form of the cone programming model, it needs to be transformed into the following form [15]:

$$u_{i,t} = \frac{V_{i,t}^2}{\sqrt{2}} \quad or \quad u_{j,t} = \frac{V_{j,t}^2}{\sqrt{2}} \tag{38}$$

$$H_{ij,t} = V_{i,t} V_{j,t} \cos \theta_{ij,t} \qquad T_{ij,t} = V_{i,t} V_{j,t} \sin \theta_{ij,t} \tag{39}$$

$$2u_{i,t} u_{j,t} \geq H_{ij,t}^2 + T_{ij,t}^2, H_{ij,t} \geq 0, ij \in \Omega_L \tag{40}$$

Replacing variables u, H, and T in the original optimization problem with the variables of cone optimization, a standard form of the second-order cone programming model for the original optimization problem is obtained as (41)–(54):

$$\min \quad \sum_{t=1}^{T} \sum_{(i,j) \in \Omega_L} P_{Loss}^{DG}(u_{i,t}; u_{j,t}; H_{ij,t}; T_{ij,t}) + \sum_{t=1}^{T} Q^{(t)}(P_{DGi,t}) \tag{41}$$

$$\sum_{(i,j)\in\Omega_L} P_{Loss}^{DG}(u_{i,t};u_{j,t};H_{ij,t};T_{ij,t}) = \sum_{(i,j)\in\Omega_L} G_{ij}(\sqrt{2}u_{i,t} + \sqrt{2}u_{j,t} - 2H_{ij,t}) = \sum_{i=1}^{N}(P_{DGi,t} - P_{Di,t}) \tag{42}$$

subject to

$$\sum_{j\in\Omega_i}(\sqrt{2}G_{ij}u_{i,t} - (G_{ij}H_{ij,t} - B_{ij}T_{ij,t})) = P_{DGi,t} - P_{Di,t} \tag{43}$$

$$\sum_{j\in\Omega_i}(\sqrt{2}B_{ij}u_{i,t} - (B_{ij}H_{ij,t} + G_{ij}T_{ij,t})) = Q_{DGi,t} - Q_{Di,t} \tag{44}$$

$$\frac{V_{i,\min}^2}{\sqrt{2}} \le u_{i,t} \le \frac{V_{i,\max}^2}{\sqrt{2}} \tag{45}$$

$$I_{ij,t}^2 = (G_{ij}^2 + B_{ij}^2)(\sqrt{2}u_{i,t} + \sqrt{2}u_{j,t} - 2H_{ij,t}) \le I_{ij,\max}^2 \tag{46}$$

$$0 \le P_{DGi,t}^2 + Q_{DGi,t}^2 \le 2 * \frac{S_{DGi,\min}}{\sqrt{2}} * \frac{S_{DGi,\max}}{\sqrt{2}}, \tag{47}$$

where $S_{DGi,\min}$ and $S_{DGi,\max}$ are respectively maximal and minimal power limit of the DG of the renewable energy system at node i of the first stage. $Q^{(t)}(P_{DGi,t})$ is the minimum value of the problem

$$\min \quad \sum_{t=1}^{T}\sum_{(i,j)\in\Omega_L} P_{Loss}^{DGEV}(u_{i,t}^*;u_{j,t}^*;H_{ij,t}^*;T_{ij,t}^*) \tag{48}$$

$$\sum_{(i,j)\in\Omega_L} P_{Loss}^{DGEV}(u_{i,t}^*;u_{j,t}^*;H_{ij,t}^*;T_{ij,t}^*) = \sum_{(i,j)\in\Omega_L} G_{ij}(\sqrt{2}u_{i,t}^* + \sqrt{2}u_{j,t}^* - 2H_{ij,t}^*)$$

$$= \sum_{i=1}^{N}(P_{DGi,t}^* - P_{Di,t} - P_{EVi,t}) \tag{49}$$

subject to

$$\sum_{j\in\Omega_i}(\sqrt{2}G_{ij}u_{i,t}^* - (G_{ij}H_{ij,t}^* - B_{ij}T_{ij,t}^*)) = P_{DGi,t}^* - P_{Di,t} - P_{EVi,t} \tag{50}$$

