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In-field failure assessment of tractor hydraulic system operation via pseudospectrum of acoustic measurements

Saurabh GUPTA[1], **Mahdi KHOSRAVY**[2,3,*], **Neeraj GUPTA**[4], **Hemant DARBARI**[5]

[1]Department of Advanced Engineering, John Deere India Pvt. Ltd., Pune, India
[2]Department of Electrical Engineering, Federal University of Juiz de Fora, Minas Gerais, Brazil
[3]Department of Electrical Engineering, University of the Ryukyus, Okinawa, Japan
[4]Department of Computer Science and Engineering, Oakland University, Rochester City, Michigan, USA
[5]Center for Development of Advanced Computing, Pune, India

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Abstract: Hydrological working conditions are critical indicators for the maintenance of tractor mechanical systems. This research presents a pseudospectrum approach to analyze the particle pollution level of the hydraulic filter in its aging process and thus on-time prediction and diagnosis of weak or faulty conditions. The practical experiments of pseudospectrum analysis are performed on oil filter sound records. For mobile application purposes, besides deployment of advanced audio recording equipment, three popular brands of cell phones are used. The soundtracks are recorded in different incremental stages of fluid contamination by pollutive particles until being choked based on the ISO4406 standard. The pseudospectrum of the oil filter acoustics leads to two numerical features: the low-band energy and 12.5 kHz relative peak energy (12.5RPE). This manuscript proposes these two features for efficient on-line monitoring of the liquid cleanliness level as well as the prediction and diagnosis of choked condition of the tractor hydraulic system.

Key words: Preventive maintenance, hydraulic systems, acoustic measurements, spectral analysis

1. Introduction

Smartphones have an increasing impact on society in moving towards higher convenience with more and more practical applications. The Internet of Things (IoT) also brings more efficiency for mobile applications. Furthermore, in complex and uncertain operating conditions, the use of a smartphone with Internet technologies could be helpful in the monitoring and detection of the operational conditions of a tractor, especially during emergency circumstances in a field far away from possible workshops.

Although fruitful research has been performed on the prognostics of mechanical and rotational equipment [1–8], there are few reports on the predictive maintenance of hydraulic system parts. In the case of a tractor, the hydraulic system is commonly subjected to regular maintenance. Namely, the oil filter protects the system's components from malfunction resulting from fluid pollution due to particles. As Damen Technical Agencies reported (https://dta.eu/hydraulics/hydraulic-filtration), on average, every minute, about one million particles with size bigger than $> 1\mu m$ enter a hydraulic system, which can severely damage the components of the system and result in costly tribological failure of the system [9, 10]. The required cleanliness level (RCL) [11] is the upper bound for the concentration of particles in a hydraulic system. When contamination surpasses the RCL, costly cleaning operations are imposed on hydrological operators. Cleanliness monitoring provides timely

*Correspondence: mahdi.khosravy@ufjf.edu.br

warning of these problems for the timely maintenance of the oil filter, thus providing a considerable increase in the lifetime of the hydraulic system.

Three modes of particle monitoring [12] are suggested in the literature as off-line, on-line, and in-line, which are respectively from a fluid sample, flow line, and pressurized line [13]. Off-line monitoring suffers from the need of expert operator skills for sampling and analysis. This is due to the need for extraction of fluid for analysis in a laboratory. Furthermore, each monitoring results in time extension, and in the results of sampling and analysis, additional pollution interferes with the results and leads to data errors. In both on-line and in-line monitoring, either automatic particle counters (APCs) [14, 15] are installed in connection with the flow or filter blockage monitors (FBMs) [16] are in use. APCs work based on the principle of light extinction, where a narrow light beam illuminates the passage of fluid and the polluting particles obscure the light beam by blockage or scattering. Corresponding electrical pulses are thus produced and their number and amplitude determine the size and density of the particles. The disadvantage of deployment of APCs is their high sensitivity to optical interfaces such as air bubbles, water droplets, additive silicone antifoam droplets, or emulsions, which result in erroneous data and wrong conclusions and thus waste maintenance efforts [17].

