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Online network coding-based multicast routing in multichannel multiradio wireless mesh networks

Leili FARZINVASH*
Faculty of Electrical and Computer Engineering, University of Tabriz, Tabriz, Iran

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Abstract: In this paper, we consider the problem of online multicast routing in multichannel multiradio wireless mesh networks (WMNs). We propose an efficient online algorithm, namely zone-based multicast routing (ZBMR), which exploits network coding and wireless broadcast advantage. In the proposed algorithm, to investigate the acceptance of an arrived session in polynomial time, the WMN is divided into some zones. The derived zones are processed sequentially, where the zone processing is defined as connecting the receivers in a given zone to the session. The main challenge in this scheme is to enable data transmission to the receivers in each zone. If a zone does not contain the source node, it should obtain data from the previously processed neighboring zones. The problem is that the data transmission fails if there is no receiver on the common border between the considered zone and its processed neighboring zones. Our solution to tackle this challenge is to add some virtual receivers to the borders of the zones. The extensive simulations show that ZBMR increases the acceptance rate by 50% in comparison to the previous approaches.

Key words: Wireless mesh network, online multicast routing, multichannel multiradio, network coding, polynomial time, wireless broadcast advantage

1. Introduction

With the emergence of multicast applications, designing effective algorithms for group communication has drawn significant attention [1]. One important domain where multicast routing is widely used is broadband wireless mesh networks (WMNs). These networks provide Internet connectivity in rural and metropolitan areas. From the popular multicast-based services in WMNs, we can point out online games, video conferencing, distance education, and online TV. The mentioned applications typically require a considerable amount of bandwidth. Therefore, it is critical to develop high-throughput multicast routing algorithms.

Recently, the network coding (NC) technique has been proposed as a bandwidth-efficient solution to perform multicast routing. In this scheme, the ingress flows to each relay node are coded with each other. This coding scheme reduces bandwidth utilization in comparison to the tree structure [2], which enhances the performance of the system. Some studies on multicast routing in WMNs have employed this technique [3–14]. The main shortcoming of these works is that they studied offline scenarios. In other words, they assumed that the multicast sessions are given in advance. Consequently, they cannot handle online multicast routing, in which the sessions arrive at the system dynamically.

To support online applications, in this paper we investigate the problem of online NC-based multicast routing in WMNs. In the proposed setting, it is assumed that each multicast session demands a specific bandwidth. In addition, the arrival time and duration of the sessions are unknown before. A session is accepted

*Correspondence: l.farzinvash@tabrizu.ac.ir

if its bandwidth requirement can be satisfied. To increase the acceptance rate, we adopt the multichannel multiradio (MC-MR) setting. In this scheme, the nodes are equipped with some radios and utilize multiple channels for data transmission [15]. Therefore, the amount of interference throughout the network decreases. The wireless broadcast advantage (WBA) is also included in our design to improve the acceptance rate as much as possible. Using this property in multicast routing, the data sent by a node using a specific channel are obtained by all of its children to which the same channel is assigned. Therefore, the amount of utilized bandwidth for multicast routing reduces substantially.

While exploiting WBA in NC-based multicast routing, the challenge is that the children of the nodes have to be determined during data transmission. The parent-child relationship between the nodes is modeled using binary variables. As a result, the optimal model for the acceptance of arrived sessions becomes mixed integer programming (MIP). Due to NP-hardness of MIP models [16], the time complexity of the intended problem is not acceptable for online services. Therefore, we design the zone-based multicast routing (ZBMR) algorithm considering the optimal model. This algorithm comprises two phases of zone formation and zone processing, which are described below.

When a multicast session arrives, the WMN is divided into disjoint zones. In the proposed zone formation scheme, the nodes are traversed in breadth-first search (BFS) order from the source node of the session. While considering a given node, the corresponding binary variables to the common links between the node and previously processed nodes are added to the current zone. The number of binary variables in each zone, which presents the parent-child relationship between the nodes regarding the arrived session, is limited to a given threshold. Accordingly, when the number of binary variables in the current zone reaches the threshold, the binary variables corresponding to the subsequent visited links are added to the next zone. This procedure continues until all nodes are processed.

After forming the zones, they are processed sequentially. The aim of the zone processing scheme is to deliver the multicast data to the included receivers. To this end, it employs a mathematical model, which is based on the optimal model. This model performs NC-based multicast routing, link scheduling, and exploitation of WBA within the assumed zone. The zone is processed successfully if there is enough bandwidth to transmit multicast data to all of its receivers. The session is accepted if all zones are processed successfully. As the number of binary variables in each zone is less than the predefined threshold, the time complexity of processing of a given zone is polynomial regarding the network parameters. Consequently, the proposed algorithm becomes polynomial time.

In the proposed zone processing scheme, the zone that contains the source node is processed first. Next, the neighboring zones of the previously processed zones are considered. This procedure continues until all zones are processed. The important issue in this scheme is that data transmission between two neighboring zones may fail if the sender zone has no receiver on its border. This is because there is no guarantee that the nonreceiver nodes obtain the multicast data completely. To tackle this problem, some nodes on the borders are converted to receiver nodes. In other words, they virtually become the receivers of the multicast session. These nodes obtain the complete set of data and can forward it to the receivers in neighboring zones.

The rest of this paper is organized as follows: Section 2 reviews the topics related to the intended problem, comprising multicast routing in MC-MR WMNs and NC-based multicast routing. The network model is presented in Section 3. In Section 4, the phases of the ZBMR algorithm, comprising zone formation and zone processing, are explained. The effectiveness of our algorithm is investigated through extensive simulations in Section 5. Finally, we conclude the paper in Section 6.

2. Related work

Recently, there have been many research studies on multicast routing in MC-MR WMNs [10–14, 17–27]. Some proposals have used the multicast tree to enable data transmission from the source node to the receivers of the multicast session [17–27]. The main concern in these works is to diminish the imposed interference on the multicast tree. The aim of [17] is to minimize interference as well as utilize bandwidth. To this end, the authors proposed an optimal model, which constructs the multicast tree and performs channel assignment simultaneously. This model is not applicable to large instances due to its high time complexity. Therefore, a heuristic algorithm was also proposed to solve the problem in a reasonable running time. In this approach, the multicast tree is constructed in some iterations. In each iteration, the path with the most weight is added to the partially constructed tree. The weight of a path is computed according to maximum imposed interference on its links and the number of its covered receivers. Next, the proper channels are assigned to the links of the chosen path. The aim of the channel assignment procedure is interference minimization and exploiting WBA.

In the proposed algorithm in [18], the imposed interference on each node is regulated based on the number of its descendant receivers. Accordingly, the least-loaded channels are assigned to the nodes with more descendant receivers. Thus, the total delivered data rate to the receivers increases. Reference [19] studied multicast tree construction in multirate MC-MR WMNs. In this work, the authors noticed that employing WBA in the multirate setting may degrade the bandwidth. Therefore, they exploited WBA such that the performance is improved.

QoS-aware multicast routing in MC-MR WMNs was considered in [20–24]. The authors of [20] designed a cross-layer algorithm, which performs channel assignment and multicast tree construction jointly. The goal of the algorithm is to maintain delay constraint, minimize the interference, and reduce bandwidth utilization together. This scheme employed the genetic algorithm to construct low-interference multicast trees. Chakraborty [21] considered the jitter constraint besides the mentioned objectives. In this algorithm, the author used the differential evolutionary scheme and genetic algorithm for tree construction and channel assignment, respectively. The problem of delay-bounded and priority-aware multicast routing was investigated in [22]. The proposed scheme guarantees that the maximum delay of data transmission to the receivers is limited to the predefined threshold. In addition, the multicast trees of the high-priority sessions incur less interference.