$$\sum_{j\in\Omega_i}(\sqrt{2}B_{ij}u_{i,t}^* - (B_{ij}H_{ij,t}^* + G_{ij}T_{ij,t}^*)) = Q_{DGi,t}^* - Q_{Di,t} - Q_{EVi,t} \tag{51}$$

$$\frac{V_{i,\min}^2}{\sqrt{2}} \le u_{i,t}^* \le \frac{V_{i,\max}^2}{\sqrt{2}} \tag{52}$$

$$(I_{ij,t}^*)^2 = (G_{ij}^2 + B_{ij}^2)(\sqrt{2}u_{i,t}^* + \sqrt{2}u_{j,t}^* - 2H_{ij,t}^*) \le I_{ij,\max}^2 \tag{53}$$

$$0 \le (P_{DGi,t}^*)^2 + (Q_{DGi,t}^*)^2 \le 2 * \frac{S_{DGi,\min}^*}{\sqrt{2}} * \frac{S_{DGi,\max}^*}{\sqrt{2}} \tag{54}$$

The coordination optimization of the DG and EV charging problem given by (41)–(54) is a convex optimization problem. Interior point methods are considered to be one of the most successful classes of algorithms for solving stochastic convex optimization problems [2]. Baha used a unified practical primal interior decomposition algorithm to solve all SSOCP models mentioned in [2]. Thus the SSOCP models in this paper are optimized using the second-order cone programming package MOSEK [16].

4. Simulation results of case study

The proposed solution method was implemented in MATLAB R2011b running on a Pentium Dual-core CPU, 3.06 GHz PC with 3 GB of RAM, and a 64-bit operating system. The toolbox used for optimization was MOSEK. The code was implemented for a 69-node radial distribution system with total load of 3.802 MW+ j2.694 MVA and it is demonstrated in Figure 2. Charging stations of EVs are installed at nodes 10, 24, 32, 38, 52, and 67. From loss sensitivity factors, the top three nodes 27, 61, and 65, which are found to be more sensitive, are chosen to install DG units in the system [17]. The DG units are solar photovoltaic and wind turbines. The total active power of DGs is 3.6 MW and the power factor of them is 0.95. A load curve of the 69-node distribution system is shown in Figure 3. To validate the proposed method, the code was also implemented on a larger scale 118-node radial distribution system without tie-lines. The total power loads are 22.709MW+j17.041MVA. Detailed data of the 118-node radial distribution system are given in [18]. Charging stations of EVs are installed at nodes 19, 44, 54, 68, 71, 78, 94, and 102 with 3000 EVs. DG units are installed at eight nodes, 25, 36, 48, 56, 75, 88, 103, and 116, which are more sensitive to loss sensitivity factor in the system [19]. The capacity of the DG of renewable energy systems is 1.5 MW and the power factor is 0.95. The curve load is also shown in Figure 3 for comparability.
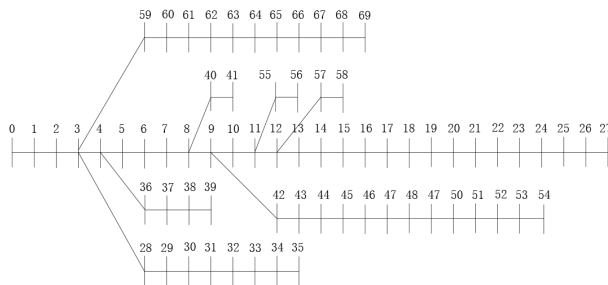
Figure 2. A distribution system with 69 nodes.
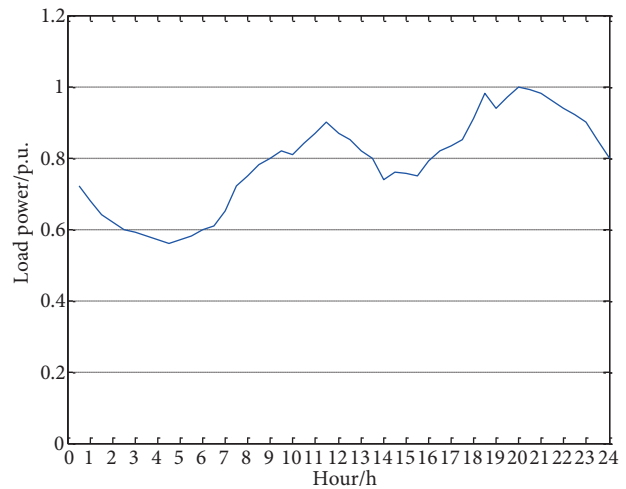
Figure 3. Daily load demand curve.

