

Robust power system state estimation by appropriate selection of tolerance for the least measurement rejected algorithm

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Abstract: Modern power systems are highly complicated and nonlinear in nature. Accurate estimation of the power system states (voltage-magnitude and phase-angle) is required for the secure operation of the power system. The presence of bad-data measurements in meters has made this estimation process challenging. An efficient estimator should detect and eliminate the effect of bad data during the estimation process. Least measurement rejected (LMR) is a robust estimator that has been found successful in dealing with various categories of bad data. The performance of LMR depends upon the proper selection of a tolerance for each measurement. This paper presents a novel approach for tolerance value selection to improve the capability of handling different single and multiple bad-data scenarios successfully. The performance of this updated LMR (ULMR) is compared with weighted least squares, weighted least absolute value, and two versions of LMR from the literature. IEEE 30-bus and 118-bus systems are used to demonstrate the robustness of the proposed estimator under different bad-measurement (single and multiple) scenarios.

Key words: State estimation, weighted least square, postprocessing, weighted least absolute value, least measurement rejected, bad data, tolerance selection

1. Introduction

State estimation (SE) is the process of estimating the values of state variables (voltage magnitudes and phase angles) of buses based on some redundant measurements, such as voltage magnitudes and power injections on buses, as well as power and current flows in branches [1]. It is an imperative process that ensures power system security by monitoring the network precisely. A conventional supervisory control and data acquisition (SCADA) system utilizes these measurements collected by remote terminal units (RTUs) installed at various substations in order to estimate the system states. For normally distributed errors, least squares or the weighted version of it are able to provide the optimal solution. Currently, postprocessed weighted least squares (WLS) is very widely used in power utilities but suffers from the long time requirement to complete the estimation task [2,3].

Several robust estimators are presented in the literature that can withstand bad-data presence. Algorithms with nonlinear objective functions like quadratic constant (QC), quadratic root (QR), and quadratic square root are proposed [4–6]. They try to minimize the function of measurement errors. Equivalent linear programming (LP) based on the least absolute value (LAV) estimator and its reweighted version is displayed in the literature [7-9]. Least median squares (LMS) and least trimmed squares (LTS) are updated versions of the

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LS algorithm [10,11]. LMS has the drawback of rejecting several good measurements along with the outliers. LTS considers the sum of squared errors for (m-K) smallest residuals only. Mixed integer linear programming (MILP) of a robust estimator is formulated in [12], which uses CPLEX as the solving tool and it is found to be time efficient for larger systems. Another robust estimator is exhibited in [13], which is designed from the concept of normal measurement rate (NMR) and the theory of uncertainty of measurements. Recently, a robust estimator is demonstrated in [14] based on maximum exponential absolute value (MEAV) and solved by the primal-dual interior-point method. One of the useful robust estimators using MILP is the least measurement rejected (LMR) algorithm, which was first proposed by Irving [15,16]. Comparison between the estimators is presented by different authors. Habiballah [17] and Caro et al. [18] compared LMR with other robust estimators. In LMR, a tolerance value needs to be provided to each measurement and in all these papers a fixed value is chosen as tolerance. In [19], the tolerance is chosen in an iterative procedure, while [20] proposes the tolerance values by relating with standard deviation values. In [21] 3% of the measurement is proposed as tolerance for measured values. The authors of [22] selected the tolerable range from the quality of the meter.

The present paper presents the regression-based estimator LMR as a robust solution to power system state estimation with an original approach of selecting the tolerance values of measurements. The developed robust estimator is compared with weighted least squares (WLS), weighted least absolute value (WLAV), and two versions of LMR from the literature. Single, multiple interacting and multiple noninteracting bad-data cases were simulated in the present work to check the robustness of the proposed technique. IEEE 30-bus and 118-bus power systems are used to prepare the test cases.

