

Topological feature extraction of nonlinear signals and trajectories and its application in EEG signals classification

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Abstract: This study introduces seven topological features that characterize attractor dynamic of nonlinear and chaotic trajectories in a phase space. These features quantify volume, occupied space, nonuniformity, and curvature of trajectory. The features are evaluated as initial point invariant measures by a practical approach, which means that a feature is only sensitive to dynamic changes. The Lorenz and Rossler system trajectories are employed in this evaluation. Moreover, the proposed features are used in a real world application, i.e. epileptic seizure electroencephalogram signal classification. As the result shows, these features are efficient in this task in comparison with others studies that used the same dataset and evaluation method.

Key words: Nonlinear attractor, feature extraction, topological feature, electroencephalogram, invariant measure, classification

1. Introduction

Signal analysis in time domain and trajectory analysis in a phase space can lead to a better understanding of the dynamic of systems. Trajectory analysis can give us valuable information about an attractor and so on about systems and their behavior. Many studies propose some quantifiers that try to describe and characterize the dynamic and behavior of signals and systems. For example, approximate [1] and sample [2] entropies try to quantify the complexity of trajectory in a phase space. In many application of nonlinear analysis, the phase space of a system is not simply achievable. Therefore, trajectory is embedded from time series into estimated phase space. There are many features and much information that can be extracted from trajectory.

In real world applications, these features help to classify or identify systems and trace their changes. For example, fractal dimensions focus on occupying space capacity in detail [3] and measure the space-filling capacity of patterns that illustrate how a fractal scales differently from the space it is embedded in [4], such as box counting dimension [5], information dimension [6], correlation dimension [7], and Higuchi dimension [8]. Lyapunov exponent quantifies the rate of separation of infinitesimally close trajectories [9]. Volumetric behavior features try to characterize stretch, folding, and occupied space of trajectory [10]. Recurrence plot [11] and its quantification, named recurrence quantification analysis (RQA) [12], quantify some features that

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are related to recurrent behavior of trajectory. Katz feature [13] characterizes the topology of trajectory in a phase space by comparing the relationship between the length of trajectory and diagonals. All of these features quantify characteristic properties of trajectories and their attractors in a phase space and can be used in many applications in engineering, physics, and finance [14–18].

This study proposes some topological quantifiers to describe an attractor from trajectory behavior in a phase space. All of these quantifiers are evaluated as initial point invariant measures and are used in a real world application. The rest of the paper is organized as follows. The proposed method is described in Section 2. Section 3 is devoted to evaluating and discussing the proposed method by comparing two nonlinear systems with different parameters and a real world application. Finally, our conclusions are stated in Section 4.

2. Materials and methods

The proposed topological quantifiers could be extracted from trajectory $\vec{x}(t)$ in d-dimensional phase space (Eq. (1)). In practice, all of the d states of the phase space are not accessible and just a single discrete time measurement $x(t)$ is available [19].

$$\vec{x}(t) = [x_1(t), x_2(t), \dots, x_d(t)] \quad (1)$$

Takens' method [20] is the most frequently used for embedding time series $x(t)$, into d-dimensional phase space by using Eq. (2).

$$\vec{x}(t) = [x(t), x(t - \tau), \dots, x(t - (d - 1)\tau)] \quad (2)$$

where d is embedding dimension and τ is delay, estimated by using the false nearest-neighbors algorithm [21] and the mutual information [22], respectively.

By considering access to trajectory $\vec{x}(t)$ in the phase space, the proposed quantifiers are introduced in the next section.

2.1. Topological quantifiers

In this section we introduce seven features that describe attractor topology using trajectory vector states. Figure 1 shows Lorenz trajectory (Eq. (3)) in a 3-dimensional phase space. This trajectory is bounded by a convex hull volume and the first feature is the volume of convex hull $V(\vec{x}(t))$ (Eq. (4)).

$$\text{Lorenz system} : \begin{cases} \frac{dX}{dt} = \sigma(Y - Z) \\ \frac{dY}{dt} = rX - Y - XZ \\ \frac{dZ}{dt} = XY - \beta Z \end{cases} \quad (3)$$

$$V(\vec{x}(t)) = \text{Vol}(\text{Conv}(\vec{x}(t))), \quad (4)$$

where $\text{Conv}(\mathbf{x})$ is convex hull of all points in \mathbf{x} and $\text{Vol}(v)$ is the volume that is occupied by v .

The next four features quantify occupied space and changes in occupied space by considering distance of all vector states in $\vec{x}(t)$ to a reference point C . These four features quantify shape and complexity of the attractor using Eqs. (5) to (10).

$$\text{Occupied space}_1 : OC_1 = \frac{1}{N} \sum_{n=1}^N \|\vec{x}(n) - C_1\| \quad (5)$$

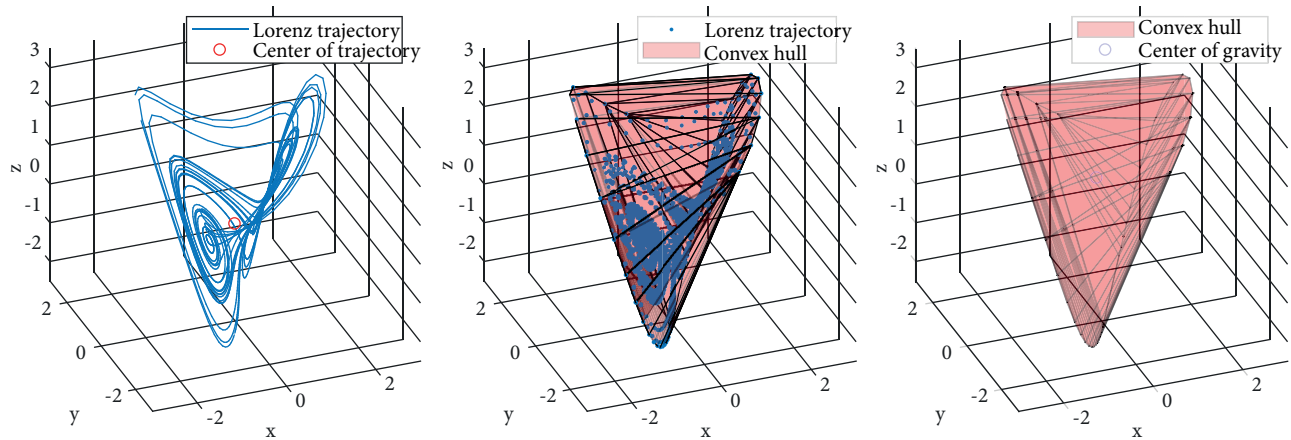


Figure 1. Trajectory of Lorenz system in $\sigma = 16$, $r = 50$, and $\beta = 4$ (left). Convex hull of the Lorenz trajectory (center) and global recurrence plot (right). Center of gravity of convex hull (right).