4.1. EV charging power

It is taken 500 EVs for the 69-node system and 3000 EVs for the 118-node as considered, which are randomly distributed at the nodes mentioned above. The total charging power of all EVs in the 69-node system in a day is shown in Figure 4 and that of the 118-node system is shown in Figure 5. The time period 0600–0800 is peak for people to travel out by much more EVs, and there are much fewer EVs going to charging stations

for recharging. Thus the charging power of EVs is only 0.5 MW in the 69-node system and only 0.7 MW in the 118-node system at the time period. The number and the charging power of EVs increase with time. It reaches maximum charging power at 2000–2200, which is the arrival time of EVs. The time period 2000–2200 is charging peak time, in which the number and the electric power of charging EVs are maximal.
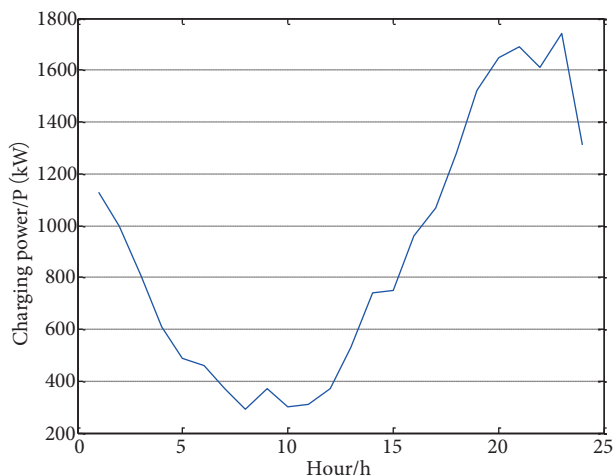
Figure 4. Daily curve for charging power of EVs for 69-node system.
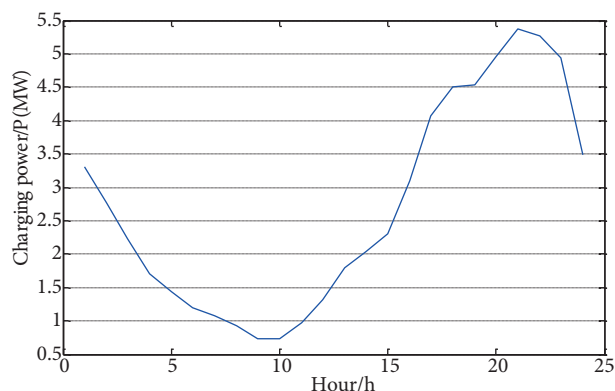
Figure 5. Daily curve for charging power of EVs for 118-node system.

