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# Assessing wind energy potential using finite mixture distributions

**Melih Burak KOCA***, **Muhammet Burak KILIÇ**, **Yusuf ŞAHİN**
Department of Business Administration, Faculty of Economics and Administrative Sciences,
Burdur Mehmet Akif Ersoy University, Burdur, Turkey

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**Abstract:** Wind has become a popular renewable energy resource in the last two decades. Wind speed modeling is a crucial task for investors to estimate the energy potential of a region. The aim of this paper was to compare the popular unimodal wind speed distributions with their two-component mixture forms. Accordingly, Weibull, gamma, normal, lognormal distributions, and their two-component mixture forms; two-component mixture Weibull, two-component mixture gamma, two-component mixture normal, and two-component mixture lognormal distributions were employed to model wind speed datasets obtained from Belen Wind Power Plant and Gökçeada Meteorological Station. This paper also provides the comparison of gradient-based and gradient-free optimization algorithms for maximum likelihood (ML) estimators of the selected wind speed distributions. ML estimators of the distributions were obtained by using Newton–Raphson, Broyden–Fletcher–Goldfarb–Shanno, Nelder–Mead, and simulated annealing algorithms. Fit performances were compared based on Kolmogorov–Smirnov test, root mean square error, coefficient of determination ($R^2$), and power density error criteria. Results reveal that two-component mixture wind speed distributions have superiority over the unimodal wind speed distributions.

**Key words:** Finite mixture distributions, wind energy, wind speed modeling, optimization algorithms

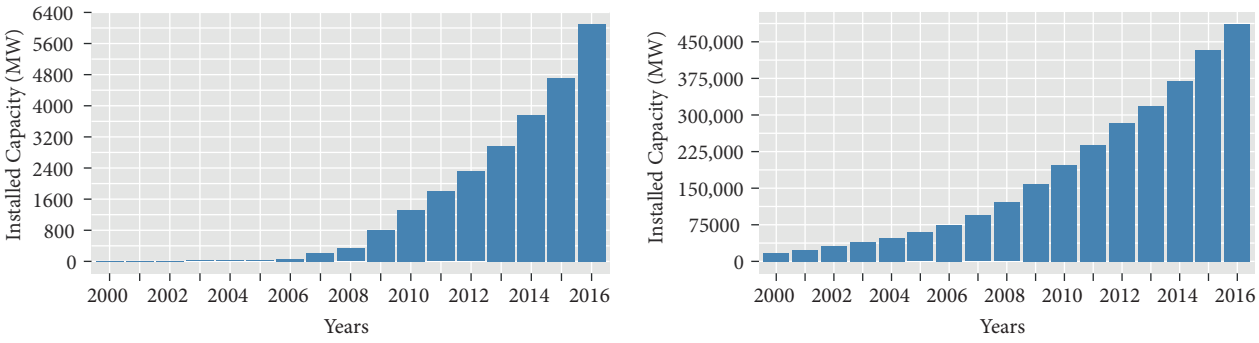
## 1. Introduction

As demand of energy increases with the population growth, renewable energy resources have started to play an important role in meeting the energy needs. Recently, the fossil fuel reserves have gradually decreased and the interest in the clean and renewable energy resources has increased [1–4]. There are several forms of renewable energy such as hydro energy, wind energy, solar energy, geothermal energy, and biomass energy. Among these renewable energy resources, wind energy has become a growing interest in recent years. There are significant increases in the number of wind energy plants installed in Turkey and the whole world. By the end of 2016, Turkey had 6106.05 MW installed wind energy capacity [5] and the wind energy potential of Turkey was predicted as 48,000 MW[1], which is more than 11% of the global installed wind energy capacity in 2016. Global wind power had an average annual growth rate of 22.1% between 1990 and 2015 [5]. Figures 1a and 1b show the cumulative installed wind energy capacity of Turkey and the world.

The different events of solar heating on the Earth's surface cause temperature and pressure differences in different regions. These differences of pressure lead air to move to other regions with low air pressure. This

*Correspondence: mbkoca@mehmetakif.edu.tr

[1]Republic of Turkey Ministry of Energy and Natural Resources. (2019). Wind [online]. Website http://www.enerji.gov.tr/en-US/Pages/Wind [accessed 25 January 2019]




(a) Turkey's installed wind energy capacity

(b) Global installed wind energy capacity

**Figure 1**. Cumulative installed wind energy capacity of Turkey and the world.

movement generates air currents which carry kinetic energy. Wind turbines convert the kinetic energy carried by air currents to mechanical or electrical energy [6]. Wind is a renewable, clean, and never-ending source of energy; however, the initial investment cost of turbines required to produce energy is high. Therefore, estimating the energy potential of the region is a critical prerequisite for entrepreneurs who are considering investment. It is also important for choosing the proper turbine model. Wind speed is the most important variable in the calculation of wind energy potential that is highly dependent on wind speed data [7, 8]. Wind speed modeling and estimation of wind energy potential have become the subject of many works in recent years.

Unimodal wind speed distributions are being highly used for wind speed modeling and wind power potential evaluation studies. Especially, two-parameter Weibull distribution is the most commonly used wind speed distribution due to its simplicity and flexibility in many wind energy studies. Beside Weibull distribution, other unimodal distributions such as Rayleigh, gamma, normal, and lognormal are also being used.

In the literature of unimodal wind speed distributions, Safari [6] compared Weibull, Rayleigh, lognormal, normal, and gamma distributions for modeling the wind speed data and found that Weibull, lognormal, and gamma distributions provide better fit for the data observed. Kaplan [9], Köse et al. [10], Çelik [11], Akpınar and Akpınar [12], and Gökçek et al. [13] employed Weibull and Rayleigh distributions to evaluate the wind energy potential in their studies. Hou et al. [7] showed that gamma and lognormal distributions performed better than Weibull distribution in some occasions. Akgül et al. [14] suggested that inverse Weibull distribution is a good alternative to the conventional Weibull distribution. Kantar and Usta showed that the minimum cross entropy [15] and the upper-truncated Weibull distribution [16] are better alternatives to the Weibull distribution. Sohoni et al. [8] compared Weibull, Rayleigh, gamma, lognormal, and inverse Gaussian distributions. They revealed that Weibull, Rayleigh, and gamma distributions represent the data better. Mert and Karakuş [17] employed 4-parameter Burr, 3-parameter generalized gamma, and Weibull distributions to analyze wind speed data obtained from Hatay, Turkey. Arslan et al. [18] proposed generalized Lindley and power Lindley distributions as alternative to Weibull distribution. Kantar et al. [19] employed the extended generalized Lindley distribution (EGLD) for wind speed analysis and they revealed that EGLD outperforms well-known unimodal distributions including Weibull, Rayleigh, gamma, and lognormal. Even though unimodal distributions are highly popular in wind speed modeling studies, they remain incapable of providing a good fit for the observations from the heterogeneous wind regime. Therefore, finite mixture distributions are also being employed.

In the literature of mixture wind speed distributions, Morgan et al. [20] stated that when Weibull distribution is compared with the more complex models such as two-component mixture Weibull (WW),

Kappa and Wakeby, it provides poor fit. Kollu et al. [21] described three mixture distribution functions including Weibull-extreme value distribution (GEV), Weibull-lognormal, and GEV-lognormal, they revealed that mixture distributions including GEV are superior to conventional Weibull, WW, gamma, and lognormal distributions. Akpınar and Akpınar [2] employed Weibull, WW, the maximum entropy principle (MEP) and the singly truncated from below normal Weibull mixture (TNW) distributions. Carta and Ramirez [22] compared Weibull distribution with TNW and WW distributions. Usta and Kantar [23] employed skewed generalized error distribution (SGED), and skewed t distribution (STD) in wind speed modeling. They showed that SGED and STD performed better than two- and three-parameter Weibull distributions. Chang [24] compared 6 distributions including Weibull, mixture of gamma and normal, NN, mixture of normal and Weibull, WW, and MEP. Akdağ et al. [25] and Jaramillo and Borja [26] showed the superiority of WW to conventional Weibull distribution in their studies. Shin et al. [27] utilized various homogeneous and heterogeneous mixture distributions, and they found that Weibull-extreme value type-one mixture distribution is the most appropriate distribution for the wind speed data observed in the UAE. Mazzeo et al. [28] proposed a mixture of two truncated normal distributions to model wind speed data obtained from several locations.

The main contribution of this paper is the comparison of maximum likelihood (ML) estimators of the unimodal and two-component mixture wind speed distributions by using Newton–Raphson (NR), Broyden–Fletcher–Goldfarb–Shanno (BFGS), Nelder–Mead (NM), and simulated annealing (SA) optimization algorithms for the datasets obtained from an actual wind farm and a meteorological station. Our motivation can be seen from Figures 2a–2c, where the empirical wind speed density functions are highly heterogeneous. To assess the fit performances of wind speed distribution functions, Kolmogorov–Smirnov (K-S) test, root mean square error (RMSE), coefficient of determination ($R^2$), and power density error (PDE) were considered as judgment criteria. Computation times were also included for a fair assessment. The rest of this paper is organized as follows: Section 2 describes the conventional unimodal and two-component mixture wind speed distributions, parameter estimation method, optimization algorithms, and model assessment criteria. A detailed information about the datasets and assessments of chosen wind speed distributions are presented in Section 3. Section 4 presents some aspects of the study. Finally, Section 5 includes the concluding remarks.

## 2. Materials and methods

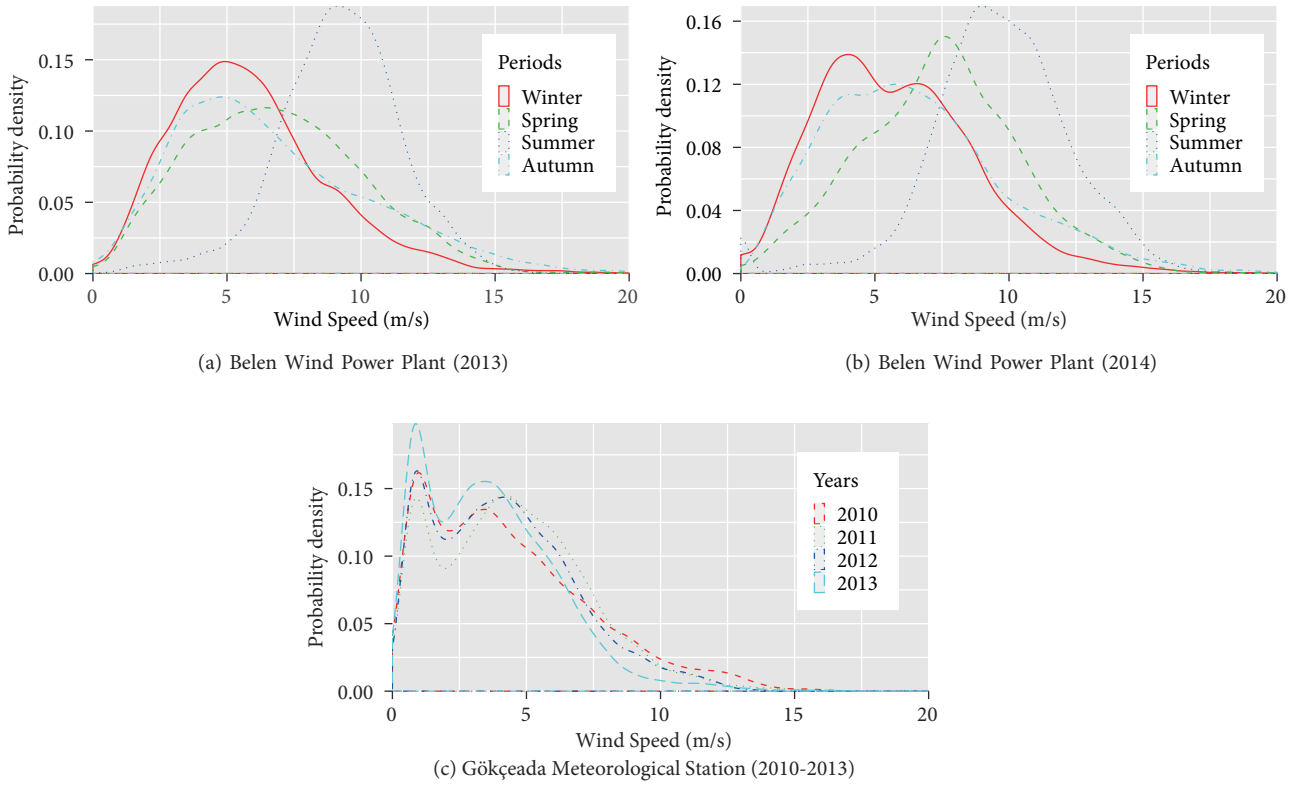
Numerous wind speed distributions have been used in previous studies. The most common ones, Weibull, gamma, normal, and lognormal distributions, and their two-component mixture forms were considered in this paper. To find the unknown parameters of wind speed distributions, we used the ML approach which is the most common estimation technique in wind energy studies. NR, BFGS, NM, and SA algorithms were used to obtain ML estimators of the selected wind speed distributions.