The presented algorithm in [23] preserves the delay constraint of the multicast sessions. This scheme constructs trees and performs channel assignment together to fully exploit the capacity of the network. The considered QoS criterion in [24] is the bandwidth of the sessions. The proposed algorithm in this work performs multicast routing in two phases. It first constructs the multicast trees with minimum utilized bandwidth. Next, the algorithm performs channel assignment such that the interference throughout the WMN decreases. The mentioned scheme achieves near-optimal performance. However, it investigated offline scenarios and could not support online multicast applications.

References [25–27] examined online multicast routing in MC-MR WMNs. The shortest path algorithm was applied to connect the receivers to the multicast tree in [25, 26]. In these works, the tree is constructed in some iterations. In each iteration, the shortest path between each unattached receiver and the partially constructed tree is determined first. Next, the receiver with the minimum cost shortest path is added to the tree. This procedure continues until all receivers are included in the tree. The cost of a given link is computed according to the amount of imposed interference on it and the possibility of exploiting WBA. In [27], the acceptance of each session is investigated in two phases. First, a loop-free mesh backbone connecting the source to the receiver nodes is constructed. The tree corresponding to the session is constructed over this mesh in the second phase. The mentioned studies utilized a tree structure for multicast routing. As shown in [2], using NC increases the performance of multicast routing substantially. Therefore, it is preferred to adopt the NC-based approach if the routers are capable of performing coding operations.

In the rest of this section, the previous NC-based multicast routing algorithms are considered [3–14]. These works presented cross-layer solutions to handle different aspects of the network jointly. Vien et al. [3] proposed a dual subgradient model, which considers multicast routing, link scheduling, energy consumption of the nodes, and network lifetime simultaneously. In this work, the network lifetime constraint is preserved via proper power control. Video streaming in single-channel wireless networks was investigated in [4]. The aim of this work is to minimize the distortion of the obtained videos by the receivers. This algorithm assumes that the MAC layer is TDMA and performs multicast routing and scheduling together. Reference [5] also aims at minimizing the distortion of the obtained videos by the receivers. This work considers the heterogeneous capabilities of mobile clients and transmits data to each client accordingly.

Some studies exploited coding techniques to reduce the utilized bandwidth as much as possible [6, 7]. These works investigated multilayer multicast routing, in which the multicast data are divided into a number of layers. Each receiver obtains some consecutive layers considering the bandwidth limitation. Through performing coding operations between different layers, the proposed algorithms in [6, 7] diminish the total required bandwidth for transmitting successive layers. Reference [8] studied NC-based multicast routing in the case of packet dropping. The loss of some coded packets disrupts the decoding operation at the receivers. Hence, the algorithm retransmits some extra coded packets considering the network status to mitigate this issue.

References [10–14] examined NC-based multicast routing in MC-MR WMNs. Zhou et al. [10] considered video transmission over MC-MR WMNs. They proposed a cross-layer solution for link scheduling, channel assignment, and multicast routing, where the objective is to minimize the distortion of the delivered videos to the receivers. Reference [11] investigated the multicast routing problem in multigateway WMNs. In this work, gateway selection is taken into account in addition to power control and multicast routing. Moreover, the authors proposed a dual subgradient model to obtain the optimal solution. The problem of NC-based multilayer multicast routing in MC-MR WMNs was investigated in [12]. This work processes the layers sequentially from the base layer to upper ones. While processing a layer, the receivers that obtain it are determined.

In [13], the authors provided a two-phase algorithm for maximizing the transmission rate of the multicast session. In the first phase, the links are partitioned into groups in such a way that the links of each group have no interference while performing multicast routing. Channel assignment is carried out in the second phase. A similar problem was considered in [14] for multirate WMNs. This work assumes that the links can support multiple data rates. Consequently, the problem is to select the proper data rate per link besides multicast routing and channel assignment. In summary, the mentioned NC-based algorithms are offline and are not suitable for managing online multicast sessions. Therefore, we propose an online algorithm for accepting multicast sessions, which exploits NC and WBA to achieve a high acceptance rate.

The considered algorithms in this section are compared in detail in Table 1.

3. Network model

This paper considers MC-MR WMNs. The network is modeled by graph $G = (V, E)$, where V and E are the sets of nodes and links, respectively. The nodes are static, and each node v is equipped with o_v radios. It is assumed that there are K orthogonal channels for data transmission. Binary variable a_v^k presents the assignment of channel k to node v. It takes value 1 if channel k is assigned to node v and 0 otherwise.

Table 1. Comparison of the studied algorithms.

Algorithm	Multicast routing method	Online/offline	MAC layer	MC-MR
Reference [3]	NC	Offline	TDMA	Single-channel
Reference [4]	NC & multi-layer	Offline	TDMA	Single-channel
Reference [5, 7]	NC & multi-layer	Offline	Wired	-
Reference [6]	NC & multi-layer	Offline	MIMO	-
Reference [8]	NC	Offline	Contention-based	Single-channel
Reference [9]	NC	Offline	Wired	-
Reference [10, 11, 13]	NC	Offline	TDMA	MC-MR
Reference [12]	NC & multi-layer	Offline	TDMA	MC-MR
Reference [14]	NC	Offline	TDMA & multi-rate	MC-MR
Reference [17, 18, 20–22]	Tree	Offline	Contention-based	MC-MR
Reference [19]	Tree	Offline	Contention-based & multirate	MC-MR
Reference [23]	Tree	Offline	Interference free	MC-MR
Reference [24]	Tree	Offline	TDMA	MC-MR
Reference [25–27]	Tree	Online	TDMA	MC-MR

The multicast session m is described using a 5-tuple $(s_m, R_m, rt_m, a_m, e_m)$. The source node, receiver set, and demand rate of the session are presented by rt_m, s_m, and R_m, respectively. The session arrives at a_m, and, in the case of acceptance, it leaves the system at e_m. In NC-based multicast routing, we have the notations of logical flow and physical flow. The logical flow of receiver $w \in R_m$ over link (u,v) is shown by g_{uv}^{mw}. The physical flow of session m over link (u,v) is denoted by f_{uv}^{m}. Moreover, r_{uv}^{mk} presents the data rate of session m over link (u,v) using channel k.

To investigate the acceptance of session m, the network is partitioned into some zones in the first phase. The ith corresponding zone to the session is denoted by Z_i^m. As has been already mentioned, some nodes of the zone are considered as virtual receivers to enable data transmission to other zones. The union of these virtual receives by the receivers in $R_m \cap Z_i^m$ forms the receiver set of the zone, which is presented by Zr_i^m. In addition, data transmission to the receivers of Zr_i^m is performed via border receivers of the previously processed neighboring zones. These receivers form the set of the source nodes of Z_i^m, which is denoted by Zs_i^m. It is obvious that the Zs set of the first zone is equal to $\{s_m\}$.

