Research Article

TÜBİTAK

QRMW: quantum representation of multi wavelength images

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Abstract: In this study, we propose quantum representation of multiwavelength images (QRMW) that gives preparation and retrieving procedures of quantum images. The proposed QRMW model represents multichannel and $2^n \times 2^m$ images. Moreover, we present image comparison and some image operations based on the QRMW model. Comparing our model with the models in the literature, the QRMW model has less time complexity. It also uses fewer qubits than existing models in the literature.

Key words: Multiwavelength quantum images, QRMW model, quantum image processing, quantum image compression, quantum image representation

1. Introduction

There have been many recent developments in quantum computing due to the advantages over classical computing. One of them is quantum digital image processing.

Digital image processing is an important field of technologies, such as defense, medical, and imaging devices. Considering the developments in today's imaging devices and their needs in this direction, the memory required for the images increases when the size of the images grows. The operations performed on images using classic calculations take a very long time, even if very powerful computers are used. Therefore, it is necessary to find high-performance methods to store and process images. These operations can be done in a very short time with the parallel processing feature of quantum-based computations. A quantum representation of the image and algorithms based on the representation models are revealed for quantum image processing, respectively.

Various quantum image representation models are proposed to store and process image information. Venegas-Andraca [1] described the qubit lattice, which is a representation of a quantum image. Venegas-Andraca and Ball [2] proposed the entangled image model, which uses entangled qubits. Latorre [3] presented the Real Ket model through image compression in a quantum context. Le et al. [4] presented a flexible representation of quantum images (FRQI) model for quantum computers. In their study, color information is kept for display of images in the form of a normalized state that collects information about colors and their locations. Zhang et al. [5] presented a novel enhanced quantum representation of digital images (NEQR) model, which uses the basis states of a qubit sequence to store the grayscale values at each pixel instead of the probability amplitude of a qubit. Zhang et al. [6] presented the quantum image representation for log-polar images (QUALPI) model.

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Sun et al. [7] proposed the RGB three-channel representation of quantum images (MCQI) model by developing the FRQI model. Yuan et al. [8] presented a simple quantum representation of infrared images (SQR) model by using qubit lattice and FRQI methods. Abdolmaleky et al. [9] introduced the RGB three-channel representation for quantum colored images (QMCR) model by using the NEQR model.

In most of the studies in this field, quantum image representations have been presented for single- or triple-color (RGB) images. It is well known that image processing is important at different wavelengths in today's technologies. In this respect, there are a wide range of applications employing hyperspectral imaging, ranging from satellite-based/airborne remote sensing and military target detection to industrial quality control and lab applications in medicine and biophysics. Therefore, it is important to create a representation model of quantum images that will include all wavelength. There is no representation model for multiwavelength images in the literature. If the existing RGB models QMCR and MCQI are developed for more than three channels, they will use too many qubits and will have more time complexity.

In this paper, a quantum representation of multiwavelength images (QRMW) model is proposed for images that can be formed for all wavelengths of light. The proposed QRMW model uses fewer qubits and has less time complexity than existing models in the literature. These advantages are explained in the following sections with comparisons.

This paper is organized as follows: in Section 2, we introduce the latest related works used in comparison with our model. In Section 3, we describe the newly proposed QRMW image representation model, which gives the procedures of preparation and retrieving of quantum image. In Section 4, image compression and some image operations based on QRMW are presented. Finally, a conclusion is given.

2. Related works
In classical computers, image pixels are stored in the classical bits through the photoelectric conversion system. In quantum computers, the storage unit is the qubit and so images must be stored in qubits. So far, proposed quantum image representation models [1–9] represent $2^n \times 2^n$ images with single or triple channels. Image retrieval is probabilistic in the imaging models that hold color information at amplitude (for example, the models based on FRQI). However, retrieval is nonprobabilistic in the imaging models where the color information is held at basis states of the qubit sequence (for example, models based on NEQR).

2.1. The MCQI model
The MCQI model was originally proposed by Sun et al. [7]. This model stores information about the colors and the position of each pixel of image at the amplitudes. This model for $2^n \times 2^n$ image can be mathematically expressed as follows [7]:

$$|I\rangle = \frac{1}{2^{n+1}} \sum_{i=0}^{2^{2n}-1} \left| C_{RGB\alpha}^i \right\rangle \otimes |i\rangle$$

$$\left| C_{RGB\alpha}^i \right\rangle = \cos\theta_R^i |000\rangle + \cos\theta_G^i |001\rangle + \cos\theta_B^i |010\rangle + \cos\theta_\alpha^i |011\rangle +$$
$$+ \sin\theta_R^i |100\rangle + \sin\theta_G^i |101\rangle + \sin\theta_B^i |110\rangle + \sin\theta_\alpha^i |111\rangle ,$$

where $|000\rangle |001\rangle \ldots |111\rangle$ are the computational basis states of three qubits, $\theta_R^i, \theta_G^i, \theta_B^i, \theta_\alpha^i$ are four angles used to encode the information of color channels of the ith pixel, and $|i\rangle$ for i = 0, 1, . . ., $2^{2n} - 1$ are $2^{2n} -$ D computational basis states [7].

