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Channel and carrier frequency offset estimation based on projection onto a bidimensional basis

Roberto CARRASCO ALVAREZ[1,*], **Ramon PARRA MICHEL**[2],
Aldo Gustavo OROZCO LUGO[3], **Marco Antonio GURROLA NAVARRO**[1]
[1]Department of Electronic Engineering, CUCEI, University of Guadalajara, Guadalajara, Mexico
[2]Department of Electrical Engineering, Telecommunications Section, CINVESTAV-IPN, Guadalajara, Mexico
[3]Department of Electrical Engineering, Communications Section, CINVESTAV-IPN, Mexico City, Mexico

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Abstract: Two of the most counterproductive effects that must be dealt with in communication systems in realistic environments are carrier frequency offset (CFO) and time-varying channels. These problems are usually addressed by using independent approaches for each one. This paper introduces an algorithm that attacks both of these effects in a joint fashion. It is based on a rough compensation of CFO, and after considering that the remaining CFO uncertainty can be seen as part of the time-varying channel a channel estimation that includes that composite channel is performed. Particularly, the channel estimation based on projection onto a bidimensional basis is shown to be adequate to perform this process. The approach suggested in this paper is illustrated for a superimposed training-based communication system. Simulation results corroborate the validity of the proposed algorithm, where the performance obtained is similar to that achieved for the time-varying channel estimators based on bidimensional basis projection when no CFO is present.

Key words: Carrier frequency offset, communication channel estimation, orthogonal basis expansion, superimposed training

1. Introduction

In digital communication systems, carrier frequency offset (CFO) is a harmful phenomenon that appears when there is a difference between the transmitter and receiver oscillators. An inadequate cancellation of it severely compromises bit error rate (BER) performance. For the correction of this problem, several authors have proposed different techniques, dealing only with time-invariant channels. In [1–5] and references therein, an explicit training sequence (multiplexed with data) is employed for estimating the CFO in orthogonal frequency-division multiplexing (OFDM) systems. For single carrier transmissions, [6] proposes the use of time multiplexed pilots in a two-step algorithm; first, a coarse estimation of the CFO is made and subsequently a fine estimation is carried out. In the case of implicit training (training sequence superimposed onto data), CFO estimation and compensation was studied in [7–9].

However, for time-varying channels, we can only cite [10], where a method is proposed that combines the use of an exponential basis (for each time-variant channel coefficient) and pilot sequences for OFDM systems. For single carrier systems, to the best of the authors' knowledge no method exists that estimates CFO along with time-varying channels. Thus, a new method is proposed capable of estimating both the CFO and the time-

*Correspondence: r.carrasco@academicos.udg.mx

varying channel for single carrier systems. Although this new method is capable of working with an explicit training sequence, superimposed training (ST) is assumed because it can deal with faster varying channels.

1.1. Objectives and contributions

This paper presents a new method for estimating both the CFO and the time-varying frequency selective communication channel for single carrier systems. To achieve this, the method assumes ST and projection onto a bidimensional basis to perform a two-step algorithm. First, a rough compensation of the CFO is implemented. Then the estimation of the communication channel is performed using a bidimensional basis expansion as in [11] but considering, during the design of such a basis, the impairments of the lack of precision of the previous step. In this way, an increment in the number of elements of the basis is avoided if the algorithm presented in [11] were implemented as reported. Moreover, a fine estimation and compensation of the CFO are also circumvented, which is typically an iterative complex computational task. The viability of the proposed method is validated through simulation results.

1.2. Paper organization

This paper is structured as follows. The CFO problem in a communication system based on ST is discussed in Section 2. In Section 3, the mathematical formulation for estimating the CFO and the communication channel is established. The performance of the algorithm presented is corroborated via simulation in Section 4. Conclusions and final comments are stated in Section 5.

1.3. Notation

Operators $(\cdot)^H$, $(\cdot)^T$, and $(\cdot)^*$ denote complex conjugate transpose, transpose, and complex conjugation, respectively. $\mathbf{I}_N$ is the $N \times N$ identity matrix. $tr(\mathbf{A})$is the trace of the square matrix $\mathbf{A}$, and $diag(\mathbf{a})$ represents the zero matrix with its main diagonal equal to the vector $\mathbf{a}$. The entry corresponding to the i-th row and j-th column of the matrix A is expressed as $[\mathbf{A}]_{i,j}$; $\lfloor x \rfloor$ represents the floor operation that returns the highest integer less than or equal to x, and $\lceil x \rceil$ is the ceil operation that returns the lowest integer greater than or equal to x. $F(\cdot)$ is the Fourier transform operator, $\langle \cdot \rangle_N$ indicates module N, $\delta(t)$ is the Dirac delta function, and finally $E(\cdot)$ represents the expectation operator.