The notations related to the zones are clarified using the illustrated example in Figure 1. In this figure, multicast session m, with source node v_4 and receiver set $\{v_3, v_6, v_{12}\}$, arrives at the WMN. Assume that the proposed algorithm partitions the network into two zones (i.e. Z_1^m and Z_2^m). The first zone contains s_m and is processed first. The sets Zs_1^m and Zr_1^m are equal to $\{v_4\}$ and $\{v_6\}$. The nodes in Zr_2^m obtain multicast data from the border receivers in Zr_1^m. Therefore, Zs_2^m is equal to $\{v_6\}$. In addition, Zr_2^m is set to $\{v_3, v_{12}\}$.

In the proposed algorithm, the multicast routing is performed considering the WBA property of the wireless medium. If node v is a nonleaf node in session m, it sends data to its children using available channels. Variable t_v^{mk} presents the duration of data transmission by node v for session m over channel k. The existence of a parent-child relationship between nodes u and v is investigated through binary variable l_{uv}^{mk}. This variable

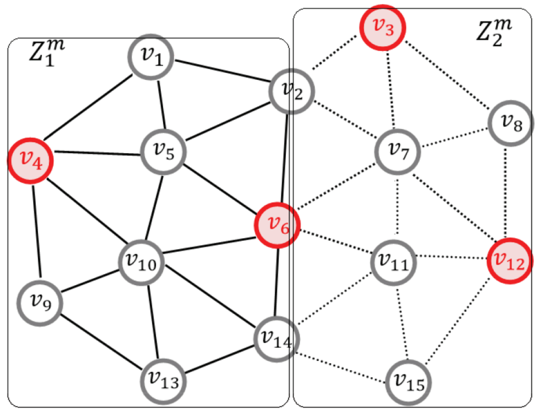


Figure 1. Network partitioning to investigate the acceptance of multicast session m, where s_m and R_m are equal to v_4 and $\{v_3, v_6, v_{12}\}$, respectively. The links of each zone are illustrated in a different style.

is set to 1 if node $u \in N_v$ is the child of node v in session m over channel k. The notation C_v^{mk} shows the children of node v in session m over channel k.

In the rest of this section, we describe the related setting to link scheduling. In the intended configuration, all nodes have equal transmission and interference ranges. Variables N_v and IN_v denote the set of the neighbors and interfering nodes of node v, respectively. In addition, cp presents the capacity of the links. In this work, we adopt the TDMA MAC layer. Therefore, the interfering transmissions should not occur at the same time. More specifically, when node v transmits the data of session m using channel k, all nodes of C_v^{mk} should be able to obtain it. The successful scheduling is achieved by preventing the interfering nodes with the nodes in C_v^{mk} from data transmission over channel k at the same time.

The reserved bandwidth for each accepted session or zone should be considered in the scheduling constraints to keep the network stable. To maintain the generated load by the accepted sessions, the following variables are defined. We present the load of node v by D_v. The amount of interference on node v over channel k is presented by I_v^k. Moreover, the load on link (u, v) over channel k is shown by L_{uv}^k. For the current multicast session m, similar variables are defined to record the load of its accepted zones. Variable Dz_v^m presents the total of ingress and egress load to/from node v, which belongs to session m. The resultant interference of the session on node v over channel k is denoted by Iz_v^{mk}. Finally, the associated data rate to the session on link (u, v) over channel k is shown by Lz_{uv}^{mk}. In the following, the size of a given set is presented by $|.|$.

4. The ZBMR algorithm

When a session arrives at the system, the aim is to allocate the required bandwidth to it. Due to incorporating binary variables for describing the parent-child relationship, the time complexity of the optimal model is not acceptable for online applications. To overcome this challenge, we design the ZBMR algorithm based on the optimal model, which manages arrived sessions in polynomial time. The idea behind this algorithm is to divide the WMN into zones and transmit the multicast data to the receivers of each zone separately. This procedure converts the problem of accepting multicast session into the simplified task of zone processing. In the following, we describe the phases of our algorithm in Sections 4.1 and 4.2. The zone formation procedure is explained in Section 4.1. Next, in Section 4.2, the mathematical model of zone processing is given. We study the time complexity of the ZBMR algorithm in Section 4.3 and prove that it is polynomial time. Finally, some issues about the distributed implementation of the online NC-based multicast routing in MC-MR WMNs are given in Section 4.4.

4.1. Zone formation

The establishment of the parent-child relationship is described using l-variables. As a consequence, the problem of accepting multicast sessions is stated as a MIP problem, which is NP-hard. To process the arrived session in polynomial time, the WMN is partitioned into zones. The number of l-variables in each zone does not exceed α, where this parameter is a predefined threshold determined by the network administrator. Introducing this threshold makes the zone processing model polynomial time regarding the network parameters. After the formation of the zones, they are processed consecutively. The first considered zone comprises source node s_m. In the following, the selected zone in each iteration should be next to one of the processed zones. Multicast session m is accepted if all zones are processed successfully.

The proposed zone formation procedure is explained using the presented example in Figure 1. In this setting, α is set to 68. Therefore, if it is assumed that each node has two common channels with each of its neighbors, each zone (except the last one) contains 34 links. As depicted in the figure, the given zones preserve this constraint. Zones Z_1^m and Z_2^m have 68 and 60 l-variables, respectively. To investigate the acceptance of the session, Z_1^m is processed first. If it is accepted, zone Z_2^m is considered. The multicast session m is accepted if there is enough bandwidth to accept this zone.

Figure 2 expounds the proposed algorithm for forming the zones of multicast session m. In the proposed scheme, the nodes are traversed in BFS order from s_m. While processing node v, the corresponding binary variables to the common links between this node and previously considered nodes are added to Z_i^m. This procedure continues until the number of binary variables of Z_i^m reaches α. At this point, a new zone is initiated, and the binary variables corresponding to the subsequent visited links are added to this zone.

To clarify the proposed zone formation algorithm, it is applied to the given WMN in Figure 1. Assume the

Input: $G = (V, E)$ and multicast session m.
Output: The zones of multicast session m.

$i \leftarrow 1$.
Traverse the nodes in BFS order from s_m.

for *each node* $v \in V$ **do**
    for *each previously considered node* $u \in N_v$ **do**
        for *each channel* $1 \leq k \leq K$ **do**
            if $a_v^k = 1$ *and* $a_u^k = 1$ **then**
                $Z_i^m \leftarrow Z_i^m \cup \{l_{vu}^{mk}, l_{uv}^{mk}\}$.

                if $|Z_i^m| = \alpha$ **then**
                    $i \leftarrow i + 1$.
                end
            end
        end
    end
end

Figure 2. Zone formation.