2.2. The QMCR model

The QMCR model was originally proposed by Abdolmalekya et al. [9]. This model is based on the NEQR. This model for $2^n \times 2^n$ image can be mathematically expressed as follows [9]:

$$|I\rangle = \frac{1}{2^n} \sum_{y=0}^{2^n-1} \sum_{x=0}^{2^n-1} |C_{RGByx}\rangle \otimes |yx\rangle$$

where the state $|C_{RGByx}\rangle$ is used to encode the information of the R, G, and B channels (2^q gray range of each channel) of the yx pixel. The state $|C_{RGByx}\rangle$ is defined as follows:

$$|C_{RGByx}\rangle = |R_{yx}\rangle |G_{yx}\rangle |B_{yx}\rangle$$

$$|R_{yx}\rangle = |r_{yx}^{q-1} r_{yx}^{q-2} \ldots r_{yx}^0\rangle, \, |G_{yx}\rangle = |g_{yx}^{q-1} g_{yx}^{q-2} \ldots g_{yx}^0\rangle, \, |B_{yx}\rangle = |b_{yx}^{q-1} b_{yx}^{q-2} \ldots b_{yx}^0\rangle$$

where r_{yx}^k, g_{yx}^k, $b_{yx}^k \in 0, 1\}$ and R_{yx}, G_{yx}, $B_{yx} \in \{0, 1, \ldots, 2^q - 1\}$ [9].

3. QRMW model

It is important to set an image model with multiple channels. In this paper, we propose a QRMW model with multiple channels (more than three) and $2^n \times 2^m$ images. In this section, the preparation and presentation of the image in the QRMW model will be explained in detail.

This new model uses the basis states of a qubit sequence to store the values at different wavelengths of each pixel of the image. QRMW uses three separate register qubit sequences to hold the position, wavelength, and color values of each pixel. QRMW keeps the whole image in the superposition of the three qubit sequences. The first qubit sequence is used to encode the color values corresponding to the respective wavelength channel of the pixels in the image, the second qubit sequence is used to encode the wavelength channel information, and the third qubit sequence is used to encode position information. Suppose that the number of wavelength channel is cn and the color scale in the channels is 2^q for the image, where 2^q represents the maximum value at any wavelength. In our model, b-qubits ($b = ceil(\log_2 cn)$) for cn-wavelength, q-qubits for color scale, and $(n + m)$-qubits for position information are required for a $2^n \times 2^m$ image. The color value (with 2^q scale) for the channel of the (y, x) position of the image can be expressed in a binary string as follows:

$$f(\lambda, y, x) = c_{\lambda_{yx}}^0 c_{\lambda_{yx}}^1 \ldots c_{\lambda_{yx}}^{q-2} c_{\lambda_{yx}}^{q-1}, \, c_{\lambda_{yx}}^k \in [0, 1], \, f(\lambda, y, x) \in [0, 2^q - 1] \tag{1}$$

The representative expression of a quantum image for a $2^n \times 2^m$ QRMW image can be written as

$$|I\rangle = \frac{1}{\sqrt{2^{b+n+m}}} \sum_{\lambda=0}^{2^b-1} \sum_{y=0}^{2^n-1} \sum_{x=0}^{2^m-1} |f(\lambda, y, x)\rangle \otimes |\lambda\rangle \otimes |yx\rangle, \tag{2}$$

where λ is channel information and yx is position information. Throughout the rest of the paper we use λ for the channel information and yx for the position information. A 2 × 2 QRMW sample image with [0–255] color scale and 4 channels is shown in Figure 1. In this sample, 12 qubits are used for color, channel, and position information. The channel and position information are stored in the superposition of second and third register qubits, respectively.
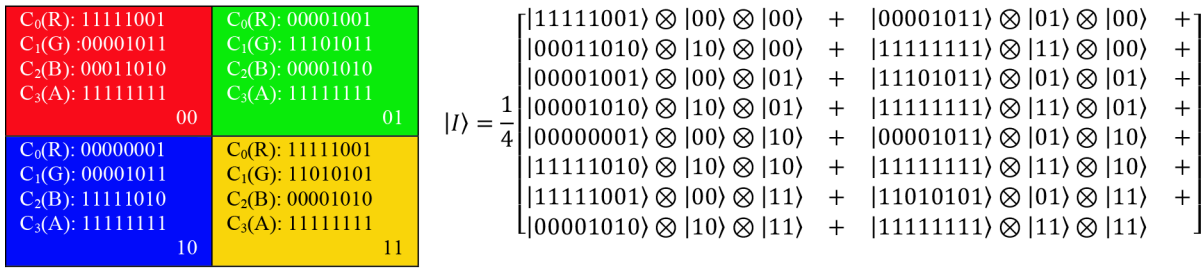
C_0(R): 11111001 C_1(G) :00001011 C_2(B): 00011010 C_3(A): 11111111 00	C_0(R): 00001001 C_1(G): 11101011 C_2(B): 00001010 C_3(A): 11111111 01
C_0(R): 00000001 C_1(G): 00001011 C_2(B): 11111010 C_3(A): 11111111 10	C_0(R): 11111001 C_1(G): 11010101 C_2(B): 00001010 C_3(A): 11111111 11