### 2.1. Unimodal wind speed distributions

#### 2.1.1. The Weibull distribution

One of the most frequently used distributions in wind speed modeling is the Weibull distribution. Let $V$ be a random variable, the probability density function (*pdf*), and cumulative distribution function (*cdf*) of the Weibull distribution are respectively given by:

$$f(v;k,c) = \frac{k}{c}\left(\frac{v}{c}\right)^{k-1} \exp\left[-\left(\frac{v}{c}\right)^{k}\right] \tag{1}$$




(a) Belen Wind Power Plant (2013)

(b) Belen Wind Power Plant (2014)

(c) Gökçeada Meteorological Station (2010-2013)

**Figure 2**. Empirical probability density functions.

and

$$F\left(v;k,c\right)=1-\exp\left[-\left(\frac{v}{c}\right)^{k}\right], \tag{2}$$

where $v$ is the wind speed data, $k$ and $c$ are the shape and scale parameters, respectively. ML estimators for $k$ and $c$ can be obtained by maximizing the log-likelihood function as shown below:

$$\ln L_{Weibull}=\sum_{i=1}^{n}\ln\{f\left(v_i;k,c\right)\}, \tag{3}$$

where $v_i$ denotes the wind speed observations and $n$ is the number of observations. The ML estimators of $k$ and $c$ are respectively given below:

$$\hat{k}=\left[\frac{\sum_{i=1}^{n}v_i^{\hat{k}}\ln\left(v_i\right)}{\sum_{i=1}^{n}v_i^{\hat{k}}}-\frac{\sum_{i=1}^{n}\ln\left(v_i\right)}{n}\right]^{-1} \tag{4}$$

and

$$\hat{c}=\left[\frac{1}{n}\sum_{i=1}^{n}v_i^{\hat{k}}\right]^{1/\hat{k}}. \tag{5}$$

However, the ML estimator of $k$ can be computed by iterative methods.

### 2.1.2. Gamma distribution

The *pdf* and *cdf* of gamma distribution are respectively given below:

$$f(v;\alpha,\beta) = \frac{v^{\alpha-1}}{\beta^{\alpha}\Gamma(\alpha)} \exp\left[-\frac{v}{\beta}\right] \tag{6}$$

and

$$F(v;\alpha,\beta) = \int \frac{v^{\alpha-1}}{\beta^{\alpha}\Gamma(\alpha)} \exp\left[-\frac{v}{\beta}\right] dv, \tag{7}$$

where $\alpha$ and $\beta$ are the shape and scale parameters respectively, and $\Gamma(\alpha)$ is the gamma function. The log-likelihood function of gamma distribution is given below:

$$\ln L_{Gamma} = \sum\nolimits_{i=1}^{n} \ln\{f(v_i;\alpha,\beta)\} \tag{8}$$

### 2.1.3. Normal distribution

Normal distribution has two parameters: $\mu$ and $\sigma$ that are called mean and variance, respectively. The *pdf* and *cdf* of normal distribution are respectively given below:

$$f(v;\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[\frac{-(v-\mu)^2}{2\sigma^2}\right] \tag{9}$$

and

$$F(v;\mu,\sigma) = \frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{v-\mu}{\sigma\sqrt{2}}\right)\right]. \tag{10}$$

The log-likelihood function of normal distribution is given by:

$$\ln L_{Normal} = \sum\nolimits_{i=1}^{n} \ln\{f(v_i;\mu,\sigma)\}. \tag{11}$$

### 2.1.4. Lognormal distribution

The *pdf* and *cdf* of lognormal distribution are respectively given by:

$$f(v;\mu,\sigma) = \frac{1}{\sigma v\sqrt{2\pi}} \exp\left[-\frac{(\ln v-\mu)^2}{2\sigma^2}\right] \tag{12}$$

and

$$F(v;\mu,\sigma) = \frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{\ln v-\mu}{\sigma\sqrt{2}}\right)\right], \tag{13}$$

where $\mu$ and $\sigma$ are the mean and variance of the $\ln v$, respectively. The log-likelihood function of lognormal distribution is given by:

$$\ln L_{Lognormal} = \sum\nolimits_{i=1}^{n} \ln\{f(v_i;\mu,\sigma)\} \tag{14}$$

### 2.2. Mixture wind speed distributions

#### 2.2.1. Two-component mixture Weibull distribution

The *pdf* and the *cdf* of WW are respectively given by:

$$f(v;p,k_1,c_1,k_2,c_2)=p\frac{k_1}{c_1}\left(\frac{v}{c_1}\right)^{k_1-1}\exp\left[-\left(\frac{v}{c_1}\right)^{k_1}\right]+(1-p)\frac{k_2}{c_2}\left(\frac{v}{c_2}\right)^{k_2-1}\exp\left[-\left(\frac{v}{c_2}\right)^{k_2}\right] \tag{15}$$

and

$$F(v;p,k_1,c_1,k_2,c_2)=p\left\{1-\exp\left[-\left(\frac{v}{c_1}\right)^{k_1}\right]\right\}+(1-p)\left\{1-\exp\left[-\left(\frac{v}{c_2}\right)^{k_2}\right]\right\}, \tag{16}$$

where $k_1$ and $c_1$ are the shape and scale parameters of the first Weibull component. Similarly, $k_2$ and $c_2$ are the shape and scale parameters of the second Weibull component. $p$ is the mixing parameter where $0<p<1$. The log-likelihood function of WW is given below:

$$\ln L_{WW}=\sum\nolimits_{i=1}^{n}\ln\left\{pf(v_i;k_1,c_1)+(1-p)f(v_i;k_2,c_2)\right\} \tag{17}$$

#### 2.2.2. Two-component mixture gamma distribution

The *pdf* and *cdf* of GG are respectively given as follows:

$$f(v;p,\alpha_1,\beta_1,\alpha_2,\beta_2)=p\frac{v^{\alpha_1-1}}{\beta_1^{\alpha_1}\Gamma(\alpha_1)}\exp\left[-\frac{v}{\beta_1}\right]+(1-p)\frac{v^{\alpha_2-1}}{\beta_2^{\alpha_2}\Gamma(\alpha_2)}\exp\left[-\frac{v}{\beta_2}\right] \tag{18}$$

and

$$F(v;p,\alpha_1,\beta_1,\alpha_2,\beta_2)=p\int\frac{v^{\alpha_1-1}}{\beta_1^{\alpha_1}\Gamma(\alpha_1)}\exp\left[-\frac{v}{\beta_1}\right]dv+(1-p)\int\frac{v^{\alpha_2-1}}{\beta_2^{\alpha_2}\Gamma(\alpha_2)}\exp\left[-\frac{v}{\beta_2}\right]dv, \tag{19}$$

where $\alpha_1$ and $\beta_1$ are the shape and scale parameters of the first gamma component, $\alpha_2$ and $\beta_2$ are the shape and scale parameters of the second gamma component respectively. $p$ is the mixing parameter where $0<p<1$. The log-likelihood function of GG is given below:

$$\ln L_{GG}=\sum\nolimits_{i=1}^{n}\ln\{pf(v_i;\alpha_1,\beta_1)+(1-p)f(v_i;\alpha_2,\beta_2)\}. \tag{20}$$

#### 2.2.3. Two-component mixture normal distribution

The *pdf* and the *cdf* of NN are respectively given by:

$$f(v;\mu_1,\sigma_1,\mu_2,\sigma_2)=p\frac{1}{\sigma_1\sqrt{2\pi}}\exp\left[\frac{-(v-\mu_1)^2}{2\sigma_1^2}\right]+(1-p)\frac{1}{\sigma_2\sqrt{2\pi}}\exp\left[\frac{-(v-\mu_2)^2}{2\sigma_2^2}\right] \tag{21}$$

and

$$F(v;p,\mu_1,\sigma_1,\mu_2,\sigma_2)=\frac{p}{2}\left[1+\operatorname{erf}\left(\frac{v-\mu_1}{\sigma_1\sqrt{2}}\right)\right]+\frac{(1-p)}{2}\left[1+\operatorname{erf}\left(\frac{v-\mu_2}{\sigma_2\sqrt{2}}\right)\right], \tag{22}$$

where $\mu_1$ and $\sigma_1$ are the mean and variance of the first normal component respectively. Similarly, $\mu_2$ and $\sigma_2$ are the mean and variance of the second normal component. $p$ is the mixing parameter and $0<p<1$. The log-likelihood function of NN is given as follows:

$$\ln L_{NN}=\sum\nolimits_{i=1}^{n}\ln\{pf(v_i;\mu_1,\sigma_1)+(1-p)f(v_i;\mu_2,\sigma_2)\} \tag{23}$$

#### 2.2.4. Two-component mixture lognormal distribution

The *pdf* and the *cdf* of LL are respectively given as follows:

$$f(v;\mu_1,\sigma_1,\mu_2,\sigma_2) = p\frac{1}{\sigma_1 v\sqrt{2\pi}}\exp\left[\frac{-(\ln v-\mu_1)^2}{2\sigma_1^2}\right] + (1-p)\frac{1}{\sigma_2 v\sqrt{2\pi}}\exp\left[\frac{-(\ln v-\mu_2)^2}{2\sigma_2^2}\right] \tag{24}$$

and

$$F(v;p,\mu_1,\sigma_1,\mu_2,\sigma_2) = \frac{p}{2}\left[1+\operatorname{erf}\left(\frac{\ln v-\mu_1}{\sigma_1\sqrt{2}}\right)\right] + \frac{(1-p)}{2}\left[1+\operatorname{erf}\left(\frac{\ln v-\mu_2}{\sigma_2\sqrt{2}}\right)\right], \tag{25}$$

where $\mu_1$ and $\sigma_1$ are the mean and variance of the first lognormal component, respectively. $\mu_2$ and $\sigma_2$ are the mean and variance of the second lognormal component, $p$ is the mixing parameter where $0<p<1$. The log-likelihood function of LL is given by:

$$\ln L_{LL} = \sum_{i=1}^{n}\ln\{pf(v_i;\mu_1,\sigma_1)+(1-p)f(v_i;\mu_2,\sigma_2)\}. \tag{26}$$

### 2.3. Maximum likelihood estimation

The main idea of the ML approach is based on the maximization of the log-likelihood function given in Eqs. (3), (8), (11), (14), (17), (20), (23), and (26). ML estimations are the solutions of the first order partial derivatives of log-likelihood functions and they can be solved numerically using optimization techniques. To obtain ML estimators of the parameters, we use maxLik function which allows to select different optimization algorithms including NR, BFGS, NM, and SA in R software [29].

#### 2.3.1. The Newton–Raphson algorithm

NR is a simple and powerful algorithm. Let $x_0$ be the initial value for a root of a differentiable function $f$, $\nabla$ is the gradient vector and $H$ is the Hessian matrix (a matrix obtained from second partial derivatives of $f$). First iteration will be

$$x_1 = x_0 - H^{-1}(x_0)\nabla(x_0) \tag{27}$$

iterations more generally

$$x_{(i+1)} = x_i - H^{-1}(x_i)\nabla(x_i), \tag{28}$$

where $i$ is the number of iterations. Iterations will stop when $||x_{(i+1)}-x_i<c||$ where $c$ is a predefined constant. NR is a fast algorithm, it converges quadratically; however, the success of the algorithm highly depends on the initial values. Another major disadvantage of NR is that it requires to compute Hessian matrix and gradient vector at each iteration.

#### 2.3.2. The Broyden–Fletcher–Goldfarb–Shanno algorithm

BFGS [30–33] is a very powerful technique in solving unconstrained optimization problems. BFGS does not explicitly compute the Hessian but it approximates; therefore, it is a fast algorithm. BFGS updates the Hessian at each iteration by using the following formula:

$$V_{k+1} = \left[I - \frac{s_k y_k^T}{s_k^T y_k}\right] V_k \left[I - \frac{y_k s_k^T}{s_k^T y_k}\right] + \frac{s_k s_k^T}{s_k^T y_k}, \tag{29}$$

where $k$ is the iteration number, $x_k$ is the starting point, $V_k \approx H_k^{-1}$, $s_k$ is the step taken from $x_k$ and $y_k = H_k s_k$. Since BFGS stores $V_k$ at each iteration, it requires sufficient computer memory.

#### 2.3.3. The Nelder–Mead algorithm

NM is a gradient-free, unconstrained nonlinear optimization algorithm that was proposed in 1965 by Nelder and Mead [34]. NM uses a simplex to find an optimal solution for the $n$-dimensional optimization problems. NM starts with a set of $n+1$ points which are the vertices of working simplex. Once the initial simplex is constructed, the objective function is evaluated at each vertices to test if the convergence criterion is satisfied, if not, NM transforms the simplex by using reflection, expansion, contraction, and shrinking operators. NM can be easily implemented and gives significant results in the first few iterations; however, it is not efficient for the large problems for which it does not guarantee an optimal solution.

#### 2.3.4. The simulated annealing algorithm

Annealing is the process of changing the internal structure of the metals by heating them up to the melting point and slowly cooling into a minimum energy crystalline structure. SA is a gradient-free, heuristic search algorithm that was inspired by the annealing process in metal works and adapted to optimization problems by Kirkpatrick et al. [35]. The algorithm does not only accept the solutions that decrease the objective function, but also accepts the solutions that increase the objective function with a probability $p = e^{-\Delta/t}$ in order to avoid getting stuck in local minima, where $\Delta$ is the increase in the objective function and $t$ is the control parameter. Probability of accepting bad solutions systematically decreases with the iterations. Similar to NM, SA tends to be slow for large problems and does not guarantee an optimal solution.

In the following section, the performances of the wind speed distributions and optimization algorithms are evaluated via model assessment criteria.

