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Design of a fractional order PID controller with application to an induction motor drive

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Abstract: In this paper, a new method for designing a fractional-order proportional-integral-derivative (FO-PID) controller with an application to an induction motor drive is proposed. In the proposed method, the motor drive is modeled using an autoregressive with exogenous input (ARX) model whose parameters are experimentally identified using real I/O data. A genetic algorithm is then used to find the FO-PID parameters. To guarantee the robustness of the controller against load variations, minimax optimization is adopted. To validate the results, the proposed controller is applied to a real-life motor drive using a hardware-in-the-loop (HIL) simulator. The experimental results show that the proposed controller significantly improves the time response of the induction motor compared to a conventional PID controller. It also shows it is robust against motor load variations.

Key words: Fractional order PID, ARX model, nonlinear optimization, Al-Alaoui operator, hardware-in-the-loop, induction motor

1. Introduction

Induction motors are widely used in industry as they are relatively inexpensive, highly efficient, robust, and very reliable [1]. As such, there is much interest in developing controllers for such electrical drives. Many closed-loop speed control techniques, such as v/f control, field-oriented control, and direct torque control [2], are currently used to improve the motor speed response. It is, however, recognized that proportional-integral-derivative (PID) controllers are by far the most dominant form of feedback control in the industry today. This is due to its design simplicity and its ability to achieve good transient and steady state performances [3]. PID controllers, however, suffer from some drawbacks such as: (1) sensitive to system parameter changes [4], (2) performance deterioration as the order of the system to be controlled increases [5], and (3) poor performance with nonlinear systems [3].

Recently, fractional-order modeling has attracted the attention of researchers and developers in the field of control system design [6,7]. It has been shown that fractional-order differential equations can describe the behavior of many real-life dynamical systems better than their integer-order counterparts [8]. Using fractional-order modeling, a generalized version of the classical PID controllers, known as fractional-order PID (FO-PID), was proposed [7].

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FO-PID controllers have a number of advantages compared to the conventional PID controllers. These include [8]: (1) more degrees of freedom (more tuning parameters), (2) better performance with high order systems, and (3) better response in the case of nonminimum phase systems.

Various methods for tuning the parameters of FO-PID controllers have been proposed in the literature. They can be categorized as analytical-based [9,10], graphical-based [11], and optimization-based [12,13]. In the optimization-based approach, the controller's parameters are computed to achieve some predefined time-domain specifications [14]. In the analytical approach, the controller's parameters are found by solving a system of analytical equations in order to achieve some prespecified frequency-domain specifications. Graphical tuning approaches are based on space plotting of the stabilizing region where some predetermined frequency domain specifications are satisfied.

Most available industrial controllers for induction motors are based on a speed observer that uses a machine model containing uncertainties due to changes in loading conditions. These modeling errors can significantly degrade the speed control performance [15]. In the present paper, an FO-PID controller is proposed to overcome the shortcomings of these conventional controllers. To guarantee the robustness of the controller, minimax optimization for the selection of the control parameters is adopted. This robustness, however, may affect the optimality of the controller parameters for a given load. The closed-loop control algorithm is implemented using a hardware-in-the-loop (HIL) platform. To the best of the authors' knowledge, this is the first work on the design and implementation of an FO-PID controller for induction motors. FO-PID has been limited to DC [16,17] and servo [18] motors. Other published works are simulation-based.

The paper has three main contributions. A new optimization-based iterative FO-PID control design method is proposed. Unlike existing methods, the proposed fractional order controller is mapped to the integer order one *within the iterative optimization process*. This distinctive feature guarantees that the controller achieves the desired performance. The proposed design process can be summarized as follows: 1). The s-domain controller's fractional transfer function is mapped to the z-domain using the Al-Alaoui operator [19], 2) the obtained (discrete-time) fractional-order transfer function is approximated by a rational integer-order transfer function, and 3) the optimal parameters of the integer-order transfer function are computed. The other contributions are the application to induction motors and the robustness to load changing.

The experimental results show that the proposed fractional-order controller outperforms the conventional integer-order one in terms of both transient and steady-state performance. It is also worth mentioning that the proposed design procedure is general enough to be applied on other electrical drive systems.