$$|I\rangle = \frac{1}{4}\begin{bmatrix} |11111001\rangle \otimes |00\rangle \otimes |00\rangle & + & |00001011\rangle \otimes |01\rangle \otimes |00\rangle & + \\ |00011010\rangle \otimes |10\rangle \otimes |00\rangle & + & |11111111\rangle \otimes |11\rangle \otimes |00\rangle & + \\ |00001001\rangle \otimes |00\rangle \otimes |01\rangle & + & |11101011\rangle \otimes |01\rangle \otimes |01\rangle & + \\ |00001010\rangle \otimes |10\rangle \otimes |01\rangle & + & |11111111\rangle \otimes |11\rangle \otimes |01\rangle & + \\ |00000001\rangle \otimes |00\rangle \otimes |10\rangle & + & |00001011\rangle \otimes |01\rangle \otimes |10\rangle & + \\ |11111010\rangle \otimes |10\rangle \otimes |10\rangle & + & |11111111\rangle \otimes |11\rangle \otimes |10\rangle & + \\ |11111001\rangle \otimes |00\rangle \otimes |11\rangle & + & |11010101\rangle \otimes |01\rangle \otimes |11\rangle & + \\ |00001010\rangle \otimes |10\rangle \otimes |11\rangle & + & |11111111\rangle \otimes |11\rangle \otimes |11\rangle & \end{bmatrix}$$

Figure 1. 2×2 image sample and expression of QRMW model.

3.1. Preparing procedure

In this section, the preparation process for QRMW images will be explained. In this model, the QRMW image is prepared in two steps.

Step 1: In the first step of the preparing procedure, q + b + n + m qubits should be prepared and these qubits should be set to $|0\rangle$ as the initial state. The initial state can be expressed as

$$|\psi_0\rangle = |0\rangle^{\otimes q + b + n + m} \tag{3}$$

The identity (I) and Hadamard (H) transforms are applied to convert the initial state $|\psi_0\rangle$ to the intermediate state $|\psi_1\rangle$. We define all quantum operations in the first step as U_1

$$U_1 = I^{\otimes q} \otimes H^{\otimes b + n + m} \tag{4}$$

where $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ [8]. By applying the U_1 transformation operator on the state $|\psi_0\rangle$, the intermediate state $|\psi_1\rangle$ is obtained as follows:

$$U_1\left(|\psi_0\rangle\right) = |\psi_1\rangle = \left(I|0\rangle\right)^{\otimes q} \otimes \left(H|0\rangle\right)^{\otimes b + n + m}$$

$$= \frac{1}{\sqrt{2^{b+n+m}}} \sum_{\lambda=0}^{2^b-1} \sum_{y=0}^{2^n-1} \sum_{x=0}^{2^m-1} |0\rangle^{\otimes q} \otimes |\lambda y x\rangle \tag{5}$$
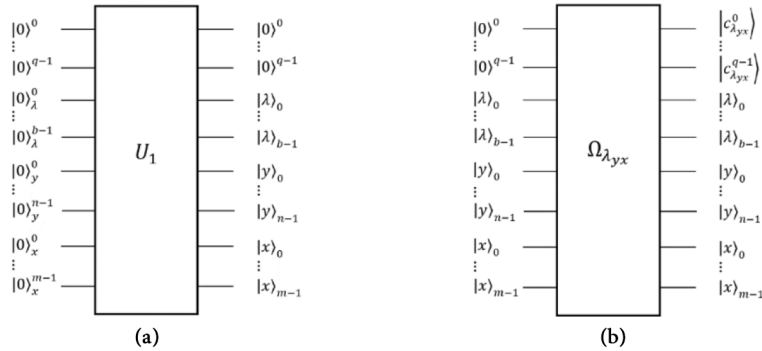
The quantum circuit of the first step is given in Figure 2a. After this step, the position and channel information of all the pixels will be stored in the QRMW quantum model. All of the pixels are stored in the superposition of the qubit sequence at the intermediate state $|\psi_1\rangle$.

Step 2: In order to set the color information of each channel of each pixel, the second step consists of 2^{b+n+m} subprocesses. The quantum subprocess $U_{\lambda_{yx}}$ of the channel (λ) of the pixel (yx) is as follows:

$$U_{\lambda_{yx}} = I^{\otimes q} \otimes \sum_{k=0}^{2^b-1} \sum_{j=0}^{2^n-1} \sum_{i=0}^{2^m-1} |kji\rangle \langle kji| + \Omega_{\lambda_{yx}}, \, for \, kji \neq \lambda yx \tag{6}$$

where the quantum operator $\Omega_{\lambda_{yx}}$, which sets the color value of the channel (λ) of the corresponding pixel (yx), is

$$\Omega_{\lambda_{yx}} = |f(\lambda, y, x)\rangle |\lambda yx\rangle \langle 0|^{\otimes q} \langle \lambda yx| \tag{7}$$

Figure 2. (a) The Black-Box circuit of the first step of the preparation procedure, (b) the Black-Box circuit of the second step which is the process of setting color to the corresponding channel of the corresponding position.

The quantum circuit of the $\Omega_{\lambda_{yx}}$ operator is given in Figure 2b.