Spectrum analysis is a way to access the frequency components of the signal. There are also other decomposition techniques, like blind source separation [18–22] and blind component pressing [23], which give the independent components, and principal component analysis [24]. Here, the spectrum analysis of the soundtrack records of the hydraulic system has been suggested for on-line monitoring and analysis. The oil filters soundtracks are recorded by mobile phone sound recorder and their pseudospectrum is estimated and transmitted via mobile phone networks for an advanced online web-based software. In either case, in order to have an on-time spectrum analysis or spectrum transmission, an efficient estimation of the spectrum with lower data length is required to avoid long length spectrum data. For this aim, the pseudospectrum of oil filter soundtracks with lower load of data is studied and analyzed for the on-line monitoring and prognostics of the hydraulic fluid pollution conditions. An approach for detection of particle pollution level as well as prediction and diagnosis of hydraulic choking is introduced. By integrating the proposed smart diagnostic algorithm in a maintenance diagnostic application for phones, even a nonexpert operator can monitor the tractor's hydraulic conditions by just locating the phone in a proper place adjacent to the oil filter and activating the application along with the working tractor engine with a particular revolutions per minute (RPM) rate. It should be noted that due to the expert system, at the back end of the developed app for mobile phones, this operation can be easily done by putting the phone close to the noise-generating part without any requirement of the knowledge of the mechanical technology of tractor operation for users. Since the proposed approach is applied to hydraulic filter sound, it directly connects to its activity and is not susceptible to interfering error such as optical errors in APCs. Besides, it does not need expensive optical equipment, and it does not require any installation or related calibrations. It is an approach to realize a user-friendly smartphone diagnostic application.

The paper's organization is as follows: Section 2 describes the process of data collection as implemented in the John Deere laboratories. In Section 3 the pseudospectrum analysis and the multiple signal classification (MUSIC) approach to it are briefly explained. Section 4 analyzes and discusses the results and finally Section 5 concludes the paper.

2. Data collection

The first and most important part of research work regarding tractor parts' fault diagnosis is data collection. Although analyzing noise-free datasets is easier and more efficient, data with noisy backgrounds better match the

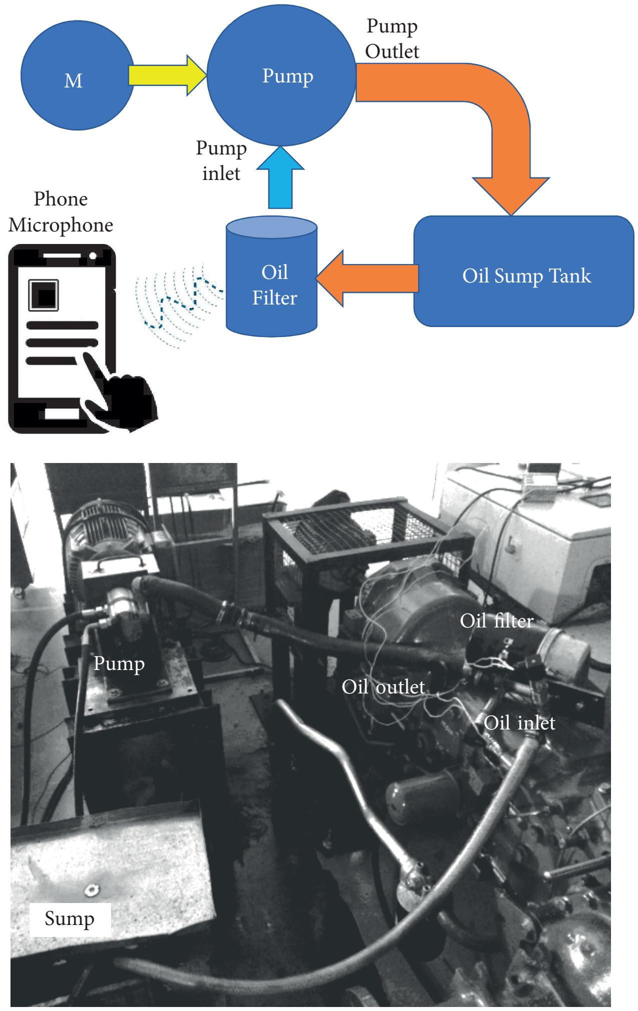

Figure 1. Schematics and laboratory setup for recording the sound of the oil filter.

practical field environment. In order to promote the user-friendly utility of the application, single-sensory sound records are acquired. This is because while feature extraction from multisensory data requires a sophisticated microphone arrangement, single-sensory records can be simply obtained by putting the smartphone in its prearranged location adjacent to the oil filter, as shown in Figure 1.

