Application of domination integrity of graphs in PMU placement in electric power networks

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Abstract: In this paper, we propose the application of the concept of power domination integrity to an electric power network. A phasor measurement unit (PMU) is used to analyze and control the power system by measuring voltage phase in electrical nodes and transmission lines. Due to the high cost of PMUs, it is necessary to minimize the number of PMUs such that the depth of observability is ensured. Placing PMUs in a network can be formulated as a graph theoretic problem of finding the minimum number of nodes (PMUs) in a graph that has a maximum number of links with other nodes. To achieve this, the concept of domination in graph theory is applied to power networks by redefining "adjacency" of a vertex as an "observed" vertex. The power domination number identifies the number of PMUs to be placed. The proposed concept of power domination integrity gives not only the minimum number of PMUs but also identifies the optimal locations for PMU placement in an electric power network.

Key words: Phasor measurement unit, optimization, domination, power domination, domination integrity, power domination integrity

1. Introduction

A phasor measurement unit (PMU) is used to measure voltage and current phasors. From this measurement, positive sequence voltage and swing angles can be estimated, which can lead to estimation of power system stability. These units are linked and synchronized with a global positioning system and send measurement data to a centralized unit. Since a PMU needs a huge investment, it is essential to optimize the number of PMUs without affecting the depth of observability of the power system. Clustering the power network is not possible, since every power network is independent of each other and has a unique identity. Hence, PMU placement becomes complicated. In this paper, we attempt to estimate the minimum number of PMUs needed to monitor a power network and their location for placement based on a graph theoretic approach.

Several methods are available for finding the optimum number and optimum placement of PMUs and they are discussed in [1–3]. Every method has its own pros and cons. Simulated annealing, tabu search, and genetic algorithm are popular methods [1]. These methods do not assure the global minimum, because if the power lines are disconnected due to abnormal operating conditions of the power system, then they do not assure optimality. Hence, there is a need for an improved method that assures a minimum number and better

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placement of PMUs, even during operation of the power system. Along with a minimum number of PMUs, we should consider the integrity of the network also.

When a PMU is placed at a node, it forms a subnetwork whose nodes are observed and it reads actual measurements or calculated pseudomeasurements of that network. Collection of these data from the subnetworks must provide the measurement of all electrical parameters at all the nodes in the network. That is, every node and edge in the network should be observed. These calculated measurements are based on Kirchhoff's and Ohm's laws. Using Ohm's law, we can calculate current in an edge linking two nodes whose voltage phasors are known or measured. If all the nodes are observed but not one, then the unknown current can be calculated using Kirchhoff's law.

In [4], Baldwin et al. proved that the maximum degree concept of nodes was not efficient at placing PMUs in optimal locations. They implemented the minimum spanning tree approach, which involves a determination of the spanning tree for finding the number and placement of PMUs. The test system used by them was the IEEE 14-bus test system. They formulated the construction of subgraph based on the following rules:

R1: Assign a current phasor measurement to each branch incident to a bus provided with a PMU.

R2: Assign a pseudocurrent measurement to each branch connecting two buses with known voltages.

R3: Assign a pseudocurrent measurement to a branch whose current can be inferred by using Kirchhoff's current law.

For $v \in V$, $N(v)$ denotes open neighborhood vertices of v, defined by $N(v) = \{u \, / \, (u, v) \in E\}$. The closed neighborhood of v is $N[v] = N(v) \cup \{v\}$. Let P be a subset of the vertex set V such that a PMU is placed at every vertex of P. The observed vertices are denoted as $M(P)$ and observed edges are denoted as $M'(P)$.

In [5], Haynes et al. presented an introduction to power domination and formulated the power network as a power system graph and defined the power dominating set and power domination number to the power network. Every electric node, bus, load, and generator of the power network were noted as vertices of a power graph, and the links between these nodes were taken as the edge in the graph. The technical term "adjacency" in graph theory is redefined as "observed." That is, if current and voltage phasor are measured by a PMU, then that node is said to be observed, using either Ohm's or Kirchhoff's law.