$$Non_uniformity_1 : \quad NU_1 = \sqrt{\frac{1}{N} \sum_{n=1}^N \left| \|\vec{x}(n) - C_1\| - OC_1 \right|^2} \quad (6)$$

$$Center\ of\ trajectory_1 : \quad C_1 = \frac{1}{N} \sum_{n=1}^N \vec{x}(n) \quad (7)$$

$$Occupied\ space_2 : \quad OC_2 = \frac{1}{N} \sum_{n=1}^N \|\vec{x}(n) - C_2\| \quad (8)$$

$$Non_uniformity_2 : \quad NU_2 = \sqrt{\frac{1}{N} \sum_{n=1}^N \left| \|\vec{x}(n) - C_2\| - OC_2 \right|^2} \quad (9)$$

$$Center\ of\ gravity : \quad C_1 = \frac{\sum_l \left(\frac{1}{d+1} \sum_{i=0}^d x_i^* v_l \right)}{V} \quad (10)$$

where N is the number of state vectors, C_1 is center of state vectors, d is dimension of phase space, x^* is the set of simplices forming the Delaunay triangulation of convex hull, l is the number of triangles in the set of Delaunay triangulation, and v is volume of each triangle.

OC_1 and OC_2 quantify the occupying space feature of trajectory by averaging distances between all of the state vectors and a reference point. OC_1 uses center of trajectory (Eq. (7)) as the reference point C_1 . OC_2 uses center of gravity (center of mass) of the convex hull as the reference point C_2 . To calculate center of gravity the convex hull must be divided into a set of Delaunay triangulation. Figure 2 shows a simple example of steps of calculating center of gravity using Delaunay triangulation. After dividing the convex hull into set of Delaunay triangulation, the volume (or surface or hyper-volume) and center of each subset must be calculated. Center of gravity is the average of the center of all subsets with the weight of their volume.

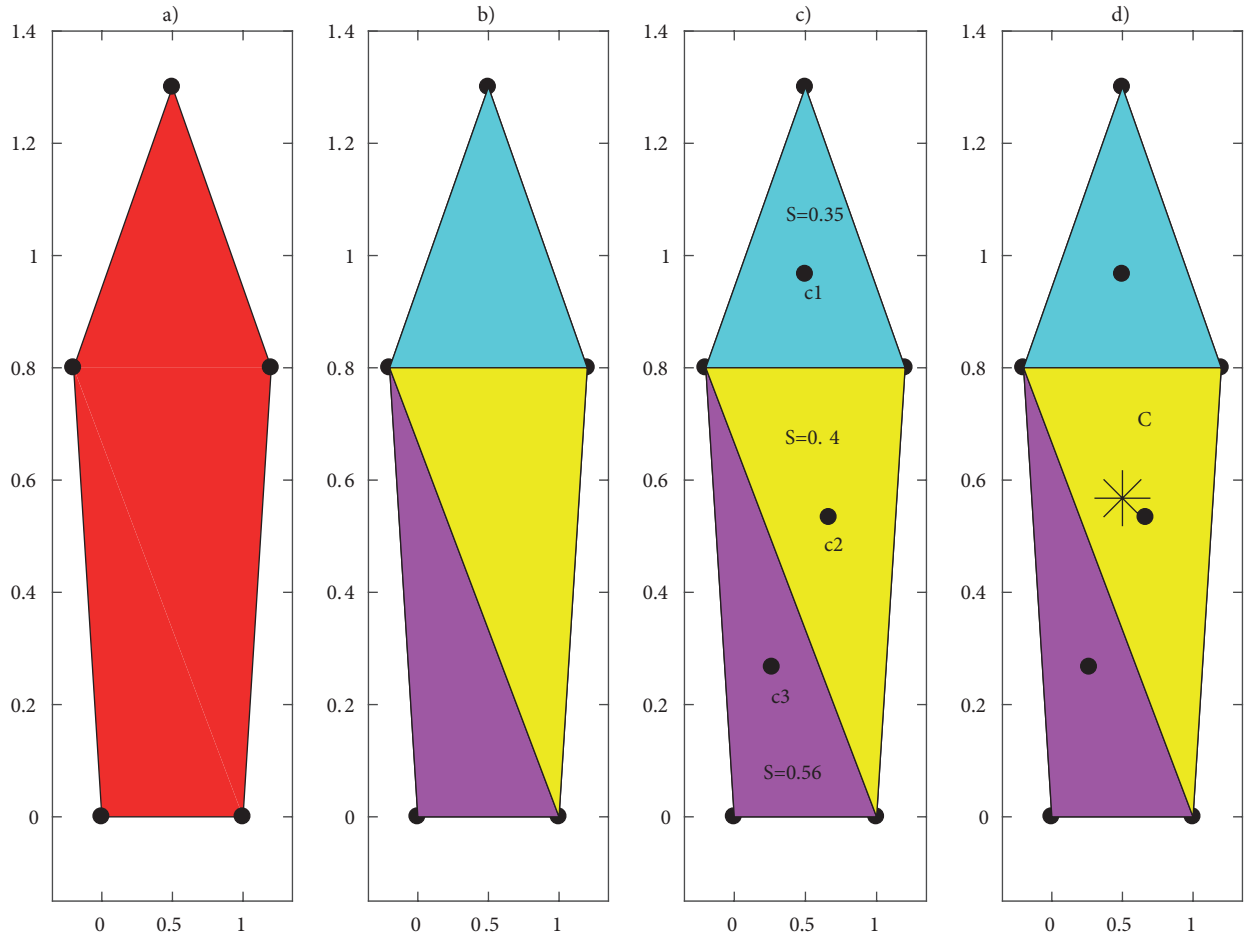


Figure 2. Simple 2-D example of calculating center of gravity. a) A 2-D convex hull. b) Delaunay triangulation of convex hull. c) Center and surface of Delaunay triangulation. d) Center of gravity by weighted average of center of Delaunay triangulation.

