

## Composite vector quantization for optimizing antenna locations

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**Abstract:** In this paper, we study the location optimization problem of remote antenna units (RAUs) in generalized distributed antenna systems (GDASs). We propose a composite vector quantization (CVQ) algorithm that consists of unsupervised and supervised terms for RAU location optimization. We show that the CVQ can be used i) to minimize an *upper bound* to the cell-averaged SNR error for a desired/demanded location-specific SNR function, and ii) to maximize the cell-averaged *effective SNR*. The CVQ-DAS includes the standard VQ, and thus the well-known squared distance criterion (SDC) as a special case. Computer simulations confirm the findings and suggest that the proposed CVQ-DAS outperforms the SDC in terms of cell-averaged “effective SNR”.

**Key words:** Distributed antenna system, antenna allocation problem, clustering, squared distance criterion

### 1. Introduction

It has recently been shown that the distributed antenna system (DAS) outperforms traditional co-located antenna systems (CASs) in terms of not only transmit power saving but also spectral efficiency for various cellular radio environments (e.g., [1–4]). Therefore, the DAS is considered a new cellular communication structure for future wireless communication networks [3]. The traditional CAS co-locates the antenna elements according to the wavelength; however, the DAS distributes its antenna elements (located at remote antenna units (RAUs)) geographically over the cell area. Indeed, one of the best possible ways to meet the exponentially increasing mobile data traffic in coming years is to bring the antennas closer to the user equipment (UE). For further information and references about the DAS, see e.g. [3].

The system performance improvements of the DAS in terms of power saving and spectral efficiency highly depend on the locations of its RAUs [3,5–7]. Several papers analyzed the performance of DASs with fixed RAU locations for various transmit strategies for uplink or downlink. The optimal RAU location in terms of “area averaged bit error probability” for linear downlink DASs is derived in [8]. An optimal radius for RAU locations of the DAS in circular layout is investigated in [9]. The authors of [4] propose an iterative algorithm to determine optimal RAU locations based on stochastic approximation theory. The so-called squared distance criterion (SDC) was proposed in [6] in order to find optimal antenna locations in a generalized DAS (GDAS) [10] in order to maximize a lower bound of the cell averaged ergodic capacity. The paper [6] converts the RAU location problem into the codebook design problem in vector quantization [11]. This implies that any clustering algorithm like Lloyd or k-means can be used to optimize the RAU locations of the DAS [6]. As a result, the

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SDC [6] received much attention within the DAS academic community, and following in the footsteps of the SDC and the analysis in [6], several other papers further investigated the SDC for different DAS scenarios. In [12], the SDC is applied to the downlink DAS with selection transmission (ST) in a single cell. The squared-distance-divided-power-criterion is proposed in [13] for linear DAS, which similarly maximizes a lower bound to the ergodic capacity. A RAU location design method for single-cell and two-cell downlink DASs with ST is presented in [5], which maximizes a lower bound of the expected signal-to-noise ratio (SNR). The results in [5] are either the same or quite close to the SDC solution. Similarly, an SNR criterion is used for a DAS with multiple-antenna ports in [14]. In [7], the authors extend the SDC results to single- and two-cell DASs with ST, maximal ratio transmission (MRT), and zero-forcing beamforming under sum power constraint by maximizing a lower bound of the expected SNR. In [15], the authors place RAUs in a circular layout and estimate the optimal radius of the antenna layout that maximizes the sum rate. RAU locations are located evenly on a circle in [16] and the authors derive some analytical expressions for the achievable rate of an arbitrarily located user. A geometric model is used to analyze the RAU communications in [17].

In all the aforementioned works in the DAS literature, a performance index like cell averaged ergodic capacity or expected SNR is optimized evenly over the whole geographical area of the DAS without any location-specific desired performance preferences. However, in many practical systems desired/demanded SNRs (and thus desired data rates) depend on locations. For example, desired/demanded average SNR in hot spot areas like schools and meeting areas is much higher than those in remote and less densely populated areas. Therefore, there is a need for optimizing the RAU locations for cases where location-specific desired SNRs are specified. This paper addresses this question. In this paper, we follow a different approach from any others mentioned above taking the location-specific desired SNR (data rates) into account. For a given location-specific desired SNR function in the geographical area of GDAS, what are the optimum RAU locations? To address this question, we propose a composite vector quantization (CVQ) algorithm consisting of unsupervised and supervised terms for RAU location optimization. We show that the CVQ i) minimizes an upper bound to the cell-averaged SNR error, provided a desired/demanded location-specific SNR function, and ii) maximizes the cell-averaged effective SNR.

The paper is arranged as follows: We present the system model in section 2. The proposed CVQ-DAS is presented and analyzed in section 3. Simulation results are shown in section 4, followed by the conclusions in section 5.

**Notation 1** *Throughout the paper, bold upper and bold lower case letters denote matrices and vectors, respectively, and superscript  $(\cdot)^T$  denotes transpose,  $\mathbf{I}_M$  is the  $M \times M$  identity matrix, and  $E\{\cdot\}$  represents the expectation.*

## 2. System model

Let us consider a GDAS [10] with  $N$  RAUs in each cell and  $M$  antenna elements in each RAU, and every UE has one antenna. We examine a noise-limited environment as in [6]. This corresponds to an isolated cell case or any frequency reuse case where the co-channel interference is small compared to the thermal noise. If the RAU includes multiple co-located antenna elements, then the channels between one RAU and the UE undergo the same large-scale fading. All channels between the antennas and the UE are assumed to be flat fading and slow fading. Let us denote the channel vector from the  $n$ th RAU to the UE as

$$\mathbf{h}_n = \sqrt{\frac{s_n}{d_n^\alpha}} [h_{n,1} h_{n,2} \cdots h_{n,M}]^T, \quad (1)$$

where  $\alpha$  is the path loss exponent,  $s_n$  is the large-scale fading (e.g., shadow fading) term (between the UE and the  $n$ th RAU) and is modeled as log-normal random variable (i.e.  $10 \log_{10}(s_n)$  is a zero mean Gaussian random variable (rv) with standard deviation  $\sigma_s$ ), and  $h_{n,m}$  ( $n = 1, \dots, N$ ,  $m = 1, \dots, M$ ) represents the small-scale fading (multipath) (e.g., Rayleigh fading) term (between the UE and the  $m$ th antenna element of the  $n$ th RAU), and is modeled as a unit-variance circularly symmetric complex Gaussian rv. Large-scale and small-scale fadings are independent. Then the  $NM \times 1$  dimensional channel vector  $\mathbf{h}$  between the UE and the DAS has the form  $\mathbf{h} = [\mathbf{h}_1^T \mathbf{h}_2^T \cdots \mathbf{h}_N^T]^T$ . In this paper, we examine the maximal ratio combining (MRC) case in uplink. Representing the RAU transmit power as  $p_{UE}$ , the received signal vector  $\mathbf{y}$  at GDAS can be written as  $\mathbf{y} = (p_{UE} \mathbf{a} \mathbf{h}) + \boldsymbol{\zeta}$ , where  $a$  is the transmitted symbol,  $\mathbf{h}$  is the channel vector, and  $\boldsymbol{\zeta}$  is a zero-mean complex additive white Gaussian noise vector whose covariance matrix is  $E\{\boldsymbol{\zeta} \boldsymbol{\zeta}^H\} = \sigma_\zeta \mathbf{I}_{NM}$  in which  $\sigma_\zeta > 0$ . The uplink expected SNR with the MRC [18] for the 2-dimensional or 3-dimensional user location  $\mathbf{x}_l$  is obtained by averaging the instantaneous SNR over the small-scale and large-scale fadings:

$$\bar{\theta}_a(\mathbf{x}_l) = \frac{p_{UE}}{\sigma_\zeta^2} \sum_{n=1}^N E_{h,s} \{s_n g_{n,\mathbf{x}}\} \frac{1}{\|\mathbf{x}_l - \mathbf{c}_n\|_2^\alpha}, \quad (2)$$

where  $E_{h,s}\{\cdot\}$  denotes the expectation with respect to  $h$  (small-scale fading) and  $s$  (large-scale fading),  $n$  is the index of the RAU and  $g_{n,\mathbf{x}} = \sum_{m=1}^M |h_{n,m}|^2$ , and  $p_{UE}$  is the transmit power of the UE, and  $n \in \{1, \dots, N\}$ . It is assumed that the expectation  $E_{h,s}\{\cdot\}$  for a particular location is calculated over all possible small-scale fading and large-scale fading realizations as done in [6] and [3]. Let location vector be  $\mathbf{x} \in \mathbb{R}^{q \times 1}$ , and an arbitrary probability distribution function (pdf) of the UE location be denoted by  $f(\mathbf{x})$ . Similarly, the locations of the RAUs  $\{\mathbf{c}_n\}_{n=1}^N \in \mathbb{R}^{q \times 1}$ . Let  $d_{\min}$  denote the minimum distance between the UE location and an RAU location, and  $d_{\max}$  be the radius of the GDAS. The system performance of the GDAS is calculated for the area denoted as  $\bar{\Omega}$

$$\bar{\Omega} : \mathbf{x} \in \Omega \ni \|\mathbf{x} - \mathbf{c}_n\|_2 \geq d_{\min}, \quad n = 1, \dots, N \quad (3)$$

Supposed that an arbitrary location-specific desired/demanded SNR, denoted as  $\bar{\theta}_d(\mathbf{x})$ , is provided. The actual average SNR is given by (2). Then the cell averaged desired/and actual SNR, denoted as  $\hat{\Gamma}_d$ , and  $\hat{\Gamma}_a$ , respectively, is derived by averaging  $\bar{\theta}_d(\mathbf{x})$  and  $\bar{\theta}_a(\mathbf{x})$  over user locations, i.e.  $\hat{\Gamma}_d = E_{\mathbf{x}}[\bar{\theta}_d(\mathbf{x})]$  and  $\hat{\Gamma}_a = E_{\mathbf{x}}[\bar{\theta}_a(\mathbf{x})]$ , where  $E_{\mathbf{x}}[\cdot]$  denotes the expectation over user locations. In what follows, we define *wasted SNR* and *effective SNR*:

**Definition 1** *Wasted SNR: For a given location  $\mathbf{x}$ , if the supplied/actual SNR  $\bar{\theta}_a(\mathbf{C}, \mathbf{x})$  is higher than the demanded/desired SNR  $\bar{\theta}_d(\mathbf{x})$ , then the excessive amount is useless, and thus is wasted. Thus,  $\bar{\theta}_{wasted}(\mathbf{x}) = \min(0, \bar{\theta}_a(\mathbf{x}) - \bar{\theta}_d(\mathbf{x}))$ .*

**Definition 2** *Effective SNR: The “effective SNR” for location  $\mathbf{x}$ , denoted as  $\bar{\theta}_{eff}(\mathbf{x})$ , is defined as the amount of SNR that is completely utilized by the users and not wasted according to a given desired SNR  $\bar{\theta}_d(\mathbf{x})$ : thus,  $\bar{\theta}_{eff}(\mathbf{x}) = \min(\bar{\theta}_d(\mathbf{x}), \bar{\theta}_a(\mathbf{x}))$ . Cell averaged effective SNR is then equal to  $E_{\mathbf{x}}[\bar{\theta}_{eff}(\mathbf{x})]$ .*

Defining a  $q \times N$  dimensional matrix  $\mathbf{C} \in \mathbb{R}^{q \times N}$ , where  $q$  is 2 or 3 and whose columns are the RAUs locations  $\{\mathbf{c}_n\}_{n=1}^N$ , the RAU location optimization problem could be formulated as

$$\max_{\mathbf{C}} (J_a(\mathbf{C}) = E_{\mathbf{x}} [\bar{\theta}_a(\mathbf{C}, \mathbf{x})]) \quad (4)$$

as in e.g. [5,7]; or

$$\min_{\mathbf{C}} (J_2(\mathbf{C}) = E_{\mathbf{x}} [(\bar{\theta}_d(\mathbf{x}) - \bar{\theta}_a(\mathbf{C}, \mathbf{x}))^2]) \quad (5)$$

In this paper, the proposed composite vector quantization (named CVQ-DAS) in the next section minimizes  $J_2(\mathbf{C})$  in (5) because i) the desired SNR is location-specific and we aim to shape the actual SNR function  $\bar{\theta}_a(\mathbf{C}, \mathbf{x})$  according to the desired SNR function  $\bar{\theta}_d(\mathbf{x})$ , and ii) the  $J_2(\mathbf{C})$  in (5) *minimizes the wasted SNR* and *maximizes the effective SNR*.

### 3. CVQ-DAS for determining GDAS RAU locations

#### 3.1. Statistical setting

Let us consider the  $J_2(\mathbf{C})$  in (5). In Appendix A, we show that the average SNR function  $\bar{\theta}_a(\cdot)$  in (2) has a global Lipschitz constant, which is denoted as  $\nu_{glob}$  for the interval  $[d_{\min}, d_{\max}]$ .