The general methodology of single-sensory data collection deployed by the authors is as follows: First, we have recorded the aging-related sound signals by manually increasing the particle pollution in the hydraulic system, which directly affects the aging stage of the filter and then failure concerning the signals. Apart from a possible defect problem with the generated vibration sound, aging and pollution produce their corresponding vibration signals that can be categorized into classes of signals for different levels of aging. In order to avoid interference of the other parts' vibration fault signals and in order to have a clear view of the concerned failure, the experiments are done on a new tractor hydraulic system in John Deere India headquarter's laboratories (https://www.deere.co.in). Here, in recording the soundtracks of the oil filter, just the normal condition of a tractor with a normal engine RPM of 1000 has been implemented to have a clear observation of the defect effect apart from the other possible harmonic interferences in the environment. The recorded data are related to the different levels of particle pollution inside the hydraulic system. These pollution levels are arranged based on

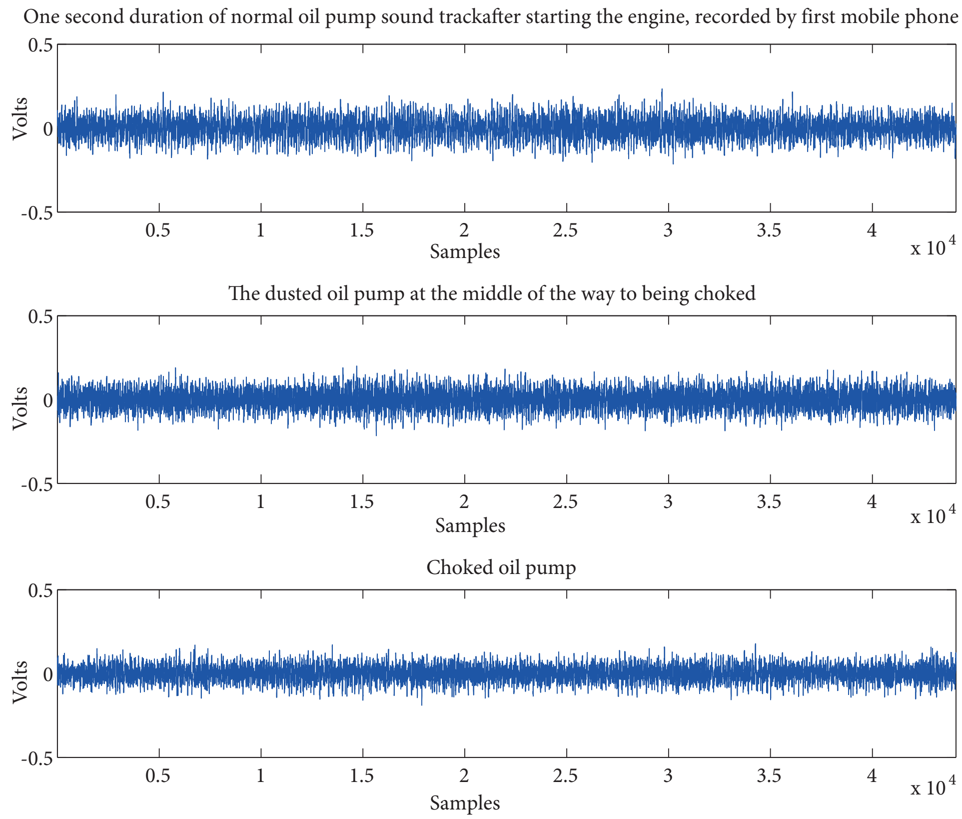

Figure 2. One second duration example of audio tracks recorded by one of the mobile phones. Top signal is the audio record after starting the engine while oil filter is in normal condition, middle signal is related to the oil filter with almost half of the dust required to be clogged, and bottom one is once the oil filter is clogged.

the ISO4406 standard [25] from zero, which means a new oil filter and clean fluid, to one level, two levels, and so on until six levels of polluting particles. After that, the oil filter is choked and that is the last level for audio recording.

In this research work, the experiment is performed on tractor oil filter fault diagnosis through audio tracks recorded with three different popular brands of cellphone. Here, to avoid mentioning the names of the brands to respect the mobile phone producers' rights, we just call them the first, second, and third cellphones.