Nonuniformity quantifiers NU_1 and NU_2 quantify the nonspherical feature of the attractor. These quantifiers characterize variation of distance of state vectors to a reference point.

To quantify the complexity of behavior of trajectory and topology of its attractor, we focus on the curvature of trajectory (Eq. (11)).

$$C\vec{u}rv(\vec{x}(t)) = \vec{x}(t+1) - 2\vec{x}(t) + \vec{x}(t-1) \tag{11}$$

The curvature vector $Curv(\vec{x}(t))$ represents how much trajectory in specified state vector $\vec{x}(t)$ is straight or curved and is curved in which direction. Figure 3 shows the curvature vector of the Lorenz trajectory in 3-D phase space. Thus, two other quantifiers are defined by using curvature values of trajectory as Eqs. (12) and (13).

$$Average\ of\ curvature\ magnitude : AC = \frac{1}{N-2} \sum_{n=2}^{N-1} \|C\vec{u}rv(\vec{x}(t))\| \tag{12}$$

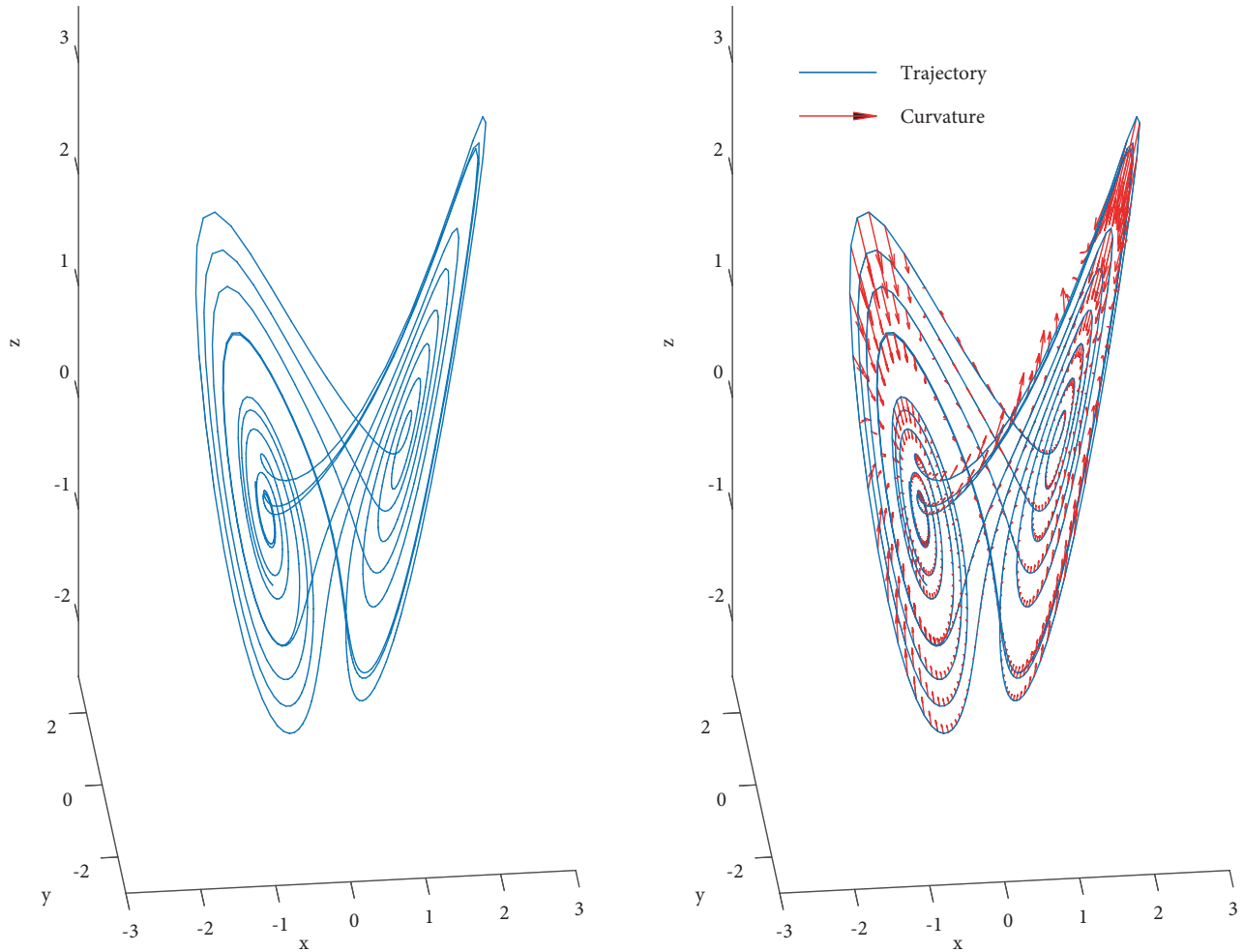


Figure 3. The sample of Lorenz trajectory in 3-dimension phase space (left) and its curvature vectors (right).

$$non_uniformity_3 : NU_3 = \sqrt{\frac{1}{N-2} \sum_{n=2}^{N-1} \left| \left\| \vec{Curv}(\vec{x}(t)) \right\| - AC \right|^2} \quad (13)$$

AC value shows the average curvature magnitude and NU_3 shows variation of curvature magnitude. A small value of AC represents that the attractor has small complexity and is flat. On the other hand, a large value of NU_3 represents that the attractor has more twisting and is more complex.

A quantifier measure in nonlinear and chaotic dynamic analysis should have at least two important properties as follows:

- a) The quantifier measure must be sensitive to dynamic changes.
- b) The quantifier measure must be invariant to initial point changes.