### 2.4. Model assessment

Wind speed distributions used in this study were evaluated based on K-S test, RMSE, $R^2$, and PDE criteria. K-S test was used for deciding whether a sample belongs to a population with a specific distribution. Let $F_0(x)$ be the population cumulative distribution and $S_N(x)$ be the observed cumulative step function of sample, then test statistic $d$ is the maximum difference between the empirical and analytical cumulative distributions [36].

$$d = \max |F_0(x) - S_N(x)|. \tag{30}$$

RMSE indicates the deviation between the observed data points and model's predicted values. $R^2$ shows how well the model fits the observed data. RMSE and $R^2$ are respectively given below:

$$RMSE = \left[ \frac{\sum_{i=1}^{n} \left( \hat{F}(X_{(i)}) - \frac{i}{n+1} \right)^2}{n} \right]^{1/2} \tag{31}$$

and

$$R^2 = 1 - \frac{\sum_{i=1}^{n} \left( \hat{F}(X_{(i)}) - \frac{i}{n+1} \right)^2}{\sum_{i=1}^{n} \left( \hat{F}(X_i) - \bar{\hat{F}}(X_i) \right)^2}, \tag{32}$$

where $\hat{F}$ is the estimated $cdf$, $X_{(i)}$ is the $i$-th order statistics, $\bar{\hat{F}} = \sum_{i=1}^{n} \hat{F}_i/n$, and $n$ is the number of wind speed observations [18]. PDE is described as follows:

$$PDE = \left| \frac{P_R - P_D}{P_R} \right| \times 100, \tag{33}$$

where $P_R$ is the wind power density computed from the actual wind speed observations. $P_D$ is the wind power density obtained from a theoretical density function. $P_R$ and $P_D$ are respectively given by:

$$P_R = \frac{1}{2}\rho \frac{1}{n} \sum_{i=1}^{n} v_i^3 \tag{34}$$

and

$$P_D = \frac{1}{2}\rho \int_0^{\infty} v^3 f_D(v) dv. \tag{35}$$

Lower K-S test statistic, RMSE and PDE values, and higher $R^2$ value indicate better fit performance.

## 3. Results

### 3.1. Seasonal wind speed analysis

The seasonal wind speed observations were gauged at 80 m height and collected on 10-min basis in Belen Wind Power Plant located in Hatay region of Turkey. Geographical coordinates of the plant is 36°28′42.2″N, 36°12′45.0″E. A preprocessing stage was not employed to the dataset. Table 1 shows the summary of wind speed for each season and year. Accordingly, summer months are more convenient for wind power generation.

**Table 1**. Descriptive statistics of the wind speed data (m/s) obtained from Belen Wind Power Plant.

| | 2013 | | | | 2014 | | | |
|---|---|---|---|---|---|---|---|---|
| Period | Mean | Std. Dev. | Min | Max | Mean | Std. Dev. | Min | Max |
| Winter | 6.83 | 3.75 | 0.40 | 24.90 | 6.58 | 3.44 | 0.40 | 22.20 |
| Spring | 6.89 | 3.18 | 0.40 | 22.50 | 7.48 | 3.01 | 0.40 | 24.40 |
| Summer | 9.20 | 2.28 | 0.60 | 18.40 | 9.79 | 2.42 | 0.70 | 19.10 |
| Autumn | 5.89 | 2.91 | 0.40 | 21.90 | 5.85 | 2.88 | 0.40 | 21.00 |
| Annual | 7.29 | 3.33 | 0.40 | 24.90 | 7.38 | 3.26 | 0.40 | 24.40 |

Figure 3 presents the frequency of wind directions and the average wind speed for each direction and calm (wind speed $<2$ m/s). Maximum occurrence is at NNW with the average wind speed of 7.83 m/s.

The parameter estimations, K-S test statistic, RMSE, $R^2$ values, and the computation times of the wind speed distributions obtained by using four different optimization algorithms are presented in Tables 2 and 3. According to the results of K-S test statistic, WW was the most successful distribution in 2013. WW provided the best fit in winter and spring by using BFGS, and in autumn by using NM. NN was the second-best distribution and provided the best fit in summer by using NR. When the results were evaluated according to the RMSE and $R^2$ criteria, WW provided the best fit in winter by using NR and in spring by using BFGS. NN and LL provided the best fit by using NR in summer and autumn, respectively. According to the evaluation based on PDE, LL using NM and SA provided better modeling in winter and autumn, respectively.

In 2014, GG provided the best fit in all months according to K-S test statistic, RMSE, and $R^2$ criteria. For the K-S test statistic, GG provided better modeling by using NM in winter and spring, and NR in summer

**Table 2**. Parameter estimations and model evaluation results for the dataset obtained from Belen Wind Power Plant in 2013.