2. System model

Figure 1 presents the discrete-time baseband-equivalent model of the communication system considered in this article, where the effect of CFO is modeled as a complex exponential signal multiplying the received sequence. In this model, the transmitted signal $s(k^{'})$ is formed by the composition of the data sequence $b(k^{'})$ and the training sequence $c\left(k^{'}\right)$. The index k' helps to enumerate the samples at the symbol rate $1/T$, where T is the symbol period. It is assumed that the data sequence comes from a random finite alphabet that possesses zero mean and variance equal to $E\left(\left|b\left(k^{'}\right)\right|^2\right) = \sigma_b^2$. On the other hand, the training sequence $c\left(k^{'}\right)$ is constructed as in [12], with power equal to $1/P \sum_{k=0}^{P-1} |c(k')|^2 = \sigma_c^2$, where P is the period of the training signal.

As demonstrated in [11], if the received signal is oversampled at a rate $1/T_r$ where $T_r = T/r$ and $r \geq 2$

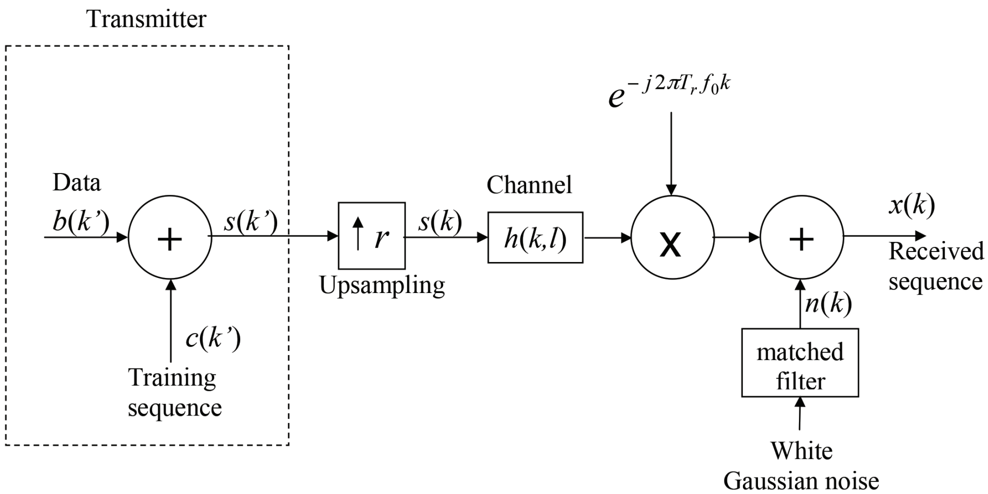

Figure 1. Digital communication system model considered.

is an oversampling factor, then it is possible to obtain an improvement in the performance of the estimation algorithm. In order to accomplish that, an upsampling block in the model is necessary to make the rates of both the transmitter and the receiver signals commensurate. The upsampled version of $s(k^{'})$ denoted as $s(k)$ is propagated through the communication channel $h(k,l)$, which is formed by the convolution of the pulse-shaping and matched filters with the propagation medium. The indexes k and l help to enumerate the time and time-delay samples, respectively. For simplicity, $h(k,l)$ can be visualized as a finite impulse response (FIR) filter made up of $l=0,\ldots,L-1$ coefficients that are varying in time, where $L=\left(\left\lfloor\frac{\tau_{max}}{T}\right\rfloor\right)\times r$ and τ_{max} is the maximum delay spread in seconds (taking into consideration both the propagation medium and filter durations). The signal distorted by the communication channel is detected in the receiver and, due to the effect of the CFO, multiplied by a discrete complex exponential with frequency f_0, where f_0 includes the difference between the transmitter and receiver oscillators. The resulting signal is contaminated with colored Gaussian noise $n(k)$ for obtaining $x(k)$. The noise $n(k)$ is produced by filtering zero-mean white Gaussian noise with variance σ_n^2 with the matched filter. A transmission by blocks is assumed, where each block is composed of N symbols and preceded by a cyclic prefix of length equal or longer than L/r symbols. This cyclic prefix is formed by repeating the last symbols to transmit. In this paper, perfect block synchronization is considered. The latter permits us, as shown in [13], to fix the period of the training sequence at $P=L/r$. In order to simplify the analysis, the length of the cyclic prefix is assumed to be equal to the period of the training signal; moreover, it is assumed that N is a multiple of P.