3. Pseudospectrum analysis

After collection of single-sensory vibration audio signal records by three different mobile phones, we process the audio tracks in connection with the density level of polluting particles. Although preprocessing of the signal [26–30] and noise cancellation are essential for the main purpose of processing, here we avoid preprocessing so as not to lose noise information concerning the polluting particles. The level of the dust inside the pump determines the age of the pump before being clogged. In order to detect the clogging stage and predict it being choked with particles, a precise feature should be chosen. For this goal, here, a pseudospectrum analysis of the audio tracks is proposed. The acquirement of a pseudospectrum is due to its lower data rate compared to a DFT-based data spectrum. The low data rate of the pseudospectrum brings the fusibility of fast analysis via a phone application or quick on-line transfer of the data through even a slow Internet connection. Here, a brief review of the pseudospectrum is given.

3.1. Pseudospectrum

A well-behaved function, $f(x)$, in an interval can be represented as expansion in a set of orthonormal functions, $P_n(x)$, as follows:

$$f(x) = \sum_{n=0}^{\infty} a_n P_n(x), \qquad x \in [a, b], \tag{1}$$

where $P_n(x)$s are orthonormal polynomials mathematically expressed as follows:

$$\int_a^b w(x) P_n(x) P_m(x) dx = \delta_{nm}, \tag{2}$$

where $w(x)$ is an appropriate weight function for the orthonormal condition. δ_{nm} is the Kronecker delta, defined as follows:

$$\delta_{nm} = \begin{cases} 1, & m = n \\ 0, & m \neq n. \end{cases} \tag{3}$$

Orthonormal polynomials as basis functions are classically well known; Bernard Shizgal has listed some of them [31].

3.2. Eigenvalue problem approach to pseudospectra

One approach to pseudospectra is the eigenvector method. The pseudospectral method can be provided by eigenvalues as follows:

$$\int_a^b k(x, y) \psi_n(y) dy = \lambda_n \psi_n(x), \tag{4}$$

where $k(x, y)$ is the kernel with the assumption of being well behaved on both sides of the equation. By using the appropriate quadrature points $\{x_i\}$, the integration is reduced to summation of a set of linear equations:

$$\sum_{i=1}^{N} W_i k(x_j, x_i) \psi_n(x_i) = \lambda_n \psi_n(x_j). \tag{5}$$

3.3. Pseudospectrum by multiple signal classification

We are looking for an estimation of a set of reliable constant features or parameters on which the audio records are dependent. There have been some classical methods pursuing the same goal, such as maximum likelihood (ML) [32, 33] and maximum entropy (ME) [34, 35]. However, the aforementioned classical methods suffer from parameter estimation sensitivity and bias of the parameters. These problems were overcome by the deployment of the covariance matrix of the signal for estimation of complex sinusoids by Pisarenko [36, 37]. Later, Schmidt, by deriving a geometric solution, developed the multiple signal classification (MUSIC) algorithm [38]. MUSIC estimates the pseudospectrum of the signal using the estimates of the eigenvectors of the correlation matrix of the input data signal. MUSIC estimation of the signal frequency content is done by deployment of eigenspace. Its assumption of the input signal is p complex exponentials together with background noise. The spanned subspace of the signal is done by the p eigenvectors corresponding to the p largest eigenvalues of $\mathbf{R}_x$, the $M \times M$ autocorrelation matrix of the input signal, while the sorting order of eigenvalues is in the decreasing direction.