Figures 6 and 7 show respectively the 24-h power loss of the base case and that with EVs in the 69-node system and 118-node system. Due to the stochastic charging load power of EVs the power loss with EVs is much larger than that of the base case, which shows that the randomness of charging power of EVs has a great impact on the power grid. The power loss may reach a very serious level especially in the EV charging peak time 2000–2200 when the peak power of the general is added to the distribution network.
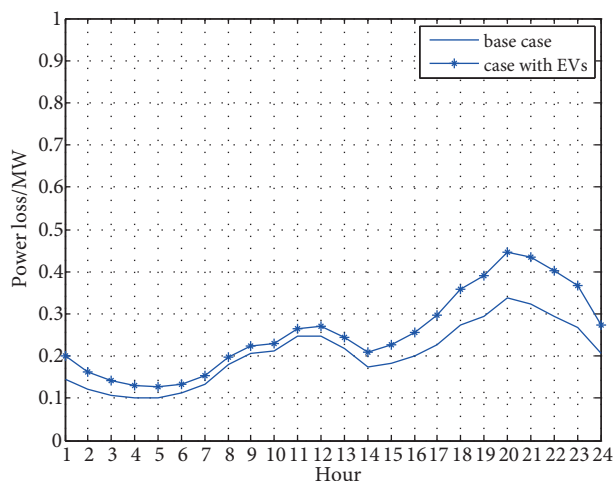
Figure 6. Daily curves of system power loss with and without EVs for 69-node system.
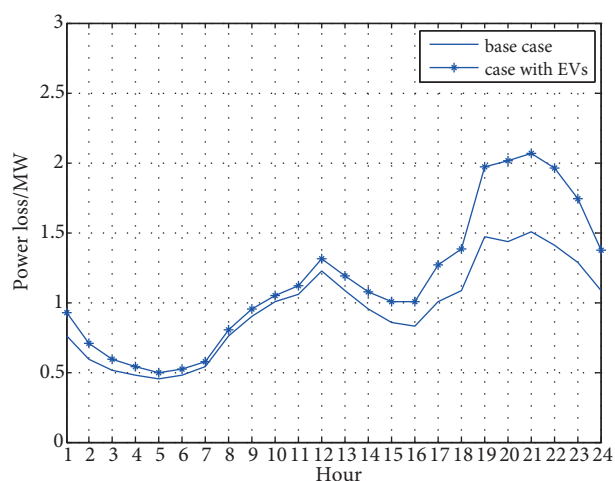
Figure 7. Daily curves of system power loss with and without EVs for 118-node system.

Table 1 and Table 2 show respectively the power flow of some nodes at three specific time periods, 0600–0800, 1400–1600, and 2000–2200, in the 69-node system and 118-node system. It can be seen from the two

tables that the voltage amplitudes and angles of some nodes in the network exceed the maximum allowable value of the voltage offset and the power loss may reach a very serious level especially in the electric vehicle charging peak time when the peak power of the general is added to the distribution network.

Table 1. Power flow of a 69-node distribution system with EVs.

Node number	0600–0800		1400–1600		2000–2200	
	Amplitude/p.u.	Angle/deg.	Amplitude/p.u.	Angle/deg.	Amplitude/p.u.	Angle/deg.
27	1.0020	0.3735	0.9973	0.4054	0.9567	0.5074
35	1.0245	0.0047	1.0437	0.0058	1.0455	0.0057
49	1.0401	-0.1536	1.0396	-0.1771	1.0379	-0.2536
41	1.0264	0.1036	1.0216	0.0969	1.0125	0.1013
53	0.9623	0.8336	0.9550	0.9418	0.9258	1.2183
54	0.9643	0.8598	0.9413	0.9643	0.9214	1.2503
69	1.0165	0.2318	1.0110	0.2432	0.9237	0.2903
Ploss/MW	0.1635		0.2192		0.4262	

Table 2. Power flow of the 118-node distribution system with EVs.

Node number	0600–0800		1400–1600		2000–2200	
	Amplitude/p.u.	Angle/deg.	Amplitude/p.u.	Angle/deg.	Amplitude/p.u.	Angle/deg.
25	1.0371	−0.0423	1.0267	−0.0632	1.0189	−0.0980
47	1.0111	−0.2784	1.0008	−0.3597	0.9846	−0.5657
54	1.0009	−0.1777	0.9878	−0.2318	0.9631	−0.4065
75	0.9777	−0.0479	0.9519	−0.1391	0.9018	−0.3729
77	0.9768	−0.0318	0.9507	−0.1179	0.9001	−0.3421
85	1.0243	−0.2280	1.0169	−0.3264	1.0042	−0.5092
113	0.9982	0.6669	0.9821	0.8304	0.9574	1.0960
Ploss/MW	0.6349		1.0288		2.0122	