The paper is organized as follows. Section 2 introduces the control problem. The proposed method for FO-PID controller design is detailed in section 3. In section 4, the experimental setup for the induction motor feedback control is described. Numerical and experimental results are presented in section 5. Concluding remarks and discussions are provided in section 6.

2. Problem statement

As mentioned above, a number of fractional-order controllers have been proposed in the literature. One of the most challenging problems associated with this type of controller is their implementations, as discussed below.

There are currently different ways of approximating fractional-order operators and systems for implementation purpose. Unfortunately, none of these can be considered universally optimum [9]. In this work, we restrict ourselves to the design of a noninteger order PID controller of the form:

$$C_{fr}(s) = K_p + \frac{K_i}{s^\gamma} + K_d s^\mu \tag{1}$$

The interest in this type of controller is due to its simplicity, derived from its similarity to the conventional PID ($\gamma = \mu = 1$). This controller has five tunable parameters ($K_p, K_i, \gamma, K_d, \mu$). This is two more than the conventional PID controller. These extra degrees of freedom can be used to achieve additional design specifications such as robustness [11].

There are different definitions for fractional-order derivatives. The most widely used is the one obtained through the extension of the Riemann and Liouville integer-order integral [8]:

$$D_c^\alpha f(t) = \frac{1}{\Gamma(k-\alpha)} \frac{d^k}{dt^k} \int_c^t (t-\tau)^{k-1-\alpha} f(\tau) d\tau, \qquad k \in N \tag{2}$$

where $\alpha \in (k-1, k]$, Γ is the Gamma function, and c is the initial time instant, often assumed to be zero. If zero initial conditions are assumed, the dynamical systems described by the fractional differential equations give rise to fractional order transfer functions using the Laplace transform [9].

The existing integer-order approximations of fractional-order operators can be classified into continuous-time and discrete-time approximations. For the continuous-time case, a number of approximations, such as continued fraction, Oustaloup, and modified Oustaloup, have been proposed [11]. For the discrete-time case, two steps are required: 1) discretization (usually using Euler and Tustin operators) of the (continuous-time domain) transfer function and 2) integer-order approximation. Integer-order approximations are usually obtained by truncating a continued fraction expansion (CFE) [11].

The Tustin operator based discretization scheme is known to exhibit large errors in high frequency ranges. A new (hybrid) method obtained by combining Euler and Tustin operators has been proposed to overcome the high frequency related error. This new operator, known as the Al-Alaoui operator, is given by [19]

$$H_{Al-Alaoui(a)}(z) = aH_{Euler}(z) + (1-a) H_{Tustin}(z) = \frac{aTz}{(1-z)} + \frac{(1-a)T}{2}\left(\frac{z+1}{z-1}\right) \tag{3}$$

where T is the sampling period and $a \in (0, 1)$ is a user-specified weight. The author in [20] claims that $a = 3/4$ gives good results. Using this value, the above equation becomes

$$H_{Al-Alaoui(3/4)}(z) = \frac{7T}{8}\left(\frac{z+1/7}{z-1}\right)$$

In general, the discretization of the fractional-order differentiator s^r ($r \in R$) can be expressed by the so-called generating function $s \approx Gen(z^{-1})$. The generating function for the fractional-order differentiator (for $a = 3/4$) is given by [21]

$$s^r = \left[\frac{8}{7T}\left(\frac{z-1}{z+1/7}\right)\right]^r \tag{4}$$

The continued fraction expansion can be used to approximate (4) with a finite-order rational transfer function. Based on [22], an approximate rational function, $D^r(z)$, can be obtained as follows:

$$D^r(z) = \left(\frac{8}{7T}\right)^r CFE\left\{\left(\frac{z+1/7}{z-1}\right)^r\right\} \approx \left(\frac{8}{7T}\right)^r \frac{A_p(z^{-1})}{B_q(z^{-1})}, \tag{5}$$

where $D^r(z)$ denotes the discrete equivalent of the Al-Alaoui fractional-order operator, and $A_p(z^{-1})$ and $B_q(z^{-1})$ are polynomials of degree p and q, respectively. Therefore, using the Al-Alaoui fractional operator given in (5), the fractional-order PID controller (1) can be approximated by an integer-order discrete-time controller, a necessary step for practical implementation. The integer order controller approximating the fractional order PID controller (1) is given by

$$C_{int}(z) = \frac{K_d D^{\gamma+\mu}(z) + K_p D^{\gamma}(z) + K_i}{D^{\gamma}(z)} \tag{6}$$

Given a linear dynamical system (plant) described by a transfer function $G_p(z)$, the problem of interest is to find a fractional controller in the form (6) that stabilizes the system while satisfying certain desired time domain specifications.