Hence, in the next section these seven quantifier measures are evaluated by trajectories of two nonlinear systems, Lorenz and Rossler, in chaotic parameters to testing these two properties. In addition, to present the potential

ability of the proposed features, these features are employed in a real world application, electroencephalogram (EEG) signal classification.

2.2. Evaluation and application

The attractor topology of a nonlinear or chaotic system is dependent on the dynamic of the system. Therefore, a topological quantifier must be sensitive to dynamic and not sensitive to trajectory detailed behavior. We use two nonlinear systems, Lorenz (Eq. (3)) and Rossler [23] (Eq. (14)), to evaluate the proposed topological quantifiers as initial point invariant measures. An efficient quantifier with this evaluation must have a few changes by changing initial point in the same system parameters and significant changes by changing system parameters.

$$\text{rossler system : } \begin{cases} \frac{dX}{dt} = -Y - Z \\ \frac{dY}{dt} = X + aY \\ \frac{dZ}{dt} = b + Z(X - c) \end{cases} \quad (14)$$

Figure 4 shows Lorenz trajectory in the phase space in $\sigma = 16$, $\beta = 4$, and different r parameter and Figure 5 shows Rossler trajectory in $a = 0.2$, $b = 0.4$, and different c parameter. Ten trajectories for each system in random initial points and different parameters are generated for evaluation. For each generated trajectory all of the seven quantifiers are extracted. The t-test method is used to evaluate the proposed quantifiers as initial point invariant measures and the results are presented in section 3.

2.2.1. EEG signal classification

Based on some studies by the community of neurophysiology researchers an electroencephalogram (EEG) signal is a multivariate time series that stems from a highly nonlinear and multidimensional system [24]. On the other hand, since epilepsy, which is among the most common neurological disturbances, is a condition related to the electrical activity of the brain, it can be studied by analyzing EEG signals [25]. In order to evaluate the proposed method, seven topological quantifiers are employed to classify EEG signals in epileptic seizure analysis as a real world application. EEG signals used in this study are single channel. Thus, these signals must be embedded to the phase space by using Takens' method (Eq. (2)). Takens' method parameters d and τ are estimated with the false nearest-neighbors algorithm [21] and the mutual information [22], respectively. After embedding EEG signals to the phase space, seven proposed features are extracted and these features are used to train and test a multiclass model for support vector machines (SVMs) [26] with Gaussian kernel function and $\gamma = 0.5$, where γ is a parameter for one-class learning. In the test procedure, 10-fold cross validation routine and prediction accuracy of the trained models are evaluated.

2.2.2. Dataset

The EEG dataset used in this study is publicly available from the University of Bonn, Germany [27]. The complete database contained five sets denoted as A–E (Table 1), with 100 samples of 23.6-s duration. The signals were recorded with the same 128-channel amplifier system and digitized at 173.61 samples per second. Sets C and D denoted as interictal data are recorded during the patients in pre-ictal. Set E, which is called ictal data, contains signals recorded during the epileptic seizure.

In different studies some combinations of classification are considered. Three cases of these combinations are focused on in this study as shown in Table 2.

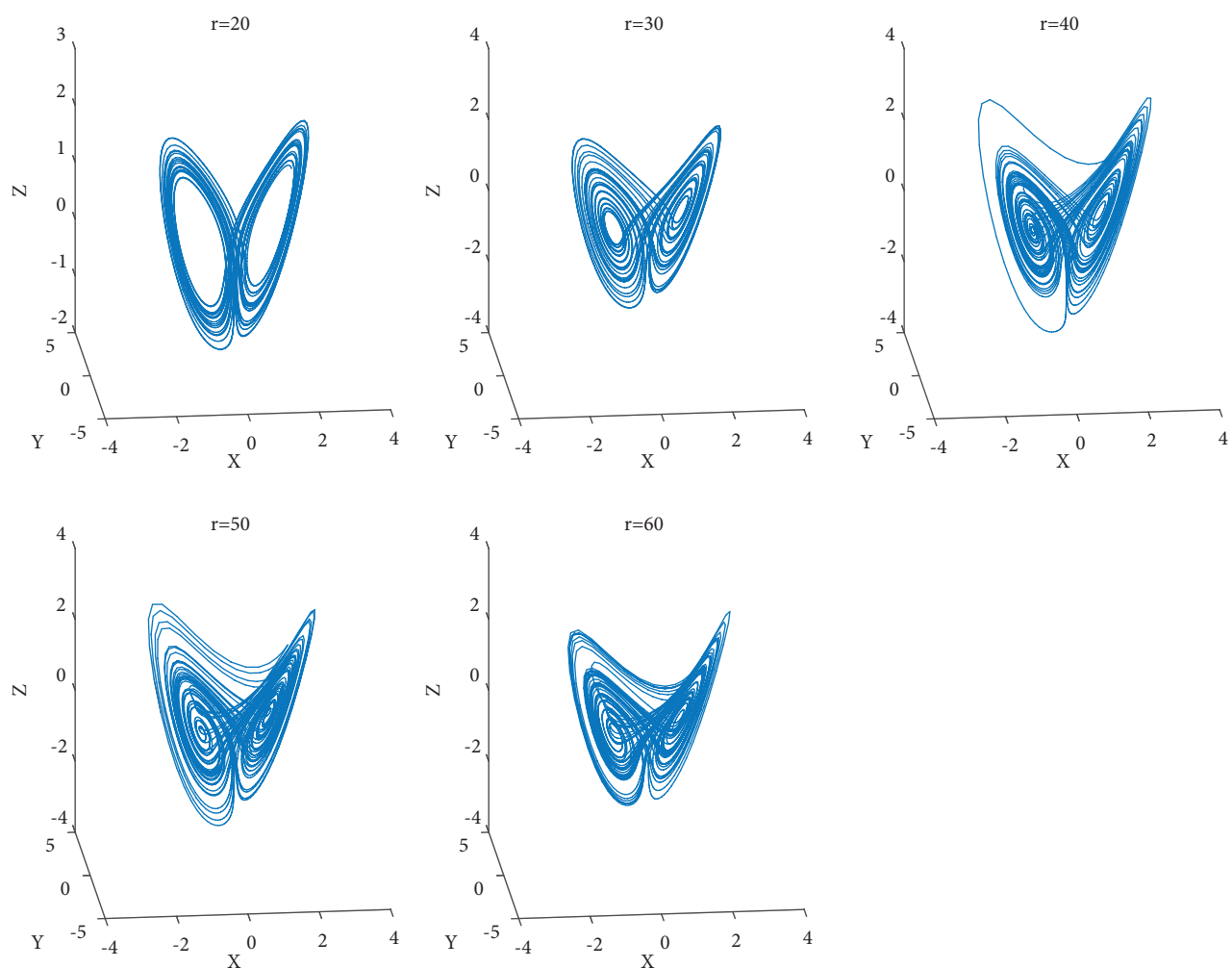


Figure 4. Trajectories of the Lorenz system in different parameters.