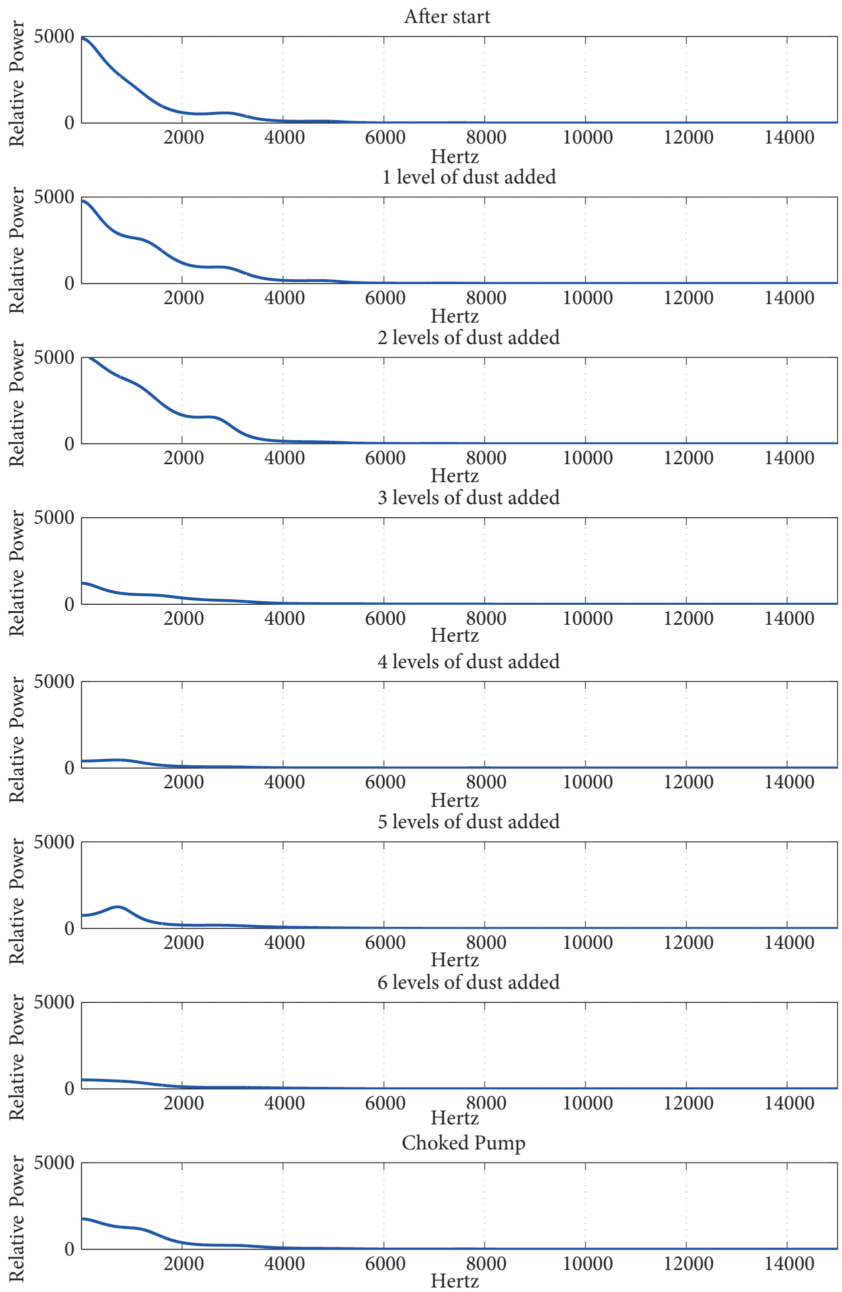

Figure 3. Pseudospectrum of the acoustic measurements by the first cellphone. Top curve: the clean hydraulic system; middle curves: increasing levels of polluting particles; lowest curve: the choked hydraulic system.

3.4. Oil pump fault diagnosis by MUSIC

Here, by application of the MUSIC algorithm to each signal record, its pseudospectrum is estimated. Thereafter, the pseudospectrum curves are compared and evaluated over the frequency range of the signals. Although thr MUSIC-estimated pseudospectrum is over normalized frequencies (in rad/sample), by having the sampling frequency of the records, we can perform the evaluation on frequencies in hertz.

