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Reducing power system model dimensions based on linearization for static analysis

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Abstract: A reliable solution for determining the effect of a group of contingencies on a power system is to simulate all of them through load flow. It is impossible to simulate any detailed cases through a complete AC load flow solution because of the high number of possible contingencies. For this reason, system dimension reduction methods, which are based on an equivalent of one part of a system, are being used. In this paper we compare our previously proposed network equivalent method with the 2 well-known equivalent methods of Ward and Ward-PV in terms of solving speed and accuracy. The proposed network equivalent method is based on linearization. In this method, first the network is divided into 3 areas: internal, boundary, and external. Next, the equations of the external and boundary areas are linearized and the external area equations are removed. By doing so, the number of equations is decreased and the speed of load flow solutions is increased. The simulations are applied on 14-, 30-, 39-, and 57-bus IEEE systems. The results of simulations show the effectiveness of our proposed method.

Key words: Static power system analysis, system dimension reduction, equivalent, linearization

1. Introduction

Modern power systems connect to each other to maintain high levels of reliability [1]. The internal system is a small part of an interconnected network that is monitored by an energy management system (EMS). The external system is the rest of network, which is not monitored by the EMS. It is necessary to have a useful model for showing the external system to monitor the internal system and evaluate its static security in an effective way [2]. The external system model should effectively represent the influence of the external system on the internal system. In some studies, such as contingencies ranking, modeling of the external system in detail requires a large amount of computation. For this reason, it is better to use reduced models for the external system. In general, network reduction techniques are divided into static and dynamic classes based on their applications. The main purpose of providing an appropriate equivalent model for power system dynamics is to produce the aggregated steady-state and dynamic characteristics of the full network. Common dynamic equivalent methods are coherency [1], modal analysis [2,3], optimal modal coherent aggregation [4,5], inertial and slow coherency aggregation [6,7], extension of balanced truncation [8], and exciter model aggregation [9]. Static reduction methods are used for power flow calculations and system operation and other static analysis. Static reduction methods include Ward [10–13], Ward-PV [11,12,14], REI [12,15,16], Kron reduction [10,13,17], and Zhukov reduction [9] equivalent methods. The Ward method is one of the best and simplest ones in terms

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of equivalents, so it is used widely. This method is used to solve the problem of transmission loss allocation in very large networks with multiple interconnected regions or countries [18]. This method is also applied for steady-state security assessment of power systems [19]. In the Ward method, the whole external system is reduced and replaced by an equivalent network, but in the Ward-PV method only PQ buses are reduced and PV buses remain unchanged. The simplicity and the high accuracy of Ward-PV in estimating active power flow in transmission lines are 2 reasons for its wide use. In this paper, both Ward and Ward-PV methods are used to evaluate the accuracy of the proposed method.

The Ward equivalent method is based on the relation between the current and voltage of buses in an operating point. External system relations are removed and thus some branches are created among boundary buses, and also between boundary buses and the ground. Furthermore, an extra injection current is created in boundary buses. This injection current and the injection power that is caused by it are assumed constant for a different situation of the internal system. In the Ward method, boundary and external system buses are regarded as constant current sources, which increase the error. As mentioned earlier, in a Ward-PV equivalent internal system, PV buses remain unchanged and the equivalent applies only to PQ buses. It reduces the errors, but error still may happen due to the assumption of the constant current sources for PQ buses.

We presented a novel method for a power system equivalent to improve the speed solution of the optimal coordinated voltage control problem [20]. We also used this method for solving the preventive security-constrained optimal power problem [21]. This method is based on linearization, in which first the external and boundary system equations are linearized and then external system equations and variables are removed. In [20] and [21], it was emphasized that the maintenance of control variables in the external and boundary systems and computation time reduction are main advantages of the proposed power network reduction; however, the performance accuracy of this method has not been addressed. Thus, the goal of the current study is to compare this method with Ward and Ward-PV in terms of solving accuracy and speed. The comparison is applied based on the error of active and reactive power flows in transmission lines for different contingencies as well as the error of bus voltages.

This paper is organized as follow: in Section 2, Ward and Ward-PV equivalent methods are investigated. Section 3 explains the suggested method for network reduction. Case studies and simulation results are discussed in Section 4, and Section 5 presents the conclusion of the paper.