4.2. Distributed generation coordination using SSCOP

In order to reduce the impact of the randomness of charging power of EVs, it is necessary to regulate the power output of the DG of renewable energy systems using SSOCP. Using the interior point method to solve the problem, the optimal results of the first stage in the two distribution systems are shown in Figures 8–11, respectively. They show that due to DGs injected the voltage offset and the power loss are smaller than that for the base case. In the second stage the load in the distribution systems contains two parts. One is the deterministic load; the other is the stochastic load of charging power of EVs. The optimal results of the second stage for the 69-node system and 118-node system are shown in Figures 12–15. The optimal output power of the DG of renewable energy systems in two different stages in the two distribution systems in the three specific time periods is shown in Tables 3 and 4, respectively. It can be seen from Tables 3 and 4 that in the second stage the output power of the DG of the renewable energy systems is greater than that in the first stage due to the randomness of EVs' charging power.
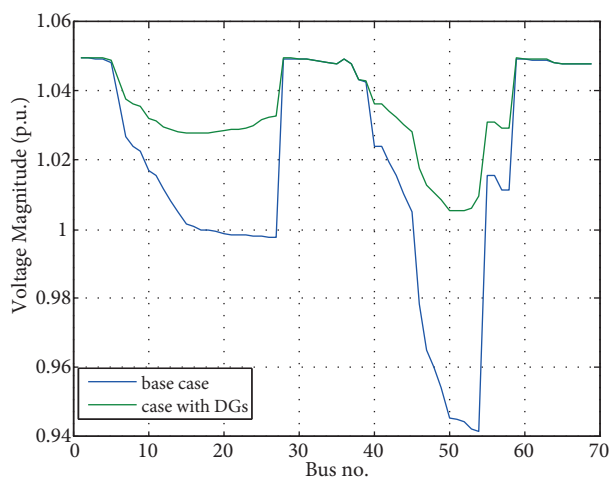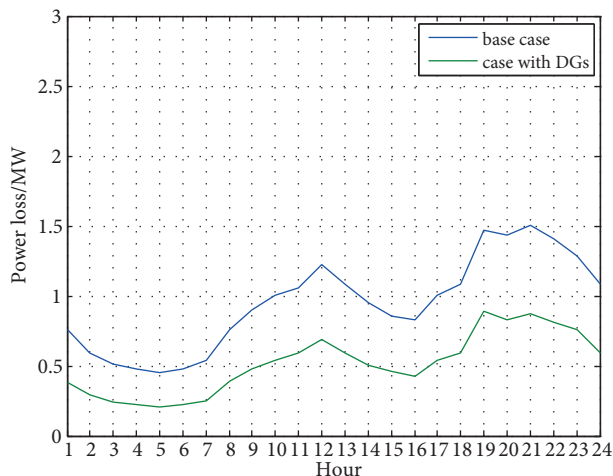
Figure 8. First stage of nodal voltage change within 2000–2200 for 69-node system.
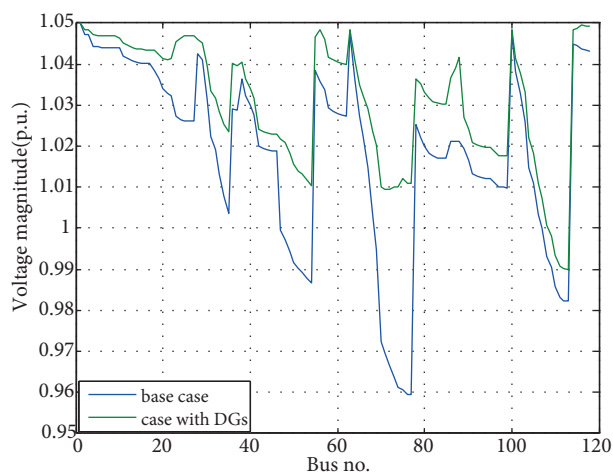
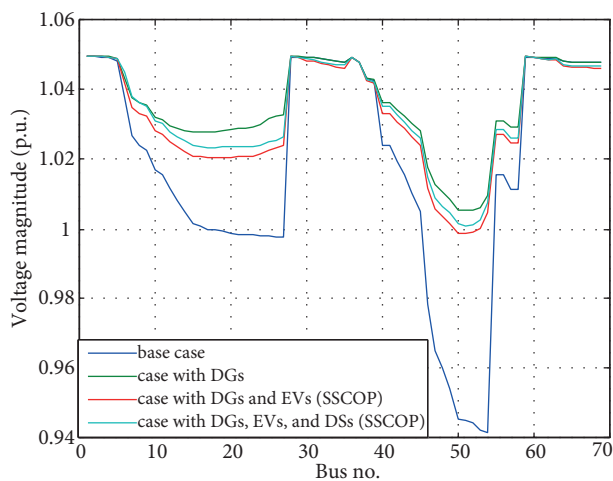**Figure 9**. First stage of nodal voltage change within 2000–2200 for 118-node system.
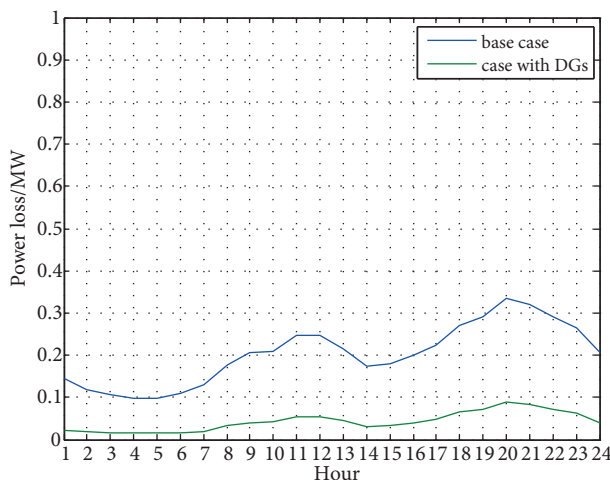
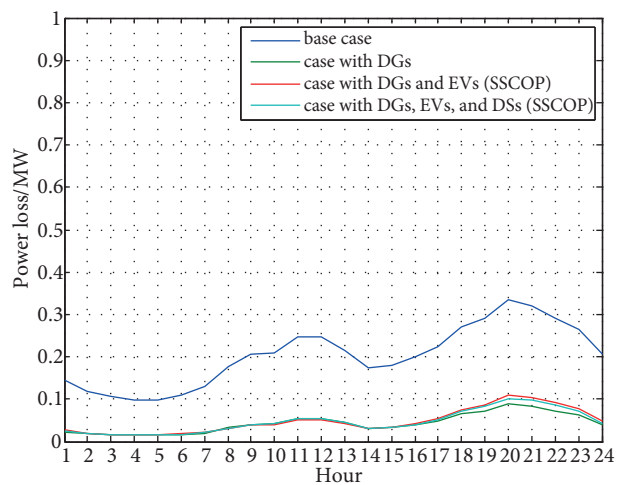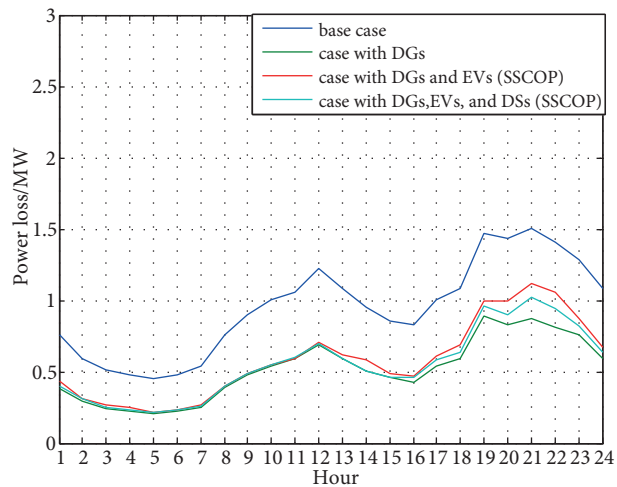**Figure 10**. First stage of daily curves for power loss of 69-node system.

