

An efficient technique based on firefly algorithm for pilot design process in OFDM-IDMA systems

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Abstract: Accurate placement of pilot tones has been a crucial task in multicarrier transmission technologies since there is a strong relation between pilot positions and channel estimation performance. Therefore, the firefly algorithm (FA) is proposed for achieving the optimal pilot distribution by optimizing the pilot positions in order to minimize the estimation errors of the least squares algorithm employed in orthogonal frequency division multiplexing-interleave division multiple access (OFDM-IDMA) systems. According to the simulation results, our proposed FA-based pilot optimizer provides a great performance increase in OFDM-IDMA systems by obtaining the most appropriate pilot distribution pattern among the considered pilot placement strategies. The upper bound of mean square error (MSE) is used as the fitness function in the optimization process for avoiding the matrix inversion operation that is needed when calculating MSE itself.

Key words: OFDM-IDMA, pilot tones design, channel estimation, firefly algorithm

1. Introduction

Orthogonal frequency division multiplexing-interleave division multiple access (OFDM-IDMA) is a unique transmission technology possessing the capability of eliminating both intersymbol interference (ISI) and multiple access interference (MAI), known as the two main impairments to be fixed in order to ensure high data rates and spectral efficiency with minimal error in wireless communications. As the combination of OFDM and IDMA, the OFDM-IDMA system has the advantages of both of these robust transmission schemes. Thanks to its hybrid nature, both ISI and MAI problems can be resolved by OFDM and IDMA layers, respectively [1]. Furthermore, unlike the conventional multiuser detection (MUD) procedure, which is impractical due to its high complexity, the usage of a very low-cost iterative chip-by-chip multiuser detection (CBC MUD) technique based on the IDMA principle brings the OFDM-IDMA system to an advantageous position [2].

On the other hand, the channel estimation process, which is an essential task for multicarrier transmission technologies, is needed at the receiver side of the OFDM-IDMA system to get channel state information required for eliminating channel fading effects. For this purpose, one of the most common and efficient ways of obtaining channel coefficients, known as pilot-based channel estimation method with comb-type pilot arrangement procedure, can be used [3,4]. The comb-type pilot arrangement is based on the strategy of placing pilot tones uniformly throughout each OFDM symbol. In this method, the distribution form of pilot tones located at certain points of each OFDM symbol has a direct effect on the channel estimation performance. In other words, there is a chance to minimize the estimation errors by optimizing the pilot positions.

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In our study, the pilot positions are optimized by using the firefly algorithm (FA) for the purpose of maximizing the performance of the least squares (LS) channel estimator employed in the OFDM-IDMA system. In order to adopt the pilot positions to the optimization process, the position values of pilot tones distributed throughout a single OFDM symbol are expressed as a D -dimensional position vector. The number of vector dimensions symbolized by D is equal to the number of pilot tones used in a single OFDM symbol and each dimension denotes the position of the related pilot tone. The optimization process starts with the generation of random position vectors equal in number to the population size determined for FA. After that, the dimension values belonging to the position vectors, each of which corresponds to one individual in the firefly population, are updated by the FA until the stopping criterion is ensured. Subsequent to ensuring the stopping criterion, the pilot position vector providing the least estimation errors among the population members is selected as the best solution. In the simulations, the proposed FA-based pilot design procedure is compared not only with conventional methods like equispaced and random-based pilot placements but also two robust pilot optimization techniques based on a genetic algorithm (GA) and particle swarm optimization (PSO) to be able to see the performance of our proposed method more clearly. According to the performance comparisons carried out with regard to bit error rate (BER) and mean square error (MSE) criteria, our proposed technique outperforms each of the considered pilot placement strategies.

The main contributions of our study to the literature are as follows:

1. The pilot optimization process is carried out with a different approach by adopting the FA to the pilot design problem.
2. The FA is integrated into the OFDM-IDMA system for the first time in order to optimize pilot locations and it outperforms both GA- and PSO-based pilot design techniques.
3. This is the first study in which the FA is used for the pilot design problem in the literature.

For the OFDM-IDMA system or any of the other transmission technologies, no study about pilot design using the FA has been carried out so far. However, some studies in which pilot tones are designed employing some other optimization techniques excluding the FA for OFDM and multiple-input multiple-output (MIMO)-OFDM schemes can be found in the literature [5–11]. In [5] and [6], optimal pilot placement was achieved by considering the MSE value of the LS channel estimator for a MIMO-OFDM system in order to minimize the estimation errors. In [7], a feedback technique based on the pilot allocation mechanism was proposed for optimizing the pilot locations. In [8] and [9], the operation of pilot design was carried out using a PSO algorithm for a MIMO-OFDM system. In [10], two different pilot design schemes based on a GA were suggested for an OFDM system. In [11], the channel equalization process carried out for a space-time block coding (STBC)-OFDM system was improved by using PSO and a GA.