2. Ward and Ward-PV equivalent methods

In order to reduce the model, the power system can be divided into 3 systems: internal, boundary, and external. The internal system is a part of the network the behavior of which is important for the operator and is modeled in detail. The boundary buses are the interface between the internal and external systems. The external system is another part of the network that is connected to the internal system through boundary buses. Because of the electrical distance between the external and internal systems, it is not necessary to model it in detail. Figure 1 shows the network division. In different methods of equivalents, the external system is replaced with a simpler system. As in references [10–12,15,16], in this paper, it is assumed that the external system data are accessible; otherwise, the external system equivalent is applied based on measured data in boundary buses [22].

2.1. Ward equivalent

For any power system the equation of voltage and current buses forms the matrix equation of Eq. (1), where internal, boundary, and external system elements are shown by i, b, and e, respectively. Y_{bb}^{e} is a diagonal

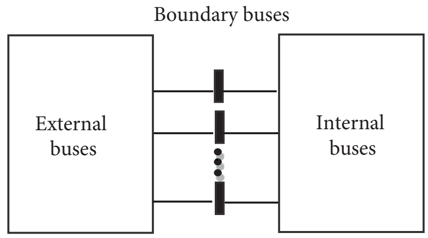

Figure 1. Network division procedure.

matrix and its elements are the sum of admittances of boundary branches connected to external buses. Y_{bb}^{i} is a matrix such that its diagonal elements include admittances of branches that are connected to boundary-boundary buses and branches that are connected to internal-boundary buses, and off-diagonal elements that include the negative of the boundary-boundary admittances. In Eq. (1), E_i and I_i, E_e and I_e, and E_b and I_b are voltage and current vectors for the internal, external, and boundary system buses, respectively.

$$\begin{bmatrix} Y_{ee} & Y_{eb} & 0 \\ Y_{be} & Y_{bb}^{e} + Y_{bb}^{i} & Y_{bi} \\ 0 & Y_{ib} & Y_{ii} \end{bmatrix} \begin{bmatrix} E_e \\ E_b \\ E_i \end{bmatrix} = \begin{bmatrix} I_e \\ I_b \\ I_i \end{bmatrix} \tag{1}$$

To remove the external system buses, voltage vector E_e has to be removed from Eq. (1). In order to do that, first, E_e is calculated from the first row of Eq. (1).

$$Y_{ee}E_e + Y_{eb}E_b = I_e \Rightarrow E_e = Y_{ee}^{-1}\left(I_e - Y_{eb}E_b\right) \tag{2}$$

Replacing Eq. (2) in the second row of Eq. (1), the second row changes as follows:

$$Y_{eq}E_b + Y_{bb}^{i}E_b + Y_{bi}E_i = I_b - I_{eq} \tag{3}$$

In which:

$$Y_{eq} = Y_{bb}^{e} - \left(Y_{be}Y_{ee}^{-1}Y_{eb}\right) \tag{4}$$

$$I_{eq} = Y_{be}Y_{ee}^{-1}I_e \tag{5}$$

Here, Y_{eq} is the reduced network admittance matrix. I_{eq} is the extra injection current to boundary buses, which is created as a result of external system outage. Y_{eq} creates a set of new branches between boundary buses, which finally is added to admittance matrix Y_{bb}^{i}. With use of Eqs. (1)–(5), the injection complex power can be computed in boundary buses. In order to do this, first the complex power resulting from the injection current is obtained.

$$S_{eq} = E_b I_{eq}^{*} \Rightarrow S_{eq} = E_b\left(Y_{be}Y_{ee}^{-1}I_e\right)^{*} \tag{6}$$

With substitution of I_e in terms of injection complex power in external buses S_e, it can be seen that:

$$\begin{aligned} I_e^{*} &= E_e^{-1}S_e \\ S_{eq} &= E_b\left(Y_{be}\right)^{*}\left(Y_{ee}^{*}\right)^{-1}E_e^{-1}S_e \end{aligned} \tag{7}$$

Here, S_e, E_e, and E_b are determined by the primary load flow. Finally, the total injection power in boundary buses is obtained by summing S_{eq} to the primary injection power of these buses.

2.2. Ward-PV equivalent

Despite its advantages, the Ward equivalent suggested in the previous section is unable to portray the accurate effect of external PV buses. Therefore, it is suggested that the Ward reduction process be applied only to external system PQ buses, while external system PV buses remain unchanged to have more accurate results. To preserve the external system PV buses, Eq. (1) is rewritten in the form of Eq. (8). In Eq. (8), Y_{QQ} and Y_{VV} are the admittance matrixes of PQ and PV buses in the external zone, respectively.