3. FO-PID controller design

In the present work, a new method is proposed to find the optimum control parameters $\gamma K_p K_i$, K_d, μ. These parameters are computed such that the controlled system exhibits some desired time-domain performances. There are several performance criteria that can be used for this purpose. Some of these are integral of absolute error (IAE), integral of squared-error (ISE), or integral of time-weighted-squared-error (ITSE) [17].

$$IAE = \int_0^\infty |y_{ref}(t) - y(t)|\,dt = \int_0^\infty |e(t)|\,dt \tag{7}$$

$$ISE = \int_0^\infty e^2(t)dt \tag{8}$$

$$ITSE = \int_0^\infty te^2(t)\,dt, \tag{9}$$

where $e(t)$ is the error between the desired profile $(y_{ref}(t))$ and the actual one $(y(t))$. IAE and ISE criteria tend to produce responses with small overshoots but large settling times. This is due to the fact that all errors are weighted uniformly over time. Although the ITSE performance criterion can overcome this drawback, it cannot ensure a desirable stability margin [14]. To achieve a better performance, an objective function is selected that combines IAE and ITSE. This new criterion is given below in the discrete-time domain:

$$J = \rho_1 J_1 + \rho_2 J_2, \tag{10}$$

where

$$J_1 = \frac{\sum_{k=1}^{n} |e(k)|}{n} \qquad J_2 = \frac{\sum_{k=1}^{n} ke^2(k)}{n}$$

n is the length of the signals $r(k)$ and $y(k)$ in samples and ρ_1 and ρ_2 are positive weighting factors. In this paper we restrict ourselves to the case $\rho_2 = 2\rho_1 = 2\rho$ to restrict the search space to one-dimensional instead of two-dimensional.

For the current application, we are interested in designing a controller that is robust against load variations of the drive system. To do this, the whole operating range is divided into a number of operating conditions: no-load, 1/4,..., full load. The robust controller must minimize the worst speed error among the different operating conditions. The proposed controller can be obtained by solving the following minimax optimization problem [25]:

$$\hat{J} = \min_{K_p, K_i, \gamma, K_d, \mu} \; \forall \max_{\substack{\text{operating} \\ \text{conditions}}} J \tag{11}$$

Thus the problem becomes that of finding control parameters that solve the above optimization problem. The weight ρ is varied from to 10 and the value that leads to the minimum cost function is selected.

The optimization problem (11) is a difficult one to solve since it is nonlinear and nonconvex. Deterministic optimization methods, gradient or nongradient based, will fail to give a feasible solution. Therefore, probabilistic optimizations, such as genetic algorithm (GA), particle swarm optimization (PSO), and simulated annealing (SA), can be used to solve it (11). In this paper, the genetic algorithm is used for this purpose The proposed controller design (i.e. calculating the optimum parameters $\gamma K_p K_i$, K_d, μ) can be summarized in the following steps:

Step 1 (Initialization): Select the stopping criterion and randomly initialize the values of the parameters $\gamma K_p K_i$, K_d, μ.

Step 2 (Transfer function approximation): Use the Al-Alaoui operator to find the approximate discrete-time integer controller $C_{int}(z)$ of the FO-PID in the form (6).