Formally, let $G : (V, E)$ be a power network graph, where vertex set V represents a set of electrical nodes and edge set E of G represents the links between the electrical nodes. A dominating set S is a subset of V such that every vertex in $V - S$ is adjacent to at least one vertex with S. Now the problem is finding a subset S of the vertex set V having a PMU, so that all the remaining vertices in $G - S$ are adjacent to at least one vertex in S. This is identified as the problem of finding the dominating set of graph G. This can be considered as a formal domination problem in graph theory. For $v \in V$, $N(v)$ denotes open neighborhood vertices of v, defined by $N(v) = \{u \, / \, (u, v) \in E\}$. The closed neighborhood of v is $N[v] = N(v) \cup \{v\}$. Let P be a subset of the vertex set V such that a PMU is placed at every vertex of P. The observed vertices are denoted as M(P) and observed edges are denoted as $M'(P)$.

The simplified version of propagation rules, which is equivalent to the original version defined in [5], is given as follows:

Domination step: Initial set $M(P) = P \cup N(P)$

Propagation step: If there exists $v \in M(P)$ such that $\mathrm{N}(v) \cap \{V - M(P)\} = \{w\}$ then $M(P) = M(P) \cup \{w\}$ Repeat the propagation step until no such vertices exist.

In [6], Brueni redefines the power domination problem using graph theory as follows:

R1. If $v \in P$ and $u \in N(v)$, then $v \in M(P)$ and $(u \in v) \in M'(P)$. A bus with a PMU and any line extending from the bus is observed.

R2. By Ohm's law: if $(u, v) \in M'(P)$ and $u \in M(P)$, then $v \in M(P)$. Any bus that is incident to an observed line connected to an observed bus is observed.

R3. By Ohm's law: if $u, v \in M(P)$ and $(u, v) \in E$, then $(u, v) \in M'(P)$. Any line joining two observed buses is observed.

R4. By Kirchhoff current: if $v \in M(P)$ and $|N(v) \cap (V - M(P))| \leq 1$, then $N[v] \subseteq M(P)$. If all the lines incident to an observed bus are observed, save one, then all the lines incident to that bus are observed.

R5. Derived: if for all $u \in N(v)$ and $(u, v) \in M'(P)$, then $v \in M(P)$ Any bus incident only to an observed line is observed.

This paper intends to study the stability and optimal usage of PMU placement by modeling the electrical power network as a simple graph. All the regular procedures study only PMU optimality. Using domination integrity, we try to give an optimal solution with good network stability.

2. Basic definitions

Let $G : (V, E)$ be a graph with nonempty set of vertices V and edge set E. We are considering G as a simple connected graph. A subset S of V is called a dominating set if the vertex in $V - S$ has at least one neighbor in S. The minimum cardinality of the dominating set has a domination number of G. The domination number is denoted by $\gamma(G)$. A subset P of V is called a power dominating set if every vertex in $G - P$ is observed by P. The power domination number is defined as the minimum cardinality of a power dominating set, and it is denoted by $\gamma_P(G)$. All the undefined terms are taken in the same sense as given in [5]. The distance between two vertices u and v is the minimum number of edges between them. The number of incident edges is known as degree of a vertex. The maximum distance between any pair of vertices is known as the eccentricity of a graph.

3. Proposed approach for placement of PMU

The proposed approach for solving the PMU placement problem is based on edge decomposition. Instead of finding the spanning tree of the given power network graph, the given graph G is decomposed into edge disjoint subgraphs, each having a power domination number of one. Reunion of the subgraphs from the least to the biggest component dynamically can be used to place a PMU in an optimum location.

3.1. Algorithm

In [5], the authors have shown that the PDS problem is an NP-complete problem. Hence, we divide the PMU placement problem into two parts: first, finding the number of PMUs to observe the network, and second, identifying the locations to place the PMUs in the network. The ultimate aim of this algorithm is to find the number of PMUs needed to observe all the vertices and edges in the given electric network. To achieve this, the given network is decomposed into an edge disjoint subgraph with a power domination number of one. The number of such edge disjoint subgraphs gives the number of PMUs required to observe the network.

The steps in the proposed approach are presented in the form of an algorithm given below.