Although there is no study about pilot design using the FA for any of the transmission technologies, some applications of the FA to the other telecommunication or engineering problems are available in the literature [12–15]. In [12], the FA-based peak-to-average power ratio reduction technique was proposed for OFDM systems. In [13], the FA-based maximum likelihood estimator was suggested for the joint estimation of channel frequency offset and channel response in an orthogonal frequency division multiple access system. In [14], the FA was employed in attaining optimal power system restoration. In [15], the FA-based fuzzy membership assigning procedure was proposed.

As can be seen from the literature review, there are various successful applications of the FA in different engineering fields. However, due to being invented more recently compared to many of the other intelligent optimization algorithms, there is still a significant number of engineering problems that the FA has not yet been applied to, including the pilot design problem. In this paper, by considering this serious gap in the literature, the FA was utilized in solving the pilot design problem of the OFDM-IDMA system for the first time and quite promising results were achieved.

The outline of the paper is as follows: in Section 2, the system framework of OFDM-IDMA and its working principle are given. In Section 3, the FA and the way of its implementation to the pilot design problem are clarified. In Section 4, the fitness function of the FA is derived. In Section 5, the simulation results are given. In Section 6, computational complexity analysis is performed, and the paper is finalized with the conclusions in Section 7.

2. System description

The block diagram belonging to the OFDM-IDMA system with K users is demonstrated in Figure 1. At the transmitter side, the data streams allocated to K users are encoded by any type of forward error correction (FEC) encoders. After the encoding process, a spreading operation in which the same spreading sequence is used for each user is carried out. Following the execution of an interleaving process through the interleavers by which the user separation is achieved, the modulation, pilot insertion, inverse fast Fourier transform (IFFT), and addition of cyclic prefix operations are performed, respectively. At the receiver side, after the operations of removing the cyclic prefix and FFT, the following $N \times 1$ signal vector $\mathbf{Y}(n)$ is obtained [16,17]:

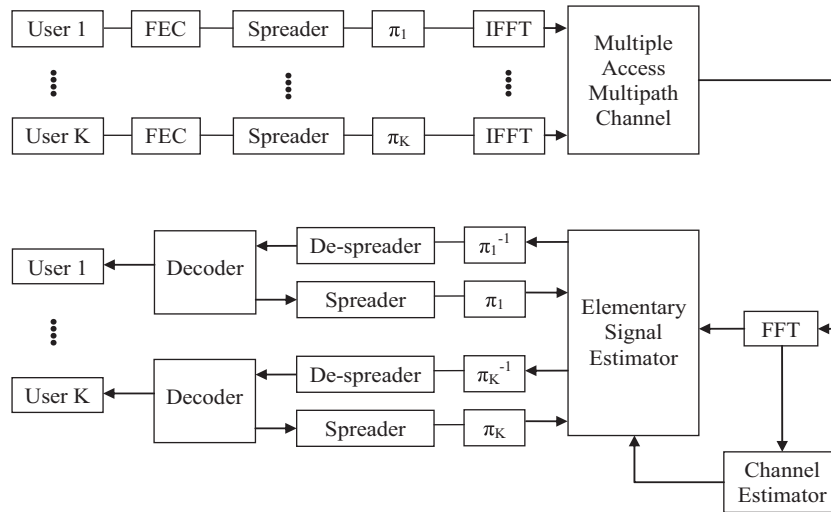


Figure 1. The OFDM-IDMA system architecture.

$$\mathbf{Y}(n) = \sum_{k=1}^K \mathbf{X}_k^{diag}(n) \mathbf{F} \mathbf{h}(n) + \mathbf{W}(n), \tag{1}$$

where $\mathbf{X}_k(n)$, $\mathbf{h}(n)$, and $\mathbf{W}(n)$ denote the vectors of $N \times 1$ transmitted symbols, $N \times 1$ channel impulse response (CIR), and $N \times 1$ additive white Gaussian noise, respectively. k and n are the user and subcarrier numbers, respectively. $(\cdot)^{diag}$ converts its input arguments to a diagonal matrix. The $N \times N$ unitary discrete