| | | Winter | | | | Spring | | | | Summer | | | | Autumn | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Model | | NR | BFGS | NM | SA | NR | BFGS | NM | SA | NR | BFGS | NM | SA | NR | BFGS | NM | SA |
| Weibull | $\hat{k}$ | 1.9251 | 1.9251 | 1.9250 | 1.9248 | 2.3031 | 2.3031 | 2.3029 | 2.3035 | 4.5081 | 4.5081 | 4.5076 | 4.5065 | 2.1396 | 2.1396 | 2.1397 | 2.1395 |
| | $\hat{c}$ | 7.7204 | 7.7204 | 7.7205 | 7.7215 | 7.7814 | 7.7814 | 7.7816 | 7.7794 | 10.0561 | 10.0561 | 10.0562 | 10.0546 | 6.6577 | 6.6577 | 6.6573 | 6.6563 |
| | K-S | 0.0400 | 0.0400 | 0.0400 | 0.0401 | 0.0145 | 0.0145 | 0.0145 | 0.0147 | 0.0387 | 0.0387 | 0.0387 | 0.0389 | 0.0361 | 0.0361 | 0.0361 | 0.0360 |
| | RMSE | 0.0196 | 0.0196 | 0.0196 | 0.0196 | 0.0057 | 0.0057 | 0.0057 | 0.0058 | 0.0194 | 0.0194 | 0.0194 | 0.0194 | 0.0163 | 0.0163 | 0.0162 | 0.0162 |
| | $R^2$ | 0.9953 | 0.9953 | 0.9953 | 0.9952 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9950 | 0.9950 | 0.9950 | 0.9950 | 0.9966 | 0.9966 | 0.9966 | 0.9966 |
| | PDE | 20.6890 | 20.6890 | 20.6890 | 20.6890 | 18.3722 | 18.3722 | 18.3722 | 18.3722 | 18.5411 | 18.5411 | 18.5411 | 18.5411 | 20.1933 | 20.1933 | 20.1933 | 20.1933 |
| | Time (s) | 0.1222 | 0.1173 | 0.0862 | 8.3316 | 0.1254 | 0.1267 | 0.0901 | 9.6447 | 0.9391 | 0.1741 | 0.0937 | 9.7885 | 0.1209 | 0.1220 | 0.0791 | 9.3253 |
| Gamma | $\hat{\alpha}$ | 3.2279 | 3.2277 | 3.2282 | 3.2296 | 4.0351 | 4.0314 | 4.0343 | 4.0424 | 12.9250 | 12.9287 | 12.9329 | 12.9499 | 3.9184 | 3.9183 | 3.9183 | 3.9151 |
| | $\hat{\beta}$ | 0.4728 | 0.4728 | 0.4728 | 0.4729 | 0.5856 | 0.5851 | 0.5855 | 0.5863 | 1.4044 | 1.4048 | 1.4055 | 1.4072 | 0.6657 | 0.6657 | 0.6656 | 0.6648 |
| | K-S | 0.0196 | 0.0196 | 0.0196 | 0.0194 | 0.0490 | 0.0491 | 0.0490 | 0.0484 | 0.0784 | 0.0784 | 0.0785 | 0.0783 | 0.0280 | 0.0281 | 0.0280 | 0.0278 |
| | RMSE | 0.0073 | 0.0073 | 0.0073 | 0.0073 | 0.0257 | 0.0257 | 0.0257 | 0.0253 | 0.0447 | 0.0447 | 0.0447 | 0.0446 | 0.0111 | 0.0111 | 0.0111 | 0.0110 |
| | $R^2$ | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9921 | 0.9921 | 0.9921 | 0.9923 | 0.9712 | 0.9712 | 0.9711 | 0.9713 | 0.9985 | 0.9985 | 0.9985 | 0.9985 |
| | PDE | 16.0788 | 16.0788 | 16.0788 | 16.0788 | 9.4608 | 9.4608 | 9.4608 | 9.4608 | 13.9351 | 13.9351 | 13.9351 | 13.9351 | 15.9369 | 15.9369 | 15.9369 | 15.9369 |
| | Time (s) | 0.2488 | 0.1668 | 0.1414 | 20.6883 | 0.2853 | 0.1933 | 0.2013 | 24.3288 | 3.4912 | 0.2416 | 0.1401 | 16.2806 | 0.2782 | 0.2020 | 0.1803 | 23.4487 |
| Normal | $\hat{\mu}$ | 6.8269 | 6.8269 | 6.8265 | 6.8248 | 6.8908 | 6.8908 | 6.8908 | 6.8903 | 9.2030 | 9.2031 | 9.2033 | 9.2022 | 5.8863 | 5.8862 | 5.8868 | 5.8853 |
| | $\hat{\sigma}$ | 3.7529 | 3.7529 | 3.7530 | 3.7513 | 3.1806 | 3.1806 | 3.1808 | 3.1815 | 2.2827 | 2.2828 | 2.2829 | 2.2822 | 2.9086 | 2.9086 | 2.9083 | 2.9077 |
| | K-S | 0.0870 | 0.0870 | 0.0870 | 0.0868 | 0.0379 | 0.0379 | 0.0379 | 0.0378 | 0.0316 | 0.0316 | 0.0316 | 0.0317 | 0.0635 | 0.0635 | 0.0635 | 0.0633 |
| | RMSE | 0.0485 | 0.0485 | 0.0485 | 0.0484 | 0.0193 | 0.0193 | 0.0192 | 0.0192 | 0.0159 | 0.0159 | 0.0159 | 0.0159 | 0.0361 | 0.0361 | 0.0362 | 0.0361 |
| | $R^2$ | 0.9711 | 0.9711 | 0.9711 | 0.9713 | 0.9957 | 0.9957 | 0.9957 | 0.9957 | 0.9967 | 0.9967 | 0.9968 | 0.9967 | 0.9835 | 0.9835 | 0.9835 | 0.9836 |
| | PDE | 24.5109 | 24.5109 | 24.5109 | 24.5109 | 20.4666 | 20.4666 | 20.4666 | 20.4666 | 18.0519 | 18.0519 | 18.0519 | 18.0519 | 23.1526 | 23.1526 | 23.1526 | 23.1526 |
| | Time (s) | 0.1042 | 0.0744 | 0.0470 | 4.7736 | 0.1251 | 0.0858 | 0.0573 | 5.7397 | 0.1130 | 0.1034 | 0.0626 | 5.7139 | 0.1110 | 0.0864 | 0.0492 | 5.4515 |
| Lognormal | $\hat{\mu}$ | 1.7580 | 1.7580 | 1.7581 | 1.7581 | 1.8012 | 1.8012 | 1.8011 | 1.8010 | 2.1804 | 2.1804 | 2.1803 | 2.1811 | 1.6396 | 1.6396 | 1.6396 | 1.6386 |
| | $\hat{\sigma}$ | 0.6036 | 0.6036 | 0.6036 | 0.6036 | 0.5502 | 0.5502 | 0.5503 | 0.5505 | 0.3071 | 0.3071 | 0.3071 | 0.3070 | 0.5466 | 0.5466 | 0.5466 | 0.5465 |
| | K-S | 0.0380 | 0.0380 | 0.0380 | 0.0380 | 0.0813 | 0.0813 | 0.0813 | 0.0814 | 0.1037 | 0.1037 | 0.1037 | 0.1027 | 0.0623 | 0.0623 | 0.0623 | 0.0630 |
| | RMSE | 0.0222 | 0.0222 | 0.0222 | 0.0222 | 0.0455 | 0.0455 | 0.0455 | 0.0456 | 0.0620 | 0.0620 | 0.0621 | 0.0616 | 0.0322 | 0.0322 | 0.0322 | 0.0326 |
| | $R^2$ | 0.9940 | 0.9940 | 0.9940 | 0.9940 | 0.9738 | 0.9738 | 0.9738 | 0.9737 | 0.9385 | 0.9385 | 0.9384 | 0.9394 | 0.9869 | 0.9869 | 0.9869 | 0.9866 |
| | PDE | 25.0429 | 25.0429 | 25.0429 | 25.0429 | 28.6518 | 28.6518 | 28.6518 | 28.6518 | 5.9794 | 5.9794 | 5.9794 | 5.9794 | 14.1455 | 14.1455 | 14.1455 | 14.1455 |
| | Time (s) | 0.1193 | 0.1098 | 0.0657 | 5.6535 | 0.1182 | 0.0977 | 0.0774 | 6.6735 | 0.1186 | 0.0973 | 0.0785 | 7.1158 | 0.1041 | 0.0997 | 0.0725 | 6.4544 |
| WW | $\hat{p}$ | 0.6904 | 0.6903 | 0.6885 | 0.6565 | 1.0000 | 0.9534 | 0.9237 | 0.9586 | 0.4404 | 0.4406 | 0.4405 | 0.4326 | 0.3106 | 0.3072 | 0.3105 | 0.2766 |
| | $\hat{k_1}$ | 2.1370 | 2.1372 | 2.1397 | 2.1815 | 2.2967 | 2.4270 | 2.2444 | 2.3469 | 7.1144 | 7.1128 | 7.1153 | 7.1145 | 2.3356 | 2.3441 | 2.3422 | 2.3732 |
| | $\hat{c_1}$ | 8.9439 | 8.9442 | 8.9549 | 9.1261 | 7.7423 | 7.9980 | 7.5717 | 7.7115 | 9.8118 | 9.8117 | 9.8120 | 9.8220 | 8.6250 | 8.6538 | 8.6344 | 8.7828 |
| | $\hat{k_2}$ | 2.7777 | 2.7775 | 2.7724 | 2.7065 | 3.1546 | 2.9055 | 4.4246 | 1.8593 | 3.7788 | 3.7787 | 3.7780 | 3.7823 | 2.4888 | 2.4870 | 2.4896 | 2.4627 |
| | $\hat{c_2}$ | 4.9376 | 4.9377 | 4.9405 | 5.0033 | 10.5488 | 3.1446 | 10.0694 | 8.7069 | 10.1833 | 10.1834 | 10.1833 | 10.1488 | 5.7477 | 5.7488 | 5.7427 | 5.7721 |
| | K-S | 0.0106 | 0.0106 | 0.0107 | 0.0115 | 0.0189 | 0.0085 | 0.0116 | 0.0183 | 0.0213 | 0.0213 | 0.0213 | 0.0232 | 0.0110 | 0.0110 | 0.0108 | 0.0132 |
| | RMSE | 0.0034 | 0.0034 | 0.0034 | 0.0035 | 0.0073 | 0.0032 | 0.0037 | 0.0078 | 0.0079 | 0.0079 | 0.0079 | 0.0085 | 0.0041 | 0.0041 | 0.0041 | 0.0048 |
| | $R^2$ | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9994 | 0.9999 | 0.9998 | 0.9993 | 0.9992 | 0.9992 | 0.9992 | 0.9991 | 0.9998 | 0.9998 | 0.9998 | 0.9997 |
| | PDE | 18.7228 | 18.7251 | 18.6919 | 18.5364 | 19.4171 | 18.4896 | 18.6196 | 19.0814 | 18.5121 | 18.5139 | 18.5101 | 18.8228 | 18.4681 | 18.4701 | 18.5150 | 19.8003 |
| | Time (s) | 1.0141 | 0.4094 | 0.4556 | 15.8725 | 3.2329 | 0.7257 | 1.0376 | 14.4229 | 2.0229 | 0.5003 | 0.3997 | 18.3971 | 1.9495 | 0.4825 | 0.6681 | 17.7581 |
| GG | $\hat{p}$ | 0.9553 | 0.9751 | 0.9922 | 0.9999 | 0.5723 | 0.5696 | 0.5760 | 0.6077 | 0.0773 | 0.0772 | 0.9562 | 0.9503 | 0.0493 | 0.0492 | 0.0539 | 0.0790 |
| | $\hat{\alpha_1}$ | 3.2865 | 3.2106 | 3.3444 | 3.2056 | 3.5466 | 3.5460 | 3.5458 | 3.5084 | 3.8828 | 3.8760 | 21.8635 | 20.3563 | 5.0682 | 5.0790 | 4.6273 | 3.6949 |
| | $\hat{\beta_1}$ | 0.4997 | 0.4657 | 0.4867 | 0.4691 | 0.6448 | 0.6462 | 0.6427 | 0.6187 | 0.7039 | 0.7031 | 2.3243 | 2.1508 | 2.6424 | 2.6509 | 2.2854 | 1.5097 |
| | $\hat{\alpha_2}$ | 57.7626 | 21.5722 | 21.9789 | 16.2252 | 12.1676 | 12.1042 | 12.2680 | 12.3076 | 23.6119 | 23.6032 | 4.4459 | 3.9443 | 4.6724 | 4.6717 | 4.6955 | 4.8628 |
| | $\hat{\beta_2}$ | 4.7475 | 5.1419 | 17.8380 | 6.9823 | 1.3904 | 1.3837 | 1.4008 | 1.4059 | 2.4823 | 2.4815 | 1.0201 | 0.8932 | 0.7670 | 0.7669 | 0.7688 | 0.7838 |
| | K-S | 0.0142 | 0.0188 | 0.0199 | 0.0186 | 0.0101 | 0.0101 | 0.0101 | 0.0108 | 0.0204 | 0.0205 | 0.0310 | 0.0266 | 0.0140 | 0.0140 | 0.0143 | 0.0178 |
| | RMSE | 0.0047 | 0.0062 | 0.0085 | 0.0070 | 0.0036 | 0.0036 | 0.0036 | 0.0040 | 0.0073 | 0.0073 | 0.0131 | 0.0117 | 0.0050 | 0.0050 | 0.0051 | 0.0060 |
| | $R^2$ | 0.9997 | 0.9995 | 0.9992 | 0.9994 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9994 | 0.9994 | 0.9979 | 0.9983 | 0.9997 | 0.9997 | 0.9997 | 0.9996 |
| | PDE | 18.5660 | 15.1842 | 16.9282 | 15.4839 | 18.1386 | 18.1233 | 18.1379 | 17.8087 | 18.1197 | 18.1182 | 18.8051 | 16.8960 | 18.8536 | 18.8542 | 18.7839 | 18.1330 |
| | Time (s) | 7.3365 | 1.8129 | 1.0878 | 16.2333 | 14.0648 | 3.3544 | 1.1282 | 39.0213 | 15.7481 | 1.6067 | 1.8752 | 24.7736 | 5.7423 | 3.3821 | 1.6819 | 35.8875 |
| NN | $\hat{p}$ | 0.4653 | 0.4653 | 0.4652 | 0.4539 | 0.7290 | 0.7290 | 0.7292 | 0.7291 | 0.7600 | 0.9785 | 0.7778 | 0.7491 | 0.3125 | 0.3125 | 0.3126 | 0.3085 |
| | $\hat{\mu_1}$ | 9.4257 | 9.4256 | 9.4241 | 9.4991 | 7.9655 | 7.9654 | 7.9652 | 7.9899 | 9.4445 | 9.3442 | 9.4607 | 9.4582 | 8.3967 | 8.3963 | 8.3962 | 8.3854 |
| | $\hat{\sigma_1}$ | 3.7102 | 3.7102 | 3.7115 | 3.6674 | 2.9467 | 2.9467 | 2.9465 | 2.9732 | 1.8571 | 2.0916 | 1.8842 | 1.8368 | 3.1343 | 3.1342 | 3.1342 | 3.1654 |
| | $\hat{\mu_2}$ | 4.5653 | 4.5652 | 4.5668 | 4.5613 | 4.0005 | 4.0003 | 3.9991 | 4.0192 | 8.4385 | 2.7803 | 8.3186 | 8.4643 | 4.7452 | 4.7452 | 4.7453 | 4.7676 |
| | $\hat{\sigma_2}$ | 1.8356 | 1.8356 | 1.8361 | 1.8500 | 1.5847 | 1.5846 | 1.5838 | 1.5680 | 3.1656 | 1.0457 | 3.1864 | 3.1582 | 1.9167 | 1.9166 | 1.9170 | 1.9151 |
| | K-S | 0.0207 | 0.0207 | 0.0208 | 0.0195 | 0.0196 | 0.0196 | 0.0196 | 0.0227 | 0.0158 | 0.0182 | 0.0173 | 0.0176 | 0.0238 | 0.0238 | 0.0238 | 0.0250 |
| | RMSE | 0.0082 | 0.0082 | 0.0082 | 0.0083 | 0.0081 | 0.0081 | 0.0081 | 0.0094 | 0.0058 | 0.0068 | 0.0066 | 0.0064 | 0.0093 | 0.0093 | 0.0093 | 0.0100 |
| | $R^2$ | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9989 | 0.9996 | 0.9994 | 0.9995 | 0.9995 | 0.9990 | 0.9990 | 0.9990 | 0.9988 |
| | PDE | 19.6350 | 19.6355 | 19.6457 | 20.3744 | 19.1657 | 19.1658 | 19.1617 | 18.1525 | 18.3813 | 18.4366 | 18.2399 | 18.2388 | 19.1776 | 19.1805 | 19.1700 | 19.3657 |
| | Time (s) | 2.1712 | 0.5648 | 0.4381 | 9.5315 | 1.2048 | 0.4292 | 0.5763 | 11.0855 | 2.5562 | 0.5225 | 0.6585 | 10.7883 | 1.8506 | 0.4270 | 0.4874 | 10.5634 |
| LL | $\hat{p}$ | 0.2472 | 0.7509 | 0.2202 | 0.2334 | 0.6165 | 0.6165 | 0.6168 | 0.3043 | 0.9236 | 0.0764 | 0.0760 | 0.9195 | 0.7111 | 0.7111 | 0.7093 | 0.2929 |
| | $\hat{\mu_1}$ | 1.2097 | 1.9388 | 1.1540 | 1.2568 | 2.0671 | 2.0671 | 2.0670 | 1.2377 | 2.2323 | 1.5534 | 1.5463 | 2.2275 | 1.8298 | 1.8298 | 1.8302 | 1.2043 |
| | $\hat{\sigma_1}$ | 0.6425 | 0.4656 | 0.6847 | 0.6987 | 0.3150 | 0.3150 | 0.3153 | 0.5444 | 0.2056 | 0.5452 | 0.5450 | 0.2044 | 0.3886 | 0.3886 | 0.3880 | 0.6009 |
| | $\hat{\mu_2}$ | 1.9382 | 1.2128 | 1.9289 | 1.9001 | 1.3737 | 1.3737 | 1.3733 | 2.0387 | 1.5534 | 2.2323 | 2.2324 | 1.5073 | 1.1714 | 1.1715 | 1.1740 | 1.8447 |
| | $\hat{\sigma_2}$ | 0.4660 | 0.6429 | 0.4758 | 0.4812 | 0.5774 | 0.5774 | 0.5774 | 0.3417 | 0.5452 | 0.2056 | 0.2056 | 0.5894 | 0.5953 | 0.5953 | 0.5954 | 0.3856 |
| | K-S | 0.0265 | 0.0265 | 0.0221 | 0.0314 | 0.0182 | 0.0182 | 0.0182 | 0.0243 | 0.0299 | 0.0299 | 0.0296 | 0.0398 | 0.0109 | 0.0109 | 0.0109 | 0.0224 |
| | RMSE | 0.0105 | 0.0105 | 0.0098 | 0.0126 | 0.0084 | 0.0084 | 0.0084 | 0.0106 | 0.0124 | 0.0124 | 0.0123 | 0.0178 | 0.0041 | 0.0041 | 0.0041 | 0.0108 |
| | $R^2$ | 0.9987 | 0.9987 | 0.9989 | 0.9982 | 0.9992 | 0.9992 | 0.9992 | 0.9987 | 0.9982 | 0.9982 | 0.9982 | 0.9962 | 0.9998 | 0.9998 | 0.9998 | 0.9986 |
| | PDE | 9.2462 | 9.2741 | 5.2775 | 7.8888 | 13.7967 | 13.7970 | 13.7676 | 13.9524 | 16.9868 | 16.9869 | 16.9765 | 18.2255 | 15.7563 | 15.7595 | 15.8758 | 12.0246 |
| | Time (s) | 0.9111 | 0.5233 | 0.6733 | 11.2895 | 1.7616 | 0.4969 | 0.5630 | 13.2933 | 2.3909 | 0.6043 | 0.8924 | 10.2937 | 2.3941 | 0.5317 | 0.7449 | 12.7604 |

**Table 3**. Parameter estimations and model evaluation results for the dataset obtained from Belen Wind Power Plant in 2014.