**Definition 3** *Winning RAU for a given location  $\mathbf{x}$ : For any location  $\mathbf{x} \in \Omega$ , we call the RAU location that is the closest to  $\mathbf{x}$  the winning RAU, denoted by  $\mathbf{c}_{n(\mathbf{x})}$ , where index  $n(\mathbf{x}) \in \{1, 2, \dots, N\}$ . In other words, for the winning RAU  $\mathbf{c}_{n(\mathbf{x})}$ :  $\|\mathbf{c}_{n(\mathbf{x})} - \mathbf{x}\| = \min \{\|\mathbf{x} - \mathbf{c}_n\|\}_{n=1}^N$ .*

**Proposition 1** *The  $J_2(\mathbf{C})$  in (5) is upper bounded by the following  $UB_2(\mathbf{C})$ :*

$$UB_2(\mathbf{C}) = (q+1) \int_{\mathbf{x} \in \bar{\Omega}} \left( \nu_{glob}^2 \|\mathbf{x} - \mathbf{c}_{n(\mathbf{x})}\|_2^2 + (\bar{\theta}_d(\mathbf{x}) - \bar{\theta}_a(\mathbf{C}, \mathbf{c}_{n(\mathbf{x})}))^2 \right) f(\mathbf{x}) d\mathbf{x}, \quad (6)$$

where  $f(\mathbf{x})$  is the probability distribution function of the UE location.

**Proof** The proof is given in Appendix B. □

#### 3.2. Deterministic setting

In section 3.1, we analyze the RAU location problem from a statistical point of view. In what follows, we derive the same upper bound in a deterministic setting in order to devise the RAU allocation algorithm by using a gradient descent approach. Let us assume that we are given  $L$  location samples from the user distribution  $f(\mathbf{x})$ , denoted by set  $\{\mathbf{x}_l\}_{l=1}^L$ . Then  $J_2(\mathbf{C})$  in (5) is approximated by these  $L$  samples as

$$J_2(\mathbf{C}) \approx \tilde{J}_2(\mathbf{C}) = \frac{1}{L} \sum_{l=1}^L (\bar{\theta}_d(\mathbf{x}_l) - \bar{\theta}_a(\mathbf{C}, \mathbf{x}_l))^2, \quad (7)$$

where  $\bar{\theta}_d(\mathbf{x}_l)$  and  $\bar{\theta}_a(\mathbf{C}, \mathbf{x}_l)$  are the desired/demanded and the actual/supplied average SNR, respectively, at location  $\mathbf{x}_l$ . Following the steps (A.1) to (B.8), we similarly obtain the following upper bound denoted as

$UB_{2,d}(\mathbf{C})$ :

$$\tilde{J}_2(\mathbf{C}) \leq UB_{2,d}(\mathbf{C}) = \frac{q+1}{L} \left( \sum_{l=1}^L \nu_{glob}^2 \|\mathbf{x}_l - \mathbf{c}_{n(l)}\|_2^2 + (\bar{\theta}_d(\mathbf{x}_l) - \bar{\theta}_a(\mathbf{C}, \mathbf{c}_{n(l)}))^2 \right) \quad (8)$$

**Corollary 1** Following the steps (5) to (B.8) for  $J_1(\mathbf{C}) \approx \tilde{J}_1(\mathbf{C}) = \frac{1}{L} \sum_{l=1}^L (\bar{\theta}_d(\mathbf{x}_l) - \bar{\theta}_a(\mathbf{C}, \mathbf{x}_l))$ , we obtain the following upper bound:

$$\tilde{J}_1(\mathbf{C}) \leq UB_{1,d}(\mathbf{C}) = \frac{1}{L} \left( \sum_{l=1}^L \sqrt{q+1} \nu_{glob} \|\mathbf{x}_l - \mathbf{c}_{n(l)}\|_2 + (\bar{\theta}_d(\mathbf{x}_l) - \bar{\theta}_a(\mathbf{C}, \mathbf{c}_{n(l)})) \right) \quad (9)$$

**Proposition 2** Defining  $\lambda_{n,\mathbf{x}} = E_{h,s} \{s_n g_{n,\mathbf{x}}\}$ ,  $\lambda_{n,\mathbf{x}}^{\max} = \max \{\lambda_{n,\mathbf{x}}\}$ ,  $\varphi'_{\max} = -\alpha d_{\min}^{-(\alpha+1)}$ , and  $J_a(\mathbf{C}) \approx \tilde{J}_a(\mathbf{C}) = \frac{1}{L} \sum_{l=1}^L \bar{\theta}_a(\mathbf{C}, \mathbf{x}_l)$  from (4), the  $\tilde{J}_a(\mathbf{C})$  is lower bounded by the following  $LB_a(\mathbf{C})$ :

$$LB_a(\mathbf{C}) = \frac{1}{L} \sum_{l=1}^L (\beta \|\mathbf{x} - \mathbf{c}_{n(\mathbf{x})}\|_2 + \theta_a(\mathbf{C}, \mathbf{c}_{n(\mathbf{x})})), \quad (10)$$

where  $\beta = \frac{pUE}{\sigma_s^2} \sum_{n=1}^N \lambda_{n,\mathbf{x}}^{\max} \varphi'_{\max} < 0$ .

**Proof** Defining  $\varphi(d) = d^{-\alpha}$ ,  $d_{n,x} = \|\mathbf{x} - \mathbf{c}_n\|_2$ , and  $d_{n,c} = \|\mathbf{c}_{n(\mathbf{x})} - \mathbf{c}_n\|_2$ , and then adding and subtracting the term  $\theta_a(\mathbf{C}, \mathbf{c}_{n(\mathbf{x})})$  from the  $\tilde{J}_a(\mathbf{C})$ , we write

$$J_a(\mathbf{C}) = \frac{1}{L} \sum_{l=1}^L \left[ \left( \sum_{n=1}^N \lambda_{n,\mathbf{x}} (\varphi(d_{n,x}) - \varphi(d_{n,c})) \right) + \theta_a(\mathbf{C}, \mathbf{c}_{n(\mathbf{x})}) \right] \quad (11)$$

Applying the mean value theorem to the function  $\varphi(d) = d^{-\alpha}$  for  $\forall d_i, d_j \in [d_{\min}, \infty)$  gives

$$\varphi(d_i) - \varphi(d_j) = (d_i^{-\alpha} - d_j^{-\alpha}) \varphi'(\mu d_i + (1-\mu)d_j), \quad (12)$$

where  $\mu \in [0, 1]$ , and  $\varphi'(\cdot) < 0$ . The derivative of  $\varphi(d)$  is  $\varphi'(d) = -\alpha d^{-(\alpha+1)} < 0$ . Hence, the minimum (negative) value of the derivative for the interval  $[d_{\min}, \infty)$  is at  $d = d_{\min}$ , and is denoted by  $\varphi'_{\max} < 0$ . Using the triangle inequality  $(d_{k,x} - d_{k,c}) \leq \|\mathbf{x} - \mathbf{c}_{k(\mathbf{x})}\|_2$ , for  $k = 1, 2, \dots, K$ , and the fact that  $\varphi'_{\max} < 0$ , we obtain a lower bound to the  $J_a(\mathbf{C})$  in (4):

$$J(\mathbf{C}) \geq \frac{1}{L} \sum_{l=1}^L (\beta \|\mathbf{x} - \mathbf{c}_{n(\mathbf{x})}\|_2 + \theta_a(\mathbf{C}, \mathbf{c}_{n(\mathbf{x})})) = LB(\mathbf{C}), \quad (13)$$

where  $\beta = \frac{pUE}{\sigma_s^2} \sum_{n=1}^N \lambda_{n,\mathbf{x}}^{\max} \varphi'_{\max} < 0$ , which gives the lower bound  $LB_a(\mathbf{C})$  in (10). This completes the proof.  $\square$