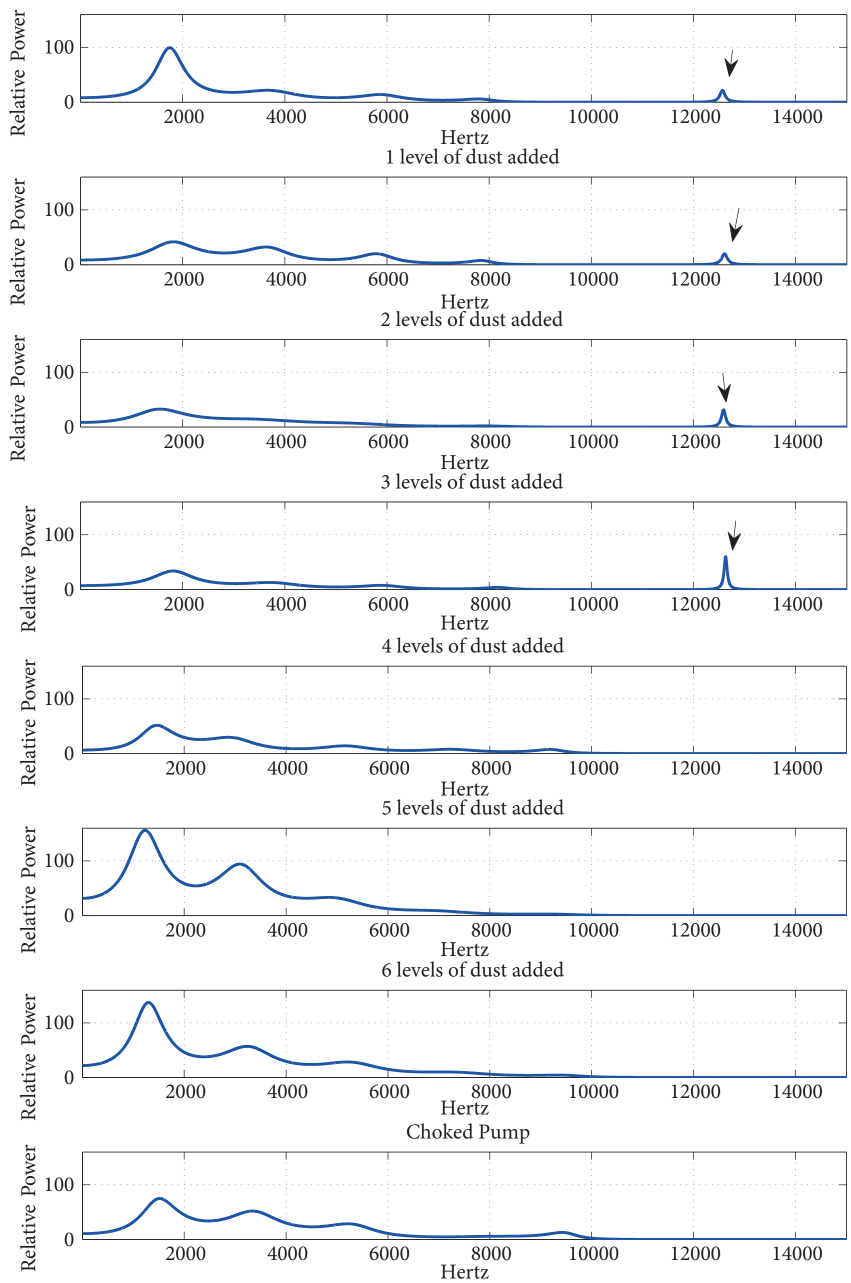

Figure 4. Pseudospectrum of the acoustic measurements by the second cellphone. Top curve: the clean hydraulic system; middle curves: increasing levels of polluting particles; lowest curve: the choked hydraulic system.

4. Results and discussion

Using MATLAB (2017b), the MUSIC algorithm has been implemented for the phones' acoustic records of the tractor oil filter sound. The MUSIC pseudospectrum of each mobile phone's sound records corresponding to eight different conditions is analyzed. The audio tracks are recorded in different conditions as follows:

Track 1: The sound of the oil pump just after starting the engine. The hydraulic system is clean and RPM is 1000.