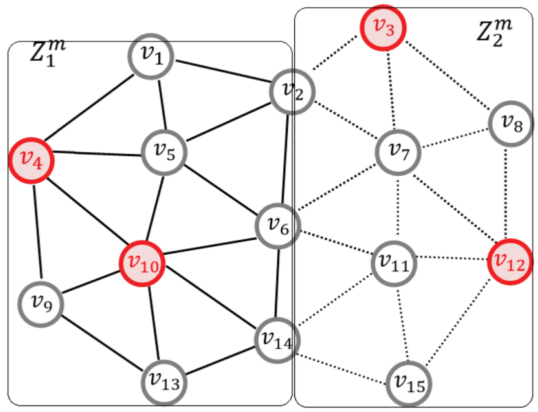


Figure 3. An example of network partitioning that fails to accept multicast session m, where $s_m = v_4$ and $R_m = \{v_3, v_{10}, v_{12}\}$.

sets of assigned channels to nodes v_4, v_1, and v_5 are equal to $\{k_1, k_3, k_4, k_8\}$, $\{k_1, k_2, k_3, k_9\}$, and $\{k_2, k_4, k_8, k_9\}$ in this configuration. As derived from the figure, the hop count distance of nodes v_1, v_5, v_{10}, and v_9 from the source node (i.e. v_4) is equal to one. Hence, these nodes are considered first. Initially, node v_1 is processed and binary variables $l_{v_4v_1}^{mk_1}$, $l_{v_1v_4}^{mk_1}$, $l_{v_4v_1}^{mk_3}$, and $l_{v_1v_4}^{mk_3}$ are added to Z_1^m. Next, node v_5 is processed and binary variables $l_{v_4v_5}^{mk_4}$, $l_{v_5v_4}^{mk_4}$, $l_{v_4v_5}^{mk_8}$, $l_{v_5v_4}^{mk_8}$, $l_{v_1v_5}^{mk_2}$, $l_{v_5v_1}^{mk_2}$, $l_{v_1v_5}^{mk_9}$, and $l_{v_5v_1}^{mk_9}$ are also added to Z_1^m. This procedure continues until all nodes are processed.

The challenge of the proposed approach is to deliver multicast data to the receivers of each zone. While processing Z_i^m, the data delivered to its receivers pass through common border nodes with previously processed zones. If these nodes are not in R_m, they may not obtain the multicast data. This condition limits the possibility of data delivery to the receivers of Z_i^m. In this situation, session m may not be accepted despite the availability of adequate bandwidth. Figure 3 shows an example of this difficulty. In this figure, s_m and R_m are set to v_4 and $\{v_3, v_{10}, v_{12}\}$, respectively. Assume the multicast data are delivered to v_{10} through paths $v_4 - v_{10}$ and $v_4 - v_5 - v_{10}$. Therefore, the border nodes of the zone (i.e. v_2, v_6, and v_{14}) do not obtain the data of session m. Hence, it is impossible to deliver multicast data to v_3 and v_{12}, which leads to the rejection of the session.

Our solution to tackle this problem is to convert some border nodes of each zone to the receivers of session m. The receiver nodes obtain multicast data completely. Therefore, putting these virtual receivers on the border of a zone guarantees that the multicast data are delivered by the receivers of the neighboring zones. For example, if node v_6 is considered as a virtual receiver in the proposed example in Figure 3, the multicast data can be transmitted to the nodes in Zr_2^m.

The remaining point is to determine the proper subset of the border nodes that should be considered as virtual receivers. Enlarging the receiver set usually increases bandwidth consumption. Therefore, the number of virtual receivers should be minimized as much as possible. The proposed algorithm for determining the proper virtual receivers is described in Figure 4. This algorithm selects the virtual receivers in Z_i^m for data transmission to Z_j^m. In the proposed scheme, the border nodes between Z_i^m and Z_j^m are considered. If border node v is in $\{s_m\} \cup R_m$ and has at least one common channel with the nonborder nodes of Z_j^m, $bRecNo$ is increased by one. Here, $bRecNo$ presents the number of (virtual) receivers on the border of Z_i^m and Z_j^m. Otherwise, it is put in the $bNodes$ set and its H set is formed. Set H_{ijv}^m consists of the neighbors of node v that are in $Z_j^m - Z_i^m$ and have no receiver neighbor on the border.

To minimize the number of virtual receivers, the border nodes with larger H-sets are selected as virtual receivers. For this purpose, the nodes in $bNodes$ are sorted according to the size of their H-sets in each iteration. The node with the largest H-set, namely node v, is added to the receiver set of Z_i^m, and $bRecNo$ is increased by one. The nodes in H_{ijv}^m are covered by node v. Therefore, they are removed from the other H-sets. To have a reasonable number of virtual receivers, $bRecNo$ is limited to the threshold $maxVRecNo$.

Input: Z_i^m and Z_j^m.
Output: The virtual receivers of Z_i^m.

$bRecNo \leftarrow 0$.
$bNodes \leftarrow \emptyset$.

for *each node* $v \in Z_i^m \cap Z_j^m$ **do**
  if $0 < \sum_{u \in N_v \cap (Z_j^m - Z_i^m)} \sum_{k=1}^{K} a_v^k a_u^k$ **then**
    if $v \in R_m$ *or* $v = s_m$ **then**
      $bRecNo \leftarrow bRecNo + 1$.
    else
      $bNodes \leftarrow bNodes \cup \{v\}$.

      for *each node* $u \in N_v \cap (Z_j^m - Z_i^m)$ **do**
        if $N_u \cap R_m \cap Z_i^m = \emptyset$ *and* $0 < \sum_{k=1}^{K} a_v^k a_u^k$ **then**
          $H_{ijv}^m \leftarrow H_{ijv}^m \cup \{u\}$.
        end
      end
    end
  end
end

while $bRecNo < maxVRecNo$ *or there is nonempty* H*-set* **do**
  Sort $bNodes$ according to the size of H-sets of its nodes.
  $v \leftarrow$ The first node of the list.
  $Zr_i^m \leftarrow Zr_i^m \cup \{v\}$.
  $bRecNo \leftarrow bRecNo + 1$.

  for *each node* $u \in bNodes$ **do**
    $H_{iju}^m \leftarrow H_{iju}^m - H_{ijv}^m$.
  end
end

Figure 4. Virtual receiver selection.

4.2. Zone processing

After forming the zones, each zone Z_i^m is processed to connect its receivers to the session. For this purpose, the data of session m are transmitted from the nodes in Zs_i^m to the receivers in Zr_i^m. In the following, the proposed model for processing zone Z_i^m is explained:

$$\min\{x + (\sum_{v \in V} \sum_{k=1}^{K} t_v^{mk})/|V|\} \tag{1}$$

$$I_v^k + Iz_v^{mk} + \sum_{u \in IN_v \cap Z_i^m} t_u^{mk} \leq x \quad \forall v \in V,\, 1 \leq k \leq K,\; a_v^k = 1 \tag{2}$$

$$\sum_{v \in Zs_i^m} \sum_{u \in N_v} g_{vu}^{mw} - \sum_{v \in Zs_i^m} \sum_{u \in N_v} g_{uv}^{mw} = rt_m \quad \forall w \in Zr_i^m \tag{3}$$

$$\sum_{u \in N_v} g_{vu}^{mw} - \sum_{u \in N_v} g_{uv}^{mw} = \begin{cases} -rt_m & v = w \\ 0 \quad v \in Z_i^m / \{Zs_i^m, Zr_i^m\} \end{cases} \quad \forall w \in Zr_i^m \tag{4}$$

$$g_{uv}^{mw} \leq f_{uv}^m \quad \forall w \in Zr_i^m,\, \forall (u,v) \in Z_i^m \tag{5}$$

$$f_{uv}^m = \sum_{k=1}^{K} r_{uv}^{mk} \quad \forall\, (u,v) \in Z_i^m \tag{6}$$

$$l_{uv}^{mk} \leq a_u^k\, a_v^k \quad \forall\, (u,v) \in Z_i^m \tag{7}$$

$$t_u^{mk} \leq a_u^k \quad \forall\, u \in Z_i^m \tag{8}$$

$$r_{uv}^{mk} = cp\, l_{uv}^{mk}\, t_u^{mk} \quad \forall (u,v) \in Z_i^m,\, 1 \leq k \leq K \tag{9}$$