The proposed composite vector quantization for DAS (CVQ-DAS) is given as

$$\mathbf{c}_{n(l)}(t+1) = \mathbf{c}_{n(l)} - \varepsilon_I(t) (\mathbf{c}_{n(l)} - \mathbf{x}_l) - \varepsilon_O(t) (\mathbf{c}_{n(l)} - \mathbf{c}_n) \tau(t), \quad (14)$$

where  $\mathbf{c}_{n(l)}$  is the winning RAU to a given location  $\mathbf{x}_l$ , and  $\tau(t) = \begin{cases} +1, & \text{if } \bar{\theta}_d(\mathbf{x}_l) > \bar{\theta}_a(\mathbf{C}, \mathbf{x}_l) \\ -1, & \text{if } \bar{\theta}_d(\mathbf{x}_l) < \bar{\theta}_a(\mathbf{C}, \mathbf{x}_l) \\ 0, & \text{if } \bar{\theta}_d(\mathbf{x}_l) = \bar{\theta}_a(\mathbf{C}, \mathbf{x}_l) \end{cases}$ , and  $\varepsilon_I(t)$

and  $\varepsilon_O(t)$  are decreasing nonnegative real numbers representing the step sizes.

**Proposition 3** For a given set of  $\{\mathbf{x}_l, \bar{\theta}_d(\mathbf{x}_l)\}_{l=1}^L$ , updating the winning RAU location  $\mathbf{c}_{n(l)}$  for location  $\mathbf{x}_l$  at step  $t$  by the CVQ-DAS in (14) minimizes the upper bound  $UB_{2,d}(\mathbf{C})$  in (8). In the CVQ-DAS in (14), the  $\varepsilon_I(t)$  and  $\varepsilon_O(t)$  are decreased at each step when  $t$  goes to infinity. The  $UB_{2,d}(\mathbf{C})$  in (8) minimizes an upper bound to  $J_2(\mathbf{C})$  in (5), and maximizes a lower bound  $LB_a(\mathbf{C})$  in (10) to  $J_a(\mathbf{C})$  in (4).

**Proof** For a given  $\{\mathbf{x}_l, \bar{\theta}_d(\mathbf{x}_l)\}$ , first defining  $e_{2,l} = \nu_{glob}^2 \|\mathbf{x}_l - \mathbf{c}_{n(l)}\|_2^2 + (\bar{\theta}_d(\mathbf{x}_l) - \bar{\theta}_a(\mathbf{C}, \mathbf{c}_{n(l)}))^2$  and  $e_{1,l} = \nu_{glob} \|\mathbf{x}_l - \mathbf{c}_{n(l)}\|_2 + (\bar{\theta}_d(\mathbf{x}_l) - \bar{\theta}_a(\mathbf{C}, \mathbf{c}_{n(l)}))$  from (8) and (9), respectively, and calculating their gradients with respect to the winning RAU location  $\mathbf{c}_{n(l)}$ , and then updating the winning  $\mathbf{c}_{n(x)}$  according to their instantaneous gradients  $\mathbf{c}_{n(l)}(t+1) = \mathbf{c}_{n(l)}(t) - \varepsilon(t) \frac{de_{2,l}(t)}{d\mathbf{c}_{n(l)}}$  and  $\mathbf{c}_{n(l)}(t+1) = \mathbf{c}_{n(l)}(t) - \varepsilon(t) \frac{de_{1,l}(t)}{d\mathbf{c}_{n(l)}}$ , respectively, gives the proposed CVQ in (14), where  $\varepsilon(t)$  is a decreasing nonnegative real number representing the step size. In the case of minimization of  $UB_{2,d}(\mathbf{C})$  in (8), the step sizes  $\varepsilon_I(t)$  and  $\varepsilon_O(t)$  are equal to

$$\begin{aligned} \varepsilon_I(t) &= \varepsilon(t) 2(q+1) \nu_{glob}^2 \\ \varepsilon_O(t) &= \varepsilon(t) \frac{2\alpha(q+1)v^2 p_{UE}}{\sigma_\zeta^2} \sum_{n=1}^N \frac{\lambda_{n,\mathbf{x}}}{\|\mathbf{c}_{n(l)} - \mathbf{c}_n\|_2^{\alpha+2}}, \end{aligned} \quad (15)$$

where  $\lambda_{n,\mathbf{x}} = E_{h,s} \{s_n g_{n,\mathbf{x}}\}$ . Similarly, in the case of minimization of  $UB_{1,d}(\mathbf{C})$  in (9), the step sizes  $\varepsilon_I(t)$  and  $\varepsilon_O(t)$  are equal to

$$\varepsilon_I(t) = \varepsilon(t) (\sqrt{q+1}) \nu_{glob} \quad \varepsilon_O(t) = \varepsilon(t) \frac{2\alpha p_{UE}}{\sigma_\zeta^2} \sum_{n=1}^N \frac{\lambda_{n,\mathbf{x}}}{z_{n,l}^{0.5\alpha+1}} \quad (16)$$

Finally, in the case of maximization of  $LB_a(\mathbf{C})$  in (10), because  $\beta < 0$  in (10), in order to maximize the  $LB_a(\mathbf{C})$ , the location matrix  $\mathbf{C}$  should both minimize  $E[\|\mathbf{x} - \mathbf{c}_{n(x)}\|_2] \approx \frac{1}{L} \sum_{l=1}^L \|\mathbf{x} - \mathbf{c}_{n(x)}\|_2$  and maximize  $E\{\theta_a(\mathbf{C}, \mathbf{c}_{n(x_l)})\} \approx \frac{1}{L} \sum_{l=1}^L \theta_a(\mathbf{C}, \mathbf{c}_{n(x_l)})$  at the same time. According to this observation, we devise a two-step iterative procedure:

**Step: 1**

$$\mathbf{c}_{n(l)}(t+1) = \mathbf{c}_{n(l)}(t) - \varepsilon(t) \frac{d\|\mathbf{x} - \mathbf{c}_{n(x)}\|_2}{d\mathbf{c}_{n(l)}} \quad (17)$$

**Step: 2**

$$\mathbf{c}_{n(l)}(t+1) = \mathbf{c}_{n(l)}(t) + \varepsilon(t) \frac{\theta_a(\mathbf{C}, \mathbf{c}_{n(x_l)})}{d\mathbf{c}_{n(l)}}, \quad (18)$$

where  $\varepsilon(t) > 0$ . Eq. (17) gives

$$\mathbf{c}_{n(l)}(t+1) = \mathbf{c}_{n(l)}(t) - \varepsilon_I(t) (\mathbf{c}_{n(l)}(t) - \mathbf{x}_l), \quad (19)$$

where  $\varepsilon_I(t) = \varepsilon(t) / \|\mathbf{c}_{n(l)} - \mathbf{x}_l\|_2$ . Eq. (18) gives

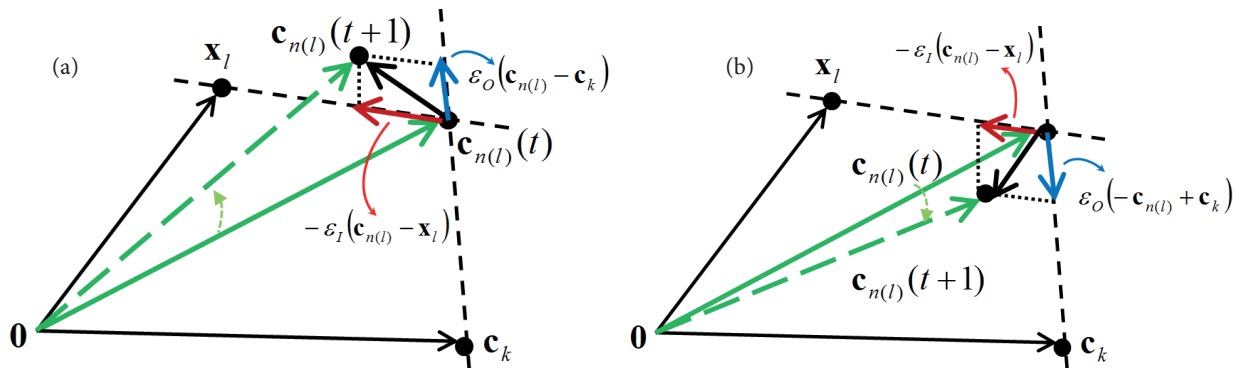
$$\mathbf{c}_{n(l)}(t+1) = \mathbf{c}_{n(l)}(t) - \varepsilon_O(t) (\mathbf{c}_{n(l)}(t) - \mathbf{c}_k), \quad (20)$$

where  $\varepsilon_O(t) = \varepsilon(t) \frac{\alpha}{2\sigma_c^2} \sum_{k=1}^N \frac{\lambda_k}{z_{k,l}^{0.5\alpha+1}}$ . Combining (19) and (20) into one step gives the proposed CVQ in (14).

From (15)–(20), by suitably decreasing the step size  $\varepsilon(t)$  (and eventually  $\varepsilon_I(t)$  and  $\varepsilon_O(t)$ ) at each step when  $t$  goes to infinity assures that the RAU locations converge, which completes the proof.  $\square$

Examining the proposed CVQ update for the winning RAU in (14), there are two terms:

- 1) The first term whose step size is  $\varepsilon_I(t)$  is nothing but the well-known Kohonen rule, where only the winning vector  $\mathbf{c}_{n(l)}$  for the input  $\mathbf{x}_l$  is updated, which realizes a vector quantization (VQ) in input space (see e.g. eq. 13.46 on pp. 13–15 in [19]). Therefore, the codebook vector  $\mathbf{c}_{k(l)}$  gets *always* closer to the input  $\mathbf{x}_l$  in an unsupervised fashion.
- 2) The other term introduces a supervised term and makes the winning RAU location  $\mathbf{c}_{k(l)}$ , at each step, either get closer to or go away from all other RAU locations depending on the desired SNR value for that particular location. If  $(\bar{\theta}_d(\mathbf{x}_l) > \bar{\theta}_a(\mathbf{c}_{k(l)}))$  then  $\tau(t) = 1$  and the winning  $\mathbf{c}_{k(l)}$  gets closer to all  $\mathbf{c}_k$ ; otherwise if  $(\bar{\theta}_d(\mathbf{x}_l) < \bar{\theta}_a(\mathbf{c}_{k(l)}))$  then  $\tau(t) = -1$  and the winning  $\mathbf{c}_{n(l)}$  goes away from  $\mathbf{c}_n$ , where  $n = 1, 2, \dots, N$ . Update of the winning codebook vector (RAU location) when  $\bar{\theta}_d(\mathbf{x}_l) > \bar{\theta}_a(\mathbf{C}, \mathbf{x}_{n(l)})$  and  $\bar{\theta}_d(\mathbf{x}_l) < \bar{\theta}_a(\mathbf{C}, \mathbf{x}_{n(l)})$  is depicted in Figures 1a and 1b, respectively.



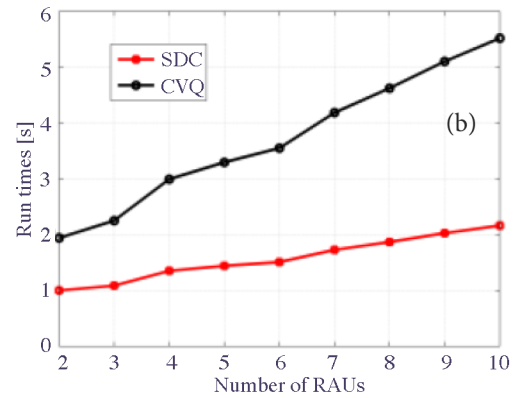
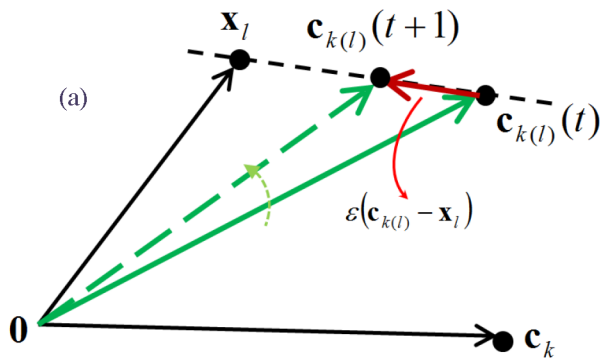
**Figure 1.** Update of the winning codebook vector when (a)  $\bar{\theta}_d(\mathbf{x}_l) > \bar{\theta}_a(\mathbf{C}, \mathbf{x}_{n(l)})$  and (b)  $\bar{\theta}_d(\mathbf{x}_l) < \bar{\theta}_a(\mathbf{C}, \mathbf{x}_{n(l)})$  by (14) for the  $N = 2$  case (e.g., if  $n(l) = 1$ , then  $k = 2$ ).