Step 3 (Output computation): Use the system model $G_p(z)$ and the FO-PID controller's transfer function $C_{int}(z)$ to track a desired speed profile of the closed loop system (see Figure 1) in the form

$$y(k) = Z^{-1}\left[Y(z)\right] = Z^{-1}\left[\frac{C_{int}(z)G_p(z)}{1 + C_{int}(z)G_p(z)} Y_{ref}(z)\right] \tag{12}$$

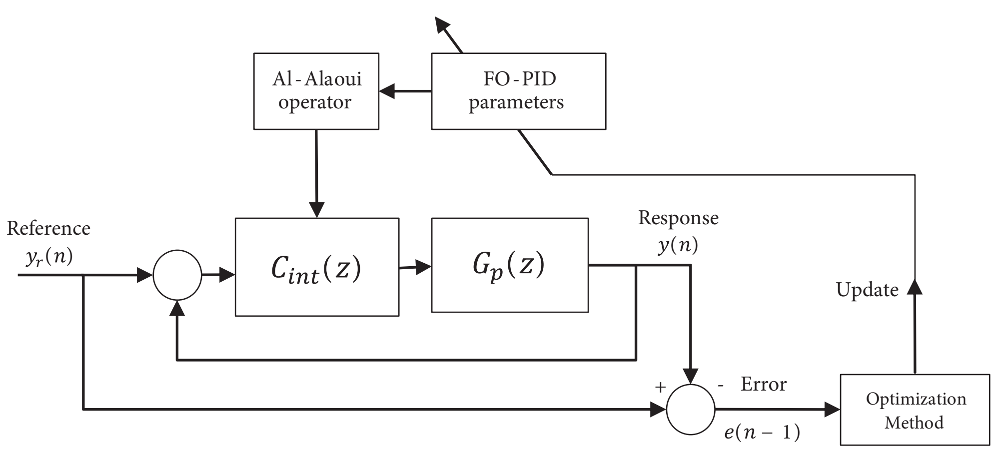

Figure 1. The proposed FO-PID design method.

Step 4 (Stopping decision): Compute the fitness function (11) until no more improvement can be achieved. Otherwise, go to step 5.

Step 5 (Parameters update): Using an optimization algorithm (see implementation section for details), update the FO-PID parameters to minimize (12).

Step 6: Go to step 2.

Traditionally, fractional order controllers are optimized for the best performance criterion and then transformed into integer controllers. This may lead to deterioration of performance due to approximations embedded in the transformations. In contrast, the proposed design method uses the Al-Alaoui operator within the iterative optimization process shown in Figure 1. This guarantees that the performance is not affected by the mapping. To validate the design method, the controller was used to control the speed of a real-life induction motor. This approach can be effectively applied to more general linear and nonlinear control systems [26].

4. Experimental setup

The experimental design involves an induction motor (AC drive inverter, four-pole squirrel-cage three-phase induction motor, and dynamometer) and an acquisition system as shown in Figure 2. The motor has a stator voltage of 120/208 V, nominal speed of 1385 rpm, and nominal current of 0.67 A. The tachometer outputs a speed signal with a sensitivity of 0.002 V/rpm. A picture of the AC drive system is shown in Figure 2. The motor frequency control is responsible for generating a pulse width modulation (PWM) pseudo-AC waveform from the microprocessor based on an analogue input signal that ranges from 0 to 10 V dc. By controlling the PWM waveform, a three-phase AC power at any desired frequency can be generated using the six switches. This signal is generated by a computer within a hardware-in-the-loop setup.