Step 1: Find the maximum length subgraph D_i (such as cycles or paths or complete graphs or stars) whose power domination number is one.

Step 2: Delete the edges in D_i. If all the adjacent edges of a vertex are in D_i, then delete that vertex also.

Step 3a: If the remaining graph is still connected, go to step 1 and find the next edge disjoint subgraph.

Step 3b: If the remaining graph is disconnected, perform steps 1 and 2 for every connected component. Repeat until the remaining graph has no edges.

Step 4: (Number of edge disjoint components + number of pendant vertices – 1) gives the number of PMUs needed to observe the entire network graph.

The location of PMU placement is done through reconstructions of the network. A PMU is placed in the component with the least cardinality and then merging the next least and adjusting the location of the PMU in order to observe the merged component. Repeating the procedure, we obtain a fully observed network. If we place a PMU in a node, it observes all the nodes adjacent to it. Hence, the next placement of a PMU may be placed in a 2-distance node. We may place a PMU in a maximum degree node. If there is no need for further placement, then we have arrived at the required number of PMUs to observe the entire network. Further placement is required if there is at least one node that is unobserved by any already placed PMU nodes. Concluding these, in placing the PMU, the following guidelines may be considered:

If v is a pendant vertex, then a PMU can be placed at most to a 2-distance vertex to observe v.

In placing a new PMU, the minimum distance between two PMU-placed vertices is two.

Instead of considering the maximum degree vertex, the vertex with second maximum degree can be considered for PMU placement.

In the next section, the algorithm is applied to IEEE 14-bus, 30-bus, and 57-bus test systems.

3.2. Implementation of the proposed algorithm for IEEE test systems

This section deals with the implementation of the proposed approach for placement of PMU. Three test systems, namely IEEE 14-bus, 30-bus, and 57-bus test systems, are considered.

3.3. IEEE 14-bus test system

The algorithm presented in the previous section is applied to the standard IEEE 14-bus test system. The line diagram of the IEEE 14-bus test system is presented in Figure 1 and its corresponding graph G is shown in Figure 2. The longest cycle, the first edge decomposition D_1, in this power model is 1-2-3-4-7-9-14-13-12-6-5-1. Since this is a cycle, a single PMU is enough to observe all the nodes and it can be placed in any node. In the remaining graph (as shown in Figure 3), the second edge decomposition, D_2, is the maximum path: 2-5-4-9-10-11-6-13. Removing corresponding edges from the graph (as shown in Figure 4), we have only one edge 2-4 (7-8 is a pendant vertex), which is the third edge decomposition D_3. Since there is only one pendant vertex, it does not disturb the number of PMUs in the network. Hence, we need 3 (number of decomposition + number of pendent vertices – 1) PMUs to observe all the vertices and edges for the IEEE 14-bus test system.

For placing PMUs, let us start with the least decomposition factor D_3 in Figure 4. To observe all the vertices and edges, we may place a PMU in vertex 2 or 4. Merging D_2 with D_3 (as shown in Figure 3), we may place PMUs at 2 and 4. If a PMU is placed at vertex 4, the second PMU to be placed at vertex 9 overlaps many observed vertices and edges one more time. Thus, place the first PMU at vertex 2. Taking 2-distance from vertex 2, the second PMU may be placed at vertex 9. Including D_1 at this stage yields the whole network, as shown in Figure 2. From vertex 9, the vertices at 2-distance are 3, 13, and 11, and we can easily verify that if a PMU is placed at 13, the whole network is fully observed. The optimum placement of PMUs is $\{2, 9, 13\}$.
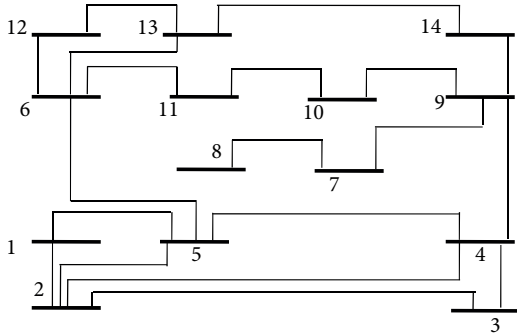
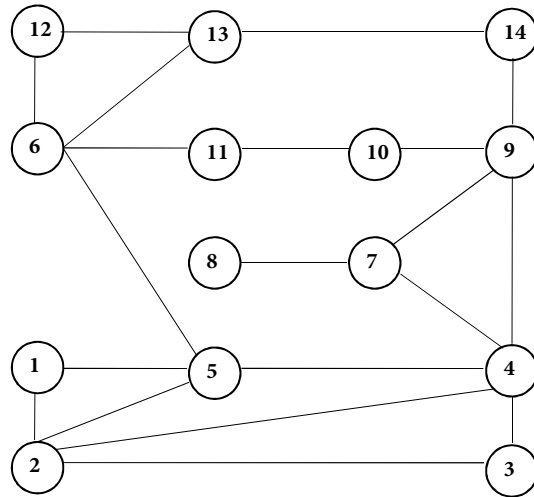**Figure 1**. Line diagram for the IEEE 14-bus test system.