$$\begin{bmatrix} Y_{QQ} & Y_{QV}Y_{QV} & Y_{Qb} & 0 \\ Y_{VQ} & Y_{VV} & Y_{Vb} & 0 \\ Y_{bQ} & Y_{bV} & Y_{bb}^{e}+Y_{bb}^{i} & Y_{bi} \\ 0 & 0 & Y_{ib} & Y_{ii} \end{bmatrix} \begin{bmatrix} E_Q \\ E_V \\ E_b \\ E_i \end{bmatrix} = \begin{bmatrix} I_Q \\ I_V \\ I_b \\ I_i \end{bmatrix} \tag{8}$$

To remove external system PQ buses, voltage vector E_Q must be removed from Eq. (8). In order to do that, E_Q is obtained from the first row of Eq. (8):

$$Y_{QQ}E_Q + Y_{QV}E_V + Y_{Qb}E_b = I_Q \tag{9}$$

$$E_Q = Y_{QQ}^{-1}\left(I_Q - \left(Y_{Qb}E_b + Y_{QV}E_V\right)\right) \tag{10}$$

By replacing E_Q in the second and third row of Eq. (8), there would be the following:

$$\left(Y_{Vb} - Y_{VQ}Y_{QQ}^{-1}Y_{Qb}\right)E_b + \left(Y_{VV} - Y_{VQ}Y_{QQ}^{-1}Y_{QV}\right)E_V = I_V - \left(Y_{VQ}Y_{QQ}^{-1}I_Q\right) \tag{11}$$

$$\left(Y_{bV} - Y_{bQ}Y_{QQ}^{-1}Y_{QV}\right)E_V + \left(Y_{bb}^{e} - Y_{bQ}Y_{QQ}^{-1}Y_{Qb}\right)E_b + Y_{bb}^{i}E_b + Y_{bi}E_i = I_b - Y_{bQ}Y_{QQ}^{-1}I_Q \tag{12}$$

According to Eqs. (11) and (12):

$$I_{eqV} = Y_{VQ}Y_{QQ}^{-1}I_Q \tag{13}$$

$$I_{eqb} = Y_{bQ}Y_{QQ}^{-1}I_Q \tag{14}$$

Here, I_{eqV} is the extra injection current in external PV buses and I_{eqb} is the extra injection current in boundary buses. Considering Eq. (8), Y_{eq} is as follows:

$$Y_{eq} = \begin{bmatrix} Y_{VV} & Y_{Vb} \\ Y_{bV} & Y_{bb} \end{bmatrix} - \begin{bmatrix} Y_{VQ} \\ Y_{bQ} \end{bmatrix} \times \left[Y_{QQ}\right]^{(-1)} \times \begin{bmatrix} Y_{QV} & Y_{Qb} \end{bmatrix} \tag{15}$$

The injection complex power in boundary buses and external PV buses could be obtained from the injection currents.

$$S_{eqb} = E_b\left(Y_{bQ}\right)^{*}\left(Y_{QQ}^{*}\right)^{-1}E_Q^{-1}S_Q \tag{16}$$

$$S_{eqV} = E_V\left(Y_{VQ}\right)^{*}\left(Y_{QQ}^{*}\right)^{-1}E_Q^{-1}S_Q \tag{17}$$

Here, S_{eqb} and S_{eqV} are the injection complex powers in boundary and PV buses, respectively.

3. Power system equivalent based on linearization method

In the proposed method, first, power balanced equations of boundary and external buses are linearized. This linearization is acceptable, since in the case of fault occurrence in the internal system the external and boundary bus variations are slight in time compared to the internal. Then the variables and equations of the external system are removed from linearized equations. This contributes to a decrease in the amount of calculations. In contrast to the previous methods, the proposed method takes into account the impact of internal system variations on the external system. It should be noted that nonlinear equations of the external system are replaced with linearized ones. The first step of power flow analysis is to write nonlinear equations of active and reactive power balanced for PQ and PV buses. These equations are as follows:

$$P_{gk} - P_{Lk} - \sum_{n=1}^{N} |V_k| \, |V_n| \, |Y_{kn}| \cos(\theta_{kn} + \delta_n - \delta_k) = 0 k = 2, \ldots, N \tag{18}$$

$$Q_{gk} - Q_{Lk} - \sum_{n=1}^{N} |V_k| \, |V_n| \, |Y_{kn}| \sin(\theta_{kn} + \delta_n - \delta_k) = 0 k = m+1, \ldots, N \tag{19}$$