After applying $U_{\lambda_{yx}}$, the intermediate state ϕ_1 turns into Eq. (8).

$$U_{\lambda_{yx}}(|\psi_1\rangle) = U_{\lambda_{yx}} \left(\frac{1}{\sqrt{2^{b+n+m}}} \sum_{k=0}^{2^b-1} \sum_{j=0}^{2^n-1} \sum_{i=0}^{2^m-1} |0\rangle^{\otimes q} |kji\rangle \right)$$

$$= \frac{1}{\sqrt{2^{b+n+m}}} \left(\sum_{k=0}^{2^b-1} \sum_{j=0}^{2^n-1} \sum_{i=0,\,kji\neq\lambda yx}^{2^m-1} |0\rangle^{\otimes q} |kji\rangle + |f(\lambda,y,x)\rangle |\lambda yx\rangle, \right) \tag{8}$$

where $\langle 0^{\otimes q}\lambda yx| \, 0^{\otimes q}\lambda yx\rangle = 1$ due to orthogonality. The $U_{\lambda_{yx}}$ suboperation only sets the color value of the corresponding channel of the pixel in the corresponding position. The operator U_2 is defined as follows:

$$U_2 = \prod_{\lambda=0}^{2^b-1} \prod_{y=0}^{2^n-1} \prod_{x=0}^{2^m-1} U_{\lambda_{yx}} \tag{9}$$

The operator U_2 includes all the suboperations that set the color values of each channel of each pixel. After the application of the U_2 quantum operator to the $|\psi_1\rangle$ intermediate state, the final state $|\psi_2\rangle$, which represents the QRMW quantum image, is obtained as follows:

$$|\psi_2\rangle = U_2(|\psi_1\rangle) = \frac{1}{\sqrt{2^{b+n+m}}} \left(\sum_{\lambda=0}^{2^b-1} \sum_{y=0}^{2^n-1} \sum_{x=0}^{2^m-1} |f(\lambda,y,x)\rangle |\lambda yx\rangle \right) \tag{10}$$

After these two steps, the preparation procedure is over. The time complexity of the quantum operator U_1 in step 1 is $O(q+b+n+m)$. The time complexities of the quantum operators $U_{\lambda_{yx}}$, $\Omega_{\lambda_{yx}}$, and U_2 in step 2 are respectively $O(q)$, $O(1)$, $O(q\times 2^{b+n+m})$. Let us compare qubit numbers and time complexity of our QRMW model with the models in the literature.

Numbers of qubits of the QRMW, MCQI, and QMCR models with 4 channels and 2^q color scale for $2^n \times 2^n$ images are shown in Table 1.

The number of qubits used in MCQI model is less than in the QRMW model for fewer channels. However, as the number of channels increases, the number of qubits used in the QRMW model is less than in the MCQI

Table 1. Numbers of qubits of the QRMW, MCQI and QMCR with 4 channels and 2^q color scale for $2^n \times 2^n$ images.

Model	Qubits	Arbitrary qubits		Total qubits
QRMW	$q+ 2 + 2n$	-		$q+ 2 + 2n$
MCQI	$3 + 2n$	-		$3 + 2n$
QMCR	$4q+ 2n$	$4q$		$8q+ 2n$

model. Furthermore, it is clear that the QRMW model uses fewer qubits than the QMCR model for all channel. The difference between QRMW and QMCR will increase as the number of channels increases.

Position information storage is the same in all the models. Color information storage is different in all the models.

In the QRMW model, $2^n \times 2^m$ rectangular images and $2^n \times 2^n$ square images can be represented, while $2^n \times 2^n$ square images can be represented in the other models.

It is seen from Table 2 that the QRMW model shows a quadratic decrease when it is compared with the MCQI model and linear decrease when it is compared with the QMCR model.

Table 2. The time complexity information for preparing procedure of QRMW, MCQI, and QMCR images.

Model	1st Step	2nd Step
QRMW	$O(q+ 2 + 2n)$	$O(q \times 2^{(2 + 2n)})$
MCQI	$O(3 + 2n)$	$O(2^{(3 + 4n)})$
QMCR	$O(4q+ 2n)$	$O(qn \times 2^{(2 + 2n)})$

3.2. Image retrieving procedure

Efficiently retrieving the image from a quantum state is an important process. Quantum measurement is a unique way to recover classical information from a quantum state. The following operations are performed to obtain pixel values from the $2^n \times 2^m$ QRMW image with 2^b wavelength channel and 2^q color scale in the channels.

The quantum measurement preparation operator $\Gamma_{\lambda yx}$, which will be used for the qubit sequence at the desired position and wavelength, is defined as follows:

$$\Gamma_{\lambda yx} = \sqrt{2^{b + n + m}} \left(I^{\otimes q} \otimes |\lambda yx\rangle \langle \lambda yx| \right) \tag{11}$$

$$|P_{\lambda yx}\rangle = |f(\lambda, y, x)\rangle |\lambda yx\rangle \tag{12}$$

The color values of the pixels are stored at the basis states of qubit sequence in the QRMW and QMCR models. The color values of the pixels are stored at the probability amplitude in the MCQI model. The color values are exactly obtained in QRMW and QMCR models. The color values are approximately obtained in the MCQI model. The color values of the pixels need to be exact for the details of image and detailed operations.