The rest of the paper is organized as follows. Different aspects of tolerance value selection are discussed in Section 2. Section 3 gives details of the proposed updated-LMR estimator. Section 4 describes the strategies of test cases preparation. Section 5 provides the simulation results and comments on them. Section 6 concludes the paper.

2. Aspects of tolerance selection

LMR tries to reduce the number of rejected measurements in each iteration and the nonrejected measurements take part in the regression process to estimate the states of the power system. As the meter readings are mixed with errors like SCADA, it needs to be given an appropriate tolerance that suits its accuracy level.

Some special features of LMR and the tolerance value selection are presented below:

- During the estimation process, the estimated value tries to fit itself within the tolerable range. Failure to do so will result in rejection of the measurement from the regression process.
- A rejected measurement in LMR does not always mean that the measurement is bad.
- High value of T may cause the rejection of a lower number of measurements during the estimation process.
- Lower value of T may cause the rejection of a higher number of measurements during the estimation process.
- T could be as low as zero for highly accurate measurements only (such as PMUs).
- Voltage magnitude meters, being the most sensitive and accurate measurements, should be provided with lower tolerance values than power flow and injection meters.

Based on the above considerations, the next section explains the method that has been devised to ensure proper selection of tolerance.

3. Proposed state estimation procedure

3.1. Initial estimation using initial tolerance (T_{ini})

After preparing the test cases, the measurements should be used to carry on the initial estimation with LMR that minimizes the total number of rejected values (p_i) [16]:

$$\min \sum_{i=1}^n p_i, \quad (1)$$

where P_i is the 0/1 binary variable for measurement i .

Each of the measurements provided with tolerance can be modeled by inequality constraints and with binary 0/1 values that represent the status of the measurement, whether it is rejected or not:

$$h_i(x) < Z_i + t_i^+ + Np_i \quad (2)$$

$$h_i(x) > Z_i - t_i^- - Np_i, \quad (3)$$

where z_i is the SCADA measurement vector.

$h_i(x)$ corresponds to a nonlinear function vector that relates measurements to states.

t_i^+ and t_i^- are the upper and lower limit obtained from the i th element in vector T added to and subtracted from the i th measurement z_i , respectively. x is the state vector.

N is the arbitrarily large positive scalar value. The value of N was selected large enough to avoid any kind of influence by the rejected values.

For the “good” measurements, which are within the tolerance limit, the p_i value is 0. Whenever it is needed to ignore or switch off a measurement, the value of p_i becomes 1.

The problem formulation of LMR described above is then converted into a mixed integer linear program (MILP) problem formulation to deal with it. Thus, the problem becomes like this:

$$\begin{aligned} & \min C^T Y \\ & \text{Subject to } A.Y \leq B, \end{aligned}$$

where $C^T = [0_{2n}, 1_{2q}]$, $0_n = [0 \dots 0]$, and $1_q = [1 \dots 0]$; n is the number of buses and q represents meter locations.

The following matrices represent the inequality constraints of the problem:

$$A = \begin{bmatrix} H & -N \\ -H & -N \end{bmatrix}; Y^T = [\Delta x, p]; H \text{ is the Jacobian matrix, and } B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} b + t \\ b - t \end{bmatrix}; b = \Delta z =$$

$z - z_{est}$, where b is the residue, which means the difference between measurement and estimated values.

SCADA meter measurements are given as input to the initial estimator to run the initial estimation. At this stage of estimation, there are no bad data present and all the measurements are subjected to white noise only.

In selecting the tolerance (T_{ini}) for the initial estimation, the idea of the standard deviation of different meter types is used. Minimum values are given to voltage magnitude meter readings. Power flows will have maximum values and the injections are in between.