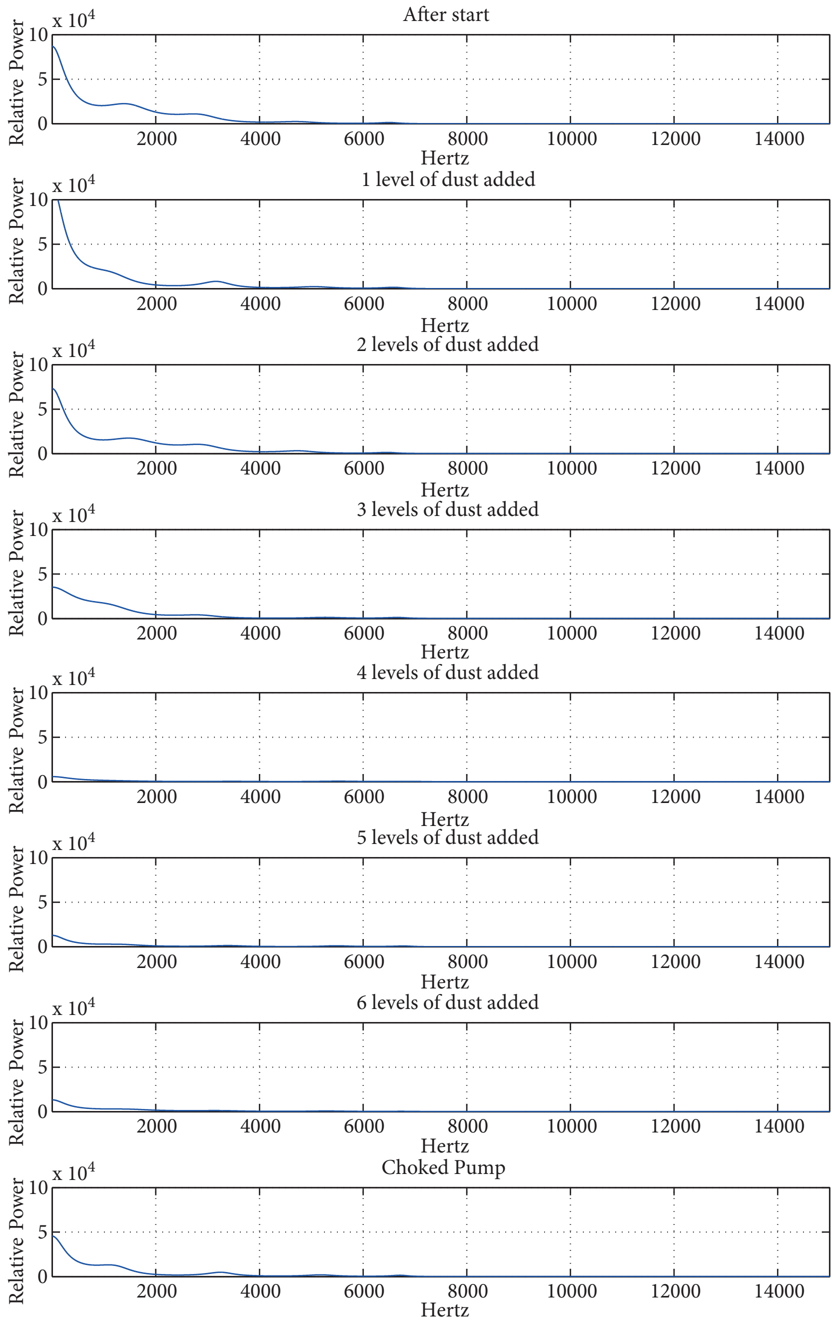

Figure 5. Pseudospectrum of the acoustic measurements by the third cellphone. Top curve: the clean hydraulic system; middle curves: increasing levels of polluting particles; lowest curve: the choked hydraulic system.

Tracks 2–7: After adding one to six levels of polluting particles according to ISO4406 [25].

Track 8: The hydraulic system is choked.

Figures 3, 4, and 5 show the MUSIC estimation of the pseudospectrum of the sound records by the first, second, and third cellphones, respectively.

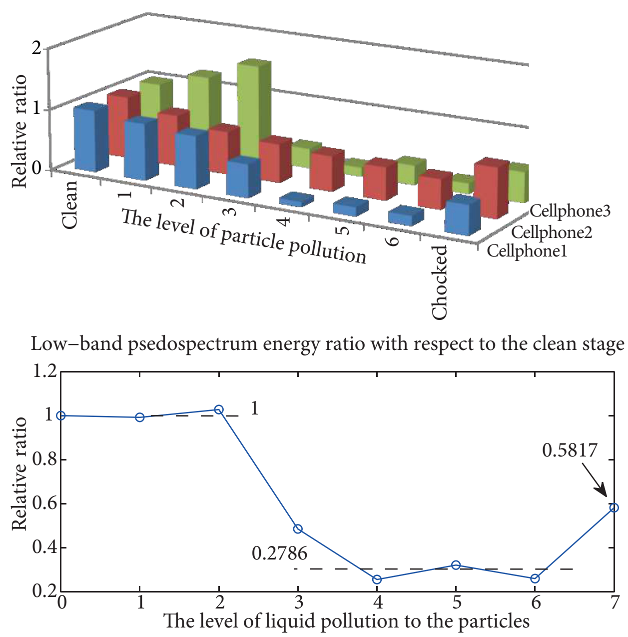

Figure 6. Top: Pseudospectrum energy of low-band section at each stage compared to pollution-free stage for the soundtracks recorded by cell phones 1, 2, and 3 in different stages of fluid pollution. Bottom: Same ratio by averaging the results of the cell phones 1, 2, and 3.