Here, g and L indexes respectively represent the generation and load. N and m are the number of all buses and generation buses, respectively. It is assumed that bus 1 is the slack bus. After dividing the system into external, boundary, and internal areas that were determined previously, the load flow equations for these 3 systems can be formulated as follows [20,21]:

$$f_i\left(x_i, x_b\right) = 0 \tag{20a}$$

$$f_b\left(x_i, x_b, x_e\right) = 0 \tag{20b}$$

$$f_e\left(x_b, x_e\right) = 0 \tag{20c}$$

x is the magnitude and angle of bus voltages. In the internal load flow equations, only the variables and parameters of internal and boundary systems exist because the external and internal systems are not connected. For the same reason, there are only variables and parameters of external and boundary systems in the external load flow equations. However, all of the variables and parameters of the 3 systems exist in the boundary. Considering the low impact of the external and boundary systems in the time of contingency in the internal system, the load flow equations of these systems could be formulated in a linear form. The external and boundary system equations are linearized as follows:

$$\frac{\partial f_e}{\partial x_b} \Delta x_b + \frac{\partial f_e}{\partial x_e} \Delta x_e = 0 \tag{21a}$$

$$\frac{\partial f_b}{\partial x_i} \Delta x_i + \frac{\partial f_b}{\partial x_b} \Delta x_b + \frac{\partial f_b}{\partial x_e} \Delta x_e = 0 \tag{21b}$$

Eq. (21) is reformulated in a matrix form.

$$A_{eb} \Delta x_b + A_{ee} \Delta x_e = 0 \tag{22a}$$

$$A_{bi}\Delta x_i + A_{bb}\Delta x_b + A_{be}\Delta x_e = 0 \tag{22b}$$

Here, Δx_e is determined from Eq. (22a):

$$\Delta x_e = -A_{ee}^{-1} \times A_{eb}\Delta x_b \tag{23}$$

By replacing Eq. (23) in Eq. (22b):

$$A_{bb}^{'}\Delta x_b + A_{bi}\Delta x_i = 0 \tag{24}$$

In which:

$$A_{bb}^{'} = A_{bb} - A_{be}A_{ee}^{-1}A_{eb}\Delta x_b \tag{25}$$

As we can see, after the equivalent, Eq. (22) is reduced to Eq. (24), in which equations and state variables of the external system are removed. The steps of the proposed algorithm are as follows:

Step 1: Solving the power flow equations in the nonreduced system for an initial operating point.

Step 2: Determining $A_{bb}^{'}$ and A_{bi} by the power flow solution given in step 1.

Step 3: Solving Eqs. (20a) and (24) in the reduced system for a new operating point.

The flowchart of the proposed method is shown in Figure 2.

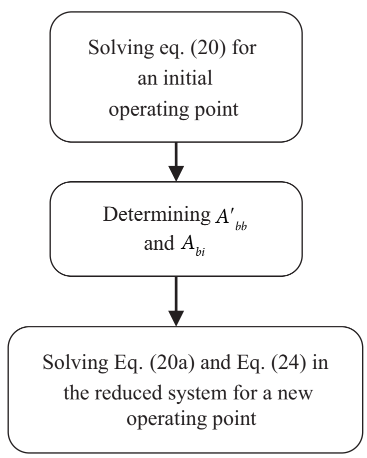

Figure 2. Flowchart of the proposed method for reducing the power system model dimensions.

The Ward equivalent method is based on the relation between the current and voltage of buses in an operating point. From all these relations, external system relations are removed and for this reason some branches are created between boundary buses and between boundary buses and the ground as well. Furthermore, an extra injection current is created in boundary buses. This injection current and the injection power that is caused by it are assumed constant for different situations of the internal system. In fact, in the Ward method, boundary and external system buses are regarded as constant current sources. This increases the error in the Ward method. As mentioned earlier, in the Ward-PV equivalent internal system PV buses remain unchanged and the equivalent applies only to PQ buses. It reduces the errors, but by assuming the constant current sources for PQ buses, the accuracy of this method is still low.

In the proposed method, first the power flow equations of boundary system and external buses become linear. This is acceptable because changes in external and boundary variables are slight in terms of time of internal system change. Then the variables and equations of the external system are removed from linearized equations. It results in a decrease in the amount of calculations. In contrast to the previous methods, in the proposed one, the impact of internal system changes on the external system is considered. However, nonlinear equations of the external system are replaced with linearized equations. In addition to the differences mentioned between the proposed method and the previous methods, in the proposed method, when a fault occurs in the internal area, we can check all of the external area bus voltages (Δx_e) and other parameters, ensuring that the external bus voltages do not surpass their limits.

4. Simulation results

In this section, the proposed method is compared with the Ward and Ward-PV methods. The comparison is applied to estimate the error of active and reactive power flows in transmission lines and error of bus voltages. The simulations are applied on 14-, 30-, 39-, and 57-bus IEEE systems. For a full-scale investigation, various partitionings and different faults are applied to the power systems; however, only the results of one of them are presented as an example. Figures 3–6 show how the 14-, 30-, 39-, and 57-bus systems are partitioned, respectively.