4. Quantum image compression and color operations

4.1. Quantum image compression

Quantum image compression (QIC) is a process that reduces the amount of computational resources used in image preparation and retrieving operations such as in classical image compression techniques. The main resources of quantum computation are the basic quantum gates and operators. This is also expressed as time complexity.

From the image preparation procedure, it is clear that the time complexity is directly related to the image size. When the image size is too large, the preparation procedure takes a long time. For QIC, first the pixels with the same color value in different positions and/or channels in the image should be grouped. Applying operations to groups with the same information instead of performing individual operations on all the pixels will reduce the resources (proportional to the number of groups). For this purpose, a Boolean expression should be obtained for the state. Methods such as Karnaugh maps and the Espresso algorithm [10] can be used to minimize Boolean expressions. In this paper, the Espresso algorithm is used to minimize Boolean expressions.

In our model, the operator $U_{\lambda_{yx}}$ is the process of setting the color value to the corresponding channel of the pixel in the corresponding position during the QRMW image preparation procedure. Thus the runtime is directly related to the number of applications of this operator.

$U_{\lambda_{yx}}$ and $\Omega_{\lambda_{yx}}$ operators have been updated to work as a single operator for the $\{y\}$ list with the same values in groups according to the minimized Boolean expressions.

$$U_{\lambda_{\{y\}x}} = I^{\otimes q} \otimes \sum_{k=0}^{2^b - 1} \sum_{j=0}^{2^n - 1} \sum_{i=0}^{2^m - 1} |kji\rangle \langle kji| + \Omega_{\lambda_{\{y\}x}} \tag{13}$$

where
$$kji \neq \lambda \{y\} x \text{ and } \{y\} = \{y_0 y_1, \ldots, y_{n-1}$$

$$\Omega_{\lambda_{\{y\}x}} = \sum_{i=0}^{n-1} |f(\lambda, y_0, x)\rangle |\lambda y_i x\rangle \langle 0|^{\otimes q} \langle \lambda y_i x| \tag{14}$$

where
$$f(\lambda, y_0, x) = f(\lambda, y_1, x) = \ldots = f(\lambda, y_{n-1}, x)$$

Let us consider an 8×4 sample image with 4 channels and different color values to illustrate the compression process by grouping pixels of the same color value in the QRMW image.

According to Figure 3, $\{y\} = \{0,1,2,3,4,5,6,7\}$. We need to use only one operator instead of eight operators for this minimization. This is the same for the other eight groups.

For the groups given in Figure 4, 9-$U_{\lambda_{yx}}$ operators are used instead of 72. If we calculate compression ratio for example, we get

$$Compression\_Ratio = \left(1 - \frac{Ops\_After\_Compression}{Ops\_Before\_Compression}\right) \times 100\% \tag{15}$$

According to the equation given in Eq. (15), the compression ratio of the QRMW is 87.5%.

Let us compare the ratio of our QRMW with the other models for four sample images given in Figure 5 with the size 4×4 .
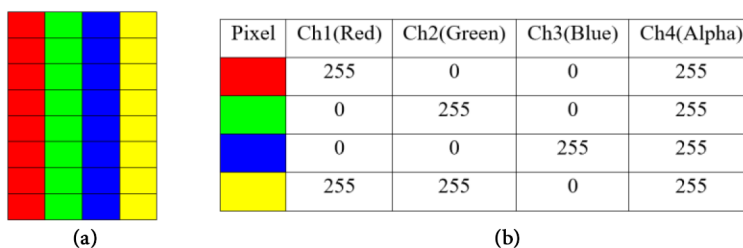
Figure 3. (a) An 8×4 sample image with 4 channels and different color values, (b) channel and color information of pixels.
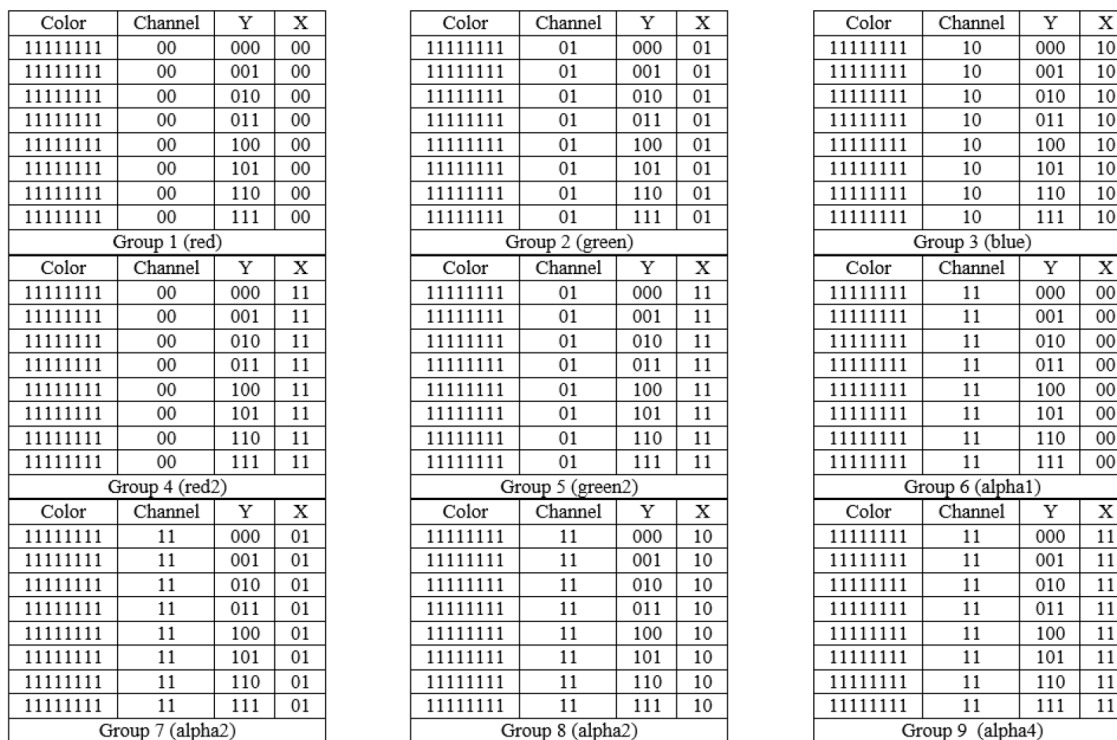
Color	Channel	Y	X
11111111	00	000	00
11111111	00	001	00
11111111	00	010	00
11111111	00	011	00
11111111	00	100	00
11111111	00	101	00
11111111	00	110	00
11111111	00	111	00
Group 1 (red)			