$$D_v + Dz_v^m + \sum_{k=1}^{K} (t_v^{mk} + \sum_{(u,v) \in Z_i^m} l_{uv}^{mk}\, t_u^{mk}) \leq o_v \quad \forall v \in Z_i^m \tag{10}$$

$$I_v^k + Iz_v^{mk} + \sum_{u \in IN_v \cap Z_i^m} t_u^{mk} \leq 1 \quad \forall v \in V,\; 1 \leq k \leq K,\; a_v^k = 1 \tag{11}$$

$$L_{uv}^k + Lz_{uv}^k + l_{uv}^{mk}\, t_u^{mk} \leq a_v^k \quad \forall (u,v) \in Z_i^m,\, 1 \leq k \leq K \tag{12}$$

$$L_{uv}^k + Lz_{uv}^k + l_{uv}^{mk}\, t_u^{mk} \leq a_u^k \quad \forall (u,v) \in Z_i^m,\, 1 \leq k \leq K \tag{13}$$

The objective function of the proposed model, which is given in Eq. (1), is twofold. It aims at keeping the network load-balanced as well as minimizing the amount of utilized bandwidth by session m. The first criterion is satisfied by minimizing the maximum load of the nodes. In this context, the load of node v over channel k is defined as the amount of its observed interference on this channel. Eqs. (1) and (2) present the linear form of the mentioned objective. The amount of utilized bandwidth by the session is divided by $|V|$ to keep both criteria at the same level.

THe constraints of Eqs. (3)–(5) model NC-based multicast routing. The logical flow balancing is expressed in Eqs. (3) and (4). In Z_i^m, the multicast data are transmitted from the nodes in Zs_i^m to the receivers in Zr_i^m. Therefore, as stated in Eq. (3), the total amount of egress logical flow of each receiver $w \in Zr_i^m$ from the nodes in Zs_i^m is equal to rt_m. The amount of ingress logical flow to receiver $w \in Zr_i^m$ is equal to rt_m as given in Eq. (4). For other nodes in Z_i^m, the amounts of ingress and egress logical flows are the same. The

Input: $G = (V, E)$, I-, D-, and L-variables, and session m.
Output: The mesh of session m, and updated I-, D-, and L-variables.

Divide the WMN into a number of zones, and sort them, as described in Section 4.1.
Set initial values of Iz-, Dz-, and Lz-variables to 0.

for *each zone* Z_i^m **do**
    Solve (1).

    if Z_i^m *is accepted* **then**
        $Iz_v^{mk} \leftarrow Iz_v^{mk} + \sum_{u \in IN_v} t_u^{mk}$ $(\forall v \in V, 1 \leq k \leq K)$.
        $Dz_v^m \leftarrow Dz_v^m + \sum_{k=1}^{K} (t_v^{mk} + \sum_{(u,v) \in Z_i^m} l_{uv}^{mk} t_u^{mk})$ $(\forall v \in V)$.
        $Lz_{uv}^{mk} \leftarrow Lz_{uv}^{mk} + l_{uv}^{mk} t_u^{mk}$ $(\forall (u,v) \in Z_i^m, 1 \leq k \leq K)$.
    end
end

if *all the zones are accepted* **then**
    $I_v^k \leftarrow I_v^k + Iz_v^{mk}$ $(\forall v \in V, 1 \leq k \leq K)$.
    $D_v \leftarrow D_v + Dz_v^m$ $(\forall v \in V)$.
    $L_{uv}^k \leftarrow L_{uv}^k + Lz_{uv}^{mk}$ $(\forall (u,v) \in E, 1 \leq k \leq K)$.
end

Figure 5. Acceptance of multicast session m.

constraint of Eq. (5) describes the relationship between the logical and physical flows. In NC-based multicast routing, the logical flows of different receivers are transmitted via physical flows. Therefore, the physical flow of session m over link (u, v) is equal to the maximum logical flow of the receivers in Zr_i^m.

The WBA is exploited by including Eqs. (6)–(9) in the model. As is formally described in Eq. (6), the physical flow of session m over link (u, v) is equal to the sum of its data rates on available channels. According to Eq. (7), if $v \in C_u^{mk}$, channel k should be assigned to both nodes u and v. Moreover, node u can transmit data over channel k if a_u^k is equal to 1. This condition is formally presented in Eq. (8). The amount of r_{uv}^{mk} is calculated by including Eq. (9) in the model. The scheduling and channel assignment constraints are expressed in Eqs. (10)–(13). The given constraint in Eq. (10) limits the total time of data transmission through node v to o_v. According to Eq. (11), the interference on node v over channel k should be less than unity. In this way, the system becomes schedulable. Constraints of Eqs. (12) and (13) state that node u can transmit data to neighboring node v on channel k if the channel is assigned to both nodes.

To investigate the acceptance of multicast session m, all of its associated zones should be processed sequentially. If there is enough bandwidth to accept Z_i^m, Iz-, Dz-, and Lz-variables are updated accordingly. After the acceptance of the session, these variables are added to the corresponding I-, D-, and L-variables. Figure 5 describes the steps of accepting multicast session m.

4.3. Time complexity analysis

The proposed scheme provides a high acceptance rate in a short time. In the following, we show that the algorithm is polynomial time and therefore is extendable to large-scale networks. The proposed model in Eq.

(1) contains binary variables to establish the parent-child relationship between the neighboring nodes. The time complexity of solving Eq. (1) is upper-bounded by $2^{\alpha}T$, where T is the time complexity of the model if l-variables are given. In this circumstance, Eq. (1) becomes a linear programming model. Thus, T will be polynomial time [28]. The total time of handling a session is of the order $O(((K|E|)/\alpha)(2^{\alpha}T))$. The key point is that parameter α, which specifies the maximum permitted number of binary variables, is constant. It is chosen by the administrator and is independent of the network parameters such as the number of the nodes and links. According to the above discussion, the time complexity of the algorithm is polynomial regarding the network parameters.

4.4. Distributed implementation of online NC-based multicast routing

Proposing a detailed and high-performance algorithm for distributed acceptance of multicast sessions in MC-MR WMNs is beyond the scope of this paper. To have an overall understanding of the distributed solution, we present a basic algorithm in the following. The suggested scheme is composed of two phases:

1) Distributed channel assignment: Using the MC-MR setting, the channels assigned to the nodes should be determined first. The proposed schemes in [29, 30] can be employed for this purpose. The main idea of these algorithms can be described in three steps: 1) Each node v assigns random channels to its radios. 2) Every node v estimates the traffic on each assigned channel k and specifies the overloaded channels. In the assumed setting, there is no traffic information in advance. Therefore, the load of node v on channel k can be estimated as the number of nodes in IN_v that select channel k (i.e. $\sum_{u \in IN_v} a_u^k$). 3) A node with an overloaded channel should switch to another channel. However, it is sufficient to change the channel of a portion of each group of interfering nodes. Therefore, the interfering nodes with overloaded channels exchange control messages to determine the node that should switch its channel.

2) Distributed NC-based multicast routing in MC-MR WMNs: The steps of this phase are described as follows:

   2-1) Distributed bandwidth estimation: The arrived session m has bandwidth requirement rt_m. To satisfy the bandwidth requirement of the session, the available bandwidth of the links should be estimated in a distributed manner. To this end, the algorithm proposed in [31] can be employed.