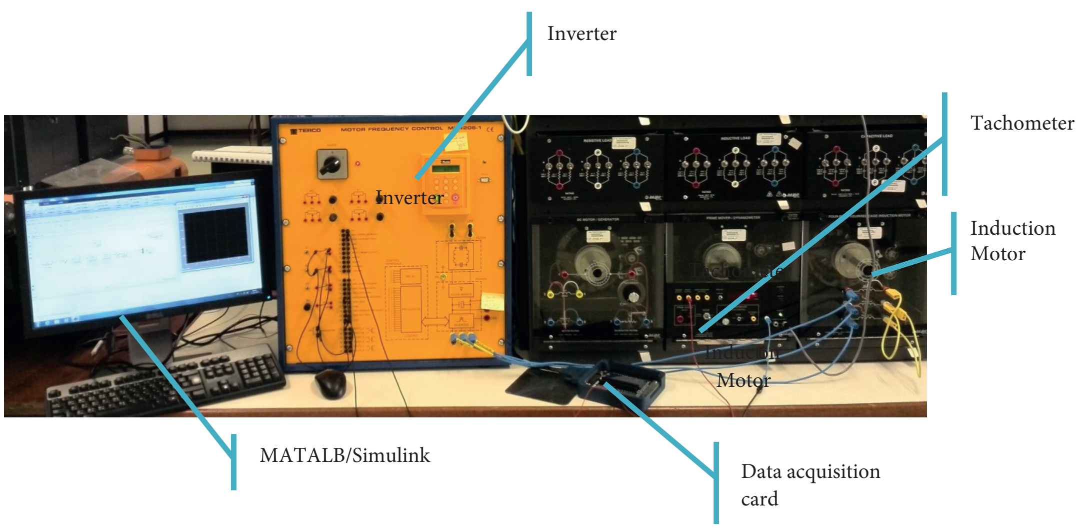

Figure 2. Lab experimental setup.

In this setup, a DAQ card NI PCIe-6323 (with 32 analogue inputs, 48 digital I/O, 4 analogue outputs, and 250 kS/s single-channel sampling rate) was used. This data acquisition system was used in collecting the input/output data for both model identification and control implementation purposes. A sampling period of 0.5 ms and output voltage level between 0 and 10 V were used in all experiments.

5. Experimental results

The objective here is to design a feedback FO-PID speed controller such that the output response tracks a desired speed profile with minimum overshoot and settling time. In order to design the controller, an experiment is first performed to identify the parameters of the model plant (modeled as an ARX). Then the proposed controller design method (detailed in section 3) is employed. Finally, the controller performance is assessed using both a theoretical model and a real-life system.

5.1. Experimental model identification

In this work, we used an autoregressive model with exogenous input [23] to model the motor. To obtain an unbiased model, the excitation (input) signal was chosen to contain a wide range of frequencies [23]. A pseudorandom binary sequence (PRBS) was used for this purpose. The PRBS levels were set to $8V \pm 11\%$, which provided a motor speed of 1000–1200 rpm for 20 s. An open loop PRBS response of the motor was recorded and is shown in Figures 3a and 3b. The PRBS voltage-speed data collected from the induction motor were used for the ARX model identification. In this paper, we used the MATLAB *arx* function to identify the parameters of the model. The model order was computed using the MATLAB functions *arxstruc* and *selstruc*.

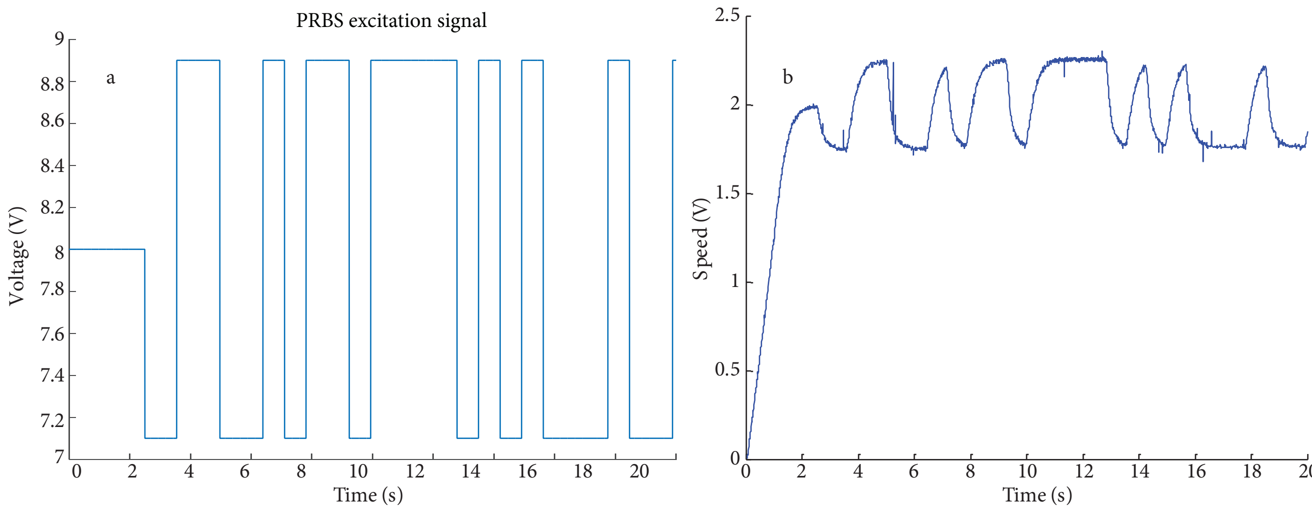

Figure 3. (a) PRBS input voltage to the motor, (b) Open-loop speed response of the motor at no load (speed is measured by a tachometer with 0.002 V/rpm sensitivity).