| | | Winter | | | | Spring | | | | Summer | | | | Autumn | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Model | | NR | BFGS | NM | SA | NR | BFGS | NM | SA | NR | BFGS | NM | SA | NR | BFGS | NM | SA |
| Weibull | $\hat{k}$ | 2.0147 | 2.0147 | 2.0146 | 2.0153 | 2.6502 | 2.6502 | 2.6503 | 2.6518 | 4.4167 | 4.4168 | 4.4174 | 4.4175 | 2.1518 | 2.1518 | 2.1517 | 2.1510 |
| | $\hat{c}$ | 7.4374 | 7.4374 | 7.4374 | 7.4381 | 8.4045 | 8.4045 | 8.4039 | 8.4052 | 10.7083 | 10.7083 | 10.7084 | 10.7085 | 6.6200 | 6.6200 | 6.6199 | 6.6189 |
| | K-S | 0.0243 | 0.0243 | 0.0243 | 0.0244 | 0.0408 | 0.0408 | 0.0408 | 0.0406 | 0.0418 | 0.0418 | 0.0418 | 0.0417 | 0.0259 | 0.0259 | 0.0259 | 0.0258 |
| | RMSE | 0.0099 | 0.0099 | 0.0099 | 0.0099 | 0.0176 | 0.0176 | 0.0176 | 0.0175 | 0.0230 | 0.0230 | 0.0230 | 0.0230 | 0.0094 | 0.0094 | 0.0094 | 0.0093 |
| | $R^2$ | 0.9988 | 0.9988 | 0.9988 | 0.9988 | 0.9961 | 0.9961 | 0.9961 | 0.9961 | 0.9930 | 0.9930 | 0.9930 | 0.9930 | 0.9989 | 0.9989 | 0.9989 | 0.9989 |
| | PDE | 19.5116 | 19.5116 | 19.5116 | 19.5116 | 18.4040 | 18.4040 | 18.4040 | 18.4040 | 18.2269 | 18.2269 | 18.2269 | 18.2269 | 19.3547 | 19.3547 | 19.3547 | 19.3547 |
| | Time (s) | 0.1203 | 0.1214 | 0.0816 | 9.2865 | 0.1650 | 0.1320 | 0.0897 | 9.5773 | 0.9527 | 0.1538 | 0.1144 | 9.6165 | 0.1221 | 0.1159 | 0.0786 | 9.2300 |
| Gamma | $\hat{\alpha}$ | 3.3492 | 3.3492 | 3.3485 | 3.3457 | 5.0710 | 5.0710 | 5.0715 | 5.0819 | 14.0980 | 14.1408 | 14.1036 | 14.1118 | 3.8038 | 3.8035 | 3.8050 | 3.8014 |
| | $\hat{\beta}$ | 0.5089 | 0.5089 | 0.5088 | 0.5087 | 0.6776 | 0.6776 | 0.6777 | 0.6791 | 1.4407 | 1.4451 | 1.4413 | 1.4422 | 0.6498 | 0.6498 | 0.6501 | 0.6498 |
| | K-S | 0.0324 | 0.0324 | 0.0324 | 0.0329 | 0.0809 | 0.0809 | 0.0808 | 0.0807 | 0.0516 | 0.0513 | 0.0516 | 0.0516 | 0.0437 | 0.0437 | 0.0437 | 0.0442 |
| | RMSE | 0.0139 | 0.0139 | 0.0140 | 0.0142 | 0.0402 | 0.0402 | 0.0402 | 0.0401 | 0.0260 | 0.0258 | 0.0260 | 0.0260 | 0.0179 | 0.0179 | 0.0179 | 0.0181 |
| | $R^2$ | 0.9977 | 0.9977 | 0.9976 | 0.9975 | 0.9791 | 0.9791 | 0.9791 | 0.9792 | 0.9909 | 0.9910 | 0.9909 | 0.9909 | 0.9962 | 0.9962 | 0.9962 | 0.9961 |
| | PDE | 12.3399 | 12.3399 | 12.3399 | 12.3399 | 9.5572 | 9.5572 | 9.5572 | 9.5572 | 15.6606 | 15.6606 | 15.6606 | 15.6606 | 13.1914 | 13.1914 | 13.1914 | 13.1914 |
| | Time (s) | 0.2783 | 0.1881 | 0.1636 | 23.4533 | 0.3366 | 0.3019 | 0.2026 | 24.4632 | 2.5824 | 0.2240 | 0.1431 | 16.6085 | 0.2761 | 0.1866 | 0.1642 | 23.2246 |
| Normal | $\hat{\mu}$ | 6.5807 | 6.5812 | 6.5801 | 6.5813 | 7.4834 | 7.4834 | 7.4831 | 7.4811 | 9.7858 | 9.7858 | 9.7853 | 9.7859 | 5.8534 | 5.8534 | 5.8530 | 5.8521 |
| | $\hat{\sigma}$ | 3.4400 | 3.4399 | 3.4397 | 3.4389 | 3.0092 | 3.0092 | 3.0092 | 3.0088 | 2.4227 | 2.4227 | 2.4227 | 2.4233 | 2.8807 | 2.8807 | 2.8809 | 2.8808 |
| | K-S | 0.0582 | 0.0582 | 0.0581 | 0.0582 | 0.0255 | 0.0255 | 0.0254 | 0.0252 | 0.0265 | 0.0265 | 0.0266 | 0.0266 | 0.0609 | 0.0609 | 0.0609 | 0.0608 |
| | RMSE | 0.0351 | 0.0351 | 0.0350 | 0.0351 | 0.0101 | 0.0101 | 0.0101 | 0.0100 | 0.0124 | 0.0124 | 0.0124 | 0.0125 | 0.0291 | 0.0291 | 0.0290 | 0.0290 |
| | $R^2$ | 0.9848 | 0.9848 | 0.9849 | 0.9848 | 0.9988 | 0.9988 | 0.9988 | 0.9988 | 0.9981 | 0.9981 | 0.9981 | 0.9981 | 0.9899 | 0.9899 | 0.9899 | 0.9899 |
| | PDE | 23.0435 | 23.0435 | 23.0435 | 23.0435 | 19.5373 | 19.5373 | 19.5373 | 19.5373 | 18.3491 | 18.3491 | 18.3491 | 18.3491 | 22.1900 | 22.1900 | 22.1900 | 22.1900 |
| | Time (s) | 0.1112 | 0.1074 | 0.0498 | 5.5525 | 0.1393 | 0.1121 | 0.0554 | 5.6607 | 0.1125 | 0.1436 | 0.0645 | 5.6534 | 0.1070 | 0.0784 | 0.0526 | 5.4856 |
| Lognormal | $\hat{\mu}$ | 1.7275 | 1.7275 | 1.7276 | 1.7294 | 1.9109 | 1.9109 | 1.9108 | 1.9127 | 2.2451 | 2.2451 | 2.2450 | 2.2453 | 1.6299 | 1.6299 | 1.6298 | 1.6309 |
| | $\hat{\sigma}$ | 0.6034 | 0.6034 | 0.6034 | 0.6014 | 0.4968 | 0.4968 | 0.4968 | 0.4975 | 0.2857 | 0.2857 | 0.2858 | 0.2870 | 0.5582 | 0.5582 | 0.5583 | 0.5571 |
| | K-S | 0.0696 | 0.0697 | 0.0696 | 0.0684 | 0.1120 | 0.1120 | 0.1121 | 0.1106 | 0.0734 | 0.0734 | 0.0735 | 0.0737 | 0.0695 | 0.0695 | 0.0695 | 0.0690 |
| | RMSE | 0.0370 | 0.0370 | 0.0370 | 0.0362 | 0.0604 | 0.0604 | 0.0605 | 0.0597 | 0.0405 | 0.0405 | 0.0406 | 0.0409 | 0.0360 | 0.0360 | 0.0360 | 0.0356 |
| | $R^2$ | 0.9826 | 0.9826 | 0.9826 | 0.9834 | 0.9490 | 0.9490 | 0.9489 | 0.9500 | 0.9763 | 0.9763 | 0.9762 | 0.9757 | 0.9841 | 0.9841 | 0.9841 | 0.9845 |
| | PDE | 35.9470 | 35.9470 | 35.9470 | 35.9470 | 21.1931 | 21.1931 | 21.1931 | 21.1931 | 10.5681 | 10.5681 | 10.5681 | 10.5681 | 21.3076 | 21.3076 | 21.3076 | 21.3076 |
| | Time (s) | 0.1402 | 0.0857 | 0.0747 | 6.4443 | 0.1187 | 0.0913 | 0.0760 | 6.7640 | 0.1577 | 0.1396 | 0.0793 | 7.1354 | 0.1171 | 0.1020 | 0.0683 | 6.4753 |
| WW | $\hat{p}$ | 0.2428 | 0.3690 | 0.5731 | 0.3613 | 0.6159 | 0.6170 | 0.3790 | 0.6220 | 0.3274 | 0.3274 | 0.3331 | 0.6806 | 0.0210 | 0.0211 | 0.0471 | 0.1034 |
| | $\hat{k_1}$ | 2.7787 | 2.4305 | 2.1328 | 2.4347 | 2.2025 | 2.2035 | 1.9701 | 2.2030 | 7.2224 | 7.2222 | 7.1284 | 4.2363 | 4.7735 | 4.7686 | 3.3984 | 2.5281 |
| | $\hat{c_1}$ | 10.8238 | 9.6997 | 8.5828 | 9.7165 | 8.1602 | 8.1609 | 7.9210 | 8.1366 | 9.6915 | 9.6915 | 9.6819 | 11.0947 | 14.0422 | 14.0374 | 11.9338 | 9.8263 |
| | $\hat{k_2}$ | 2.2149 | 2.2205 | 2.2923 | 2.1981 | 4.4243 | 4.4303 | 3.5409 | 4.4506 | 4.2286 | 4.2286 | 4.2320 | 7.3039 | 2.2639 | 2.2640 | 2.2836 | 2.2827 |
| | $\hat{c_2}$ | 6.3252 | 6.0983 | 5.9072 | 6.1469 | 8.6574 | 8.6575 | 8.6273 | 8.6798 | 11.1259 | 11.1259 | 11.1299 | 9.6967 | 6.4468 | 6.4467 | 6.3349 | 6.2413 |
| | K-S | 0.0134 | 0.0137 | 0.0136 | 0.0136 | 0.0189 | 0.0189 | 0.0221 | 0.0197 | 0.0193 | 0.0193 | 0.0205 | 0.0201 | 0.0285 | 0.0285 | 0.0272 | 0.0272 |
| | RMSE | 0.0051 | 0.0052 | 0.0052 | 0.0050 | 0.0063 | 0.0063 | 0.0078 | 0.0065 | 0.0077 | 0.0077 | 0.0078 | 0.0079 | 0.0111 | 0.0111 | 0.0114 | 0.0112 |
| | $R^2$ | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9995 | 0.9995 | 0.9993 | 0.9995 | 0.9993 | 0.9993 | 0.9993 | 0.9992 | 0.9986 | 0.9986 | 0.9985 | 0.9985 |
| | PDE | 18.3865 | 18.3640 | 17.6479 | 18.1555 | 18.4039 | 18.4035 | 18.3994 | 18.6131 | 18.4820 | 18.4821 | 18.6760 | 18.7295 | 18.4640 | 18.4640 | 18.7802 | 18.2710 |
| | Time (s) | 9.8877 | 0.8490 | 1.1513 | 17.7242 | 4.8934 | 0.7341 | 0.5463 | 18.5091 | 3.0688 | 0.6534 | 1.1056 | 18.3954 | 6.5147 | 1.3582 | 1.0788 | 15.0804 |
| GG | $\hat{p}$ | 0.9087 | 0.9108 | 0.0828 | 0.9149 | 0.4010 | 0.4059 | 0.4092 | 0.3991 | 0.0384 | 0.0384 | 0.0374 | 0.0455 | 0.1283 | 0.4088 | 0.1717 | 0.1257 |
| | $\hat{\alpha_1}$ | 4.5011 | 4.4941 | 3.8512 | 4.5318 | 3.1632 | 3.2124 | 3.2244 | 3.1700 | 4.4921 | 4.4917 | 4.5806 | 3.8710 | 36.3630 | 4.1660 | 4.4317 | 35.8537 |
| | $\hat{\beta_1}$ | 0.6411 | 0.6414 | 1.7794 | 0.6444 | 0.5270 | 0.5347 | 0.5356 | 0.5244 | 0.9062 | 0.9058 | 0.9387 | 0.6912 | 4.5974 | 1.0953 | 1.5824 | 4.5600 |
| | $\hat{\alpha_2}$ | 3.7700 | 3.7912 | 4.4011 | 4.2120 | 13.9460 | 14.0916 | 14.2064 | 14.1160 | 20.1128 | 20.1139 | 20.0885 | 20.5663 | 3.5699 | 7.9258 | 5.5297 | 3.5714 |
| | $\hat{\beta_2}$ | 1.6744 | 1.6980 | 0.6318 | 2.0045 | 1.6399 | 1.6594 | 1.6718 | 1.6751 | 2.0156 | 2.0157 | 2.0134 | 2.0643 | 0.6431 | 1.0899 | 0.8505 | 0.6410 |
| | K-S | 0.0130 | 0.0131 | 0.0128 | 0.0173 | 0.0167 | 0.0169 | 0.0166 | 0.0176 | 0.0148 | 0.0148 | 0.0149 | 0.0189 | 0.0135 | 0.0191 | 0.0252 | 0.0146 |
| | RMSE | 0.0046 | 0.0046 | 0.0049 | 0.0059 | 0.0058 | 0.0054 | 0.0054 | 0.0057 | 0.0050 | 0.0050 | 0.0050 | 0.0063 | 0.0043 | 0.0074 | 0.0095 | 0.0044 |
| | $R^2$ | 0.9997 | 0.9997 | 0.9997 | 0.9996 | 0.9996 | 0.9996 | 0.9997 | 0.9996 | 0.9997 | 0.9997 | 0.9997 | 0.9995 | 0.9998 | 0.9993 | 0.9989 | 0.9998 |
| | PDE | 17.3679 | 17.6386 | 17.6837 | 16.7456 | 17.5673 | 18.2757 | 18.2748 | 18.7118 | 18.2395 | 18.2395 | 18.2197 | 18.9532 | 17.6531 | 18.7034 | 16.2830 | 17.2923 |
| | Time (s) | 3.6823 | 1.6390 | 2.7104 | 35.2676 | 4.4158 | 0.9351 | 1.1065 | 39.2366 | 5.3488 | 0.9858 | 1.5408 | 24.3152 | 16.4054 | 3.3602 | 2.6232 | 30.2917 |
| NN | $\hat{p}$ | 0.3565 | 0.3565 | 0.3564 | 0.3507 | 0.0017 | 0.0486 | 0.9861 | 0.9713 | 0.9910 | 0.7311 | 0.2678 | 0.2456 | 0.6832 | 0.6832 | 0.6721 | 0.6854 |
| | $\hat{\mu_1}$ | 9.4063 | 9.4068 | 9.4078 | 9.5321 | 8.0844 | 7.6800 | 7.3873 | 7.3710 | 9.8518 | 9.9724 | 9.2774 | 9.2342 | 6.9581 | 6.9580 | 6.9228 | 6.9630 |
| | $\hat{\sigma_1}$ | 3.4838 | 3.4834 | 3.4830 | 3.4604 | 1.9950 | 0.5410 | 2.8770 | 2.8763 | 2.3309 | 2.6715 | 1.4400 | 1.3524 | 2.7447 | 2.7447 | 2.7476 | 2.7355 |
| | $\hat{\mu_2}$ | 5.0156 | 5.0149 | 5.0158 | 5.0404 | 7.9800 | 7.4732 | 14.2944 | 11.0238 | 2.5037 | 9.2793 | 9.9714 | 9.9712 | 3.4711 | 3.4711 | 3.5071 | 3.4496 |
| | $\hat{\sigma_2}$ | 2.1895 | 2.1891 | 2.1893 | 2.2151 | 2.9656 | 3.0822 | 4.1130 | 4.5222 | 0.7109 | 1.4415 | 2.6699 | 2.6487 | 1.2809 | 1.2809 | 1.3349 | 1.2832 |
| | K-S | 0.0283 | 0.0282 | 0.0283 | 0.0300 | 0.0881 | 0.0171 | 0.0214 | 0.0217 | 0.0298 | 0.0196 | 0.0196 | 0.0208 | 0.0299 | 0.0298 | 0.0247 | 0.0302 |
| | RMSE | 0.0114 | 0.0114 | 0.0114 | 0.0114 | 0.0562 | 0.0072 | 0.0081 | 0.0081 | 0.0136 | 0.0062 | 0.0062 | 0.0064 | 0.0119 | 0.0119 | 0.0110 | 0.0121 |
| | $R^2$ | 0.9985 | 0.9985 | 0.9985 | 0.9985 | 0.9615 | 0.9994 | 0.9992 | 0.9992 | 0.9978 | 0.9995 | 0.9995 | 0.9995 | 0.9982 | 0.9982 | 0.9985 | 0.9982 |
| | PDE | 18.9663 | 18.9597 | 18.9556 | 17.6700 | 7.1065 | 19.5770 | 18.6841 | 19.2629 | 18.5332 | 18.1906 | 18.2049 | 18.1562 | 19.9719 | 19.9724 | 21.6751 | 19.8518 |
| | Time (s) | 1.0685 | 0.2884 | 0.4920 | 10.6138 | 0.7993 | 0.5216 | 0.5474 | 7.2749 | 2.2495 | 0.6073 | 0.3860 | 10.8732 | 2.0263 | 0.5863 | 0.6385 | 10.2689 |
| LL | $\hat{p}$ | 0.7190 | 0.7190 | 0.7186 | 0.7246 | 0.7258 | 0.7261 | 0.7255 | 0.7600 | 0.9495 | 0.9495 | 0.0505 | 0.9456 | 0.5306 | 0.5306 | 0.9673 | 0.6781 |
| | $\hat{\mu_1}$ | 1.9465 | 1.9465 | 1.9464 | 1.9302 | 2.0915 | 2.0914 | 2.0915 | 2.0945 | 2.2793 | 2.2793 | 1.6011 | 2.2742 | 1.9273 | 1.9273 | 1.6620 | 1.8750 |
| | $\hat{\sigma_1}$ | 0.4111 | 0.4111 | 0.4113 | 0.4094 | 0.2836 | 0.2837 | 0.2837 | 0.2812 | 0.2206 | 0.2206 | 0.5150 | 0.2252 | 0.3325 | 0.3325 | 0.4836 | 0.3719 |
| | $\hat{\mu_2}$ | 1.1670 | 1.1670 | 1.1680 | 1.1631 | 1.4328 | 1.4321 | 1.4333 | 1.3764 | 1.6014 | 1.6014 | 2.2793 | 1.7622 | 1.2937 | 1.2937 | 0.2403 | 1.1544 |
| | $\hat{\sigma_2}$ | 0.6530 | 0.6530 | 0.6532 | 0.6540 | 0.6101 | 0.6100 | 0.6101 | 0.5975 | 0.5148 | 0.5148 | 0.2206 | 0.6434 | 0.5708 | 0.5708 | 0.3469 | 0.5540 |
| | K-S | 0.0184 | 0.0184 | 0.0183 | 0.0251 | 0.0214 | 0.0215 | 0.0215 | 0.0310 | 0.0242 | 0.0242 | 0.0241 | 0.0249 | 0.0234 | 0.0234 | 0.0755 | 0.0341 |
| | RMSE | 0.0065 | 0.0065 | 0.0065 | 0.0110 | 0.0081 | 0.0081 | 0.0081 | 0.0106 | 0.0080 | 0.0080 | 0.0080 | 0.0093 | 0.0100 | 0.0100 | 0.0356 | 0.0135 |
| | $R^2$ | 0.9995 | 0.9995 | 0.9995 | 0.9986 | 0.9992 | 0.9992 | 0.9992 | 0.9987 | 0.9992 | 0.9992 | 0.9992 | 0.9990 | 0.9988 | 0.9988 | 0.9858 | 0.9979 |
| | PDE | 12.1990 | 12.1990 | 12.1603 | 16.0787 | 14.5424 | 14.5610 | 14.5200 | 15.2620 | 17.3824 | 17.3824 | 17.3786 | 14.6096 | 14.2777 | 14.2781 | 8.9095 | 12.1171 |
| | Time (s) | 2.3842 | 0.4844 | 0.5362 | 12.6163 | 1.9030 | 0.3617 | 0.4368 | 12.8923 | 2.1394 | 0.8918 | 0.6876 | 10.0233 | 1.4879 | 0.5074 | 0.7520 | 12.3006 |