Mathematically, after the cyclic prefix is removed, the received signal can be expressed as

$$x\left(k\right)=e^{-j2\pi T_r f_0 k}\sum_{l=0}^{L-1}\left(h(k,l)s\left(\langle k-l\rangle_{N_r}\right)\right)+n(k) \tag{1}$$

for $k=0,\ldots,N_r-1$, where $N_r=N\times r$ is the number of samples taken in the receiver.

3. CFO and channel estimation based on a bidimensional basis

As discussed previously, CFO and channel estimation have been considered as different problems. Thus, the CFO is first estimated and then compensated, and subsequently an estimation of the communication channel is obtained. In this paper, CFO and channel estimation are considered as a single problem. From (1), it is possible to observe that if the exponential term and the time-varying channel $h(k,l)$ are conjoined then the result is also a time-varying channel $h_{cfo}(k,l) = e^{-j2\pi T_r f_0 k} h(k,l)$, whose frequency spectrum is a shifted version of the spectrum of $h(k,l)$. Thus, it is possible to represent this new channel as a weighted sum of the elements of a basis as in [11] and references therein, the only restriction being that the basis utilized must span the full frequency spectrum of the new channel. To clarify this point, consider that the maximum Doppler frequency of $h(k,l)$ is f_d; due to the CFO, the power spectrum of $h_{cfo}(k,l)$ is contained in the frequency interval $[f_0 - f_d\,,\, f_0 + f_d]$, which makes it necessary to obtain a basis that spans said interval. If the approximation proposed in [11] is utilized, then the discrete prolate spheroidal (DPS) sequences used for expanding the time domain must be constructed in a way that takes into account the maximum frequency $|f_0| + |f_d|$.

It could be noted that if f_0 is several times larger than f_d then the number of DPS sequences increases considerably, which implies a reduction in the channel estimation performance [11]. In order to minimize this phenomenon, it is possible to carry out a coarse estimation of the CFO as in [6] and [9]. This coarse estimation serves to counteract the effect of the CFO in a rough fashion, with the error between the original CFO and the coarse estimation approximated together with the channel via the projection onto the basis. The coarse estimation used in the present paper is based on the maximization of frequency bins method, as proposed in [9], which is supported by the fact that the received sequence in the absence of CFO has a periodic sequence embedded with period $P_r = P \times r$ due to the transmission of the training sequence $c(k)$. Therefore, the DFT of $x(k)$ presents significant frequency components at the positions iN/P for $i = 0; \ldots, P-1$. If the CFO is present, then these significant frequency components are just cyclically shifted in function of f_0. Taking this into account, in order to accomplish the rough estimation of the CFO it is necessary to obtain

$$\hat{i} = \sum_{j=0}^{P_r - 1} X\left(i + j\left(\frac{N}{P}\right)\right);\; i = 0, \ldots, \left(\frac{N}{P}\right) - 1, \tag{2}$$

where $X(k) = F(x(k))$ is the DFT of the received signal and the rough CFO estimation is given by

$$\hat{f}_0 = \begin{cases} \hat{i}\frac{1}{N \times T} & \text{for } \hat{i} < \left\lceil \frac{N}{2P} \right\rceil \\ \left(\langle \hat{i} \rangle_{\left\lceil \frac{N}{2P} \right\rceil} - \left\lceil \frac{N}{2P} \right\rceil\right)\frac{1}{N \times T} & \text{for } \hat{i} \geq \left\lceil \frac{N}{2P} \right\rceil \end{cases} \tag{3}$$

Once $\hat{f}_0$ is obtained, it is possible to correct roughly the CFO as follows:

$$\tilde{x}(k) = e^{j2\pi T_r \hat{f}_0 k} x(k). \tag{4}$$

Substituting (1) into (4) yields

$$\tilde{x}(k) = \sum_{l=0}^{L-1} \left(\tilde{h}_{cfo}(k,l)\, s\left(\langle k - l \rangle_{N_r}\right)\right) + n(k), \tag{5}$$

where $f_0 = f_0 - \hat{f}_0$ and $\tilde{h}_{cfo}(k,l) = e^{-j2\pi T_r (\Delta f_0) k} h(k,l)$ is the communication channel to be estimated, which includes the effects of the CFO that could not be counteracted by the rough estimation.