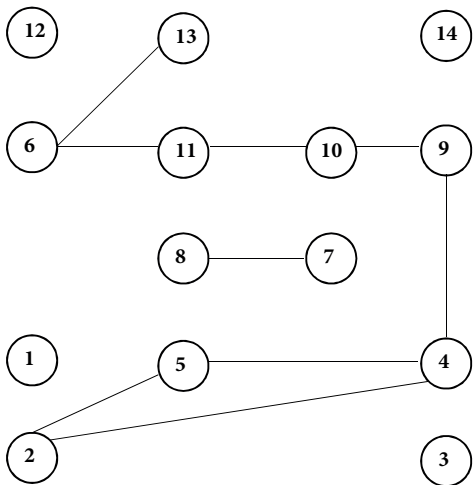**Figure 2**. Graph network model of the IEEE 14-bus test system.

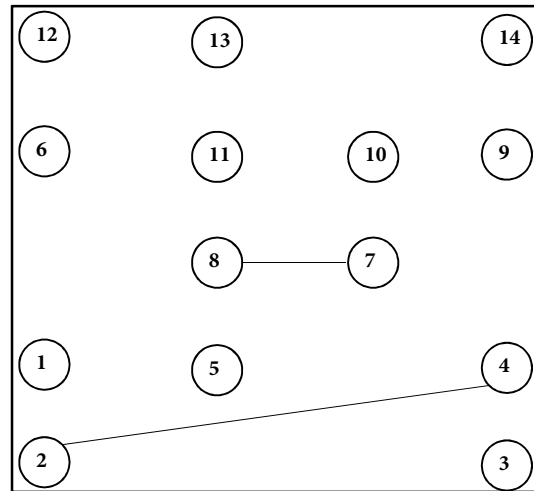**Figure 3**. $G - D_1$, IEEE14 graph after deleting D_1.

Figure 4. IEEE14 graph after deleting D_1 and D_2.

If we take 2-distance from the first PMU placed at vertex 2, vertex 6 can be also considered for placing the third PMU. Therefore, another possibility is $\{2, 6, 9\}$. Hence, the PMUs can be placed at either $\{2, 6, 9\}$ or $\{2, 9, 13\}$.

3.4. IEEE 30-bus test system

The line diagram of the IEEE 30-bus test system and the corresponding network graph are given in Figures 5 and 6, respectively. The same algorithm is applied for the IEEE 30-bus test system. The first edge decomposition, D_1, is a cycle: 1-2-5-7-6-8-28-27-25-24-23-15-18-19-20-10-17-16-12-4-3-1. Removing corresponding edges from the graph, we get 4 components (Figure 7). The vertex set of the first component C_1 is $\{12, 13, 14, 15\}$. The vertex set of the second component C_2 is $\{27, 29, 30\}$.