Fourier transform matrix expressed by \mathbf{F} is given below:

$$\mathbf{F} = \frac{1}{\sqrt{N}} \begin{bmatrix} 1 & 1 & \dots & 1 \\ 1 & e^{-j2\pi/N} & \dots & e^{-j2\pi(N-1)/N} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & e^{-j2\pi(N-1)/N} & \dots & e^{-j2\pi(N-1)(N-1)/N} \end{bmatrix}. \quad (2)$$

If vector $\mathbf{X}_k(n)$ is expressed by the sum of data and pilot tones,

$$\mathbf{X}_k(n) = \mathbf{S}_k(n) + \mathbf{P}_k(n), \quad (3)$$

where $\mathbf{P}_k(n)$ and $\mathbf{S}_k(n)$ are the $N \times 1$ pilot and data vectors, respectively, then Eq. (1) can be restated as follows:

$$\mathbf{Y}(n) = \sum_{k=1}^K \mathbf{S}_k^{diag}(n) \mathbf{F} \mathbf{h}(n) + \sum_{k=1}^K \mathbf{P}_k^{diag}(n) \mathbf{F} \mathbf{h}(n) + \mathbf{W}(n). \quad (4)$$

The simplified expression of Eq. (4) is given below:

$$\mathbf{Y} = \mathbf{G} \mathbf{h} + \mathbf{A} \mathbf{h} + \mathbf{W}, \quad (5)$$

where $\mathbf{G} = \mathbf{S}^{diag} \mathbf{F}$ and $\mathbf{A} = \mathbf{P}^{diag} \mathbf{F}$ are $N \times N$ matrixes. $N \times 1$ CIR vector \mathbf{h} can be written in the following way:

$$\mathbf{h} = [h_1, h_2, h_3, \dots, h_N]^T, \quad (6)$$

where $(\cdot)^T$ indicates the transpose operation. Thus, CIRs of the fading channel can be achieved with the help of the LS channel estimator as follows:

$$\hat{\mathbf{h}} = \mathbf{G}^t \mathbf{Y} = \mathbf{h} + (\mathbf{G}^H \mathbf{G})^{-1} \mathbf{G}^H \mathbf{W} = \mathbf{h} + \mathbf{G}^t \mathbf{W}, \quad (7)$$

where $\hat{\mathbf{h}}$ is the estimated CIR. $(\cdot)^H$ and $(\cdot)^t$ signify the Hermitian and pseudoinverse matrix, respectively [5,6,8].

Following the estimation of CIRs, the received signals are given to the elementary signal estimator (ESE) input to initiate the CBC MUD process in which the estimated channel coefficients are employed for eliminating the fading effects caused by the wireless channel. The CBC MUD process starts with production of extrinsic log-likelihood ratio (LLR) values by the ESE block for each user and keeps going with deinterleaving and despreading operations of the LLR streams. Subsequent to feeding the despreader outputs to the decoder (DEC) inputs, the signals taken from the DECs are respread and reinterleaved, respectively, and one cycle of the CBC MUD process ends with applying the reinterleaved signals to the ESE block. The operations performed for one cycle are repeated for a certain iteration number, and for each iteration LLR values and DEC outputs are recalculated [1,2].

The MSE of the LS channel estimator employed in the OFDM-IDMA system can be calculated by the expression derived below:

$$MSE = \frac{1}{N} \varepsilon \left\{ \left\| \hat{\mathbf{h}} - \mathbf{h} \right\|^2 \right\} = \frac{1}{N} \varepsilon \left\{ \left\| \mathbf{G}^t \mathbf{W} \right\|^2 \right\} = \frac{1}{N} tr \left\{ \mathbf{G}^t \varepsilon \{ \mathbf{W} \mathbf{W}^H \} \mathbf{G}^{tH} \right\}, \quad (8)$$

where $\varepsilon(\cdot)$ and $tr(\cdot)$ signify the expectation and trace operators, respectively. Assuming that the channel noise is a zero-mean white Gaussian noise, it is possible to write $\sigma^2 \mathbf{I}_m$ in place of $\varepsilon\{\mathbf{W}\mathbf{W}^H\}$, where σ^2 and \mathbf{I}_m denote the noise variance and an $M \times M$ identity matrix, respectively. By doing so, Eq. (8) can be simplified as follows:

$$MSE = \frac{1}{N} tr \{(\mathbf{G} \mathbf{G}^H)^{-1}\}. \quad (9)$$

The minimum MSE can be attained in the case of ensuring the equivalence of $\mathbf{G} \mathbf{G}^H = P \mathbf{I}_N$, where P corresponds to the fixed power assigned to the pilot tones. The eventual expression of minimum MSE can be derived as follows [5,6,8]:

$$MSE = \frac{\sigma^2}{P} \quad (10)$$

3. Firefly algorithm

The FA, which is known as a population-based metaheuristic optimization algorithm, was developed in consequence of being inspired by the communication behaviors of fireflies through their light-emitting capabilities in 2007 by Yang [18]. The FA has three distinct idealized rules based on some fundamental light emission characteristics of natural fireflies [18,19]:

1. All fireflies are assumed as unisex and each firefly moves towards brighter individuals without making sex discrimination.
2. The attractiveness level of a firefly is proportional to the amount of brightness it has. Therefore, each firefly tends to move towards brighter members. The brightness level of a firefly through the eyes of another member in the population declines as the distance between the two individuals increases due to the light absorption tendency of the air existing in the environment. If there is no other firefly that is brighter than a particular firefly in the population, this brightest member moves randomly.
3. The light intensity of a firefly is determined by the value of the fitness function belonging to the problem. The light intensity is proportional to the value of the fitness function for the maximization problems.

In the FA, the light intensity changes with respect to the inverse square law as follows:

$$I(r) = \frac{I_s}{r^2}, \quad (11)$$

where I_s and r correspond to the light intensity at the source and the distance between the two fireflies, respectively. Light intensity I in an environment with fixed absorption coefficient γ varies with respect to the distance r as follows:

$$I(r) = I_0 \cdot e^{-\gamma r^2}, \quad (12)$$

where I_0 represents the original light intensity. According to this, the attractiveness function of a firefly can be expressed as follows:

$$\beta(r) = \beta_0 \cdot e^{-\gamma r^2}, \quad (13)$$

where β_0 denotes the first attractiveness value at $r=0$. The distance between the i th and j th fireflies at positions x_i and x_j , respectively, can be defined as the Cartesian or Euclidean distance as follows:

$$r_{ij} = \|x_i - x_j\| = \sqrt{\sum_{k=1}^d (x_{i,k} - x_{j,k})^2}, \quad (14)$$

where $x_{i,k}$ specifies the k th component of the x_i coordinate belonging to the i th firefly. d is the dimension number of each solution vector (firefly). The movement of the i th firefly towards the more attractive (brighter) j th firefly is expressed with the equation given below:

$$x_i^{t+1} = x_i^t + \beta_0 \cdot e^{-\gamma r_{ij}^2} \cdot (x_j^t - x_i^t) + \alpha \cdot \left(rand - \frac{1}{2} \right). \quad (15)$$

The first term on the right side of Eq. (15) denotes the current position of the firefly, the second term corresponds to the amount of attractiveness, and the third term is used for the random movements of the fireflies. Whereas α symbolizes the randomization parameter determined with respect to the type of problem to be optimized, $rand$ represents a uniformly distributed random number that is generated in the range of $[0,1]$. The pseudocode of the FA is given in Figure 2 [18,19].

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➤ Generate initial positions  $x_i (i=1,2,\dots,n)$  for each firefly
➤ Put the initial positions into the fitness function and calculate the light intensity (fitness value) of each firefly  $f(x)=I_i$ 
➤ Define the light absorption coefficient  $\gamma$ 
  While ( $t < \text{maximum iteration number}$ )
    for  $i=1:n$  all fireflies from 1 to  $n$ 
      for  $j=1:n$  all fireflies from 1 to  $n$ 
        if ( $I_i < I_j$ ) move the “ $i$ ”th firefly towards the “ $j$ ”th firefly; end if
      • Update the light intensity by calculating the fitness value of new position
        end for  $j$ 
      end for  $i$ 
      ○ Find the global best solution vector “ $\mathbf{g}_{\text{best}}$ ” by arraying the fireflies with respect to their light intensities
      ○  $t=t+1$ 
    end while
➤ Choose the best position vector “ $\mathbf{g}_{\text{best}}$ ” as a solution

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Figure 2. Pseudocode of the firefly algorithm.

3.1. Firefly algorithm for pilot design

In our study for pilot design in the OFDM-IDMA system, candidate solutions corresponding to the positions of fireflies are represented by a D -dimensional vector like $\mathbf{X}_i = (x_i^1, x_i^2, x_i^3, \dots, x_i^D)$ where the dimensions of the solution vector correspond to the pilot positions to be optimized. In the simulations, these dimensions belonging to the solution vectors are restricted by the upper and lower bounds defined in Table 1. Since the numbers of

Table 1. Upper and lower bounds of the i th solution vector.