Color	Channel	Y	X
11111111	01	000	01
11111111	01	001	01
11111111	01	010	01
11111111	01	011	01
11111111	01	100	01
11111111	01	101	01
11111111	01	110	01
11111111	01	111	01
Group 2 (green)			

Color	Channel	Y	X
11111111	10	000	10
11111111	10	001	10
11111111	10	010	10
11111111	10	011	10
11111111	10	100	10
11111111	10	101	10
11111111	10	110	10
11111111	10	111	10
Group 3 (blue)			

Color	Channel	Y	X
11111111	00	000	11
11111111	00	001	11
11111111	00	010	11
11111111	00	011	11
11111111	00	100	11
11111111	00	101	11
11111111	00	110	11
11111111	00	111	11
Group 4 (red2)			

Color	Channel	Y	X
11111111	01	000	11
11111111	01	001	11
11111111	01	010	11
11111111	01	011	11
11111111	01	100	11
11111111	01	101	11
11111111	01	110	11
11111111	01	111	11
Group 5 (green2)			

Color	Channel	Y	X
11111111	11	000	00
11111111	11	001	00
11111111	11	010	00
11111111	11	011	00
11111111	11	100	00
11111111	11	101	00
11111111	11	110	00
11111111	11	111	00
Group 6 (alpha1)			

Color	Channel	Y	X
11111111	11	000	01
11111111	11	001	01
11111111	11	010	01
11111111	11	011	01
11111111	11	100	01
11111111	11	101	01
11111111	11	110	01
11111111	11	111	01
Group 7 (alpha2)			

Color	Channel	Y	X
11111111	11	000	10
11111111	11	001	10
11111111	11	010	10
11111111	11	011	10
11111111	11	100	10
11111111	11	101	10
11111111	11	110	10
11111111	11	111	10
Group 8 (alpha2)			

Color	Channel	Y	X
11111111	11	000	11
11111111	11	001	11
11111111	11	010	11
11111111	11	011	11
11111111	11	100	11
11111111	11	101	11
11111111	11	110	11
11111111	11	111	11
Group 9 (alpha4)			

Figure 4. The nine groups in positions with the same color information before quantum image preparation.
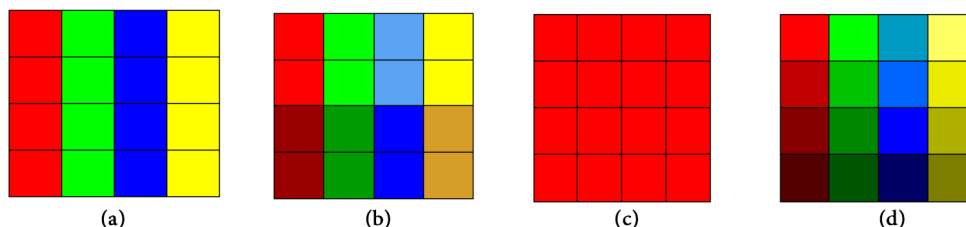

Figure 5. 4×4 Sample images with 256 color scale and 3 channels.

It is seen from Table 3 that the compression ratio of the QRMW model is more than that of the MCQI and QMCR models.

Table 3. Compression ratios of QRMW, MCQI, and QMCR models for (a)(d) sample images in Figure 5.

Model	(a)	(b)	(c)	(d)	AVG
QRMW	91.67%	83.34%	97.92%	0%	68.23%
	4/48	8/48	1/48	48/48	
MCQI	75%	50%	93.75%	0%	54.69%
	512/2048	1024/2048	128/2048	2048/2048	
QMCR	75%	50%	93.75%	0%	54.69%
	96/384	192/384	24/384	384/384	

4.2. Quantum image color operations

In this section, the operations on the images will be presented in three categories: complete color operations, partial color operations, and position operations.