The resulting transfer function for the system under study is

$$G_p(z) = \frac{-0.001473z^3 + 0.002844z^2 - 0.001529z + 0.002449}{z^4 - 0.9471z^3 + 0.2428z^2 - 0.2336z - 0.05288} \tag{13}$$

5.2. FO-PI speed control

The proposed design process, detailed in section 3, was employed to compute the parameters of the fractional order feedback speed controller for the induction motor. As many applications using induction motors only use PI controllers [15], we opted for, in this section, a fractional order proportional-integral speed controller ($K_d = 0$). Therefore, only three controller's parameters ($\gamma K_p\, K_i$, and) were computed. To identify these parameters, the following optimization methods were tested: particle swarm, firefly, and genetic algorithms. The numerical results showed that GA gave the results and, therefore, was adopted in this work.

Two parameters of the Al-Alaoui operator were preset, namely the number of truncations and the sampling period T, as shown in (5). They were selected to be respectively 5 ms and 3 as they were found to achieve a good compromise between the accuracy and the complexity of the model. The controller parameters were found to be $K_p = 3.929$, $K_I = 4.310$, and $\gamma = 0.797$. The obtained integer-order discrete-time controller is given by

$$C_{int}(z) = \frac{3.929D^{0.797}(z) + 4.310}{D^{0.797}(z)} = \frac{7.575z^6 - 19.38z^5 + 17.42z^4 - 6.213z^3 + 0.5316z^2 + 0.0951z - 0.01198}{1.9z^6 - 4.887z^5 + 4.412z^4 - 1.58z^3 + 0.01362z^2 + 0.02415z - 0.003061} \quad (14)$$

The performance of the controller (14) was evaluated in Simulink using the system model (13). The designed controller was then applied to the real-life induction motor using HIL, as shown in Figure 2. The controller was tested using a step and multitrapezoidal speed inputs. Figure 4a shows the closed-loop speed response of the real-life system. The response, which is very similar to that obtained using the mathematical model, shows an almost zero overshoot. Good speed response was also obtained (as shown in Figure 4b) when a multitrapezoidal speed profile was used as an input. The induction motor was able to successfully track the speed profile with small settling time (50% less than the conventional PI controller) and very small overshoot. Figures 5a and 5b show that the output signal is contaminated with impulsive measurement noise. This type of noise can be easily filtered using a median filter at the expense with an extra small amount of delay in the system [24].