Therefore, the main difference between the proposed CVQ and the standard SDC is as follows: while the SDC takes only the mobile locations into account, the CVQ takes not only the mobile locations but also the location-specific desired SNR  $\bar{\theta}_d(\mathbf{x})$  into account. That is why the CVQ outperforms the VQ in maximizing the effective SNR. From (14), although the computational complexity of the proposed CVQ is higher than that of

the standard VQ, the calculations and simulation campaigns are carried out only once and offline to determine the RAU locations. Once the RAU locations are determined and fixed, then there is no computation to be performed any more.

**Corollary 2** *If  $\bar{\theta}_d(\mathbf{x})$  is chosen such that  $\bar{\theta}_d(\mathbf{x}) \geq \bar{\theta}_a(\mathbf{C}, \mathbf{x})$ , for any  $\mathbf{x} \in \bar{\Omega}$ , and thus  $\tau(t) = +1$  in (14), then for maximization of  $J_a(\mathbf{C})$  in (5), the winning RAU location always both gets closer to the user location and gets closer to other RAU locations.*

There are both unsupervised and supervised terms in (14), and that is why we call it composite vector quantization (CVQ). In order to give an insight into the update rule of the CVQ-DAS in (14), we sketch the CVQ in Figure 1 for the  $K = 2$  case (e.g., if  $k(l) = 1$ , then  $k = 2$ ). The standard update of the Kohonen rule-based VQ, which corresponds to the SDC in [6], is shown in Figure 2 for the same case for comparison reasons. The Kohonen-based VQ update is given by



**Figure 2.** (a) Update of the standard VQ by (21) (and thus the SDC criterion in [2]) for the  $K = 2$  case (e.g., if  $k(l) = 1$ , then  $k = 2$ ), (b) runtimes of SDC and CVQ in Example 1.

$$\mathbf{c}_{n(l)}(t+1) = \mathbf{c}_{n(l)} - \varepsilon(t) (\mathbf{c}_{n(l)} - \mathbf{x}_l), \quad (21)$$

where  $\varepsilon(t) > 0$ , and  $n(l)$  is the index of the winning codebook vector. The Kohonen-based VQ and the SDC in [6] minimize the following VQ cost function:

$$J_{SDC}(\mathbf{C}) = \frac{1}{L} \sum_{l=1}^L \|\mathbf{x}_l - \mathbf{c}_{n(l)}\|_2^2 \quad (22)$$

Comparing (8), (19), and (20) with (21) and (22), we see that the first step of the proposed CVQ-DAS is nothing but the SDC in [2]. If the second step (20) is omitted, then the proposed CVQ-DAS becomes equal to the SDC in [6].

#### 4. Simulation results

**Example 1 (High SNR scenario)** *Without loss of generality, a direct-sequence (W)CDMA wireless network is considered in all examples of the GDAS. For link gain modeling, attenuation factor  $2 \leq \alpha \leq 6$ , the log-normally distributed  $s_n$  in (1) is generated according to the model in [20], and the lognormal variance is 6 dB. Without*



loss of generality, the sum of the RAU transmit powers is equal to 1 W for all simulations. The minimum distance between the RAU and UE is  $d_{\min} = 2$  m. Here we examine a two-dimensional GDAS scenario as in [6]. The MS locations are drawn from a PPP process [21] whose density is 0.03. Without loss of generality, the location-specific target SNR is chosen to reduce from 80 dB to 20 dB with respect to the norm of the difference between the MS location and the center of the GDAS. We plot the clusters found by the SDC [6] and the CVQ-DAS for the very same initial conditions in Figure 3 for  $N = 6$ . Figure 3 shows that the RAU locations found by the CVQ-DAS are closer to each other as compared to the SDC solutions, which yields the clusters in Figure 3. The effective SNR with respect to number of RAUs is shown in Figure 4 for the SDC and the CVQ-DAS. Figure 4 shows that the proposed CVQ-DAS outperforms the SDC in terms of the average effective SNR, and the gain is about 1 dB in most cases. This gain is obtained at the expense of computational complexity in the clustering algorithm. Figure 2a compares the runtimes of the SDC [2] and the CVQ-DAS for a 100-step clustering process and shows that both evolve almost linearly with respect to number of RAUs. Although the CVQ-DAS is slower, it pays off because the off-line clustering is done only once and gives about 1-dB SNR gain in our scenarios.

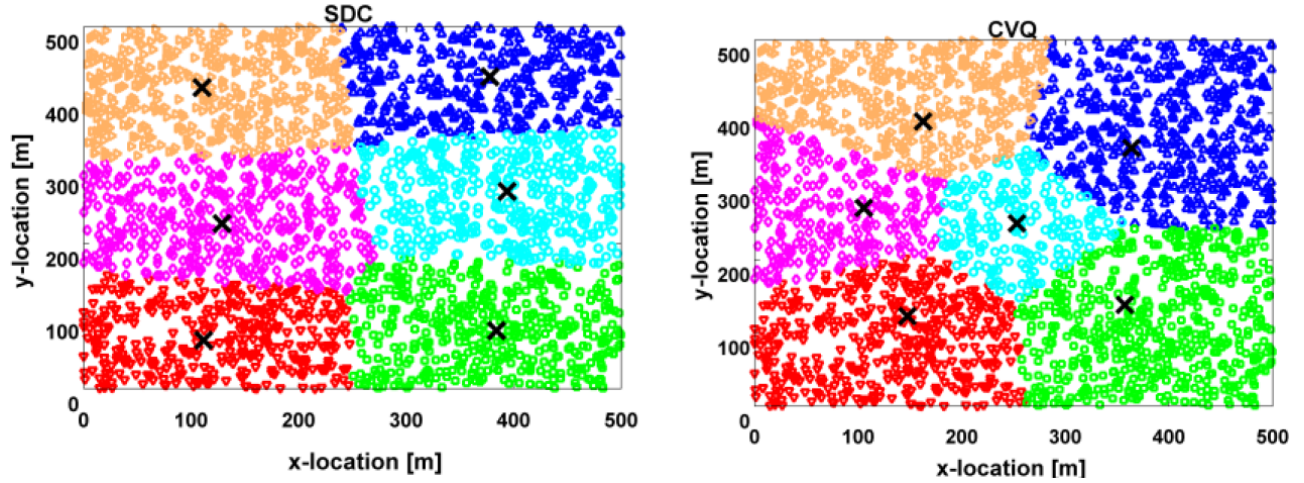


Figure 3. Clusters and RAU locations found by (a) SDC and (b) CVQ-DAS for  $N = 6$ .