Table 1. Description of five data sets.

Subjects	Five healthy volunteers		Five epileptic patients		
	Set A	Set B	Set C	Set D	Set E
Patient state	Eyes open	Eyes closed	Preictal	Preictal	Ictal
Electrode types	Surface	Surface	Intracranial	Intracranial	Intracranial

Table 2. Description of three considered cases.

Cases	Sets	Description
Cases 1	Set A, B versus Set C, D versus Set E	Healthy, interictal and ictal
Cases 2	Set A, B, C, D versus Set E	Nonseizure and seizure
Cases 3	Set D versus Set E	Interictal and ictal

There are 100 samples for each class and we use 10-fold cross validation. Thus, in cases 1 and 2 there

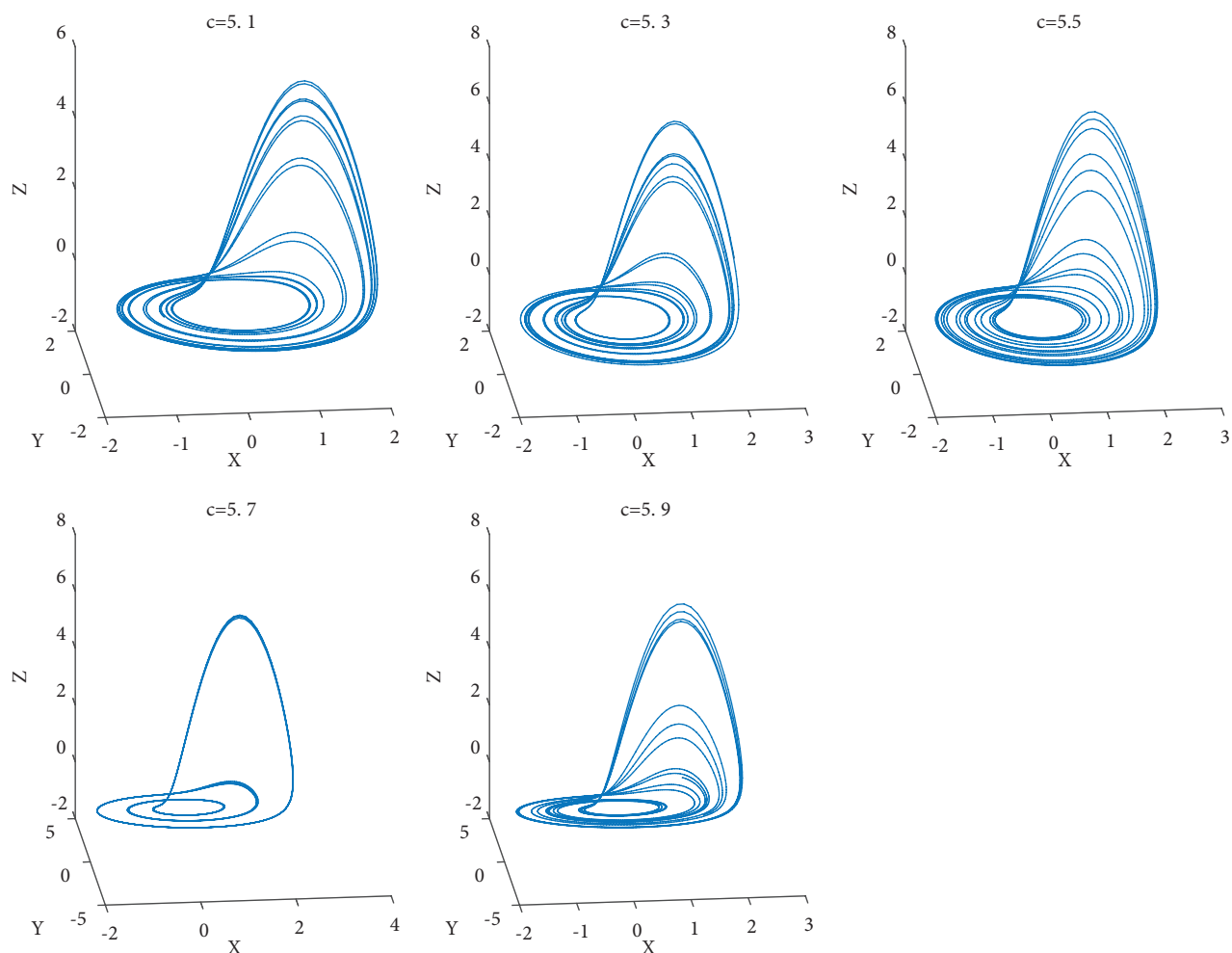


Figure 5. Trajectories of the Rossler system in different parameters.

are 450 samples for training and 50 samples for testing in each repetition. In case 3 there are 180 samples for training and 20 samples for testing in each repetition.