   2-2) Distributed NC-based multicast routing: This phase can be performed using the method proposed in [32]. This algorithm was designed for wired networks, where the interference is not a concern. As the available bandwidth of the links is estimated using [31], our problem is converted to NC-based multicast routing in wired networks.

5. Simulation results

In this section, we deal with the effectiveness of the ZBMR algorithm. First, the impact of parameter α is considered. The performance of the algorithm is measured against different amounts of this parameter to determine its most suitable value. The criterion for performance investigation is the multicast acceptance rate. Next, the impact of varying the network parameters, comprising K and the size of receiver sets of the sessions, are investigated. To this end, we compare the proposed approach to LMTR [25] and [27] under different network configurations. In the following, the algorithm presented in [27] is referred to as cross-layer multicast routing

(CLMR). The mathematical models are solved using CPLEX. The results illustrated in the diagrams are the average of 5 experiments. The transmission and interference range parameters are set to 250 m and 500 m, respectively. Parameters cp, o_v, and $maxVRecNo$ are assumed to be 54 Mbps, 4, and 1, respectively. In addition, the source nodes and receiver sets of the sessions are chosen randomly. The rate requested by each session is set to $0.012 \times cp$.

We consider some configurations with different numbers of channels and receivers. The number of channels is set to 4, 8, and 12 in the performed simulations. In addition, the multicast sessions are divided into three groups based on the size of their receiver sets. In 40-node WMNs, the multicast sessions are divided into three groups, in which the sizes of receiver sets are restricted to be fewer than 15, 15–30, and more than 30, respectively. In the same way, the multicast sessions in 60-node WMNs are partitioned into three groups, where the sizes of receiver sets are fewer than 20, 20–40, and more than 40, respectively. Table 2 lists the amounts of employed parameters in this paper.

Table 2. The amounts of employed parameters in this paper.

Parameter	Value
$\|V\|$	40, 60
Network dimensions	1500 m×1500 m ($\|V\| = 40$)
	2000 m×2000 m ($\|V\| = 60$)
Transmission range	250 m
Interference range	500 m
cp	54 Mbps
o_v	4
K	4, 8, 12
Number of sessions	200
$\|R_m\|$	[5–15], [16–30], [31–40] ($\|V\| = 40$)
	[5–20], [21–40], [41–60] ($\|V\| = 60$)
rt_m	$0.012 \times cp$
$MaxVRecNo$	1

5.1. Impact of parameter α

As shown in Section 4.3, the time complexity of ZBMR depends on α. This parameter affects the time complexity as well as the acceptance rate. Therefore, it should be determined such that these measures become reasonable. In the performed simulations in Figure 6, we examine different values of α to find its most appropriate value in various network configurations. The number of channels is set to 12 in this set of simulations. From the reported results, we can see that increasing α raises the acceptance rate. For example, consider the multicast sessions in 40-node WMNs, in which $|R_m| \leq 15$ (Figure 6a). In this case, the acceptance rate increases by 11% when α is raised from 700 to 800. In addition, as shown in Figure 6b, this criterion increases by 8% when α is raised from 600 to 700 under the assumption that $15 < |R_m| \leq 30$. Moreover, increasing α from 400 to 500 raises the acceptance rate by 4% for multicast sessions in which $30 < |R_m|$ (Figure

6c). Similar outcomes are derived for 60-node WMNs. More precisely, as depicted in Figure 6d, increasing α from 600 to 800 raises the acceptance rate by 8% for multicast sessions with less than 20 receivers in 60-node WMNs. This criterion increases by 10% when α is increased from 400 to 600 under the assumption that $20 < |R_m| \leq 40$, as shown in Figure 6e. It is also seen from Figure 6 that dense scenarios are less sensitive to the variations of α. As shown in Figures 6c and 6f, the acceptance rate improves by 6% when α is increased from 400 to 600.

Table 3 presents the running time of the reported simulations in Figure 6. This table indicates that raising α increases the time complexity of the algorithm to some extent. For example, the average time for investigating the acceptance of multicast sessions with less than 15 receivers in 40-node WMNs increases by 0.14 s when α is raised from 600 to 800. In addition, the running time increases from 0.31 s to 0.57 s for multicast sessions with $20 < |R_m| \leq 40$ in 60-node WMNs. In the following, we set α to the minimum examined value in each configuration.

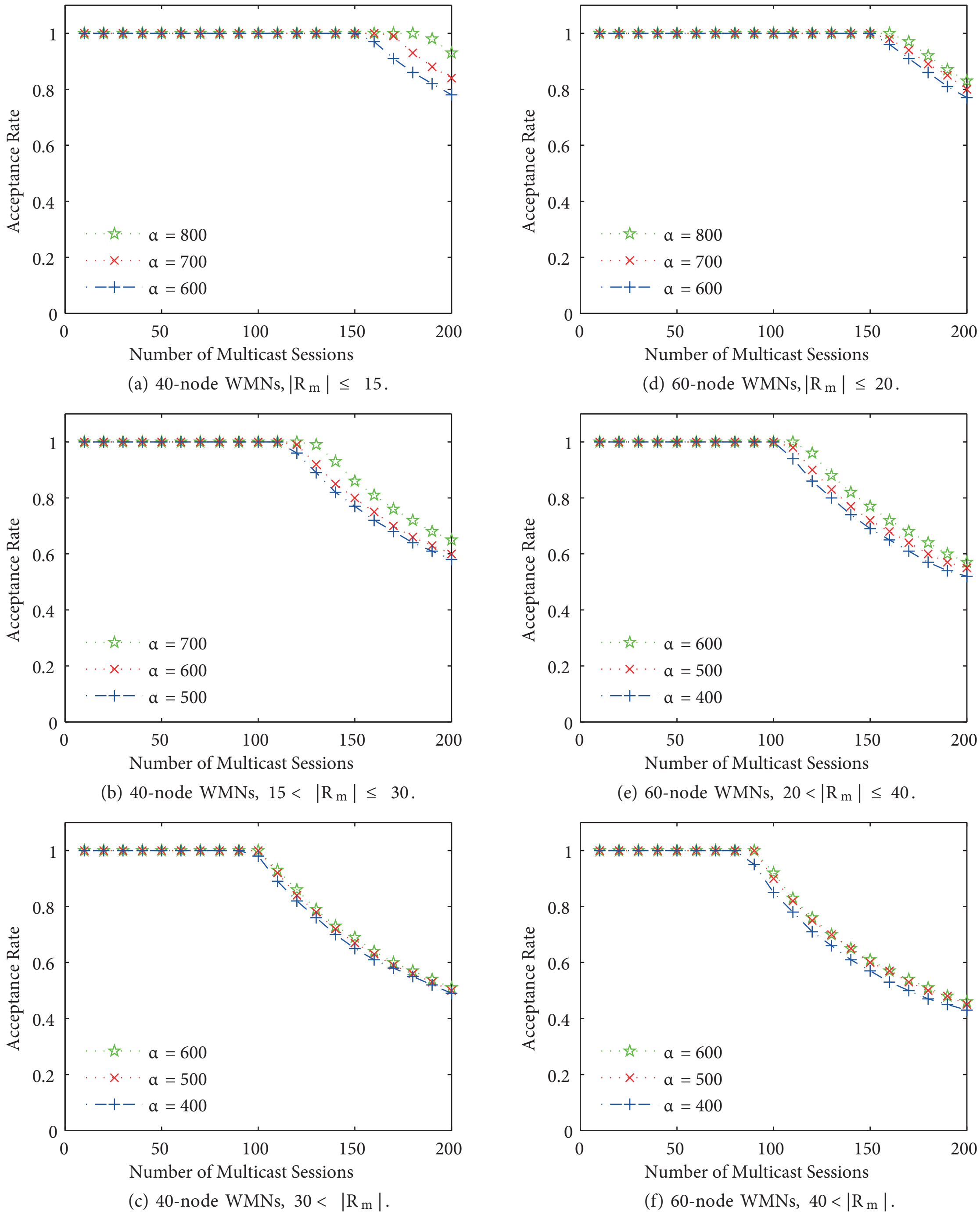
Table 3. Running time of the simulations of Figure 6.