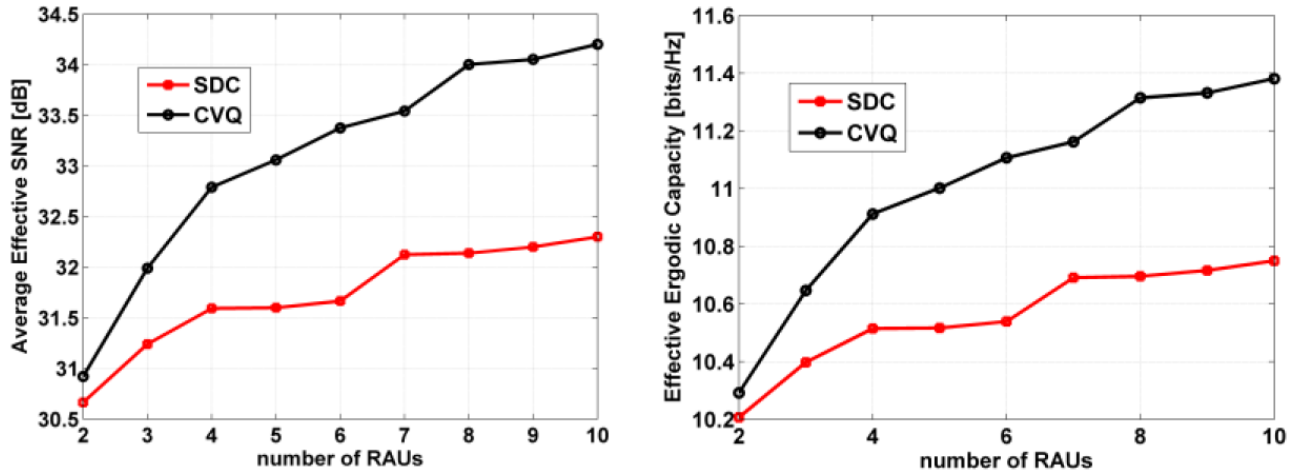
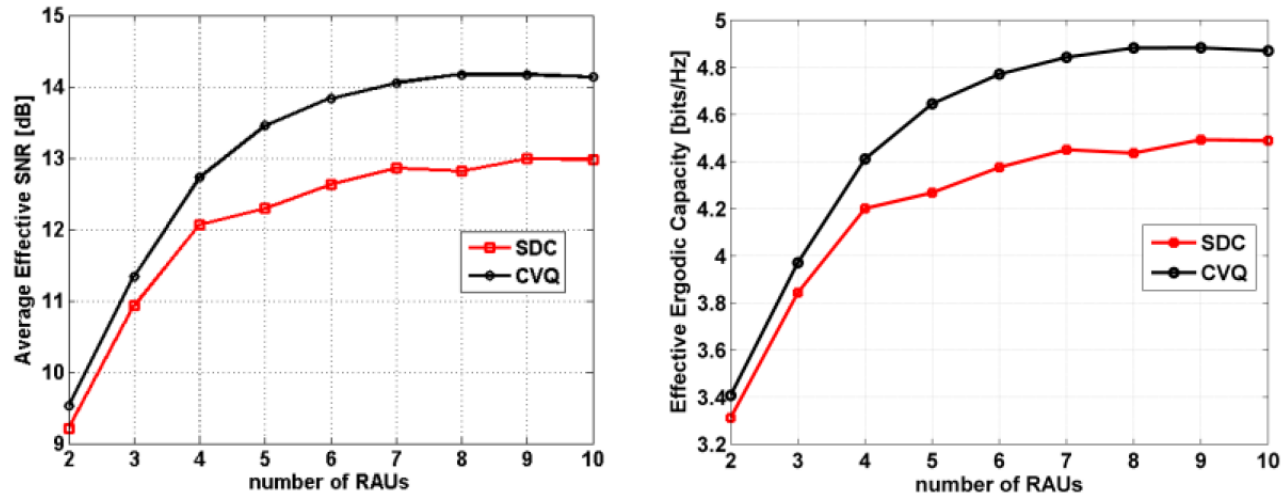


Figure 4. (a) Effective SNR, and (b) effective ergodic capacity for the high-SNR case.

**Example 2** (*Medium SNR scenario*) To generate a medium SNR scenario, the sum of the RAU transmit powers is reduced to  $0.001\text{ W}$ . The minimum distance between the RAU and UE is increased to  $d_{\min} = 20\text{ m}$ . Other parameters are the same as in Example 1. The effective SNR and effective ergodic capacity with respect to number of RAUs is shown in Figures 5a and 5b, respectively. Figure 5 shows that the proposed CVQ-DAS outperforms the SDC in terms of effective SNR.



**Figure 5.** (a) Effective SNR, and (b) effective ergodic capacity with respect to number of RAUs for the medium SNR scenario.

## 5. Conclusions

In this paper, we propose a composite vector quantization (CVQ) algorithm for the location optimization problem of RAUs in generalized DAS. The proposed CVQ minimizes an *upper bound* to the cell-averaged SNR error, provided a desired/demanded location-specific SNR function, and maximizes the cell-averaged *effective* SNR. It converts the RAU location optimization problem into a codebook design problem in vector quantization, and includes the SDC in [6] as a special case. The CVQ is composed of two terms: one unsupervised and one supervised. The unsupervised term is related to the standard Kohonen rule, which is a realization of the standard vector quantization. This unsupervised part is equal to the SDC in [6]. The other term, which is supervised, makes the winning codebook vector (RAU location), at each step, either get closer to or go away from the rest of the RAU locations depending on the desired SNR value for that location. Computer simulations confirm the findings and suggest that the proposed CVQ-DAS outperforms the SDC in terms of cell-averaged “*effective*” SNR, and gives comparable performance with respect to the SDC in terms of cell-averaged SNR.