Bounds \ Dimensions	x_i^1	x_i^2	x_i^3	x_i^4	x_i^5	x_i^6	x_i^7	x_i^D
Lower bound (Lb)	1	9	17	25	33	41	49	$8D-7$
Upper bound (Ub)	8	16	24	32	40	48	56	$8D$

subcarriers and pilot tones are defined as 128 and 16 for our system, the dimension number of the candidate solution vectors will be equal to 16, as well.

In the first stage of the pilot optimization using the FA, D -dimensional random initial solutions are generated without exceeding the upper and lower bounds specified in Table 1. After that, the fitness values (light intensities) of the initial solutions, each of which corresponds to the pilot tone position vector to be optimized, are determined according to the fitness function in Eq. (19). Since the pilot tone optimization is a minimization process in which the estimation errors are minimized, in contrast to maximization problems, it is assumed that the light intensity increases as the fitness value (MSE) that defines the estimation error decreases. Subsequent to calculating the light intensities, the light intensity of the first firefly is compared one by one to the light intensities of the remaining members in the population. For each comparison, if the light intensity of the first firefly is less than the light intensity of the other firefly compared with the first one, the first firefly moves towards this individual through Eq. (15). Following each movement, the light intensity is updated by calculating the fitness value of the new position. The operations carried out for the first firefly are repeated for all members of the population and thus the first iteration is completed. These operations performed for a single iteration are maintained until ensuring that the stopping criterion is specified as 100 iterations. After meeting the stopping criterion, the solution vector that has the most intensive light in the population is determined as the optimal pilot position vector.

4. Fitness function of the firefly algorithm

The use of MSE in Eq. (10) as a fitness function of the FA is feasible. However, the requirement for matrix inversion operation when computing MSE increases the computational load of the FA-based pilot optimizer. Therefore, in order to eliminate this computational load, the upper bound of the MSE is achieved by taking advantage of the Gershgorin circle theorem [20] since matrix \mathbf{G} is full-rank and $\mathbf{G}\mathbf{G}^H$ possesses real and positive eigenvalues. The upper bound of MSE is derived as follows:

$$\text{tr} \{(\mathbf{G}\mathbf{G}^H)^{-1}\} = \sum_{i=1}^N \frac{1}{\lambda_i} \leq \begin{cases} \frac{N}{P-R_{\max}} & , P > R_{\max} \\ +\infty & , P \leq R_{\max} \end{cases}, \quad (16)$$

where λ_i ($i = 1, 2, \dots, N$) correspond to the eigenvalues of matrix $\mathbf{G}\mathbf{G}^H$. In Eq. (16), $R_{\max} = \max(\mathbf{R}_i)$ indicates the maximum radius of the Gershgorin disc, where \mathbf{R}_i ($i = 1, 2, \dots, N$) specifies the sum of the i th row's off-diagonal components in matrix $\mathbf{G}\mathbf{G}^H$ expressed as:

$$\mathbf{G}\mathbf{G}^H = \begin{bmatrix} P & x_{1,2} & x_{1,3} & \cdots & x_{1,N} \\ x_{2,1} & P & x_{2,3} & \cdots & x_{2,N} \\ x_{3,1} & x_{3,2} & P & \cdots & x_{3,N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_{N,1} & x_{N,2} & x_{N,3} & \cdots & P \end{bmatrix}. \quad (17)$$

In Eq. (17), the diagonal elements denoted by P are equal to each other. \mathbf{R}_i can be defined as follows:

$$\mathbf{R}_i = \sum_{j=1, j \neq i}^N |x_{ij}|, \quad (18)$$

where x_{ij} ($i = 1, 2, \dots, N; j = 1, 2, \dots, N$) represents the elements possessed by matrix $\mathbf{G}\mathbf{G}^H$. The final form of the fitness function for the FA is acquired as follows [8]:

$$\text{fitness function} = \frac{R_{\max}}{P}. \quad (19)$$

5. Simulation results

In this study, the FA-based pilot design procedure we propose for an OFDM-IDMA system is compared to two robust pilot design techniques based on evolutionary and swarm intelligence-based optimization algorithms like GA and PSO, respectively, as well as the classical methods like equispaced and random-based pilot placements. The comparisons are carried out with regard to MSE and BER criteria. In the simulations, the FEC coding operation is performed by using convolutional encoders with a rate of 1/2 and a (171, 133)₈ generator polynomial. The spreading operation is carried out through the 1/8 rate spreaders and the channel model used for the simulations is ITU “Vehicular A” with [0 310 710 1090 1730 2510] ns relative delays and [0 -1 -9 -10 -15 -20] dB power paths. The other parameters regarding the OFDM-IDMA scheme are given in Table 2. The control parameters of the standard GA, FA, and PSO algorithms are determined as in Table 3.