3.2. Finding intermediate tolerance (T_{inter})

The estimated values by the initial estimation are used to find out the difference between estimated and measured values for each meter. Each meter is assigned a unique and new tolerance equal to the corresponding ($|Estimated-Measurement|$) value. This is considered the intermediate tolerance (T_{inter}) for the meters. The value of ($|Actual_i-Measurement_i|$) reflects the accuracy of i th measurement, which is very close to the value of ($|Estimated_i-Measurement_i|$) if the estimation is good. Higher values of T_{inter} reflect that the meter readings of those locations are not good and vice versa. Comparison between the actual and the initially estimated values of state variables for the IEEE 30-bus and 118-bus systems are shown in Figures 1 and 2 below.

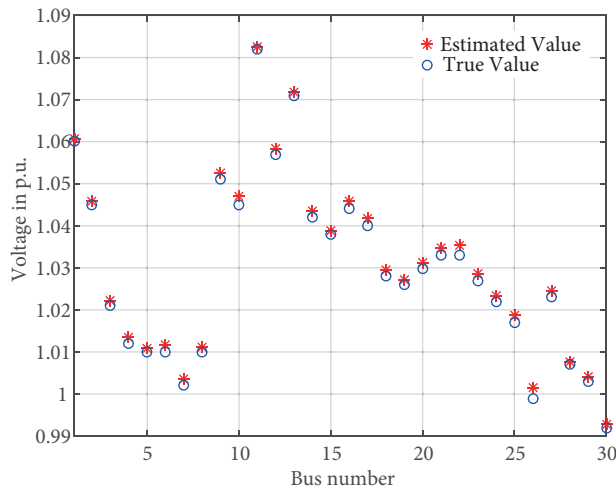


Figure 1. Estimated vs. true values of V_m for the 30-bus system in initial estimation.

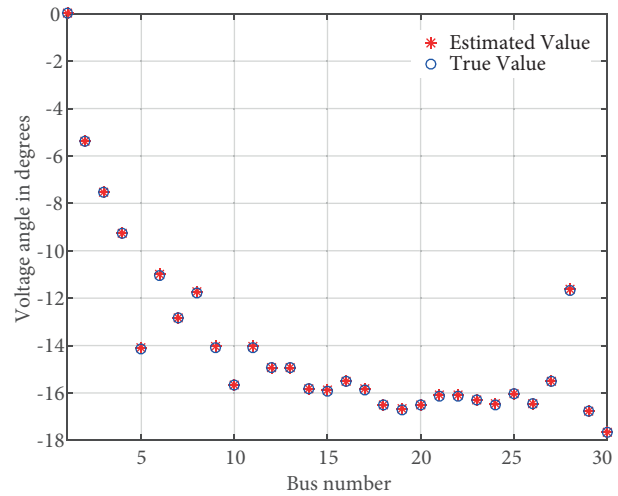


Figure 2. Estimated vs. true values of V_a for the 30-bus system in initial estimation.

3.3. Sorting the measurements

In this stage, the available measurements will be sorted according to their corresponding values of intermediate tolerance in a descendent way. The purpose of the sorting is to identify the “better” meter locations in terms of accuracy.

Let the total number of available measurements be “p”. Select the “q” number of measurements in accordance with the following conditions:

- q is equal to $2N - 1$, where N is the number of buses available in the system. The selected number of measurements must make the system observable.
- The q measurements should be among the lowest corresponding ($|estimated-measurement|$) values.

3.4. Selecting new tolerance (T_{new})

Force the tolerance of the “q” meters in step “b” to be zero. This enforcement is done due to the fact that the selected “q” meters are the best among the “p” available meters. The remaining meters will keep the intermediate tolerance (T_{inter}) as the ones obtained in step “a”. Both the revised “q” and remaining “p-q” tolerances are referred to T_{new} .

T_{new} is a vector with a dimension of $(p \times 1)$ that contains all the revised tolerances for each meter reading.

3.5. Final estimation

After the proper selection of the tolerance values, the final estimation will be carried out by LMR with T_{new} .