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## A. Appendix

To examine the global Lipschitz constant of the average SNR function  $\bar{\theta}_a(\cdot)$  in (2) for the interval  $[d_{\min}, \infty)$ , where  $d_{\min}$  is the minimum distance between the user and any RAU, we first show that the path loss function  $\varphi(d) = d^{-\alpha}$  for the interval  $[d_{\min}, \infty)$  has a Lipschitz constant as  $\vartheta = \alpha/d_{\min}^{(\alpha+1)}$ , where  $\alpha$  is the path loss exponent. Because  $\varphi(d) = d^{-\alpha}$  is a differentiable function in  $[d_{\min}, \infty)$ , we apply the mean value theorem in (12) and obtain

$$|\varphi(d_i) - \varphi(d_j)| \leq \vartheta |d_i - d_j|, \quad (\text{A.1})$$

where  $\vartheta = \alpha/d_{\min}^{(\alpha+1)}$  is a Lipschitz constant of the path loss function  $\varphi(d)$ . It is assumed that large-scale and small-scale fading random variables  $s_n$  and  $g_{n,\mathbf{x}}$  are independent, and the average large-scale fading  $E_s \{s_n\}$  is RAU location-specific. Denoting the average small-scale fadings at locations  $\mathbf{x}_i$  and  $\mathbf{x}_j$  as  $\bar{g}_{n,\mathbf{x}_i} = E_h \{g_n(\mathbf{x}_i)\}$  and  $\bar{g}_{n,\mathbf{x}_j} = E_h \{g_n(\mathbf{x}_j)\}$ , respectively, we define

$$\gamma = \left| \frac{\max \left\{ \bar{g}_{n,\mathbf{x}_i} \|\mathbf{x}_i - \mathbf{c}_k\|_2^{-\alpha}, \bar{g}_{n,\mathbf{x}_j} \|\mathbf{x}_j - \mathbf{c}_k\|_2^{-\alpha} \right\}}{\left| \|\mathbf{x}_i - \mathbf{c}_k\|_2^{-\alpha} - \|\mathbf{x}_j - \mathbf{c}_k\|_2^{-\alpha} \right|} \right| \quad (\text{A.2})$$

Using Eq. (A.2) and with  $\varphi(d) = d^{-\alpha}$  as a decreasing function, we observe that

$$\left| \bar{g}_{n,\mathbf{x}_i} \|\mathbf{x}_1 - \mathbf{c}_k\|_2^{-\alpha} - \bar{g}_{n,\mathbf{x}_j} \|\mathbf{x}_2 - \mathbf{c}_k\|_2^{-\alpha} \right| \leq \gamma \left| \|\mathbf{x}_1 - \mathbf{c}_k\|_2^{-\alpha} - \|\mathbf{x}_2 - \mathbf{c}_k\|_2^{-\alpha} \right| \quad (\text{A.3})$$

From the average SNR function  $\bar{\theta}_a(\cdot)$  in (2), we have

$$\left\| \bar{\theta}_a(\mathbf{x}_i) - \bar{\theta}_a(\mathbf{x}_j) \right\|_1 = \frac{1}{\sigma_\zeta^2} \sum_{k=1}^K \bar{s}_k \left\| \bar{g}_{k,\mathbf{x}_i} \varphi(\|\mathbf{x}_i - \mathbf{c}_k\|_2) - \bar{g}_{k,\mathbf{x}_j} \varphi(\|\mathbf{x}_j - \mathbf{c}_k\|_2) \right\|_1 \quad (\text{A.4})$$

Using (A.3) and (A.4), and applying the triangular rule and considering the definition of the  $l_1$ -norm of a vector, we obtain

$$\left\| \bar{\theta}_a(\mathbf{x}_i) - \bar{\theta}_a(\mathbf{x}_j) \right\|_1 \leq \nu_{glob} \|\mathbf{x}_i - \mathbf{x}_j\|_2, \quad \forall \mathbf{x}_i, \mathbf{x}_j \in \bar{\Omega}, \quad (\text{A.5})$$

where  $\nu_{glob} = \alpha\gamma \left( \sum_{k=1}^K p_k \bar{s}_k \right) / \left( \sigma_\zeta^2 d_{\min}^{(\alpha+1)} \right)$ , in which  $\alpha$  is the path loss exponent,  $\gamma$  is related to the average small-scale fading as defined in (A.2),  $p_k$  is the transmit power of the  $k$ th RAU,  $\bar{s}_k$  is the average large-scale fading coefficient related to the  $k$ th RAU,  $\sigma_\zeta^2$  is the average noise power, and  $d_{\min}$  is the minimum distance between any UE and RAU. From (A.5),  $\theta_a(\cdot)$  in (2) has a global Lipschitz constant  $\nu_{glob}$  for the interval  $[d_{\min}, \infty)$ .

## B. Appendix Proof of Proposition 1

Adding and subtracting the term  $\bar{\theta}_a(\mathbf{C}, \mathbf{c}_{n(\mathbf{x})})$  from the argument of the expectation in (5) and taking the absolute value gives

$$J_2(\mathbf{C}) \leq \int_{\mathbf{x} \in \bar{\Omega}} \left( \begin{array}{c} |\bar{\theta}_a(\mathbf{C}, \mathbf{c}_{n(\mathbf{x})}) - \bar{\theta}_a(\mathbf{C}, \mathbf{x})| \\ + \bar{\theta}_d(\mathbf{x}) - \bar{\theta}_a(\mathbf{C}, \mathbf{c}_{n(\mathbf{x})}) \end{array} \right)^2 f(\mathbf{x}) d\mathbf{x} \quad (\text{B.6})$$

Some of the steps in the proofs of Appendices A and B are partly inspired by the analysis in [22,23]. Examining the average SNR function  $\bar{\theta}_a(\cdot)$  in (2), we prove in Appendix A that the  $\bar{\theta}_a(\cdot)$  has a global Lipschitz constant  $\nu_{glob}$  for the interval  $[d_{\min}, \infty)$ :

$$\|\bar{\theta}_a(\mathbf{x}_i) - \bar{\theta}_a(\mathbf{x}_j)\|_2 \leq \nu_{glob} \|\mathbf{x}_i - \mathbf{x}_j\|_2, \quad \forall \mathbf{x}_i, \mathbf{x}_j \in \bar{\Omega}, \quad (\text{B.7})$$

where  $\nu_{glob} = \alpha\gamma p_{UE} \left( \sum_{n=1}^N \bar{s}_n \right) / \left( \sigma_\zeta^2 d_{\min}^{(\alpha+1)} \right)$ , in which  $\alpha$  is the path loss exponent,  $\gamma$  is related to the average small-scale fading as defined in (A.2) in Appendix A,  $p_{UE}$  is the transmit power of the user,  $\bar{s}_n$  is the average large-scale fading coefficient related to the  $n$ th RAU,  $\sigma_\zeta^2$  is the average noise power, and  $d_{\min}$  is the minimum distance between user location and any RAU. Thus, using the mean value theorem and the Lipschitz constant of  $\bar{\theta}_a(\cdot)$  in (2) and the fact that the  $l_1$ -norm of a  $(q+1)$  dimensional vector is not greater than  $\sqrt{q+1}$  times its  $l_2$  norm gives

$$J_2(\mathbf{C}) \leq UB_2(\mathbf{C}), \quad (\text{B.8})$$

where  $UB_2(\mathbf{C})$  is given by (6). This completes the proof.