Table 2. The simulation parameters of the OFDM-IDMA scheme.

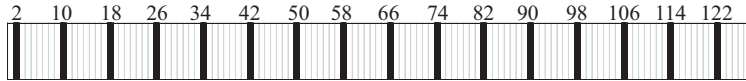
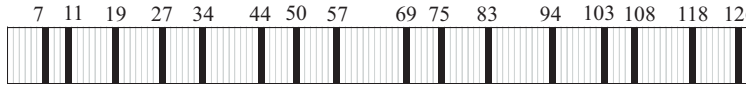
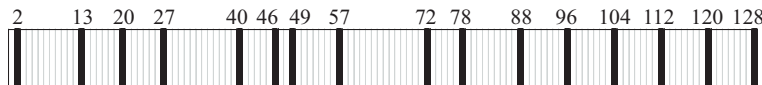
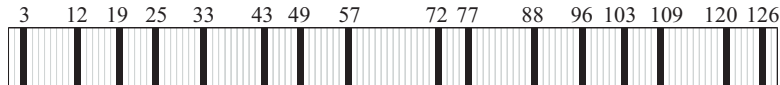
Number of subcarriers	128
Size of FFT	128
Number of pilot tones	16
Sampling frequency	3.5 MHz
Sampling period (T_s)	285.71 ns
Symbol part duration (TFFT)	$128T_s = 36.57 \mu s$
Size of cyclic prefix	$\text{FFT}/4 = 32$
Duration of cyclic prefix	$\text{TFFT}/4 = 36.57/4 = 9.14 \mu s$
Type of modulation	QPSK

The pilot placements considered in this paper are as follows:

- 1) Random placement.
- 2) Equispaced placement given in Figure 3.
- 3) Optimized placement through GA given in Figure 4.
- 4) Optimized placement through PSO given in Figure 5.
- 5) Optimized placement through FA given in Figure 6.

Table 3. The control parameters determined for GA, PSO and FA.

Genetic algorithm	
Population size (n)	10
Crossover rate (Cr)	0.8
Mutation rate (Mr)	0.005
Number of iterations (t)	100
Particle swarm optimization	
Population size (n)	10
Learning factor ($C1, C2$)	$C1 = 1.5, C2 = 2$
Number of iterations (t)	100
Firefly algorithm	
Population size (n)	10
Randomization parameter (α)	0.15
Absorption coefficient (γ)	1
Attractiveness at $r=0$ (β_0)	1
Number of iterations (t)	100

**Figure 3.** Equispaced pilot placement.**Figure 4.** Optimized placement of pilot tones through GA.**Figure 5.** Optimized placement of pilot tones through PSO.**Figure 6.** Optimized placement of pilot tones through FA.

In Figure 7, our proposed pilot design procedure based on the FA and the other methods used for benchmarking are evaluated with regard to their contribution rates to the growth of BER performance in the OFDM-IDMA scheme. In the simulations, the user number of the OFDM-IDMA system is appointed as 6. The other parameters determined for the simulations are as in Table 2 and Table 3, respectively. It is evidently viewed from Figure 7 that our proposed pilot design procedure based on the FA ensures the best BER performance among the considered strategies by providing lower BER values at each E_b/N_0 compared to the other considered methods. For instance, at 6 dB E_b/N_0 value, whereas the BER of the FA-based pilot optimizer is 4.58×10^{-4} ,

the BER values of the PSO, GA, equispaced, and random-based pilot placement strategies are 1.62×10^{-3} , 3.69×10^{-3} , 1.33×10^{-2} , and 1.28×10^{-1} , respectively.

In Figure 8, the performance analysis of the related pilot design methods is carried out based on another well-known criterion called MSE. Thanks to the MSE graph obtained by calculating the estimation errors at a certain Eb/No interval for each method, it becomes possible to investigate how much influence each pilot design strategy has on the estimation performance of the LS algorithm employed in the OFDM-IDMA scheme. In Figure 8, our proposed FA-based pilot optimizer has the lowest estimation errors among the heuristic approaches that seem to be clearly separated from the classical methods.