3. Results and discussion

A practical evaluation is used to test the proposed quantifiers as initial point invariant measures. Figures 6 and 7 show box-plots of extracted features from Lorenz and Rossler trajectories, respectively. Each box describes the distribution of specified quantifier values for the mentioned parameters and ten times random initial points. With this representation, a quantifier is an invariant measure if the boxes in related the subfigure are significantly separable from each other. This significant separability is tested by two-sample t-test method. The two-sample t-test returns a test decision for the null hypothesis that two distributions come from independent random samples from normal distributions with equal means and equal but unknown variances. This test needs two distributions to come from normal distribution. Hence, firstly the normality of two distributions must be tested. Table 3 shows the result of the Jarque–Bera test, which is a test for the null hypothesis that the data comes from a normal distribution with an unknown mean and variance [28]. Table 3 show the P-values of the Jarque–Bera test for each feature value in each parameter. A P-value > 0.05 shows that the feature values come from normal

distribution. As it can be seen in most cases P-values are larger than 0.05 and so the t-test can be applied to features values. Figures 8 and 9 present results of the two-sample t-test at the 5% significance level. Each black pixel shows feature values that cause pixels to be independent. This means two concerned boxes are significantly separated. V and OC_1 in both Lorenz an Rossler systems are efficiently sensitive to parameters changes and not sensitive to initial points changes. The other five quantifiers are mostly successful in detecting parameter changes. The whole results in this practical evaluation show that the proposed features can be used as invariant measures to quantify dynamical behavior of nonlinear systems. Therefore, it can be used in real world application of nonlinear signal analysis.

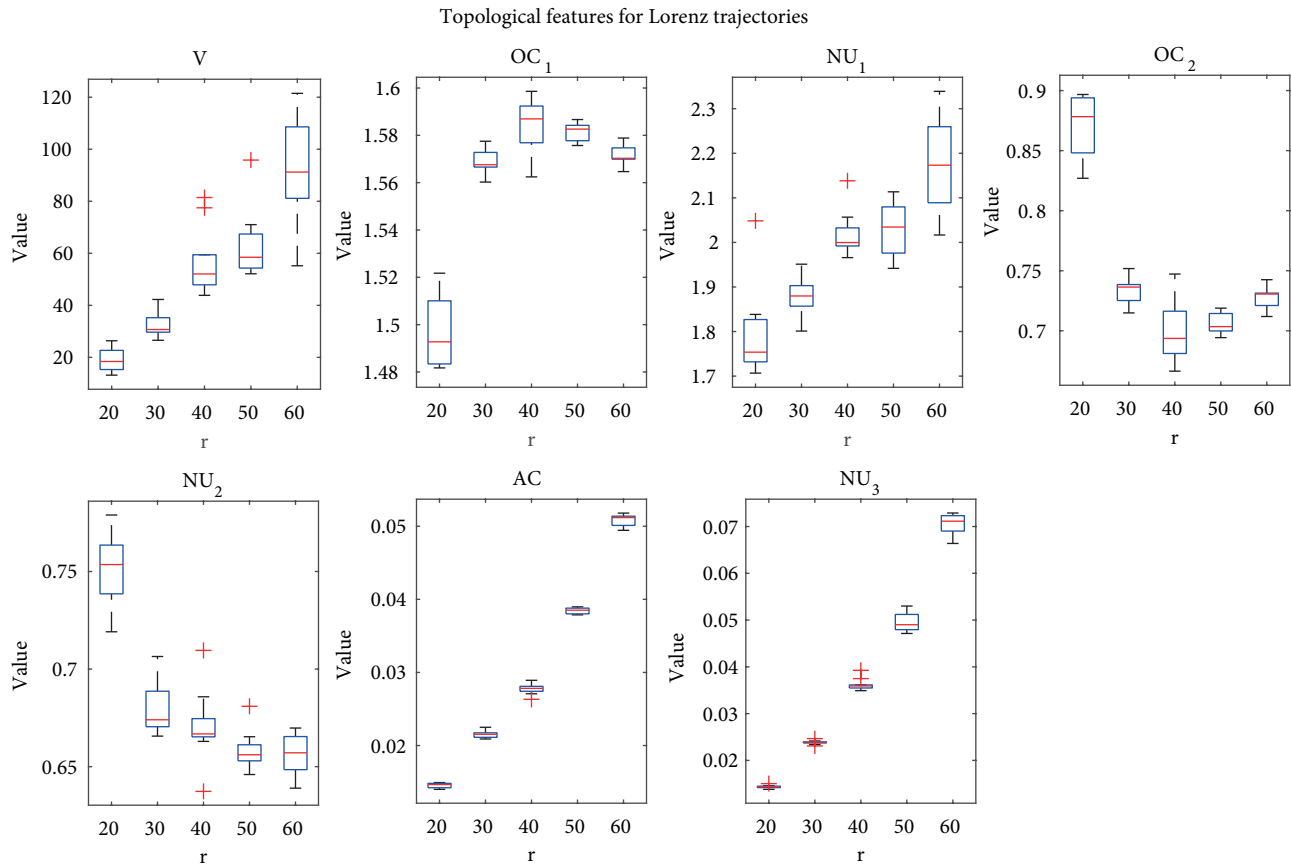


Figure 6. Box plot of the seven proposed feature values for ten time random initial points and different parameters extracted from Lorenz trajectories.

3.1. Results of EEG signal classification

The EEG signals in the dataset used are single channel. Thus, to use the proposed method Takens' method with help of the false nearest-neighbors algorithm and the mutual information is used for EEG signal embedding. The false nearest-neighbors algorithm estimates the dimension of $d = 6$ and the mutual information method estimates delay of $\tau = 5$. After extracting all proposed features from embedded signals and using multiclass SVM classifier for all three cases, the results of 10-fold cross-validation are presented. Table 4 shows the results of the proposed method in comparison with some other studies that use the same dataset, same cases, and same validation method. The best result in each case is bolded and it can be seen that in two cases, cases 2 and 3, the proposed method has the best accuracy. Case 1 is epileptic seizure prediction. In this case, the proposed