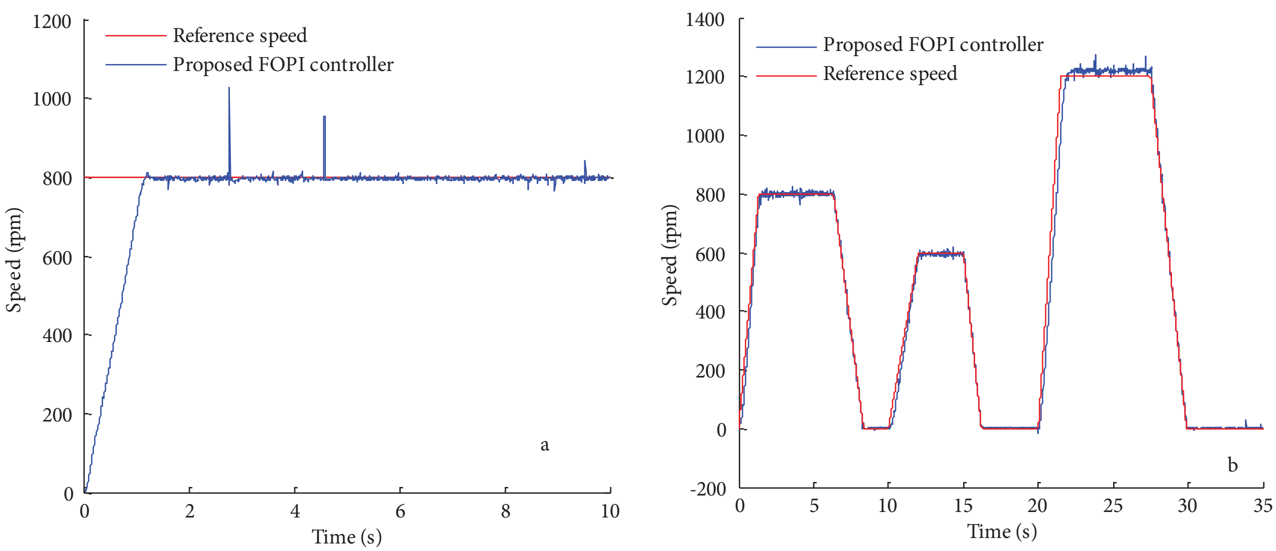

Figure 4. Speed response of a real-life induction motor when the reference signals is: (a) a step input, (b) a multitrapezoidal input.

To show the advantages of using a fractional order controller over an integer order one, a conventional PI was designed for the same induction motor using two different tuning methods, namely Ziegler–Nichols (ZN) and Astrom–Hugglund (AH). In addition, a genetic algorithm similar to the one used to identify the parameters of the FO-PI was also used to tune the integer-order PI controller. All controllers were tested on the real-life system using HIL and the reference signals mentioned above. The results are illustrated in Figures 5a and 5b. It is clear from the figure that the FO-PI outperforms the integer PI in terms of settling time and percent overshoot.

As shown in Figure 5a, although the ZN-PI controller has the fastest response among the integer-order controllers, its percent overshoot exceeds 50%. The AH controller overcomes the overshoot problem but at the expense of a larger settling time (almost twice that of the FO-PI one). Finally, as shown in Figure 5b, the GA-based PI controller's response is too slow, to a level that the induction motor fails to track the desired speed

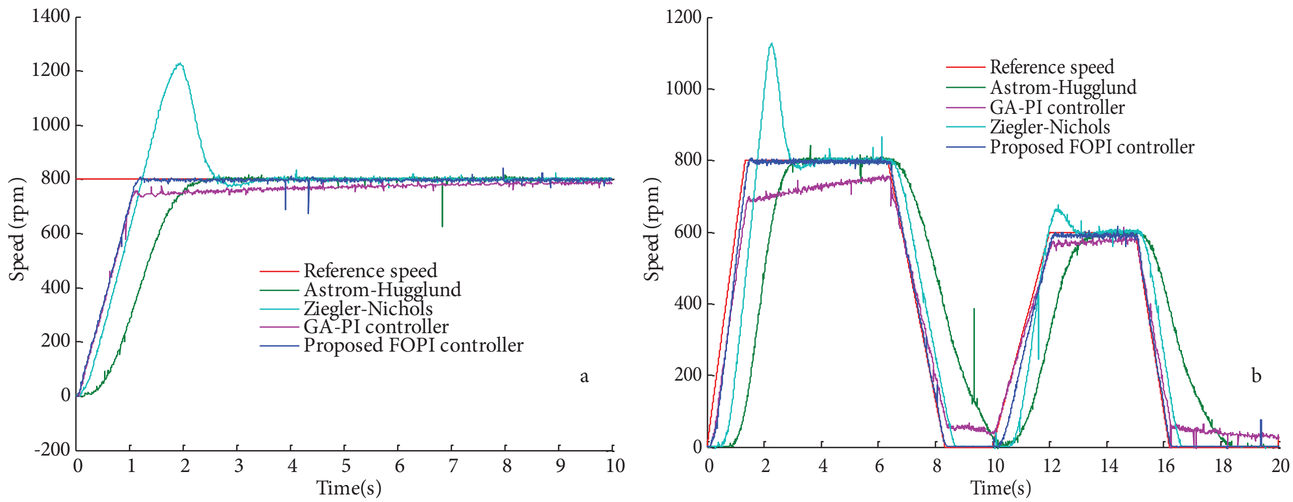

Figure 5. Speed response for the different PI controllers to: (a) step response, and (b) multitrapezoidal response.

reference. Table 1 shows a comparison of the controllers' performances in terms of settling time and overshoot percentage.