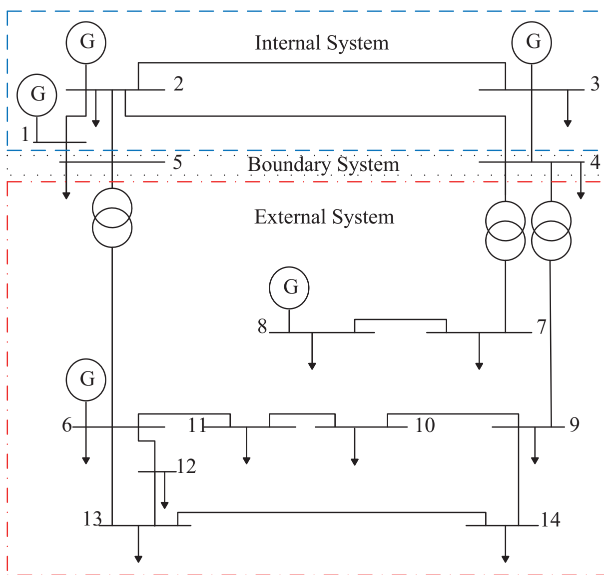

Figure 3. Network division procedure of 14-bus IEEE system.

Table 1 shows the results of the mentioned equivalent methods on the 14-bus IEEE system. The applied fault is line 2–4 outage. As we can see, the estimated error percentage of active power in all three methods is acceptable, but the estimated error percentage of reactive power is greater than those of the active power. The maximum rate for active and reactive power flows in transmission lines is 6.61% and 12.55%, respectively. The estimated error of voltage magnitude is about 0.1%, and minimum error occurred with the Ward-PV equivalent method.

Figure 4 portrays partitioning of the 30-bus IEEE system. Table 2 represents the results of the 3 mentioned equivalent methods. The applied fault is line 2–5 outage. As we can see, the estimated error percentage of active and reactive power flows in transmission lines is acceptable in each of the 3 methods. Maximum error rate for active and reactive power is 1.97% and 5.081%, respectively. The estimated error of voltage magnitude is very low, and the minimum error occurred in the Ward-PV equivalent.

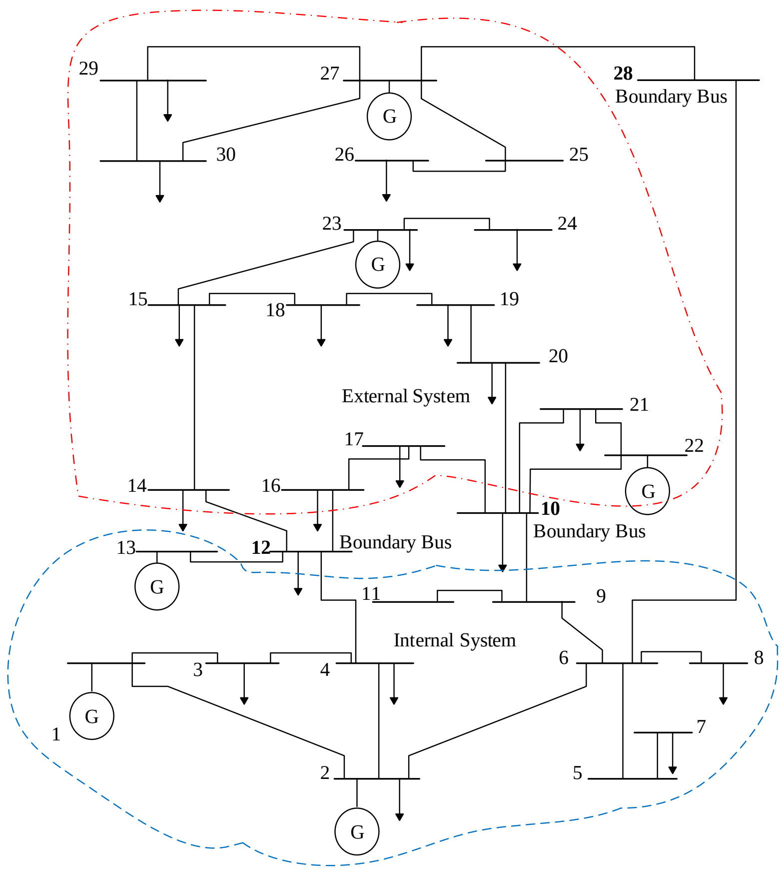

Figure 4. Network division procedure of 30-bus IEEE system.