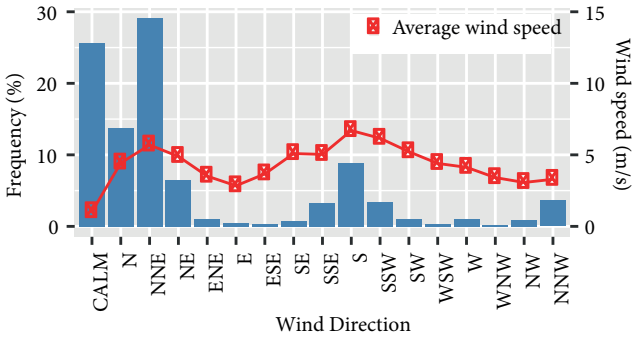



**Figure 3**. Wind direction frequency distributions of Belen Wind Power Plant.

and autumn. According to RMSE and $R^2$ criteria, GG provided the best fit by using BFGS in winter, NM in spring and summer, and NR in autumn. LL provided the best fit in winter and autumn by using NM according to the PDE criterion. Density fit plots are demonstrated in Figure 4.

In summary, mixture wind speed distributions outperformed the unimodal wind speed distributions according to K-S test statistic, RMSE, and $R^2$ criteria. Unimodal distributions performed better only in 3 seasons according to the PDE criterion. NM and BFGS were the fastest optimization algorithms for the parameter estimations of unimodal and mixture distributions, respectively. SA was the slowest algorithm for all types of distributions. Overall comparison of the distributions and the optimization algorithms is presented in Table 4.

**Table 4**. Overall comparison of the distributions and the optimization methods for the dataset obtained from Belen Wind Farm.

| Date | | KS | | RMSE | | $R^2$ | | PDE | |
|---|---|---|---|---|---|---|---|---|---|
| Year | Season | Model | Method | Model | Method | Model | Method | Model | Method |
| 2013 | Winter | WW | BFGS | WW | NR | WW | NR | LL | NM |
| | Spring | WW | BFGS | WW | BFGS | WW | BFGS | Gamma | SA |
| | Summer | NN | NR | NN | NR | NN | NR | Lognormal | SA |
| | Autumn | WW | NM | LL | NR | LL | NR | LL | SA |
| 2014 | Winter | GG | NM | GG | BFGS | GG | BFGS | LL | NM |
| | Spring | GG | NM | GG | NM | GG | NM | NN | NR |
| | Summer | GG | NR | GG | NM | GG | NM | Lognormal | BFGS |
| | Autumn | GG | NR | GG | NR | GG | NR | LL | NM |

### 3.2. Annual wind speed analysis

Wind speed dataset observed at Gökçeada Meteorological Station was used for the annual wind speed analysis. Observations were gauged at 10 m height on hourly basis from 2010 to 2013. Geographical coordinates of the station is 40°11′27.6”N, 25°54′27.0”E. The dataset was provided by the Turkish State Meteorological Service. Table 5 presents the summary statistics for the dataset.

Wind direction frequency and the average wind speed for each direction and calm are presented in Figure 5. The maximum frequency of occurrence is at NNE with the average wind speed of 5.69 m/s.

The parameter estimations, K-S test statistic, RMSE, $R^2$ values, and the computation times of the wind speed distributions obtained by using four different optimization methods are presented in Table 6.

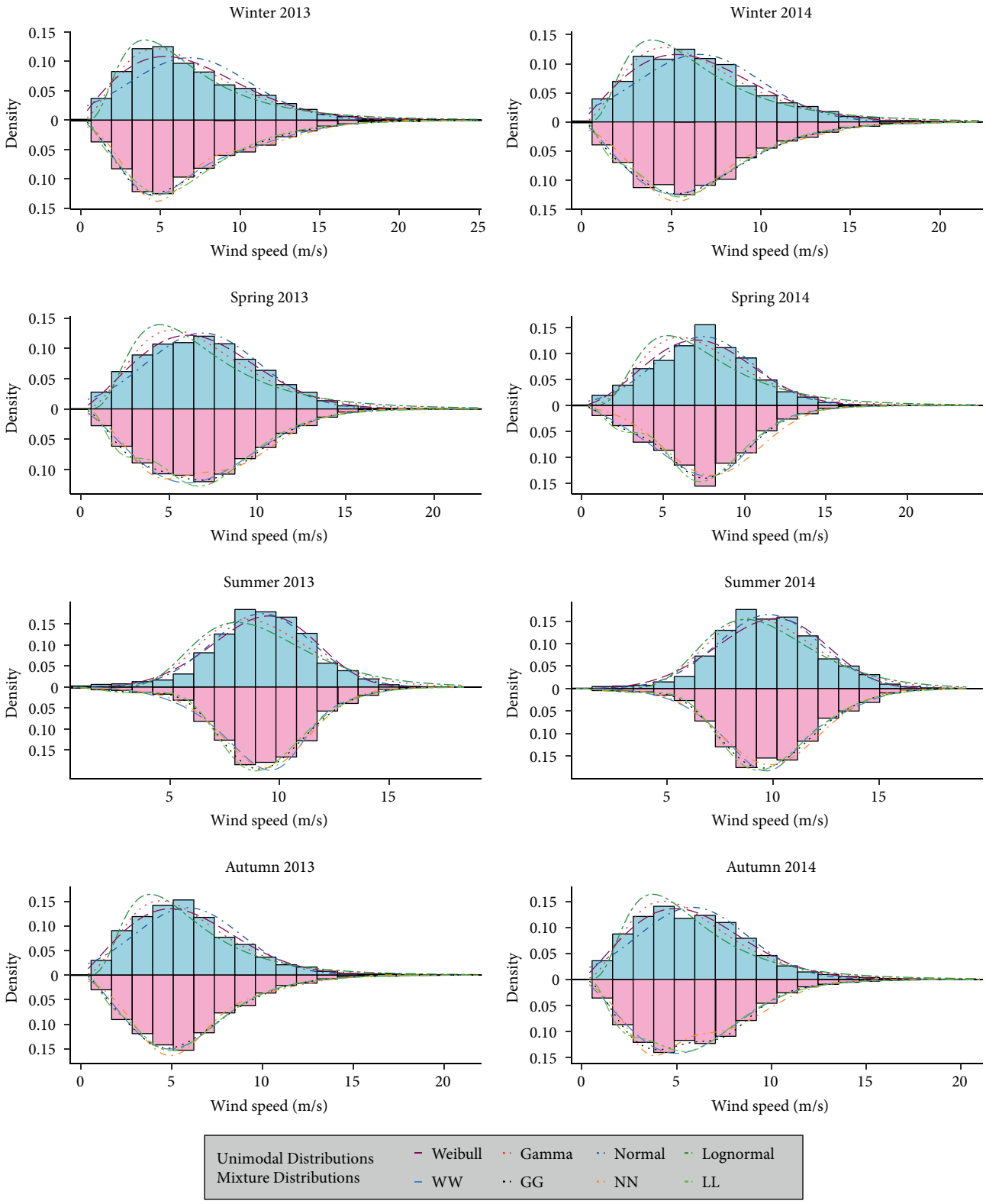



**Figure 4**. Unimodal and mixture density fits for the dataset obtained from Belen Wind Power Plant. The upper sides of the mirrored histograms present the unimodal density fits where the opposite sides present the mixture density fits.

**Table 5**. Descriptive statistics of the wind speed data (m/s) of Gökçeada Meteorological Station.

| | Years | | | |
|---|---|---|---|---|
| | 2010 | 2011 | 2012 | 2013 |
| Mean | 4.40 | 4.50 | 4.18 | 3.69 |
| Std. Dev. | 3.08 | 2.74 | 2.64 | 2.47 |
| Min | 0.10 | 0.10 | 0.20 | 0.10 |
| Max | 16.90 | 16.90 | 14.90 | 16.90 |

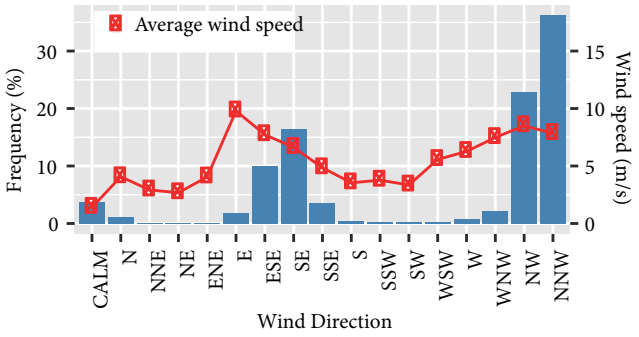



**Figure 5**. Wind direction frequency distributions of Gökçeada Meteorological Station.

GG is the most successful distribution which provides the best fit in 2010, 2011, and 2013 according to the K-S test statistic, RMSE, and $R^2$ criteria. According to the K-S test statistic, GG provided better modeling by using NM in 2010, SA in 2011, and BFGS in 2013. For the RMSE and $R^2$ criteria, GG provided the most efficiency by using SA in 2010 and NR in 2011 and 2013. WW by using NM was the second-best distribution and provided the best fit in 2012 according to the K-S test statistic, RMSE, and $R^2$ criteria. LL was the most successful distribution based on the PDE criterion, it used NM in 2010 and 2012, SA in 2011, and NR in 2013.