4.2.1. Complete color operations

The U_{CC} operator is defined as

$$U_{CC} = X^{\otimes q} \otimes I^{\otimes b + n + m} \tag{16}$$

The U_{CC} operator applies quantum X gate to each qubit of the qubit sequence in the first register in which the color information is stored for the $|I\rangle$ quantum image. This operator reverses all qubits in the color qubit sequence, setting the current color values to their opposite values. Hence, all the colors in the whole image will change. Application of the U_{CC} operator to the $|I\rangle$ quantum image is given as follows:

$$U_{CC} |I\rangle = \frac{1}{\sqrt{2^{b+n+m}}} \left(\sum_{\lambda=0}^{2^b - 1} \sum_{y=0}^{2^n - 1} \sum_{x=0}^{2^m - 1} |2^q - 1 - f(\lambda, y, x)\rangle \, |\lambda y x\rangle \right) \tag{17}$$

The quantum circuit of the U_{CC} operator and the changes in the sample image are shown in Figure 6. If we apply Eq. (17) to sample image Figure 6b, we get image Figure 6c.
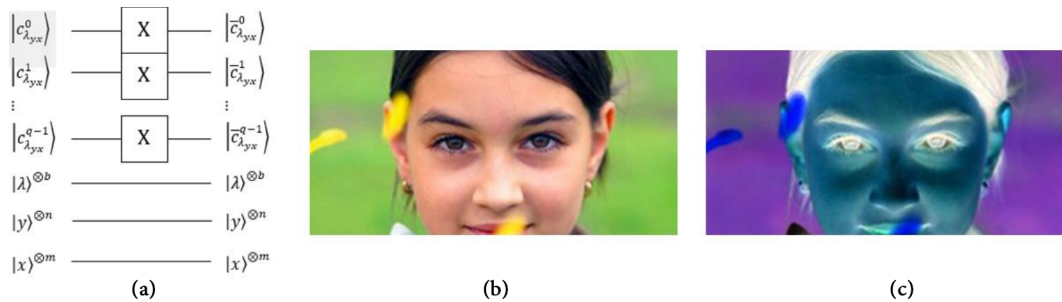
Figure 6. (a) Quantum circuit of U_{CC} operator for QRMW model, (b) sample image with 3 channels, (c) the image after U_{CC} is applied.

The U_{CC} operator can be expanded for combinations of different numbers and positions of q-qubits in the first register in which color information is stored. For example, $U_{CC} = X \otimes I^{\otimes \alpha} \otimes X \otimes I^{\otimes q - \alpha - 2} \otimes I^{\otimes b + n + m}$, where α is the desired arbitrary number for first register, with a maximum value of $q - 2$.

4.2.2. Partial color operations

The channel-controlled $U_{(PC)_k}$ operator, which is prepared to process colors in a particular channel, is given as follows:

$$U_{(PC)_k} = (b - CNOT)_k^{\otimes q} \otimes I^{\otimes b + n + m} \tag{18}$$
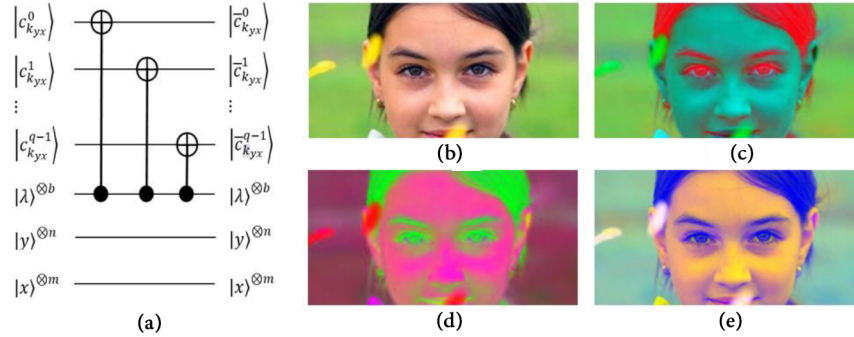
$$U_{(PC)_k} |I\rangle = \frac{1}{\sqrt{2^{b+n+m}}} \left(\sum_{\lambda=0, \lambda \neq k}^{2^b-1} \sum_{y=0}^{2^n-1} \sum_{x=0}^{2^m-1} |f(\lambda, y, x)\rangle |\lambda yx\rangle + |2^q - 1 - f(k, y, x)\rangle |kyx\rangle \right) \tag{19}$$

The U_{CH} and $U_{(CH)_k}$ operators, which are prepared for changing the color values of all or some of the channels in the quantum image, are given as follows:

$$U_{CH} = I^{\otimes q} \otimes X^{\otimes b} \otimes I^{\otimes n + m} \tag{20}$$

$$U_{(CH)_k} = I^{\otimes q} \otimes (k - CNOT)^{\otimes b} \otimes I^{\otimes n + m} \tag{21}$$

$$U_{CH} |I\rangle = \frac{1}{\sqrt{2^{b+n+m}}} \left(\sum_{\lambda=0}^{2^b-1} \sum_{y=0}^{2^n-1} \sum_{x=0}^{2^m-1} |f(\lambda, y, x)\rangle |2^b - 1 - \lambda)\rangle |yx\rangle \right) \tag{22}$$

Figure 7. (a) Quantum circuit of $U_{(PC)_k}$ operator for QRMW model, (b) sample image with 3 channels, (c) the image after $U_{(PC)_0}$) is applied, (d) the image after $U_{(PC)_1}$ is applied, (e) the image after $U_{(PC)_2}$ is applied.