Table 1. Performance comparison of the proposed method with Ward and Ward-PV equivalent methods in 14-bus IEEE system.

Equivalent methods	Checked line	Estimated error percentage of power	Checked buses	Estimated error of voltage magnitude
Ward	1–2 1–5 2–5 3–4 4–5	0.108% + j0.0107% 0.05% + j0.86% 0.314% + j0.434% 4.85% + j12.55% 6.61% + j6.88%	4 5	0.098% 0.098%
Ward-PV	1–2 1–5 2–5 3–4 4–5	0.101% + j0.098% 0.04% + j0.73% 0.284% + j0.124% 3.72% + j6.002% 4.23% + j3.078%	4 5	0.098% 0.098%
Proposed method	1–2 1–5 2–5 3–4 4–5	0.225% + j0.0038% 0.0495% + j0.209% 0.65% + j0.34% 5.04% + j6.03% 6.14% + j5.36%	4 5	0.102% 0.101%

Table 3 shows the results of the 3 mentioned equivalent methods on the 39-bus IEEE system. The applied fault is line 6–7 outage. It can be seen that the estimated error rate of active power is acceptable for all methods,

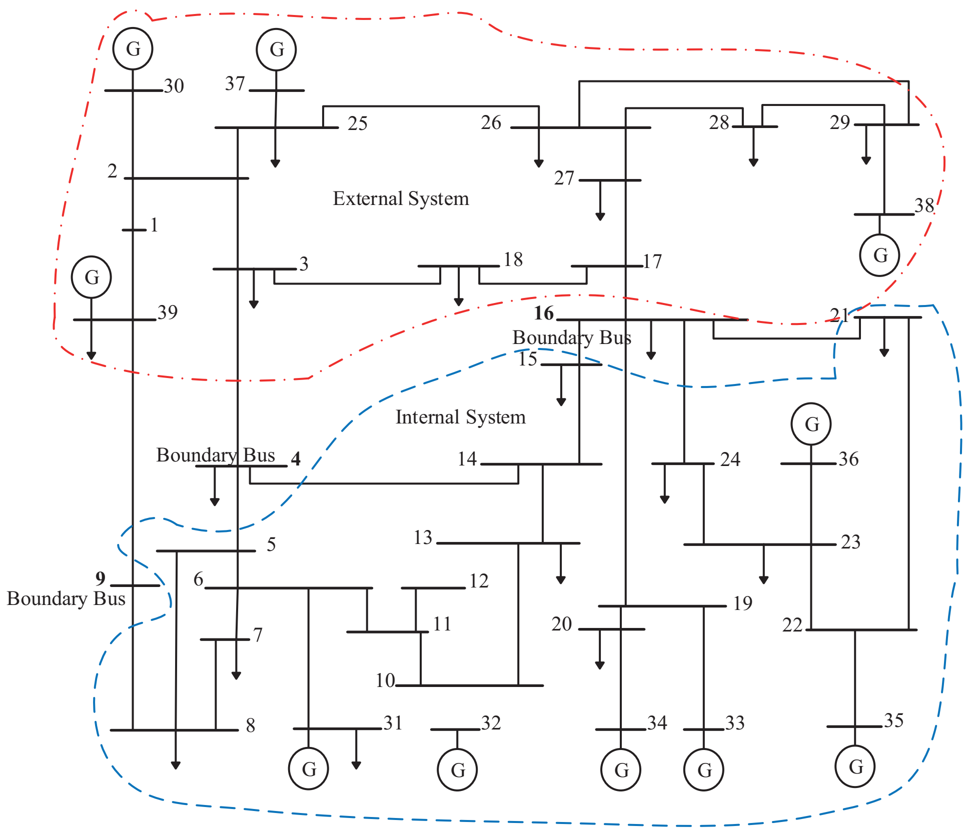

Figure 5. Network division procedure of 39-bus IEEE system.

Table 2. Performance comparison of the proposed method with Ward and Ward-PV equivalent methods in 30-bus IEEE system.