For estimating the new communication channel $\tilde{h}_{cfo}(k,l)$ it is possible to approximate the channel by the sum

$$\tilde{h}_{cfo}(k,l) = \sum_{m=0}^{M-1} \rho_m W_m(k,l) + e_p(k,l), \tag{6}$$

where $W_m(k,l)$ are the elements of the bidimensional basis, ρ_m are the weights of the basis in the approximation, and $e_p(k,l)$ is the approximation error due to the fact that the basis used does not fill all the space of $\tilde{h}_{cfo}(k,l)$. As detailed in [11], the elements of the bidimensional basis are formed by the outer product of two one-dimensional bases, e.g., DPS [14] for the time domain expansion and universal basis (UB) [11] for the time-delay domain expansion. In the case of the estimation of $\tilde{h}_{cfo}(k,l)$, where the Doppler frequency is $f_d + f_0$, the elements of the DPS basis are the $Q_1 = \lceil 2N_r(f_d + f) \rceil + 1$ eigenvectors of the matrix $[\mathbf{C}]_{k,m} = \sin(2\pi(f_d + f_0)T_R(k-m))/((k-m)\pi)$ whose eigenvalues are greater. The elements of the UB are the Q_2 eigenvectors whose eigenvalues are greater, which are calculated from a generic autocorrelation matrix $\mathbf{R}_u$. This autocorrelation function is constructed from the composite impulse response of the pulse-shaping and matched filters and a generic power delay profile [11].

Once the bidimensional basis is stated, an estimate of the values ρ_m that weight this basis in (6) is given by [11] as

$$\hat{\mathbf{r}} = \dot{\mathbf{A}}^{-1}\mathbf{A}^H\tilde{\mathbf{x}}, \tag{7}$$

where $[\hat{\mathbf{r}}]_m = \hat{\rho}_m$ and $[\tilde{\mathbf{x}}]_k = \tilde{x}(k)$ are the vector form of the estimated weights ρ_m and the corrected received signal, respectively; $\dot{\mathbf{A}} = \mathbf{A}^H\mathbf{A}$ and $[\mathbf{A}]_{k,q} = \sum_{l=0}^{L-1} \left(W_q(k,l)\, c\left(\langle k-l \rangle_{N_r}\right)\right)$ is a matrix whose columns correspond to the circular convolution of the training sequence with each function of the basis. Substituting $\hat{\rho}_m$ in (6), the estimated channel $\hat{\tilde{h}}_{cfo}(k,l)$ is given by

$$\hat{\tilde{h}}_{cfo}(k,l) = \sum_{m=0}^{M-1} \hat{\rho}_m W_m(k,l) \tag{8}$$

4. Simulation results

In this section the numerical performance of the proposed algorithm is evaluated. The parameters established for obtaining the evaluation are as follows. Two different scenarios are considered; each one consists of a time and frequency selective wide sense stationary uncorrelated scattering Rayleigh propagation medium that presents a Doppler power spectrum with a Jakes shape for all delays, with maximum Doppler frequencies $f_d = [10, 120]$ Hz, respectively. The realizations of each medium are convolved with the pulse-shaping and matched filters and the resulting communication channels are sampled at a rate T_r, and scaled to guarantee unitary energy. For all the propagation mediums, a discrete power delay profile is considered, consisting of a train of impulses located in $[0,\ 2,\ 5,\ 15,\ 20,\ 30]\,\frac{25}{32}\mu s$, with values $[-3,\ 0, -2, -6, -8, -10]$ dB. Both the matched filter and the pulse-shaping filter are considered to be square root raised cosine with a roll-off factor equal to 0.5 and the convolution of both lasts $6T$ s. A QPSK modulation is considered for generating symbols at a rate of 40 KBd (corresponding to a symbol period equal to $T = 25$ μs); these symbols are grouped in blocks of $N = 512$ symbols preceded by a cyclic prefix of length $P = 8$ symbols. An oversampling rate equal to $r = 2$ is used in the receiver. The power of both the transmitted data and the training sequences is fixed at $\sigma_b^2 = 0.8$ and $\sigma_c^2 = 0.2$ respectively. Therefore, the training-to-information ratio is $TIR = \frac{\sigma_c^2}{\sigma_b^2} = 0.25$. For constructing the basis, it is considered that the DPS basis contains $Q_1 = \lceil 2N_r(f_d + \ f)\rceil + 1$ sequences, where f is the maximum expected error during the rough estimation of the CFO, which for this simulation is fixed at $f = 100$ Hz (that is similar to the resolution of the DFT $1/(NT) = 78.12$ Hz), and the UB contains $Q_2 \leq L$ sequences corresponding to the sequences whose normalized eigenvalues are greater than 1×10^{-3}, forming a bidimensional basis with $M = Q_1 \times Q_2$ elements. For each scenario considered, Monte Carlo runs are carried out for each value of *SNR* = [0, 5, 10, 15, 20, 25] dB considered. In each Monte Carlo run, the CFO is generated randomly such that $-P/2 \leq f_0 \leq P/2$. The normalized error between the real channel realization that has roughly compensated the CFO $\tilde{h}_{cfo}(k,l)$ and the estimated channel $\hat{\tilde{h}}_{cfo}(k,l)$ is named the normalized channel MSE (NCMSE), and defined in [12] as