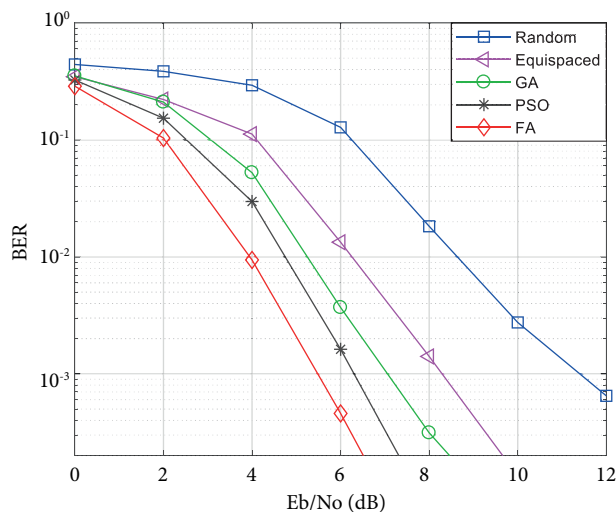


Figure 7. BER versus Eb/No for the considered strategies.

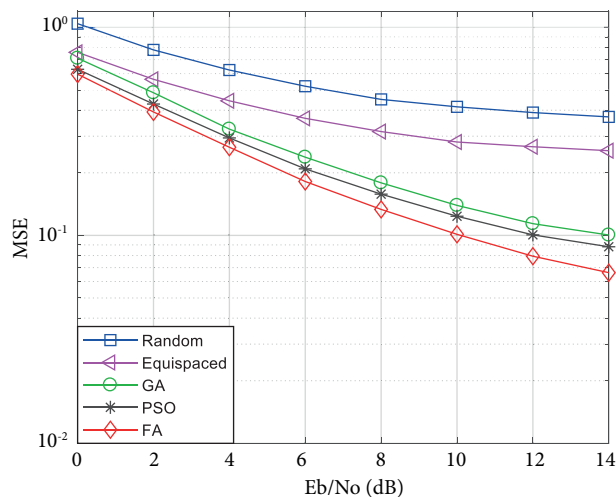


Figure 8. MSE versus Eb/No for the considered strategies.

In Figure 9, the considered intelligent optimization techniques are compared to each other with regard to their convergence performance. It can be seen from Figure 9 that GA and PSO suffer from premature convergence by not being able to improve their solutions much from the 40th iteration while the FA reaches the lowest MSE value at the end of the iterations by improving its solutions consistently throughout the optimization process thanks to its high global exploration capability.

In Figure 10, the BER performance of the OFDM-IDMA scheme employing the related heuristic-based pilot optimizers is investigated for 6, 7, and 8 users, respectively. According to Figure 10, depending on the enhancement in user number, the BER values of the OFDM-IDMA scheme increase at all values of Eb/No for each pilot optimizer. For instance, if only the BER performance of the FA-based pilot design technique at 4 dB is taken into consideration for observing the effect of varied user numbers on the system performance, BER values of the OFDM-IDMA system employing our proposed technique will be 9.42×10^{-3} , 6.35×10^{-2} , and 2.01×10^{-1} for 6, 7, and 8 users, respectively. Such BER increments affiliated with the increasing number of users arise because more users lead to further parameters to be estimated and processed, which can have a negative impact on the system performance.

As can be clearly observed from the BER, MSE, and convergence performance analyses, our proposed FA-based pilot optimizer has established definite superiority over both GA- and PSO-based pilot optimizers. Our proposed technique performing better than the other considered heuristic approaches is due to the two main advantages of the FA:

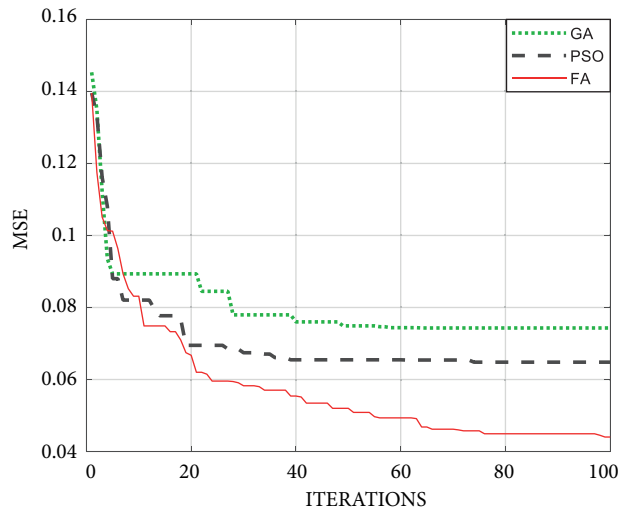


Figure 9. The convergence performance of the considered optimization techniques.