NM was the fastest optimization algorithm for unimodal wind speed distributions while BFGS and NM were the fastest algorithms for mixture distributions. SA is the slowest algorithm for all types of datasets. Density fit plots of the distributions are presented in Figure 6. Overall comparison of the distributions and optimization algorithms is presented in Table 7.

## 4. Discussion

In this paper, we presented a comparison between unimodal and two-component mixture wind speed distributions. The Weibull, gamma, normal, lognormal distributions, and their two-component mixture forms were employed to model the wind speed observations obtained from Belen Wind Power Plant and Gökçeada Meteorological Station.

Similar studies have been conducted in wind speed modeling; however, the aspects that make this study different are that it has a detailed seasonal and annual analysis and gives opportunity to compare the optimization algorithms for ML. Additionally, it also allows to compare the results of the wind speed datasets obtained from an actual wind farm and a random meteorological station. Finally, we would like to emphasize that the number of mixture components is fixed. In future works, we will focus on an unknown number of mixture components for wind speed modeling.

**Table 6**. Parameter estimations and model evaluation results for the dataset obtained from Gökçeada Meteorological Station.

| Model | | 2010 | | | | 2011 | | | | 2012 | | | | 2013 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | NR | BFGS | NM | SA | NR | BFGS | NM | SA | NR | BFGS | NM | SA | NR | BFGS | NM | SA |
| Weibull | $\hat{k}$ | 1.4415 | 1.4415 | 1.4416 | 1.4412 | 1.6522 | 1.6522 | 1.6523 | 1.6521 | 1.5969 | 1.5969 | 1.5971 | 1.5973 | 1.5138 | 1.5138 | 1.5140 | 1.5134 |
| | $\hat{c}$ | 4.8474 | 4.8474 | 4.8481 | 4.8437 | 5.0224 | 5.0224 | 5.0228 | 5.0198 | 4.6550 | 4.6550 | 4.6553 | 4.6535 | 4.0909 | 4.0909 | 4.0910 | 4.0917 |
| | K-S | 0.0527 | 0.0527 | 0.0527 | 0.0525 | 0.0626 | 0.0626 | 0.0627 | 0.0626 | 0.0598 | 0.0598 | 0.0598 | 0.0598 | 0.0622 | 0.0622 | 0.0622 | 0.0621 |
| | RMSE | 0.0176 | 0.0176 | 0.0175 | 0.0176 | 0.0306 | 0.0306 | 0.0306 | 0.0307 | 0.0261 | 0.0261 | 0.0261 | 0.0262 | 0.0255 | 0.0255 | 0.0255 | 0.0255 |
| | $R^2$ | 0.9965 | 0.9965 | 0.9965 | 0.9965 | 0.9893 | 0.9893 | 0.9893 | 0.9892 | 0.9923 | 0.9923 | 0.9923 | 0.9924 | 0.9926 | 0.9926 | 0.9926 | 0.9926 |
| | PDE | 14.7781 | 14.7781 | 14.7468 | 14.9359 | 14.1506 | 14.1506 | 14.1333 | 14.2765 | 14.2001 | 14.2007 | 14.1961 | 14.3093 | 16.8868 | 16.8867 | 16.8936 | 16.7972 |
| | Time (s) | 0.0869 | 0.0551 | 0.0593 | 6.4154 | 0.0640 | 0.0560 | 0.0491 | 6.3087 | 0.0838 | 0.0619 | 0.0493 | 6.0657 | 0.0602 | 0.0553 | 0.0472 | 6.0144 |
| Gamma | $\hat{\alpha}$ | 1.7900 | 1.7900 | 1.7905 | 1.7915 | 2.1161 | 2.1161 | 2.1161 | 2.1216 | 2.0519 | 2.0519 | 2.0518 | 2.0599 | 1.9313 | 1.9313 | 1.9312 | 1.9301 |
| | $\hat{\beta}$ | 0.4072 | 0.4072 | 0.4074 | 0.4072 | 0.4701 | 0.4701 | 0.4701 | 0.4709 | 0.4912 | 0.4912 | 0.4911 | 0.4927 | 0.5237 | 0.5237 | 0.5237 | 0.5235 |
| | K-S | 0.0560 | 0.0560 | 0.0560 | 0.0563 | 0.0842 | 0.0842 | 0.0842 | 0.0835 | 0.0699 | 0.0699 | 0.0698 | 0.0692 | 0.0639 | 0.0639 | 0.0640 | 0.0641 |
| | RMSE | 0.0247 | 0.0247 | 0.0247 | 0.0245 | 0.0455 | 0.0455 | 0.0454 | 0.0452 | 0.0379 | 0.0379 | 0.0379 | 0.0377 | 0.0354 | 0.0354 | 0.0355 | 0.0355 |
| | $R^2$ | 0.9932 | 0.9932 | 0.9931 | 0.9932 | 0.9759 | 0.9759 | 0.9759 | 0.9762 | 0.9837 | 0.9837 | 0.9837 | 0.9839 | 0.9858 | 0.9858 | 0.9858 | 0.9858 |
| | PDE | 2.8365 | 2.8502 | 2.8936 | 2.6763 | 4.2095 | 4.2095 | 4.2315 | 4.2467 | 1.5693 | 1.5693 | 1.5983 | 1.4527 | 4.3311 | 4.3311 | 4.3658 | 4.3463 |
| | Time (s) | 0.1791 | 0.2266 | 0.1433 | 21.3163 | 0.1417 | 0.1293 | 0.1203 | 19.7096 | 0.1406 | 0.1292 | 0.1149 | 17.5208 | 0.1739 | 0.1526 | 0.1370 | 19.2330 |
| Normal | $\hat{\mu}$ | 4.3956 | 4.3951 | 4.3956 | 4.3937 | 4.5014 | 4.5015 | 4.5017 | 4.5020 | 4.1776 | 4.1776 | 4.1777 | 4.1787 | 3.6880 | 3.6881 | 3.6880 | 3.6867 |
| | $\hat{\sigma}$ | 3.0777 | 3.0779 | 3.0778 | 3.0782 | 2.7361 | 2.7360 | 2.7364 | 2.7366 | 2.6392 | 2.6392 | 2.6390 | 2.6391 | 2.4715 | 2.4715 | 2.4716 | 2.4717 |
| | K-S | 0.0904 | 0.0904 | 0.0904 | 0.0905 | 0.0636 | 0.0636 | 0.0636 | 0.0636 | 0.0728 | 0.0728 | 0.0728 | 0.0727 | 0.0835 | 0.0835 | 0.0835 | 0.0836 |
| | RMSE | 0.0482 | 0.0482 | 0.0482 | 0.0481 | 0.0243 | 0.0243 | 0.0243 | 0.0243 | 0.0319 | 0.0319 | 0.0319 | 0.0320 | 0.0372 | 0.0372 | 0.0371 | 0.0370 |
| | $R^2$ | 0.9720 | 0.9720 | 0.9720 | 0.9721 | 0.9930 | 0.9930 | 0.9930 | 0.9930 | 0.9881 | 0.9881 | 0.9881 | 0.9880 | 0.9832 | 0.9832 | 0.9832 | 0.9833 |
| | PDE | 26.9569 | 26.9683 | 26.9554 | 26.9996 | 23.1635 | 23.1642 | 23.1449 | 23.1301 | 23.8453 | 23.8453 | 23.8478 | 23.8108 | 27.0874 | 27.0849 | 27.0866 | 27.1260 |
| | Time (s) | 0.0616 | 0.0432 | 0.0414 | 3.8305 | 0.0459 | 0.0407 | 0.0345 | 3.7858 | 0.0417 | 0.0342 | 0.0312 | 3.6151 | 0.0388 | 0.0514 | 0.0352 | 3.5609 |
| Lognormal | $\hat{\mu}$ | 1.1760 | 1.1760 | 1.1760 | 1.1773 | 1.2499 | 1.2499 | 1.2499 | 1.2512 | 1.1667 | 1.1667 | 1.1666 | 1.1653 | 1.0244 | 1.0244 | 1.0243 | 1.0248 |
| | $\hat{\sigma}$ | 0.8624 | 0.8624 | 0.8623 | 0.8638 | 0.8086 | 0.8085 | 0.8087 | 0.8094 | 0.8089 | 0.8089 | 0.8089 | 0.8081 | 0.8306 | 0.8306 | 0.8305 | 0.8316 |
| | K-S | 0.0915 | 0.0915 | 0.0915 | 0.0909 | 0.1279 | 0.1279 | 0.1279 | 0.1273 | 0.1124 | 0.1124 | 0.1124 | 0.1131 | 0.1092 | 0.1092 | 0.1092 | 0.1089 |
| | RMSE | 0.0500 | 0.0500 | 0.0500 | 0.0496 | 0.0712 | 0.0712 | 0.0712 | 0.0709 | 0.0624 | 0.0624 | 0.0624 | 0.0628 | 0.0594 | 0.0594 | 0.0594 | 0.0592 |
| | $R^2$ | 0.9710 | 0.9710 | 0.9710 | 0.9713 | 0.9374 | 0.9374 | 0.9374 | 0.9379 | 0.9540 | 0.9540 | 0.9539 | 0.9535 | 0.9589 | 0.9589 | 0.9589 | 0.9591 |
| | PDE | 235.3815 | 235.3815 | 235.2770 | 324.9410 | 221.5232 | 221.2855 | 221.7317 | 150.8699 | 198.5046 | 198.5046 | 198.3469 | 195.4514 | 197.4127 | 197.4127 | 197.1361 | 200.1437 |
| | Time (s) | 0.0471 | 0.0438 | 0.0364 | 4.5039 | 0.0488 | 0.0883 | 0.0371 | 4.4468 | 0.0483 | 0.0429 | 0.0385 | 4.2808 | 0.0450 | 0.0414 | 0.0351 | 4.2160 |
| WW | $\hat{p}$ | 0.8856 | 0.8856 | 0.8857 | 0.8858 | 0.8816 | 0.8816 | 0.8815 | 0.8840 | 0.8805 | 0.8805 | 0.8804 | 0.8871 | 0.8671 | 0.8671 | 0.8671 | 0.8692 |
| | $k_1$ | 1.6993 | 1.6993 | 1.6996 | 1.6916 | 2.0779 | 2.0779 | 2.0780 | 2.0918 | 1.9635 | 1.9635 | 1.9633 | 1.9642 | 1.8337 | 1.8337 | 1.8333 | 1.8581 |
| | $c_1$ | 5.4544 | 5.4544 | 5.4549 | 5.4713 | 5.6464 | 5.6464 | 5.6464 | 5.6235 | 5.2365 | 5.2365 | 5.2363 | 5.2291 | 4.6579 | 4.6579 | 4.6575 | 4.6817 |
| | $k_2$ | 3.9286 | 3.9287 | 3.9293 | 3.7766 | 3.6760 | 3.6760 | 3.6767 | 3.8149 | 3.9739 | 3.9739 | 3.9730 | 4.2543 | 3.8023 | 3.8023 | 3.8011 | 3.7690 |
| | $c_2$ | 0.8928 | 0.8928 | 0.8924 | 0.8808 | 0.8712 | 0.8712 | 0.8713 | 0.8613 | 0.8630 | 0.8630 | 0.8630 | 0.8512 | 0.8432 | 0.8432 | 0.8430 | 0.8391 |
| | K-S | 0.0197 | 0.0197 | 0.0199 | 0.1140 | 0.0177 | 0.0177 | 0.0178 | 0.0171 | 0.0159 | 0.0159 | 0.0159 | 0.0193 | 0.0195 | 0.0195 | 0.0196 | 0.0193 |
| | RMSE | 0.0071 | 0.0071 | 0.0071 | 0.0700 | 0.0073 | 0.0073 | 0.0074 | 0.0064 | 0.0058 | 0.0058 | 0.0058 | 0.0068 | 0.0084 | 0.0084 | 0.0085 | 0.0080 |
| | $R^2$ | 0.9994 | 0.9994 | 0.9994 | 0.9267 | 0.9993 | 0.9993 | 0.9993 | 0.9995 | 0.9996 | 0.9996 | 0.9996 | 0.9994 | 0.9991 | 0.9991 | 0.9991 | 0.9992 |
| | PDE | 18.8335 | 18.8334 | 18.8131 | 17.5064 | 19.0260 | 19.0260 | 19.0339 | 20.2983 | 18.6556 | 18.6556 | 18.6587 | 18.4192 | 20.4788 | 20.4788 | 20.4794 | 20.3727 |
| | Time (s) | 0.4618 | 0.3016 | 0.2496 | 10.8753 | 0.5212 | 0.2984 | 0.2712 | 10.8305 | 0.4377 | 0.2836 | 0.2529 | 10.0249 | 0.4468 | 0.2893 | 0.2610 | 10.4915 |
| GG | $\hat{p}$ | 0.8399 | 0.8377 | 0.8376 | 0.8364 | 0.8271 | 0.8271 | 0.8270 | 0.8255 | 0.8214 | 0.8214 | 0.8214 | 0.8192 | 0.7886 | 0.7887 | 0.7890 | 0.7860 |
| | $\hat{\alpha_1}$ | 2.9505 | 2.9496 | 2.9531 | 2.9446 | 4.7069 | 4.7071 | 4.7079 | 4.8994 | 4.1296 | 4.1298 | 4.1311 | 4.1141 | 4.0417 | 4.0414 | 4.0384 | 4.0489 |
| | $\hat{\beta_1}$ | 0.5801 | 0.5806 | 0.5813 | 0.5759 | 0.8960 | 0.8960 | 0.8962 | 0.9315 | 0.8440 | 0.8440 | 0.8443 | 0.8452 | 0.9090 | 0.9089 | 0.9086 | 0.9078 |
| | $\hat{\alpha_2}$ | 7.0349 | 7.0425 | 7.0367 | 7.0796 | 6.3887 | 6.3906 | 6.3921 | 6.4194 | 7.2228 | 7.2239 | 7.2218 | 7.3011 | 5.8614 | 5.8633 | 5.8686 | 5.8163 |
| | $\hat{\beta_2}$ | 8.1529 | 8.1579 | 8.1449 | 8.1488 | 7.0627 | 7.0651 | 7.0695 | 7.0404 | 8.1465 | 8.1480 | 8.1440 | 8.2312 | 6.8267 | 6.8295 | 6.8410 | 6.8209 |
| | K-S | 0.0146 | 0.0142 | 0.0141 | 0.0148 | 0.0169 | 0.0169 | 0.0169 | 0.0164 | 0.0180 | 0.0180 | 0.0180 | 0.0217 | 0.0170 | 0.0170 | 0.0170 | 0.0188 |
| | RMSE | 0.0054 | 0.0051 | 0.0052 | 0.0050 | 0.0059 | 0.0059 | 0.0059 | 0.0060 | 0.0065 | 0.0065 | 0.0065 | 0.0083 | 0.0055 | 0.0055 | 0.0055 | 0.0057 |
| | $R^2$ | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9995 | 0.9995 | 0.9995 | 0.9992 | 0.9996 | 0.9996 | 0.9996 | 0.9996 |
| | PDE | 13.8675 | 14.4054 | 14.4661 | 12.7871 | 17.2897 | 17.2950 | 17.3044 | 18.6944 | 15.7552 | 15.7594 | 15.7643 | 17.1319 | 20.0751 | 20.0746 | 20.0910 | 19.6858 |
| | Time (s) | 3.7934 | 0.8155 | 0.7546 | 29.3916 | 1.4280 | 0.7962 | 0.8397 | 29.3626 | 3.6747 | 0.6748 | 0.6841 | 28.7441 | 1.9053 | 0.7094 | 0.6679 | 29.0660 |
| NN | $\hat{p}$ | 0.1594 | 0.1594 | 0.1594 | 0.1565 | 0.1270 | 0.1270 | 0.1271 | 0.1325 | 0.1413 | 0.1413 | 0.1413 | 0.1403 | 0.1714 | 0.1714 | 0.1713 | 0.1821 |
| | $\hat{\mu_1}$ | 0.8957 | 0.8957 | 0.8956 | 0.9048 | 0.8285 | 0.8285 | 0.8285 | 0.8311 | 0.8427 | 0.8427 | 0.8428 | 0.8323 | 0.8292 | 0.8292 | 0.8291 | 0.8283 |
| | $\hat{\sigma_1}$ | 0.3019 | 0.3019 | 0.3018 | 0.2977 | 0.2556 | 0.2556 | 0.2555 | 0.2666 | 0.2576 | 0.2576 | 0.2577 | 0.2425 | 0.2720 | 0.2720 | 0.2719 | 0.2602 |
| | $\hat{\mu_2}$ | 5.0595 | 5.0596 | 5.0590 | 5.0937 | 5.0360 | 5.0360 | 5.0354 | 5.0487 | 4.7262 | 4.7262 | 4.7262 | 4.6901 | 4.2793 | 4.2793 | 4.2790 | 4.2860 |
| | $\hat{\sigma_2}$ | 2.9133 | 2.9133 | 2.9135 | 2.9446 | 2.5134 | 2.5134 | 2.5133 | 2.5105 | 2.4433 | 2.4433 | 2.4435 | 2.4834 | 2.3057 | 2.3057 | 2.3055 | 2.2775 |
| | K-S | 0.0719 | 0.0719 | 0.0719 | 0.0767 | 0.0447 | 0.0447 | 0.0446 | 0.0436 | 0.0496 | 0.0496 | 0.0496 | 0.0467 | 0.0547 | 0.0547 | 0.0547 | 0.0503 |
| | RMSE | 0.0400 | 0.0400 | 0.0400 | 0.0422 | 0.0235 | 0.0235 | 0.0234 | 0.0233 | 0.0273 | 0.0273 | 0.0273 | 0.0261 | 0.0298 | 0.0298 | 0.0298 | 0.0285 |
| | $R^2$ | 0.9786 | 0.9786 | 0.9786 | 0.9760 | 0.9929 | 0.9929 | 0.9930 | 0.9930 | 0.9904 | 0.9904 | 0.9904 | 0.9911 | 0.9879 | 0.9879 | 0.9880 | 0.9890 |
| | PDE | 24.5809 | 24.5795 | 24.5853 | 22.4655 | 22.2318 | 22.2318 | 22.2572 | 22.3702 | 22.4336 | 22.4336 | 22.4294 | 22.4474 | 24.8756 | 24.8757 | 24.8851 | 26.4505 |
| | Time (s) | 0.5265 | 0.2063 | 0.3920 | 7.4275 | 0.3589 | 0.1905 | 0.1953 | 7.0412 | 0.2909 | 0.1863 | 0.1812 | 6.8505 | 0.4233 | 0.1990 | 0.3578 | 7.1505 |
| LL | $\hat{p}$ | 0.7232 | 0.7232 | 0.7234 | 0.7322 | 0.7642 | 0.7640 | 0.7630 | 0.7796 | 0.7349 | 0.7349 | 0.7351 | 0.7169 | 0.6945 | 0.6945 | 0.6945 | 0.6271 |
| | $\hat{\mu_1}$ | 1.6055 | 1.6055 | 1.6054 | 1.5697 | 1.6238 | 1.6239 | 1.6245 | 1.6208 | 1.5681 | 1.5681 | 1.5683 | 1.5733 | 1.4759 | 1.4759 | 1.4756 | 1.5184 |
| | $\hat{\sigma_1}$ | 0.4937 | 0.4936 | 0.4937 | 0.5062 | 0.4159 | 0.4157 | 0.4152 | 0.4266 | 0.4291 | 0.4291 | 0.4293 | 0.4185 | 0.4298 | 0.4298 | 0.4299 | 0.4010 |
| | $\hat{\mu_2}$ | 0.0538 | 0.0539 | 0.0535 | 0.0241 | 0.0382 | 0.0389 | 0.0410 | 0.0202 | 0.0536 | 0.0537 | 0.0530 | 0.0992 | 0.0021 | 0.0021 | 0.0024 | 0.1394 |
| | $\hat{\sigma_2}$ | 0.5558 | 0.5559 | 0.5555 | 0.5476 | 0.5396 | 0.5400 | 0.5419 | 0.5348 | 0.5219 | 0.5219 | 0.5220 | 0.5559 | 0.5668 | 0.5668 | 0.5659 | 0.6584 |
| | K-S | 0.0191 | 0.0191 | 0.0191 | 0.0302 | 0.0242 | 0.0242 | 0.0243 | 0.0210 | 0.0212 | 0.0212 | 0.0211 | 0.0256 | 0.0190 | 0.0190 | 0.0189 | 0.0238 |
| | RMSE | 0.0082 | 0.0082 | 0.0082 | 0.0138 | 0.0088 | 0.0088 | 0.0088 | 0.0085 | 0.0080 | 0.0080 | 0.0080 | 0.0103 | 0.0069 | 0.0069 | 0.0069 | 0.0091 |
| | $R^2$ | 0.9992 | 0.9992 | 0.9992 | 0.9978 | 0.9991 | 0.9991 | 0.9991 | 0.9992 | 0.9992 | 0.9992 | 0.9993 | 0.9988 | 0.9994 | 0.9994 | 0.9994 | 0.9990 |
| | PDE | 6.8260 | 6.8287 | 6.7891 | 10.4233 | 12.9855 | 13.0224 | 13.1499 | 8.4434 | 11.3367 | 11.3367 | 11.2233 | 15.3941 | 16.7736 | 16.7739 | 16.8052 | 21.6140 |
| | Time (s) | 0.6121 | 0.2256 | 0.2457 | 8.6990 | 0.3988 | 0.2620 | 0.2159 | 8.6035 | 0.3102 | 0.2058 | 0.2464 | 8.2431 | 0.5255 | 0.2119 | 0.4040 | 8.4335 |