The problem formulation is like the initial estimation but the new values of tolerances (T_{new}) will be used in this step. Eqs. (2) and (3) used for initial estimation are updated as follows:

$$h_i(x) - Np_i < Z_i + t_{new_i}^+ \quad (4)$$

$$h_i(x) - Np_i < Z_i + t_{new_i}^- \quad (5)$$

Since the measurements now have properly chosen tolerance, the sorted “better” measurements will take part in the regression and will improve the accuracy of the estimation. For a specific meter arrangement of the power system, steps (a–c) should be done only once. Whenever there is a change in the meter configuration, the process of initial estimation, sorting, and determination of T_{new} (steps a – c) will be repeated once again to get the revised T_{new} . Otherwise, the estimator will directly go to step e and run the estimation with T_{new} .

4. Test case preparation

To examine the validity of the proposed LMR estimator, several bad-data cases are considered along with the white noise (WN) mixed measurement case. Simulation is carried out on IEEE 30- and 118-buses to investigate the performance of the estimators with different bad-data scenarios.

In the cases of simulating bad data for power-flow meters or power-injection meters, the value is selected to have the opposite sign of the original one. For the case of voltage-magnitude meters, five times the sigma value is added to or subtracted from the meter reading such that it could easily be considered an outlier. Specific locations of the bad data for each of the test cases are listed in Table 1 below.

Table 1. Different bad-data locations in 30- and 118-bus systems.

Bad-data types	Bad-data locations	
	30-bus	118-bus
SBD-1	QG_14	QG_13
SBD-2	PF_1–3	PF_5–6
SBD-3	Vm_10	Vm_51
MNI	QG_14, PT_2–1, PF_15–18, Vm_1, Vm_24	QG_13, PF_50–57, PF_4–11, QF_14–15, Vm_3, Vm_24, Vm_30
MI	PG_5, PT_2–1, PF_12–14, QF_12–14, Vm_5	QG_9, PF_4–11, PF_12–14, QF_12–14, Vm_9, Vm_4, Vm_5,

Table 1 shows three single bad-data (SBD) scenarios: power-injection, power-flow, and voltage-magnitude. Five bad-data meters are simulated in the 30-bus system for both multiple noninteracting (MNI) and multiple interacting (MI) cases. A total of seven bad-data meters are simulated in the 118-bus system for both MNI and MI cases.

The 30- and 118-bus test cases are made with voltage-magnitude (V_m), power-injections (PG and QG), real power flows (PF and PT), and reactive power flows (QF and QT) with the redundancy level of around 2.

The values of the standard deviation for WLS and WLAV need to be chosen carefully. Since the weight applied to a meter measurement is the inverse of the corresponding sigma value, it is required to assign the

maximum value of standard deviation to reactive values and then to real values, and the minimum values are assigned to the voltage magnitudes [1]. The tolerance values for the initial estimation were chosen using the same pattern. The values of sigma in WLS and T_{ini} are taken from [22].

Simulation is carried out in MATLAB. All test cases are performed on a laptop with Intel Core i5, 2.30 GHz CPU, and 8 GB RAM. The value of N for the LMR estimator is taken as 50,000. This value is large enough to guarantee that there will be no effect of the rejected values during the estimation process. The estimators of references [16] and [20] are considered for the purpose of comparison and denoted as LMR-1 and LMR-2, respectively.

The performance of the estimators is evaluated based on the cumulative estimated error (CEE) indicator, which is the summation of all the absolute difference between actual values and estimated values. Moreover, the error indicator for two state variables: voltage-magnitude (V_m) and phase-angle (V_a), are provided for the 118-bus system. V_m indicator and V_a indicator are calculated by summation of all the differences between the true values and the estimated values of the voltage-magnitudes and phase-angles, respectively. The lower the CEE, V_m , or V_a indicator is, the better the performance of the estimator and vice versa.

5. Simulation results

5.1. Results of CEE error indicator

IEEE 30-bus and 118-bus systems are used to investigate the performance of the proposed robust estimator. For performance under white noise measurements, three scenarios of single and two of multiple bad data are investigated. Tables 2 and 3 show the performance of the proposed ULMR estimator against four estimators in terms of CEE indicator.