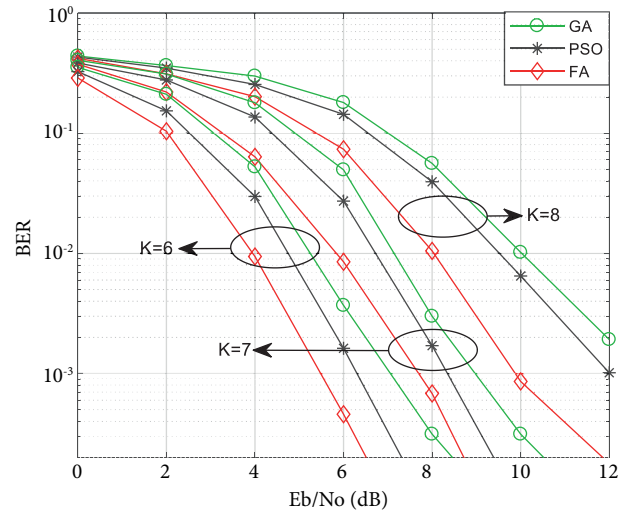


Figure 10. BER analysis of the OFDM-IDMA scheme employing the related heuristic pilot optimizers for different user numbers.

1. Automatic subdivision ability: As emphasized in Section 3, each member of the FA population tends to move towards the brighter individuals and the brightness level declines as the distance between the two members increases because of the light absorption tendency of the air existing in the environment. This mechanism leads to automatic subdivision of the firefly population into subgroups and these groups swarm around the local optimums of the optimization problem. Therefore, the best solution corresponding to the global optimum can be easily found among these local optimums swarmed around by these subgroups. The average distance between the adjacent groups is controlled by the expression of $1/\sqrt{\gamma}$. If γ is made equal to zero, the firefly population will not subdivide into subgroups [19].
2. Capability of coping with multimodality: In the case of selecting the population size sufficiently bigger than the number of local optimums, this automatic subdivision of the whole population into subgroups leads to simultaneous detection of all local optimums and thus it becomes possible to deal with nonlinear multimodal optimization problems [19].

6. Computational complexity analysis

Nearly all metaheuristic optimization algorithms are simple with regard to computational complexity. If big O notation is utilized for determining the complexities of the considered optimization algorithms, the searching complexities of the GA and PSO will be defined as $O(t \times n)$, where t and n denote the iteration number and population size, respectively. Since the FA has two inner loops in the length of population number n and one outer loop in the length of iteration number t , the complexity of the FA is calculated as $O(t \times n^2)$ for the extreme case [19]. In the case of assigning a small value to n and a large value to t ($n = 10$, $t = 100$ in this paper), as preferred mostly when determining the parameter values of optimization problems, the computational cost of the FA will be low due to the fact that the complexity of the algorithm varies linearly with t . Namely, for any optimization problem, the majority of the computational cost is determined by the fitness function evaluations [19]. For this reason, in this paper, in order to decrease the computational load of the FA, the

upper bound of the MSE is obtained in Section 4 to be used as a fitness function instead of the MSE in order to get rid of the matrix inversion process that is needed when computing the MSE. On the other hand, the significant superiorities of the FA over the other optimization techniques such as automatic subdivision ability and the capability of coping with multimodality make the FA more efficient in searching the solution space. For this reason, the aforementioned superiorities bring the FA to an advantageous position among the heuristic approaches.

7. Conclusions

In this paper, an advanced procedure based on the FA is proposed for the pilot design process in an OFDM-IDMA system and the estimation errors that are directly affected by the pilot tones' distribution form are reduced to a minimum level by optimizing the pilot positions. In the simulations, our proposed FA-based pilot optimizer, the other heuristic approaches based on PSO and GA considered in this paper, and the classical methods called equispaced and random-based pilot placements are compared to each other with regard to their contributions to the MSE and BER performance of the OFDM-IDMA scheme. According to the simulation results, our new proposal surpasses not only the classical pilot placement techniques but also the considered two robust heuristic approaches by providing the maximum improvement of MSE and BER performance in the OFDM-IDMA system.

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