Figure 11. First stage of daily curves for power loss of 118-node system.

Although the stochastic load of EVs adds to the distribution systems in the second stage, the power loss for each hour of a day and nodal voltage at the peak period 2000–2200 do not have much change. The reason is that in the second stage the output power of the DG of the renewable systems increases with the stochastic increasing charging power of EVs, which will greatly decrease power loss and also greatly improve the voltage level. Especially in the case with DGs, EVs, and DSs, no matter how much the charging power of EVs is, with the distributed power supply to the power grid, the voltage levels are greatly improved and the power loss is greatly decreased. From Figures 4 and 5 and Tables 3 and 4 we can see that the increased output power of the DG of renewable energy systems in the second stage is almost equivalent to the stochastic load of EVs' charging power. Especially in the EV charging peak time 2000–2200 the result is the same. This means that the randomness plays an important role in the problem and one can reduce power loss and save considerable energy in the distribution systems by using the stochastic model.

The simulate results of SSCOP in the case with DGs, EVs, and DSs are compared with the results

Figure 12. Second stage of nodal voltage change within 2000–2200 of 69-node system.

Figure 13. Second stage of daily curves for power loss of 69-node system.
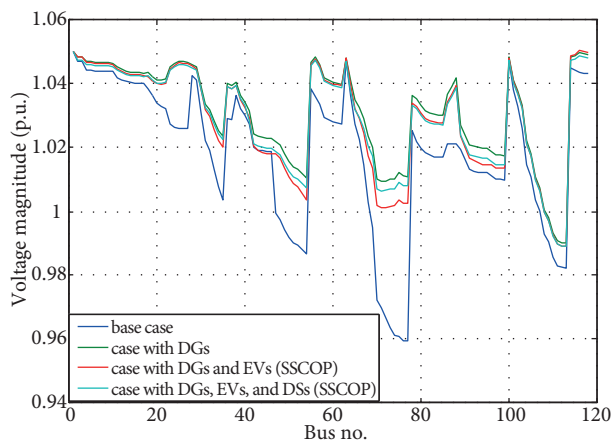
Figure 14. Second stage of nodal voltage change within 2000–2200 of 118-node system.

Figure 15. Second stage of daily curves for power loss of 118-node system.

obtained by the classical methods PSO and combined GA/PSO also in the case with DGs, EVs, and DSs. The voltage profile curves of the 118-node system with different methods are compared in Figure 16. It is observed that the voltage magnitude simulated by SSCOP is the most highly improved compared with PSO and combined GA/PSO. Figure 17 shows the power loss in each branch of the 118-node system simulated with different methods. From Figure 17, it is observed that the loss at each line is highly reduced after the DG systems' installation at optimum locations while the proposed solution SSCOP prominently reduces the loss at each line compared with the other two classical algorithms. Table 5 shows the methods that are compared regarding minimum node voltage magnitude in the 118-node system.

From the above compared results, we can confirm that with the application of the SSCOP the voltage level is improved and loss reduced compared with the results of PSO and combined GA/PSO. It also shows the advantages of dividing the load into two stages with the SSCOP.

Table 3. Optimal output power of DGs for the 69-node system.

Stage	Node number	0600–0800		1400–1600		2000–2200	
		Active power/MW	Reactive power/Mvar	Active power/MW	Reactive power/Mvar	Active power/MW	Reactive power/Mvar
Firststage	27	0.3	0.0986	0.4	0.1315	0.5	0.1644
	61	0.48	0.1578	0.53	0.1743	0.565	0.1858
	65	0.65	0.2137	0.75	0.2466	0.8	0.2630
Secondstage	27	0.5	0.1644	0.7	0.2301	1.2	0.3946
	61	0.75	0.2460	0.8	0.2630	1.2	0.3946
	65	0.8	0.2630	0.9	0.2959	1.2	0.3946
Increased output of DGs		0.62	0.2033	0.72	0.2366	1.735	0.5706

Table 4. Optimal output power of DGs for the 118-node system.

Stage	Node number	0600–0800		1400–1600		2000–2200	
		Active power/MW	Reactive power/Mvar	Active power/MW	Reactive power/Mvar	Active power/MW	Reactive power/Mvar
Firststage	25	0.58	0.19	0.68	0.22	0.88	0.29
	48	0.63	0.21	0.87	0.28	0.93	0.31
	56	0.93	0.30	1.16	0.38	1.21	0.50
	75	0.35	0.11	0.73	0.24	0.90	0.29
	88	0.32	0.10	0.45	0.15	0.86	0.28
	103	0.40	0.13	0.56	0.18	0.67	0.22
	116	0.37	0.12	0.48	0.15	0.64	0.21
Secondstage	25	0.59	0.19	0.92	0.24	1.3	0.32
	48	0.70	0.23	1.05	0.34	1.5	0.49
	56	0.95	0.31	1.21	0.39	1.5	0.49
	75	0.36	0.11	0.97	0.31	1.5	0.49
	88	0.68	0.22	0.95	0.31	1.5	0.49
	103	0.40	0.13	0.87	0.28	1.35	0.44
	116	0.56	0.18	1.10	0.36	1.5	0.49
Increased output of DGs		0.66	0.21	2.14	0.51	4.06	1.11