$\|V\|$	$\|R_m\|$	α (running time)		
40	$\|R_m\| \leq 15$	600 (0.2)	700 (0.25)	800 (0.34)
	$15 < \|R_m\| \leq 30$	500 (0.24)	600 (0.41)	700 (0.67)
	$\|R_m\| \geq 30$	400 (0.27)	500 (0.38)	600 (0.7)
60	$\|R_m\| \leq 20$	600 (0.29)	700 (0.32)	800 (0.42)
	$20 < \|R_m\| \leq 40$	400 (0.31)	500 (0.37)	600 (0.57)
	$\|R_m\| \geq 40$	400 (0.34)	500 (0.54)	600 (0.73)

5.2. Impact of number of channels

One important feature of the proposed routing algorithms in the MC-MR setting is their ability to exploit channel resources. Therefore, we study the performance of the ZBMR algorithm versus different numbers of channels in this section. Figure 7 illustrates the acceptance rate of ZBMR, LMTR, and CLMR using 4, 8, and 12 channels. In this set of simulations, the size of receiver sets is chosen arbitrarily. From this figure, we can see that increasing K raises the obtained acceptance rate by the proposed algorithm. For example, as derived from Figures 7a and 7b, increasing K from 4 to 8 raises the acceptance rate of ZBMR by 45% in 40-node WMNs. In addition, raising K from 8 to 12 increases this measure by 29% (Figures 7b and 7c). Similar results are derived for 60-node WMNs. These results indicate that our algorithm uses the channel resources efficiently. In addition, from these outcomes, we can see that the increase of K from 4 to 8 has a considerable impact on the acceptance rate. On the other hand, the increase of K from 8 to 12 is less effective on the acceptance rate measure.

The obtained results in Figure 7 also indicate that ZBMR outperforms previous approaches. More precisely, as derived from Figures 7a and 7d, it improves the acceptance rate by 146% and 68% for WMNs with 4 channels in comparison to LMTR and CLMR, respectively. Similar results are obtained for WMNs with 8 channels. In this setting, our algorithm increases the acceptance rate by 135% and 53% in comparison to LMTR and CLMR, respectively (Figures 7b and 7e). From Figures 7c and 7f, we can see that the achieved acceptance rate by ZBMR is 110% and 40% more than that of LMRT and CLMR for WMNs with 12 channels. On average, ZBMR increases the acceptance rate by 130% and 55% in comparison to LMTR and CLMR, respectively.



(a) 40-node WMNs, $|R_m| \leq 15$.

(d) 60-node WMNs, $|R_m| \leq 20$.

(b) 40-node WMNs, $15 < |R_m| \leq 30$.

(e) 60-node WMNs, $20 < |R_m| \leq 40$.

(c) 40-node WMNs, $30 < |R_m|$.

(f) 60-node WMNs, $40 < |R_m|$.

Figure 6. Multicast acceptance rate versus different values of α.

5.3. Impact of the size of receiver set

In the following, the impact of the size of receiver set on the acceptance rate of ZBMR is analyzed. For this purpose, we compare our algorithm to LMTR and CLMR by varying the sizes of receiver sets. In addition, the number of channels is set to 12. The obtained results are illustrated in Figure 8. As seen from this figure, the