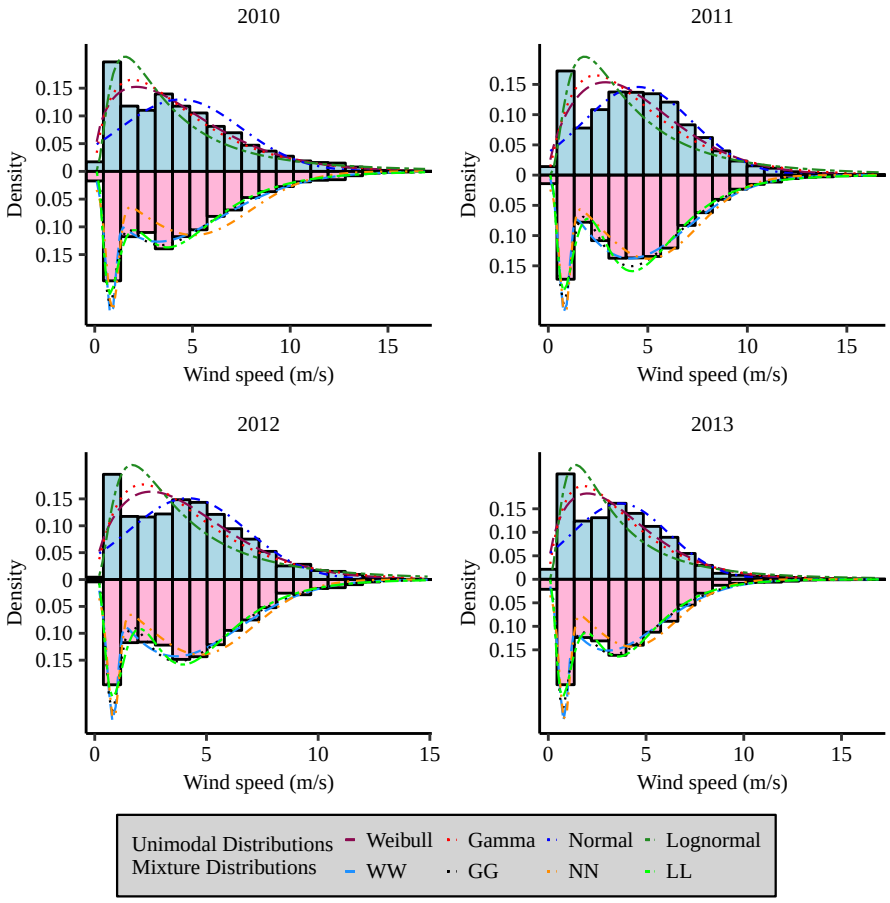



**Figure 6**. Unimodal and mixture density fits for the dataset obtained from Gökçeada Meteorological Station. The upper sides of the mirrored histograms present the unimodal density fits where the opposite sides present the mixture density fits.

**Table 7**. Overall comparison of the distributions and optimization methods for the dataset obtained from Gökçeada Meteorological Station.

| | KS | | RMSE | | $R^2$ | | PDE | |
|---|---|---|---|---|---|---|---|---|
| Date | Model | Method | Model | Method | Model | Method | Model | Method |
| 2010 | GG | NM | GG | SA | GG | SA | LL | NM |
| 2011 | GG | SA | GG | NR | GG | NR | LL | SA |
| 2012 | WW | NM | WW | NM | WW | NM | LL | NM |
| 2013 | GG | BFGS | GG | NR | GG | NR | LL | NR |

## 5. Conclusion

Main results of this study can be summarized as follows:

1. Two-component mixture distributions have superiority over the unimodal distributions for the seasonal and annual wind speed data according to the assessment criteria including K-S test statistic, RMSE, $R^2$, and PDE.

2. For the dataset obtained from Belen Wind Power Plant, WW performed better than other distributions in 2013 and GG performed better than others in 2014.
3. GG was the most efficient distribution for the dataset obtained from Gökçeada Meteorological Station.
4. Best model chosen for each time period shows more variability for the dataset obtained from Belen Wind Power Plant; therefore, single distribution may not be sufficient for modeling the wind speed data observed at wind farms.
5. NM is the fastest algorithm in parameter estimation of unimodal wind speed distributions.
6. BFGS and NM are the fastest algorithms in parameter estimation for the mixture wind speed distributions.
7. SA is the slowest algorithm for all datasets.

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