4.2.3. Position operations

The operators U_{PO} and $U_{(PO)_{yx}}$ for position operations are shown as

$$U_{PO} = I^{\otimes q + b} \otimes X^{\otimes n + m} \tag{23}$$

$$U_{(PO)_{yx}} = I^{\otimes q + b} \otimes (yx - CNOT)^{\otimes n + m} \tag{24}$$

For the $|I\rangle$ quantum image, the U_{PO} operator applies the X gate to all qubits of the qubit sequence in the third register where the position information is stored and the $U_{(PO)_{yx}}$ operator applies yx-$CNOT$ gate to the corresponding qubits in the third register. The U_{PO} operator reverses the qubits in the qubit sequence in which
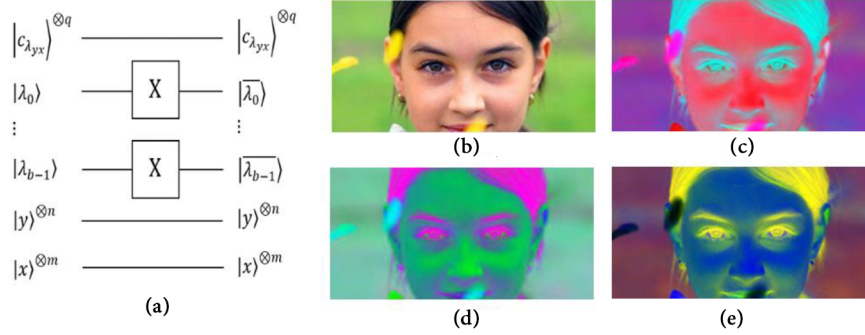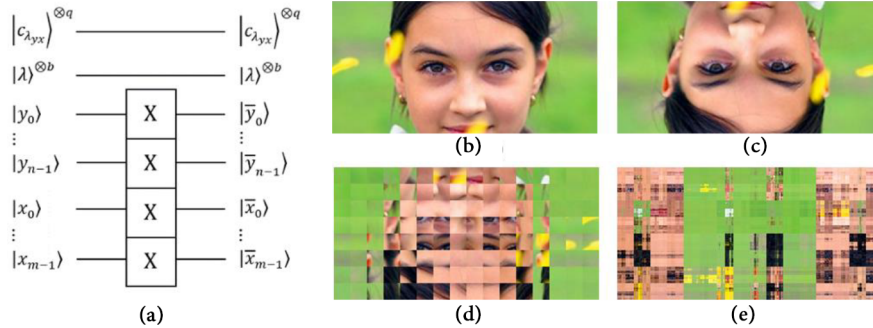
Figure 8. (a) Quantum circuit of U_{CH} operator for QRMW model, (b) sample image with 4 channels, (c) the image after U_{CH} is applied, (d) the image after $U_{(CH)_1}$ is applied, (e) the image after $U_{(CH)_2}$ is applied.

the position information is stored. Thus, all of the pixels have been relocated in the image. $U_{(PO)_{yx}}$ will change only the specified row/column pixels, not all the pixels. Application of the U_{PO} operator to the $|I\rangle$ quantum image is given as follows:

$$
U_{PO}\,|I\rangle \;=\; \frac{1}{\sqrt{2^{b+n+m}}}\left(\sum_{\lambda=0}^{2^{b}-1}\sum_{y=0}^{2^{n}-1}\sum_{x=0}^{2^{m}-1}\big|\,f(\lambda,y,x)\rangle\,|\lambda\rangle\,\big|\,2^{b}-1-(yx)\rangle\big\rangle\right)
\tag{25}
$$

The quantum circuit of the U_{PO} operator and the changes in the sample image are shown in Figure 9. The $U_{(PO)_{yx}}$ operator is the extended version of the U_{PO} operator for combinations of different numbers and positions of the $(n+m)$-qubits in the third register in which position information is stored.

Figure 9. (a) Quantum circuit of U_{PO} operator for QRMW model, (b) sample image (c) the image after U_{PO} is applied, (d) the image after $U_{(PO)_{\{1,3,5\}\{7,8,9\}}}$ is applied, (e) the image after $U_{(PO)_{\{1,3,5\}\{8,10,12\}}}$ is applied.

5. Conclusion

Images with different wavelengths of light are now used in many fields, for example, satellite images. Quantum-based image processing is an effective and important approach to reduce the high real-time computing requirements of classical image processing. In this respect, quantum representation of images is an important step.

In this paper, we present a QRMW model with multiple wavelengths and some image processing for quantum representation. QRMW has been compared with the existing quantum representation models of

image. Furthermore, the advantages/disadvantages of our model have been presented. The model has also been simulated in MS-Liquid, which is a quantum programming language developed by Microsoft [11].

An important issue is the speed of data processing, as well as data transfer and processing in any information processing. Therefore, the QRMW model offers faster processing performance with fewer qubit sources than other quantum representation models.

Acknowledgments

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