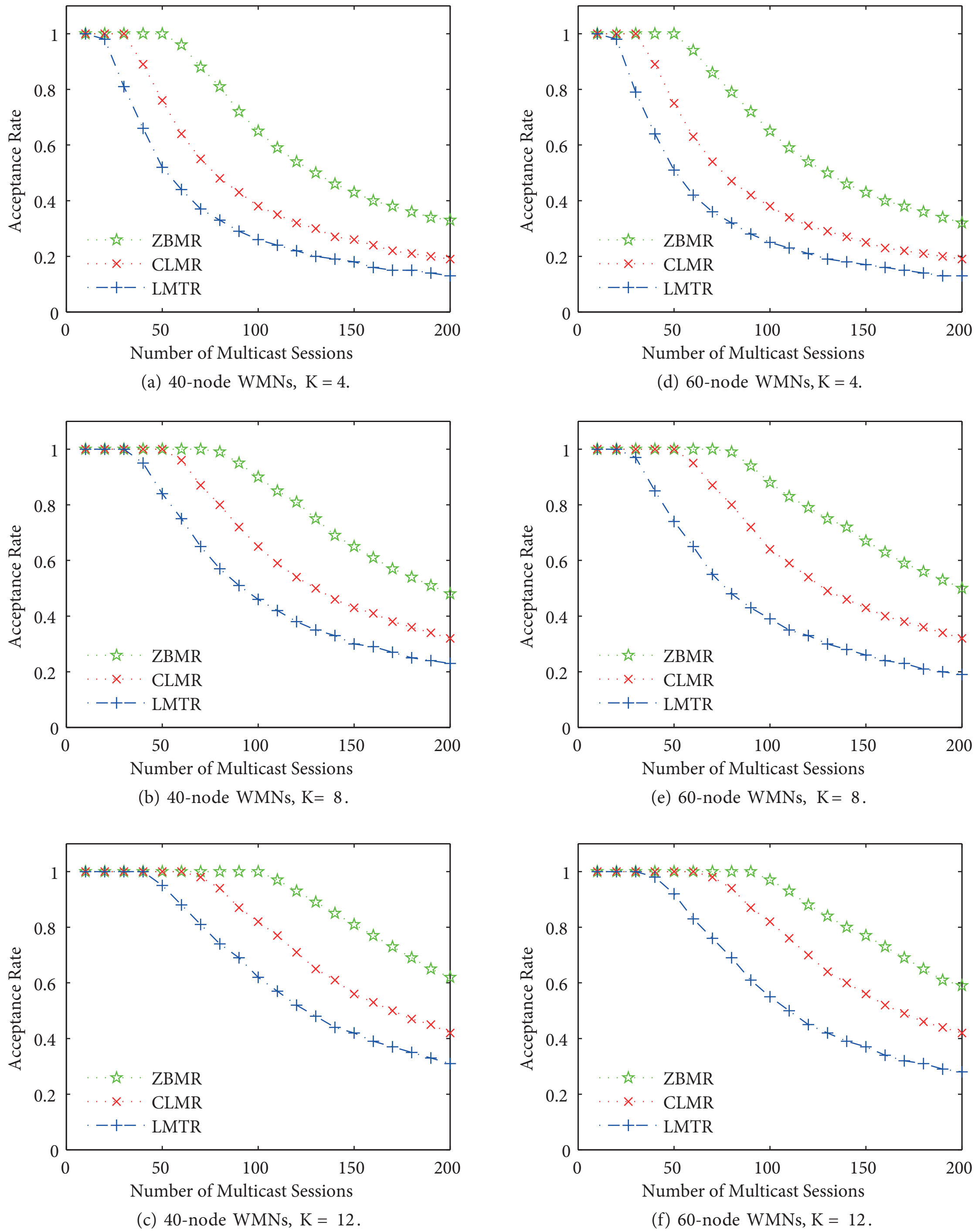
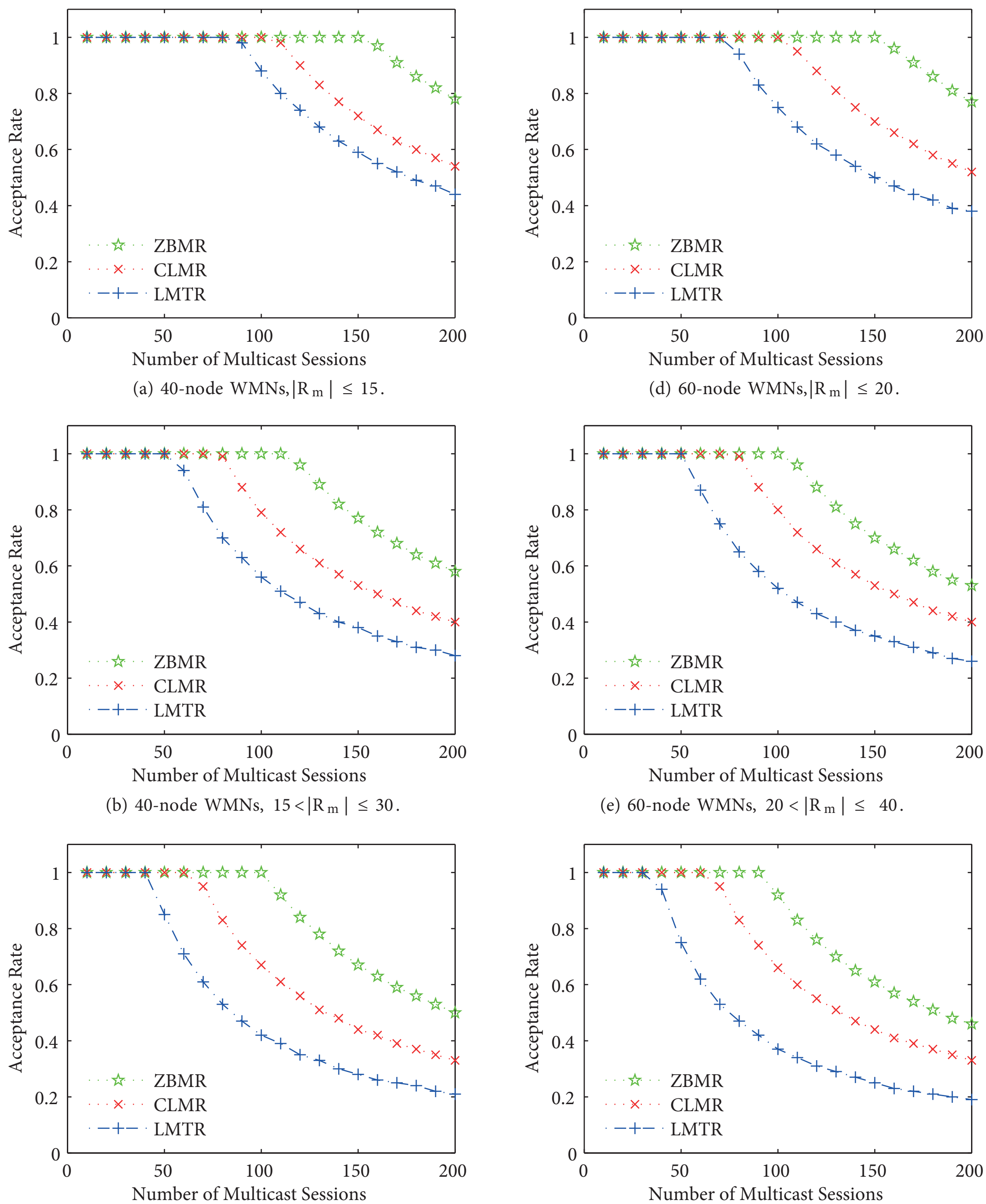


Figure 7. Multicast acceptance rate versus various number of channels.

ZBMR algorithm improves the acceptance rate considerably in comparison to the previous approaches. More precisely, it improves the acceptance rate in 40-node WMNs under the assumption that $|R_m| \leq 15$ by 78% and 44% in comparison to LMTR and CLMR, respectively (Figure 8a). From Figure 8b, it is seen that the achieved acceptance rate by ZBMR in 40-node WMNs under the assumption that $15 < |R_m| \leq 30$ is 138% and



(a) 40-node WMNs, $|R_m| \leq 15$.
(d) 60-node WMNs, $|R_m| \leq 20$.
(b) 40-node WMNs, $15 < |R_m| \leq 30$.
(e) 60-node WMNs, $20 < |R_m| \leq 40$.
(c) 40-node WMNs, $30 < |R_m|$.
(f) 60-node WMNs, $40 < |R_m|$.

Figure 8. Multicast acceptance rate versus various number of receivers.

51% more than that of LMTR and CLMR, respectively. The achieved improvement in 40-node dense WMNs is equal to 145% and 56% in comparison to the mentioned algorithms (Figure 8c).

Similar results are derived for 60-node WMNs (Figures 8d–8f). As seen from Figure 8d, our proposed algorithm outperforms the competitive algorithms in sparse scenarios. In this case, it yields 102% and 48%

more acceptance rate compared to LMTR and CLMR, respectively. These values increase to 103% and 33% under the assumption that $20 < |R_m| \leq 40$ (Figure 8e). In addition, according to Figure 8f, ZBMR increases the multicast acceptance rate by 142% and 39% in comparison to LMTR and CLMR, for multicast sessions with more than 40 receivers in 60-node WMNs. On average, the ZBMR algorithm raises this measure by 105% and 43% compared to LMTR and CLMR, respectively. The results of this section indicate that our algorithm, relative to the existing solutions, increases the acceptance rate greatly.

6. Conclusion and future work

This paper investigated online NC-based multicast routing in MC-MR WMNs, where WBA is exploited to increase the acceptance rate. Due to the high complexity of the optimal solution, we designed a polynomial time algorithm, namely ZBMR, in which the WMN is partitioned into some zones. Each zone is considered separately to investigate the possibility of data delivery to its receivers. This approach converts the time-consuming task of accepting the multicast session to some simpler tasks of zone processing. The problem that emerges in this scheme is that it may be impossible to transmit data between the zones if there is no receiver in their borders. This challenge is mitigated by selecting some nodes on the borders as virtual receivers. The experimental results validated the superiority of our algorithm in comparison to the previous approaches in the literature.

As part of our future work, we extend the ZBMR algorithm to satisfy other QoS requirements of multicast sessions including delay and jitter. This improvement is especially important for supporting multimedia applications. In addition, we plan to propose a distributed algorithm for online NC-based multicast routing in MC-MR WMNs.

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