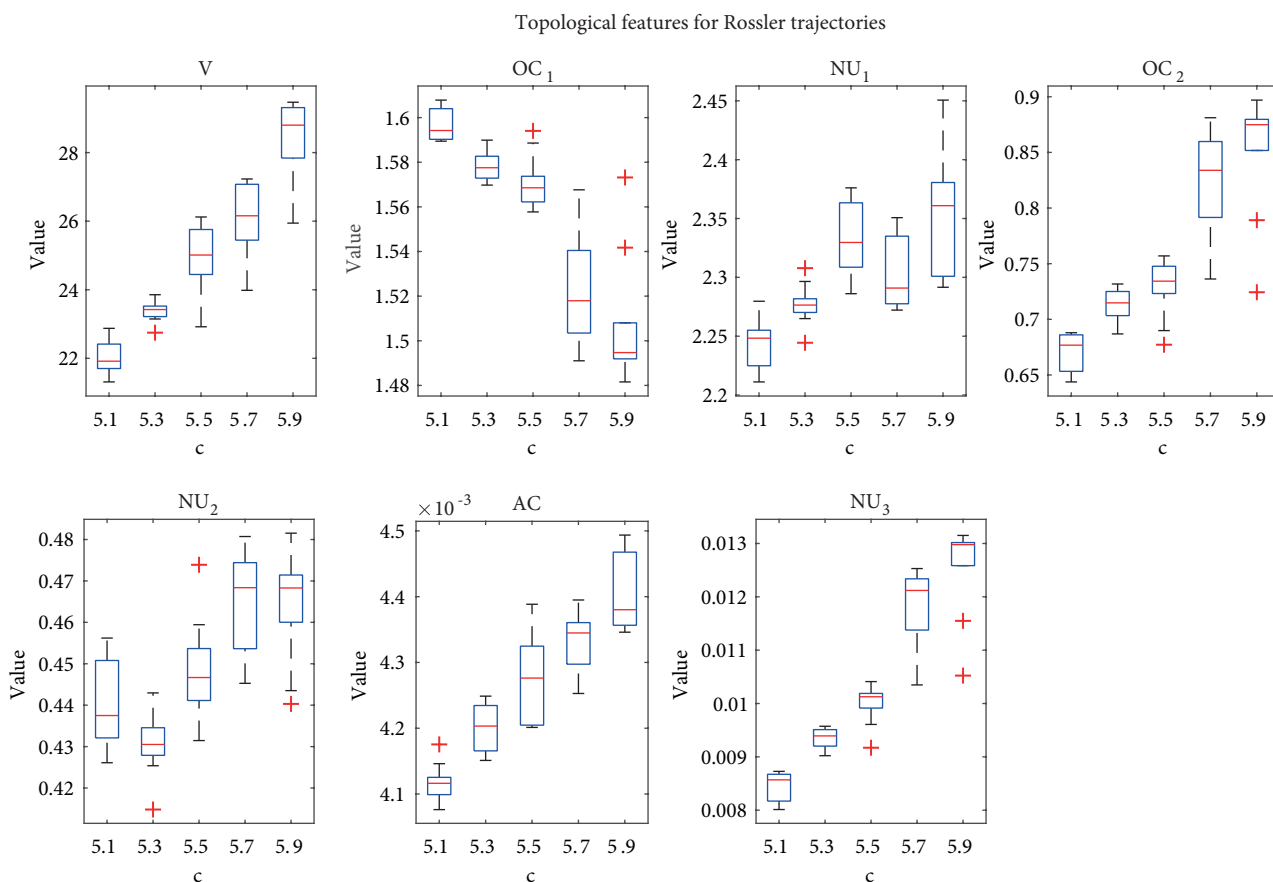


Figure 7. Box plot of the seven proposed feature values for ten time random initial points and different parameters extracted from Rossler trajectories.

Table 3. The P-values of features values for normality test. The bold values are greater than 0.05, which means feature values come from normal distribution.

	Lorenz					Rossler				
V	0.50	0.50	<i>0.01</i>	0.30	0.13	0.11	0.28	<i>0.04</i>	0.08	<i>0.02</i>
OC ₁	0.27	0.50	0.50	0.18	0.20	0.42	0.46	0.50	0.50	0.14
NU ₁	0.42	0.21	0.07	<i>0.005</i>	0.50	0.50	0.50	0.22	0.36	0.06
OC ₂	0.24	0.50	0.21	0.39	0.50	0.15	0.42	0.12	0.39	<i>0.02</i>
NU ₂	<i>0.003</i>	0.50	0.01	0.37	0.50	0.50	0.50	0.40	0.15	0.50
AC	0.23	0.50	0.27	0.41	0.50	0.14	0.40	0.10	0.28	<i>0.01</i>
NU ₃	0.50	0.07	0.35	0.05	0.50	0.33	0.50	0.22	0.33	0.26

method archives accuracy of 93%, which is greater than that of two of the three other studies. In two other cases that are epileptic seizure detection, the accuracy of the proposed method is greater than that of other studies. However, these evaluations are performed to show the potential applicability of the proposed features in a real world application, but also it can be seen how these features with help of an efficient classifier can achieve a good result.