The vertex set $\{25, 26\}$ of the third component C_3 is K_2, a complete graph on two vertices. The vertex set of fourth component C_4 is $\{2, 4, 6, 9, 10, 11, 21, 22, 24, 28\}$.
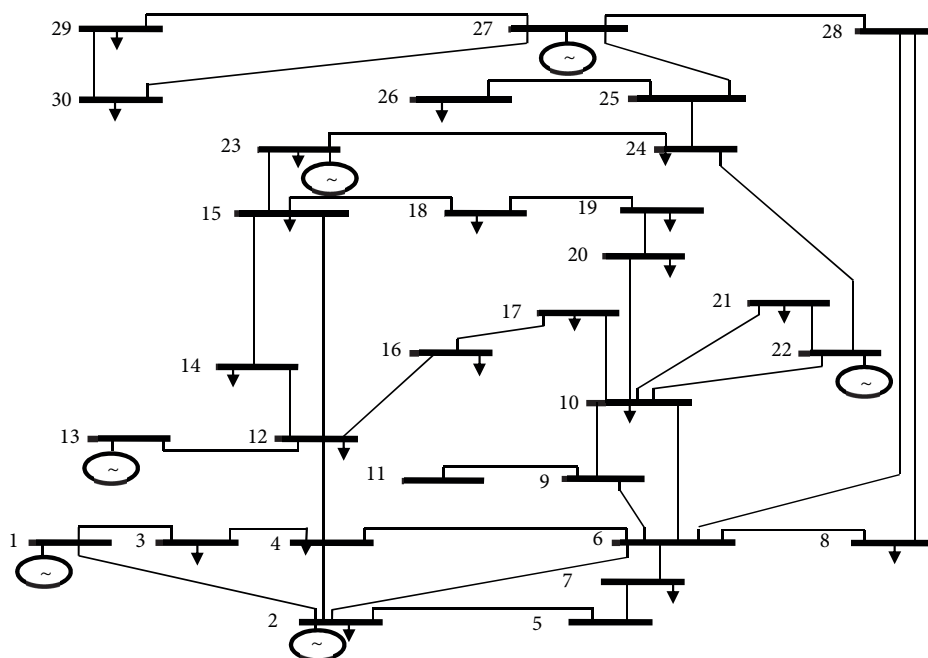
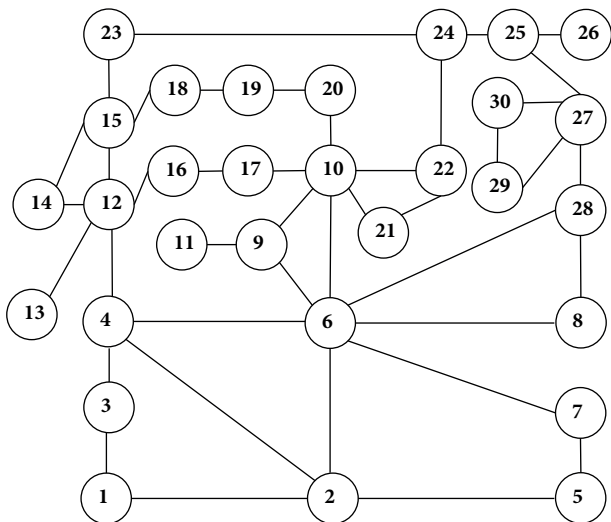**Figure 5**. Line diagram for the IEEE 30-bus test system.

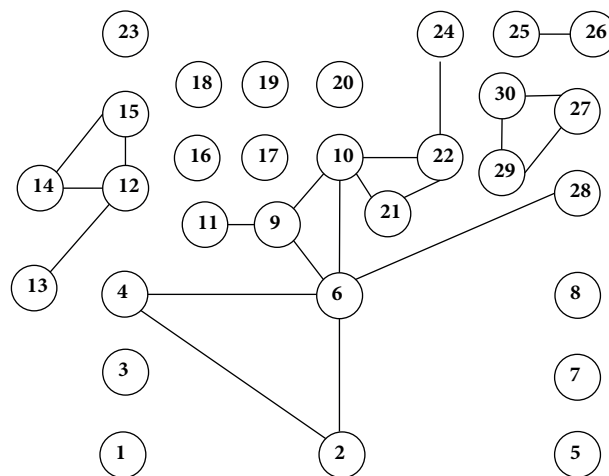**Figure 6**. Graph network model of the IEEE 30-bus test system.

Figure 7. Graph after removing D_1.