By observing the pseudospectrum curves of the three cellphones' acoustic records of the oil filter, the following points are concluded:

- The lower bands of the pseudospectrum curves for the clean stage of the hydraulic system and one level and two levels of particle pollution have similar power with less than 5.5% difference.
- As pollution reaches the third level, the lower band energy (LBE) of the pseudospectrum decreases 3.5893 times as it drops from an average of 1 in the first three levels to an average of 0.2786 in the next four levels.
- The LBE remains relatively steady with the addition of the pollutive particles after the third level , oscillating between 0.2352 and 0.3245.
- By being choked, the LBE of the pseudospectrum increases almost twice by jumping from an average of 0.2786 to 0.5817.

The top of Figure 6 shows the average energy of the lower band of the pseudospectrum of the acoustic records by each phone, and the resultant average for all three phones is demonstrated in the bottom of Figure 6. As is observed, the LBE remains the same as indicated by the ratio of 1 at the first two levels of pollution. For higher pollution levels, the LBE drops to 27.86% on average. As the system is choked, the LBE increases to 58.17% of its value in the clean stage. In brief, the LBE drops to almost one-third due to pollution, and after that, as the hydraulic system is choked it becomes twice the original value.

By observing the pseudospectrum curves of the second mobile phone's records, in addition to the points mentioned above, another point is clear: the pseudospectrum's sharp peak at higher frequencies. This feature is not observable in the records obtained by the two other phones due to possible phone-integrated antinoise filters or different qualities of the microphones. As pollutive particles increase, the peak starts to grow. As the hydraulic system is choked, the peak disappears. We have measured the 12.5 kHz relative peak energy (12.5RPE) in the range of 12–13 kHz with respect to the clean stage, as shown in Figure 7. The 12.5RPE can be used as the second indicator of the clogging stages in the hydraulic system, where its continuous rise indicates the relative increment of particle pollution. As 12.5RPE reaches values close to 2, it predicts that the system will soon be choked. Its drop to zero indicates that the hydraulic system has already choked. While the 12.5RPE feature is observable for one phone and is not observable for the two others, this does not mean that it can be omitted as a feature. It can be used as a secondary feature, but it may not be a generally reliable feature for a phone app.

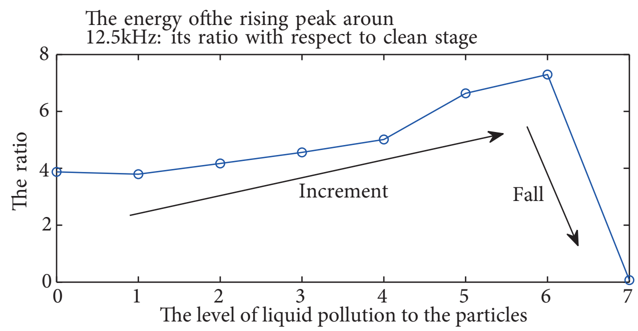

Figure 7. The averaged low-band pseudospectrum energy ratio with respect to the clean stage.

5. Conclusion

The fluid pollution monitoring of the tractor engine hydraulic system with the prediction and detection of its choked stage by a user-friendly oil filter sound analyzer smartphone application is a crucial and economic approach. For on-line data transfer, monitoring, and fast diagnosis, pseudospectrum data transfer and analysis have been suggested instead of a bulky DFT-based spectrum. Here, tractor oil filter sound records by three popular brands of cell phones are inspected by MUSIC-based pseudospectrum analysis. The records are related to eight various conditions: (track 1) clean fluid and new oil filter, (tracks 2–7) one to six levels of particle pollution according to the ISO4406 standard, and (track 8) choked condition of the hydraulic system. The pseudospectrum analysis of the soundtracks leads to two practical features: lower band energy (LBE) of the pseudospectrum as a generally reliable feature and 12.5 kHz relative peak energy (12.5RPE) to the clean stage, as a secondary feature observable by some phones. The LBE feature is observable with all three phones. This indicates that as pollution goes further than a certain level, the LBE drops to about one-third of the clean stage's value, and as the hydraulic system is choked, it increases to two-thirds. The 12.5RPE is just observed by the second cellphone, which is a benefit of the wider bandpass of both the microphone and integrated filter.

It indicates that as the particle pollution gradually increases, the 12.5RPE shows its stepwise increment. As it reaches close to twice the clean stage, the choked condition is predicted as the next early stage where the 12.5RPE drops to zero by choking occurrence.

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