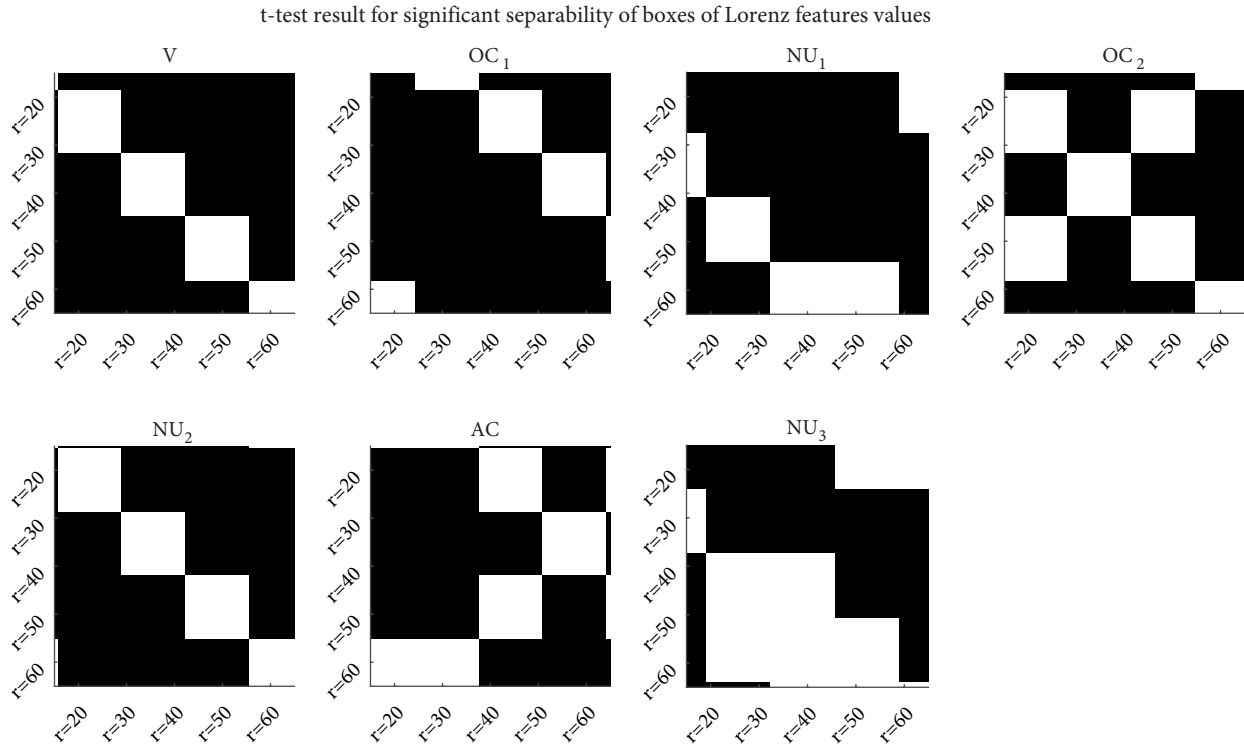


Figure 8. The result of t-test between all pair of parameters for each of the features extracted from Lorenz trajectories. The black color shows that the feature values for pair parameter are independent.

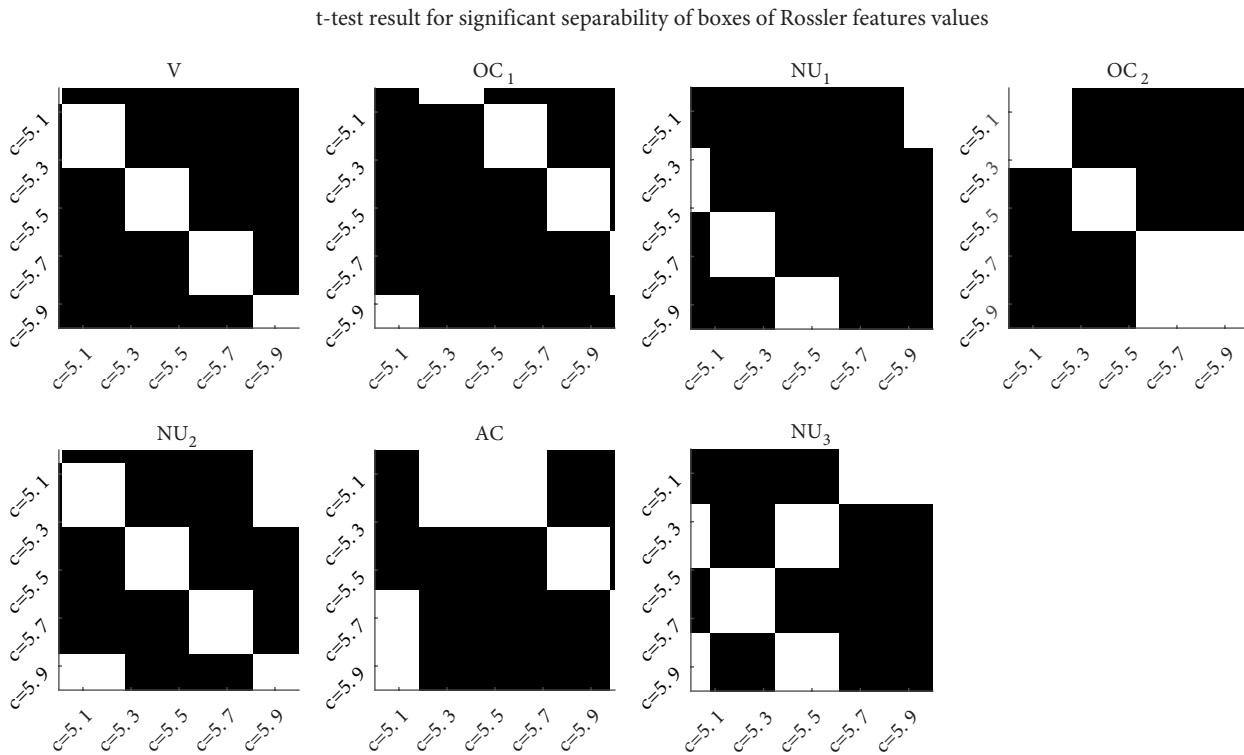


Figure 9. The result of t-test between all pair of parameters for each of the features extracted from Rossler trajectories. The black color shows that the feature values for pair parameter are independent.

Table 4. Performance comparison with some existing methods that use the same data sets.

Authors/method	Year	Accuracy (%)
Case 1 AB/CD/E		
Alam et al. [29]	2013	80.0
Riaz et al. [30]	2016	82.5
Niknazar et al. [31]	2013	98.67
Acharya et al. [32]	2012	99.0
Das et al. [33]	2016	96.28
The proposed method		93
Case 2 ABCD/E		
Zhu et al. [34]	2014	95.4
Riaz et al. [30]	2016	95.4
Kumar et al. [35]	2014	97.38
The proposed method		98.2
Case 3 D/E		
Zhu et al. [36]	2014	95.4
Riaz et al. [30]	2016	95.6
Kumar et al. [35]	2013	97.38
Alam et al. [29]	2013	100
The proposed method		100

4. Conclusion

This study introduces a topological features extraction method for nonlinear trajectories and attractors analysis. The proposed features quantify the topological dynamic of trajectories in a phase space and can be used in studies about changing dynamic of an attractor. Moreover, these features can be used in signal classification applications that would be applied on nonlinear and chaotic signals. The seven proposed features are evaluated with two approaches of a practical and real world application. The result shows the ability of the features as initial point quantifiers and a feature extraction method. These quantifiers consider some topological properties of trajectories in a phase space that are new in the literature. As the results show, the proposed features can significantly trace changes in dynamic. On the other hand, each of the proposed features is not enough to trace the changes in dynamic and should be considered alongside other quantifiers to increase efficiency.

In future studies, the method can be applied to some other types of signals in different classification tasks in electrical engineering, financial time series, biological signals, and any others concerning nonlinear and chaotic signals.

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