Except the fourth component, C_1 and C_2 have a power domination number of one. Hence, they can be considered second- and third-edge decompositions C_2 and C_3. The third component is a pendant vertex.

At this stage, we have three decompositions, and each has a power domination number of one. Now the fourth component should be decomposed further. Consider the longest path in the fourth component, in Figure 8, which is the next decomposition D_4: 2-4-6-9-10-21-22-24.

Removing these vertices from the fourth component C_4, we get two more components, C_5 with the vertex set $\{2, 6, 10, 22, 28\}$ and C_6 with the set $\{9–11\}$. The component C_5 has a power domination number of
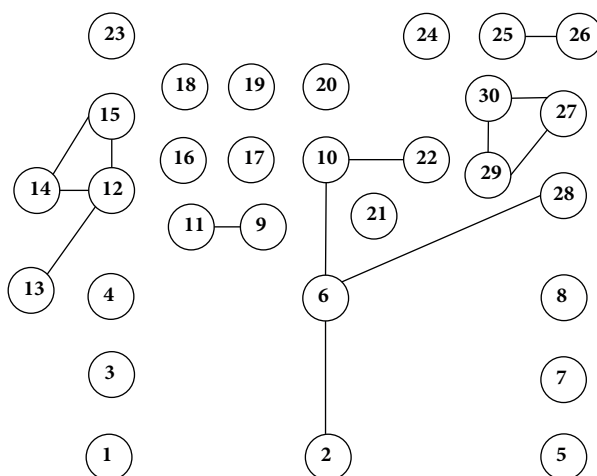
Figure 8. Graph after removing D_1 and D_4.

one and it may be considered the fifth decomposition. The component C_6 is a pendant vertex. Thus, we have 5 decompositions and three pendent vertices. Hence, $5 + 3 - 1 = 7$ PMUs are needed to observe the entire network.

Among the five edge-disjoint decompositions, the least components D_2 and D_3 are disjoint and both need a single PMU. Let us consider the next least components, D_4 and D_5, which can merge into a single component. In this, 11 is a pendant vertex. For this vertex, the 2-distance vertices are 10 and 6. Since vertex 6 has a maximum degree, we place the first PMU in vertex 10. The next PMU may be placed in either vertex 2 or vertex 4, which is 2-distance and not an end vertex. At this stage, the two PMUs may be placed either in $\{2, 10\}$ or $\{4, 10\}$.

Next, we merge the remaining component D_1 with the subgraph that implies the whole power network. The remaining pendant vertices are 13 and 26. Considering vertex 13, the 2-distances are 3, 6, 16, and 15. With the placement of $\{2, 10\}$ or $\{4, 10\}$, all the remaining vertices are observed except vertex 15. Thus, place the third PMU at 15. Now we have PMUs at $\{2, 10, 15\}$ or $\{4, 10, 15\}$. The next pendant vertex is 26. The 2-distances from 26 are 24 and 27. For the component D_3, the PMU is not yet placed and it can be kept at 27. At this stage, the PMUs are at $\{2, 10, 15, 27\}$ or $\{4, 10, 15, 27\}$.

Still, we have to place the remaining 3 PMUs. The next maximum degree vertex is 12. Therefore, let us place the next PMU in vertex 12. For the set $\{2, 10, 12, 15, 27\}$, the 2-distance vertices from vertex 12 are 3 and 18. Placing the PMU in these vertices, the full network is observed. For the set $\{4, 10, 12, 15, 27\}$, the remaining PMUs are placed at vertices 2 and 18, which are 2-distance from vertex 12. Thus, the PMUs may be placed at $\{2, 3, 10, 12, 15, 18, 27\}$ or $\{2, 4, 10, 12, 15, 18, 27\}$.

3.5. IEEE 57-bus test system

In the IEEE 57-bus test system, we may find 12 edge disjoint decomposition. This network has no pendant vertex. Thus, using step 4 in algorithm 3.1, we can say 11 PMUs are sufficient to monitor the entire network. The edge disjoint decompositions are given in Table 1.