Equivalent methods	Checked line	Estimated error percentage of power	Checked buses	Estimated error of voltage magnitude
Ward	3–4 9–11 7–5	0.023% + j2.334% 0.0002% + j0.0001% 1.83% + j4.26%	12 6 7 3	1.25% 0.0088% 1.47% 0.66%
Ward-PV	3–4 9–11 7–5	0.011% + j1.52% 0.0002% + j0.0001% 1.39% + j3.021%	12 6 7 3	1.04% 0.0078% 1.249% 0.351%
Proposed method	3–4 9–11 7–5	0.17% + j3.37% 0.00096% + j0.0003% 1.97% + j5.081%	12 6 7 3	1.79% 0.0106% 1.738% 1.041%

while it is significantly larger for reactive power. Maximum error rate for the active power is 4.77%, and it is 14.01% for the reactive power. The estimated error of voltage magnitude is about 8%. In this scenario, the lowest error belongs to the proposed method.

Table 4 demonstrates the results of the 3 mentioned equivalent methods on the 57-bus IEEE system. In this scenario line 15–14 is removed. As can be seen, the proposed method has a lower error of active/reactive

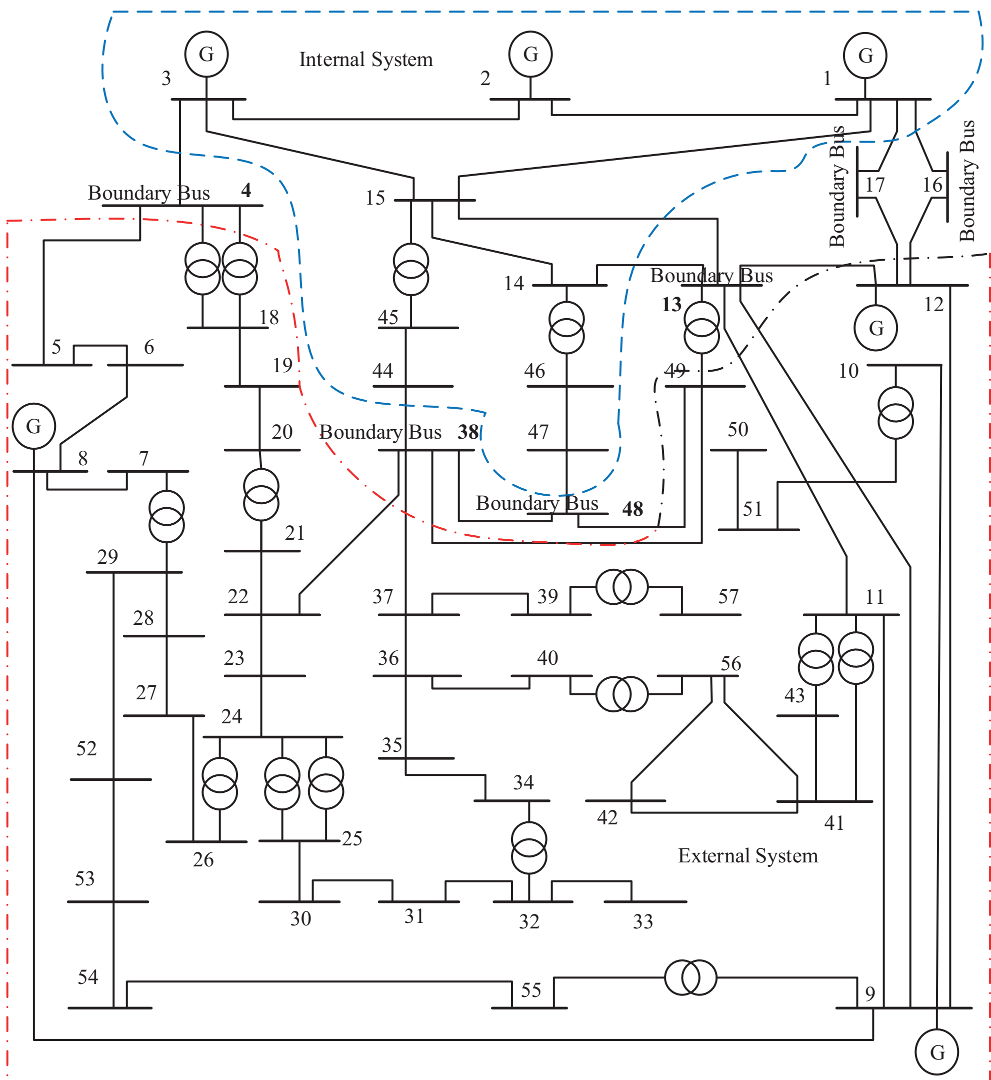

Figure 6. Network division procedure of 57-bus IEEE system.

power flows in transmission lines and voltage magnitude compared with the other two methods. The solution times of the 3 methods are compared in Table 5. According to the results, the proposed method has the lowest solution time for the 30-, 39-, and 57-bus IEEE systems.