Table 5. Worst voltage for the 118-node distribution system.

Method	SSCOP	GA/PSO	PSO
Worst voltage(p.u.)	0.9892	0.9772	0.9756
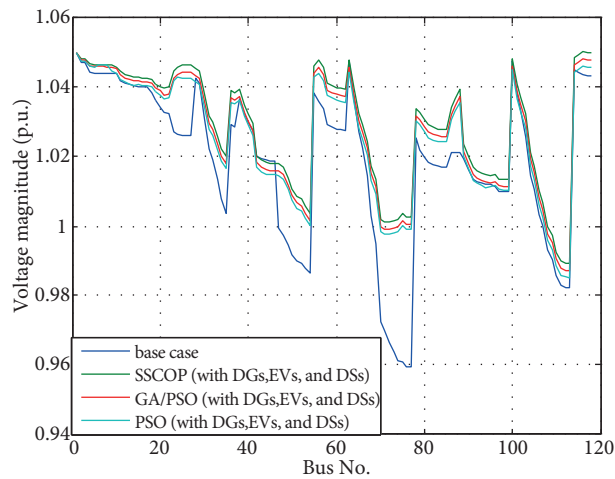
Figure 16. Comparison of voltage magnitude of 118-node system with different methods.
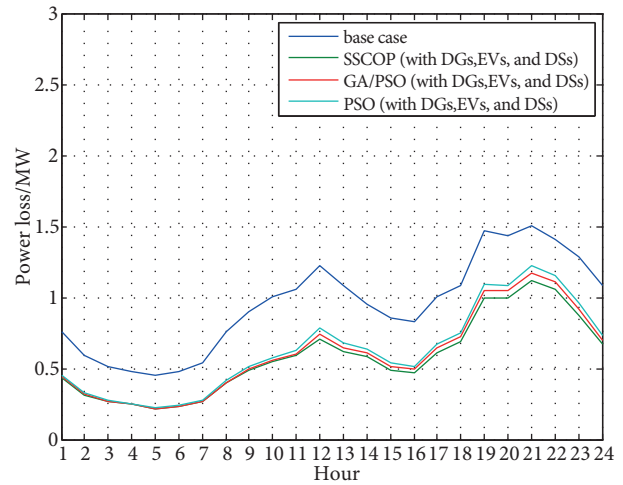
Figure 17. Comparison of voltage magnitude of 118-node system with different methods.

5. Conclusion

EVs and DGs are the factors that should be considered in the development of power systems. As a new power supply, the DG of renewable energy systems will bring great benefit to the grid. With EVs, as a special kind of load, their randomness of charging power will bring significant impact on the distribution network. Therefore, the uncertainty of EV charging is considered when optimizing the output power of the DG of renewable energy systems. In this paper, as a new approach for solving the problem with uncertain data, SSCOP is presented to solve the random load of EV charging. The proposed scheme is tested on 69-node and 118-node large-scale distribution systems to minimize the losses and to improve the voltage profile. The simulated results in the case with DGs, EVs, and DSs obtained using SSCOP are compared with the results of the classical methods PSO and GA/PSO. Test results indicate that the SSCOP algorithm is efficiently minimizing the losses and improving the voltage profile compared to GA/PSO and PSO. The performance of the proposed technique shows that due to optimum coordination of power output of the DG of renewable energy systems considering the uncertainties of EV charging, the power loss of each hour of a day is greatly decreased, and the voltage profile at all buses is also greatly improved. The simulated results demonstrate the feasibility and effectiveness of SSCOP.

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