Merging the least component to the next big component, we may observe all nodes in the network. The components D_{12}, D_{11}, D_{10}, D_9, D_8, and D_5 are independent components, and so by placing a single PMU

Table 1. Edge decomposition of the IEEE 57-bus test system.

D_i	Vertex sequence	D_i	Vertex sequence
$i = 1$	1-2-3-4-18-19-20-21-22-23-24-25-30-31-32-33-34-35-36-37-38-48-47-49-50-51-10-9-11-13-12-17-1.	$i = 7$	8-9-13-14-46-47
$i= 2$	12-16-1-15-45-44-38-49	$i = 8$	11-41-42
$i= 3$	4-5-6-8-7-29-28-27-26-24	$i = 9$	4-6-7
$i= 4$	10-12-9-55-54-53-52-29	$i = 10$	11-43
$i = 5$	37-39-57-56-40-36, 41	$i = 11$	22-38
$i = 6$	3-15-13-49-48, 14	$i = 12$	32-34

in every component at any vertex, we can observe all the nodes. Merging D_4, D_6, and D_7, using 2 PMU at nodes 13 and 54, we may observe all the vertices. Now for the remaining 3 decomposed components, 3 PMUs are needed. Thus, for 12 components, 11 PMUs are enough to observe all the nodes. These PMUs may be placed at nodes 1, 5, 13, 19, 25, 29, 32, 38, 51, 54, and 56.

4. Power domination integrity

From the above discussion, it is evident that PMU placement in a network is not unique. Therefore, we need a methodology to find the best placement that also preserves network stability. The vulnerability parameters are integrity, toughness, tenacity, rupture, scattering number, and binding number. Among these, integrity deals with a set of vertices and the remaining components after deleting that set of vertices.

Formally, **integrity** is defined, by Barefoot et al. in [7], as

$$I(G) = \min \{|S| + m(G - S)/ S \subset V\}$$

where $m(G-S)$ denotes the order of the largest component in the graph $(G-S)$. An I-set of G is any (strict) subset S of $V(G)$ for which $I(G) = |S| + m(G - S)$.

The **domination integrity** of a graph is defined [8] as

$$DI(G) = \min \{|S| + m(G - S)/ S \, is \, a \, dominating \, set \, of \, G\},$$

where $m(G-S)$ denotes the order of the largest component in the graph GS. A DI-set of G is any dominating set S, which is a subset of $V(G)$ such that $DI(G) = |S| + m(G - S)$.

4.1. Definition

A set of nodes in which PMUs are placed so as to observe the entire network is known as a power dominating set. The power domination integrity is defined as

$$\gamma_{PDI} = \min\{|P| + m(G-P) / P \, is \, a \, minimal \, power \, dominating \, set \, of \, G\}$$ where $m(G-P)$ denotes the order of the largest component of $G - P$. A power dominating set P, whose order equals γ_{PDI}, is known as a power domination integrity set, denoted as PDI-set. Here we are strictly considering the minimum number of PMUs to be placed. Thus, we have to consider only minimal power domination set. However, in [8], they consider all the power dominating sets, instead of considering the minimal set.

4.2. Example

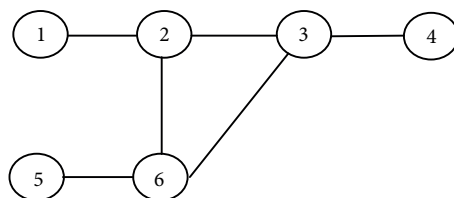Consider the graph G presented in Figure 9, which is a model of a 6-bus power network.

Figure 9. Graph G with 6 vertices.

The graph in Figure 9 consists of three minimal power dominating sets, namely $\{2\}$, $\{3\}$, and $\{4\}$. Thus $\gamma_P(G) = 1$. However, all three sets of graphs have the same power dominating integrity value of 5. Thus $\gamma_{PDI}(G) = 5$.

In [5], Haynes et al. prove that $\gamma_P(G) = 1$, where G is a path P_n with n vertices. This implies any vertex in that path P_n is a power dominating set of G and a single PMU is enough to measure all n vertices. Let $v_1, v_2, v_3, ..., v_n$ be the path vertex sequence of P_n. Let us assume that a PMU is placed in v_1. Suppose there is a breakdown in v_2, and now the PMU can only measure v_1. Even though v_3 to v_n vertices are linked and are in working condition, the PMU fails to take measurements. Thus, there is a need for domination integrity to place the PMU in a correct position so that we can provide better stability.