The numbers of power flow equations of the 14-, 30-, 39-, and 57-bus IEEE systems for the Ward, Ward-PV, and proposed methods and for internal, boundary, and external areas are shown in Table 6. Table 7 indicates the number of equations in the nonreduced power system.

It can be observed that, in all methods, the numbers of internal and boundary area equations are equal. In these methods, the internal area power flow equations are nonlinear. The boundary area power flow equations for the Ward and Ward-PV methods are also nonlinear, but they are linearized in the proposed methods. The external area power flow equations in the Ward and proposed methods are removed, but in the Ward-PV method the power flow equations of the external area PV buses remain unchanged.

According to simulation results, the proposed method in comparison with the other mentioned methods has 2 advantages, as follows:

Table 3. Performance comparison of the proposed method with Ward and Ward-PV equivalent methods in 39-bus IEEE system.

Equivalent methods	Checked line	Estimated error percentage of power	Checked buses	Estimated error of voltage magnitude
Ward	11–10 13–14 19–20 22–23	4.22% + j6.49% 4.7% + j14.01% 0.043% + j0.31% 2.3% + j3.88%	4 5 10 14	4.79% 5.31% 7.27% 7.812%
Ward-PV	11–10 13–14 19–20 22–23	3.71% + j3.48% 2.79% + j7.88% 0.028% + j0.0912% 2.02% + j2.075%	4 5 10 14	4.45% 4.98% 6.62% 7.39%
Proposed method	11–10 13–14 19–20 22–23	2.57% + j3.987% 2.98% + j8.49% 0.018% + j0.096% 1.048% + j2%	4 5 10 14	3.77% 4.75% 6.02% 6.82%

Table 4. Performance comparison of suggested method with Ward and Ward-PV equivalent methods in 57-bus IEEE system.

Equivalent methods	Checked line	Estimated error percentage of power	Checked buses	Estimated error of voltage magnitude
Ward	2–3 15–13 14–46 48–47	3.63% + j33.98% 10.924% + j14.4% 5.56% + j10.33% 7.862% + j21.24%	15 17 46	9.82% 10.85% 7.94%
Ward-PV	2–3 15–13 14–46 48–47	0.24% + j15.48% 8.21% + j10.056% 4.75% + j5.37% 6.29% + j16.95%	15 17 46	9.71% 9.043% 7.68%
Proposed method	2–3 15–13 14–46 48–47	6.61% + j10.34% 4.68% + j6.32% 3.027% + j6.14% 4.4% + j8.23%	15 17 46	8.53% 7.41% 6.32%

Table 5. Comparison of solution time for the 3 equivalent methods.

IEEE system (buses)	Proposed method (s)	Ward-PV method (s)	Ward method (s)
14	0.63293	0.29883	0.29647
30	0.17683	0.28501	0.23447
39	0.16799	0.58783	0.52374
57	0.12789	0.47882	0.46013

Table 6. Number of power flow equations in reduced model of power systems.

Test system	Ward				Ward-PV				Suggested method			
	Internal	Boundary	External	Sum	Internal	Boundary	External	Sum	Internal	Boundary	External	Sum
14-bus	2	4	0	6	2	4	2	8	2	4	0	6
30-bus	18	4	0	22	18	4	3	25	18	4	0	22
39-bus	37	6	0	43	37	6	4	47	37	6	0	43
57-bus	14	12	0	26	14	12	3	29	14	12	0	26

Table 7. Number of power flow equations in nonreduced model of power systems.

Test system	Nonreduced			
	Internal	Boundary	External	Sum
14-bus	2	4	16	22
30-bus	18	4	29	51
39-bus	37	6	24	67
57-bus	14	12	81	107

1. It has the lowest running time in power flow solution (more speed).
2. It has the lowest error percentage in estimating the active and reactive power flow from transmission lines and buses voltages (more accuracy).
3. It is possible to check external bus voltages and other parameters by Eq. (23) for new operating points.

5. Conclusion

In this paper, the solution accuracy of our previously proposed power network equivalent method was investigated. This method is based on the linearization of boundary and external equations and the removing of external equations and variables. This method was compared with the Ward and Ward-PV methods for 14-, 30-, 39-, and 57-bus IEEE systems. Our simulations showed that in all the tested power systems, the proposed method has acceptable results for the estimation of active and reactive power flows in transmission lines and bus voltages. In the 14- and 30-bus IEEE systems, the Ward-PV method has a lower error rate in comparison with the proposed method. In the 39- and 57-bus IEEE systems, the proposed method shows the lowest error rate. The time of load flow solution of the systems that are equivalent by the proposed method is lower for the IEEE systems with larger bus numbers.

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