Table 1. Comparison of PI controllers' performances.

Control method	Settling time (s)	Overshoot percentage (%)
ZN-PI	3.2	51%
AH-PI	2.4	0%
GA-PI	8	0%
Proposed FO-PI	1.2	0%

It is worth mentioning here that an FO-PID controller was designed using the proposed method and tested on the real-life system. The experimental results showed no significant improvement, compared to the FO-PI, in terms of either the transient or steady states responses of the machine in the case of no load. However, the FO-PID performed better in terms of robustness against changing load conditions.

5.3. Robustness

In order to assess robustness against load variation of the proposed controllers, different loads were applied to the motor and the motor speed was recorded at each loading level. Figures 6a and 6b show the speed response at different load levels ranging from no-load to full-load when using both FO-PI and FO-PID controllers. The results reveal that adding the fractional D term to the FO-PI controller results in a more robust controller against load variation. Table 2 shows the steady state error at different loading level using FO-PI and FO-PID controllers. It can be noted that the FO-PID controller improves the steady state error by 50% in most of the loading levels.

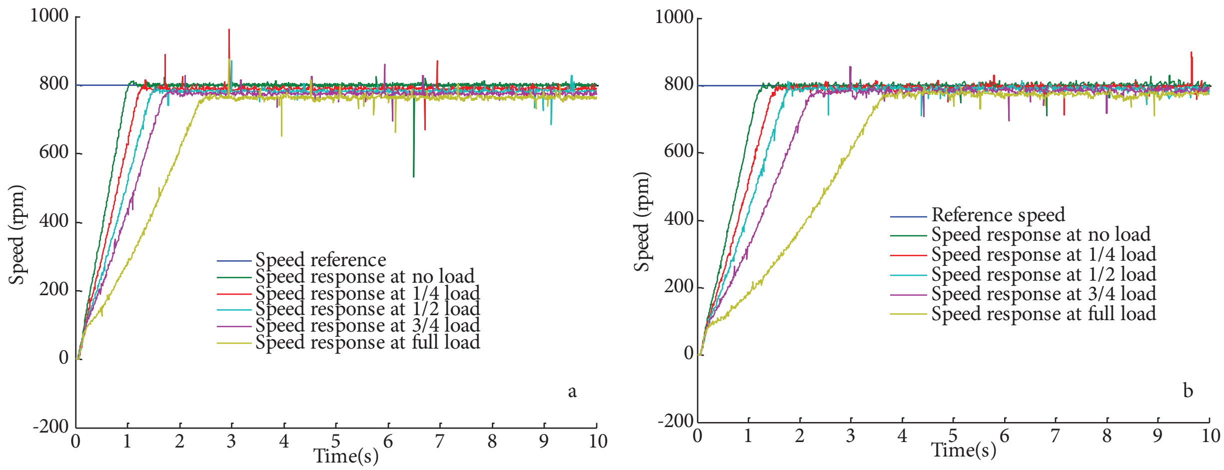

Figure 6. Speed response to step input at different loading conditions when using (a) FO-PI controller and (b) FO-PID controller.

Table 2. Comparison of FO-PI and FO-PID performance in terms of steady-state error at different loading conditions.

Loading condition	Steady-state error (%)	
	FO-PI	FO-PID
No load	0.25	0.25
¼load	1.25	0.65
$^1/_2$ load	2.5	1.25
$^3/_3$ load	3.5	1.875
Full load	4.75	3.125

6. Conclusions

This paper presents a new approach to the design of fractional-order PID used to control the speed of an induction motor. The transient and steady-state performances of the proposed fractional-order controlled were compared to those of a conventional integer-order controller both theoretically (simulation) and using a real-life system. The results show that the proposed controller performs nicely for different reference signals and shows robustness to load variations. The experimental results also showed that the FO-PID controller does not significantly improve the dynamic response of the system compared with the FO-PI controller. However, robustness test results for both controllers revealed that the FO-PID controller is more robust against load variation than the FO-PI controller.

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