4.3. Implementation of power domination integrity for IEEE test system

For the IEEE 14-bus test system, the minimal power dominating sets are $S_1 = \{2, 6, 9\}$, $S_2 = \{2, 9, 13\}$, and in [9] we may have found $S_3 = \{3, 6, 9\}$. The power domination integrity number is $\gamma_{PDI} = \min \{3 + 6, 3 + 10, 3 + 6\} = 9$. Thus, the minimal power domination integrity sets are S_1 and S_3, which have a power domination integrity number of 9.

For the IEEE 30-bus test system, the minimal power dominating sets are $S_1 = \{2, 3, 10, 12, 15, 18, 27\}$, $S_2 = \{2, 4, 10, 12, 15, 18, 27\}$, and in [9] we may have found $S_3 = \{1, 2, 10, 12, 15, 19, 27\}$ and $S_4 = \{3, 5, 10, 12, 15, 18, 27\}$. The power domination integrity number is $\gamma_{PDI} = \min \{7 + 8, 7 + 7, 7 + 9, 7 + 9\} = 14$. Thus, the minimal power domination integrity set is S_2, which has a power domination integrity number of 14.

5. Results and discussion

The proposed method of PMU placement using power domination integrity determines the minimum number of strategic bus locations where a PMU must be placed for complete observability of the power system. The algorithm has been tested on IEEE 14-bus, 30-bus, and 57-bus test systems to validate the effectiveness of the proposed method. The optimal number of PMUs and the location of placement by the proposed method are given in Table 2.

The broad criteria used for selection of PMU locations in the perspective of power system engineering are (i) locations at the buses where the transmission line has large difference of phase voltage angles at their ends, (ii) locations near large generating station/critical nodes, and (iii) buses where the apparent power in particular is high.

Table 2. Optimal nodes for placing PMU and power domination integrity values.

Methods	PMU location		
	IEEE 14	IEEE 30	IEEE 57
Power domination method	{2, 6, 9}, {2, 9, 13}	{2, 3, 10, 12, 15, 18, 27}, {2, 4, 10, 12, 15, 18, 27}	{1, 5, 13, 19, 25, 29, 32, 38, 51, 54, 56}
Topology transformation [10]	{2, 6, 9}	{1, 7, 10, 12, 19, 24, 30}	{1, 6, 13, 19, 25, 29, 32, 38, 51, 54, 56}
Immunity genetic algorithm [1]	{2, 6, 9}	{1, 5, 10, 12, 18, 24, 30}	{1, 6, 13, 19, 25, 29, 32, 38, 51, 54, 56}
Genetic algorithm [11]	{2, 6, 9}	{1, 5, 10, 12, 15, 20, 27}	-
Binary particle swarm optimization [12]	{2, 6, 9}	{1, 7, 10, 12, 19, 24, 27}	{1, 4, 9, 14, 19, 22, 25, 29, 32, 38, 51, 54, 56}

The results of the proposed method have been compared with the results of topology transformation [10], immunity genetic algorithm [1], genetic algorithm [11], and binary particle swarm optimization [12]. Table 3 shows the comparative results of minimum number of PMUs required for the complete observability of different systems.

Table 3. Number of PMUs in various methods.

Method	IEEE 14	IEEE 30	IEEE 57
Power domination (proposed method)	3	7	11
Topology transformation [10]	3	7	11
Immunity genetic algorithm [1]	3	7	11
Genetic algorithm [11]	3	7	12
Binary particle swarm optimization [12]	3	7	13

6. Conclusions

A new methodology based on power domination integrity for optimal PMU placement is presented in this paper. The domination parameter optimizes the number of PMUs and the integrity parameter provides the vulnerability of the network model. Hence the power domination integrity parameter gives an optimal integrity value for the power network. The overall optimal solution obtained is sufficient to take care of system observability under normal operating conditions.

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