$$NCMSE = \frac{\sum_{i=1}^{M}\sum_{k=0}^{N_r-1}\sum_{l=0}^{L-1}\left\|\hat{\tilde{h}}^i_{cfo}(k,l) - \tilde{h}^i_{cfo}(k,l)\right\|^2}{\sum_{i=1}^{M}\sum_{k=0}^{N_r-1}\sum_{l=0}^{L-1}\left\|\tilde{h}^i_{cfo}(k,l)\right\|^2}, \tag{9}$$

where $\hat{\tilde{h}}^i_{cfo}(k,l)$ and $\tilde{h}^i_{cfo}(k,l)$ are the estimated and original channel, respectively, in the i-th Monte Carlo run.

The simulation results for all Doppler frequencies stated are presented in Figure 2. There it is possible to contrast the plots of NCMSE when the method proposed in this paper is used, against the compensation of CFO using [9], and then the estimation of the channel using bidimensional basis projection is carried out. Additionally, the performance of the channel estimator is presented using bidimensional basis projection when no CFO is present and $Q_1 = \lceil 2N_r(f_d + \ f)\rceil + 1$ and $Q_1 = \lceil 2N_r(f_d)\rceil + 1$ DPS sequences are used, respectively. It is possible to observe that the method proposed here performs exactly as the estimator does when no CFO is considered, but the number of DPS sequences is augmented. This performance is less favorable compared to that exhibited by the estimator when no CFO is stated and the exact number of DPS sequences is utilized.

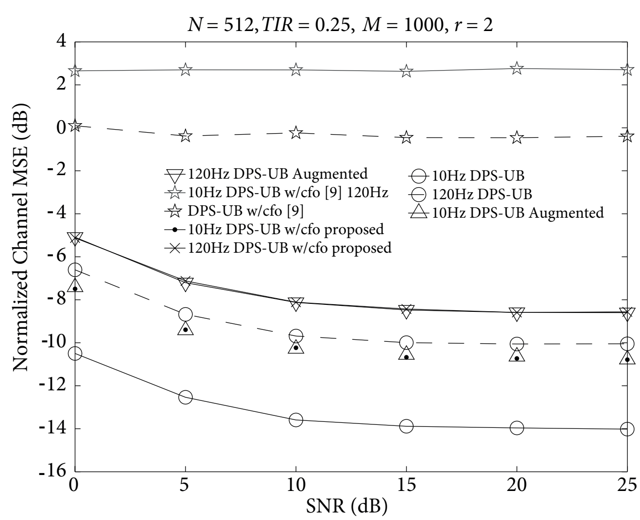

Figure 2. NCMSE simulation results with $f_d = [10, 120]$ Hz for different procedures of CFO compensation.

This phenomenon is due to the fact that the frequency band-pass of the DPS is enlarged to absorb the possible errors during the rough estimation of the CFO, and consequently this increases the number of DPS sequences to be used, but this penalty is acceptable considering the gain obtained when this method is compared with the one that first corrects CFO using [9] and then estimates the communication channel using bidimensional basis projection. According to the figure presented, the gain between both methods is at least 5 dB for $f_d = 120$ Hz, and 10 dB when $f_d = 10$ Hz. At some critical values, this gain approaches 14 dB.

5. Conclusions

Joint CFO and channel estimation was presented in this paper. The procedure proposed consists of a two-step algorithm, where a rough estimation and compensation of the CFO is accomplished as a first step, and then a bidimensional basis projection is utilized for estimating the time-varying channel together with the possible error produced in the previous step. It is demonstrated via simulation results that the performance is identical to the estimator's when no CFO is considered, but the basis is augmented, which leads to a reduction in performance compared with the estimator that utilizes the exact amount of basis. This penalty, however, is acceptable considering the performance obtained when the CFO is compensated as in the state of the art and then the communication channel is estimated, producing a gain of at least 5 dB.

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