Table 2. CEE indicator results for the 30-bus test case.

	No. of bad data	WLS	WLAV	LMR-1	LMR-2	ULMR
WN	0	0.64851	1.2108	2.0150	0.9565	0.41044
SBD-1	1	0.72292	1.3813	2.0437	1.1791	0.41192
SBD-2	1	7.1658	1.2108	2.1018	0.8595	0.4292
SBD-3	1	2.1232	1.2108	2.0161	1.1803	0.41044
MNI	5	14.578	5.6407	2.1155	1.4653	0.4316
MI	5	13.368	1.2108	2.0732	1.1162	0.42207

Table 3. CEE indicator results for the 118-bus test case.

	No. of bad data	WLS	WLAV	LMR-1	LMR-2	ULMR
WN	0	5.2016	9.6026	9.6489	6.3942	4.4702
SBD-1	1	5.3595	9.8085	10.236	6.4435	4.6799
SBD-2	1	21.7130	9.6026	13.19	6.5129	4.4702
SBD-3	1	5.7296	9.6026	9.8911	6.4120	4.4725
MNI	7	22.5090	10.0740	11.803	6.9023	4.5088
MI	7	15.8460	9.6968	9.814	6.8915	4.5048

It is seen that for each type of scenario, whether the bad data are applied or not, the proposed ULMR shows better performance than the other approaches. WLS fails to perform well in most of the bad-data cases. WLAV, LMR-1, and LMR-2 show superior performance over WLS for almost all bad-data cases as they are robust in nature. CEE indicator values of those are still very high compared to the ULMR. It outperforms the other four approaches with an extremely robust nature.

5.2. Estimation of the state variables

Better estimation of the state variables (voltage-magnitude and phase-angle) reflects the better estimation of all the power system variables. Tables 4 and 5 show the error indicator values for two state variables of the 118-bus system.

Table 4. Voltage-magnitude indicator results for the 118-bus test cases.

Cases	WLS	WLAV	LMR-1	LMR-2	ULMR
WN	0.107021	0.279181	0.1772	0.1732	0.171396
SBD-1	0.18706	0.279181	0.18907	0.1751	0.171396
SBD-2	0.174059	0.283155	0.26436	0.1760	0.15483
SBD-3	0.185041	0.279181	0.2308	0.1744	0.171559
MNI	0.585942	0.307835	0.3635	0.1821	0.172872
MI	0.415389	0.332007	0.33483	0.1798	0.174868

Table 5. Voltage-angle indicator results for the 118-bus test cases.

Cases	WLS	WLAV	LMR-1	LMR-2	ULMR
WN	5.323958	6.564508	8.2371	5.5342	4.837947
SBD-1	23.91264	6.564508	8.4519	5.6719	4.837947
SBD-2	5.444693	6.907321	8.9741	5.6025	4.397536
SBD-3	7.335471	6.564508	11.083	5.7208	4.839507
MNI	28.77546	6.202573	12.122	5.8804	4.805811
MI	16.59588	7.66958	11.378	5.9045	5.080243

It is seen from the tables above that the proposed estimator results in a lower V_a indicator among all for all the cases. For the V_m indicator, WLS outperforms the proposed estimator for the WN case only but the situation changes completely in the presence of bad data.

5.3. Estimation of the bad-data locations

It is observed that the WLS estimator is very sensitive to the presence of bad data. Robust estimators (WLAV, LMR-1, LMR-2), on the other hand, performed better in most of the bad-data cases. Table 6 shows the details of a multiple bad-data occurrence case for the 118-bus system and reflects clearly how the estimators behave in the bad-data locations.

A total of 7 bad data are applied in interacting locations and it is encouraging to see that the proposed ULMR can estimate the values for all of them. WLS failed in several cases. WLAV successfully estimated the bad-data locations except for the case of QF 12–13. The failure cases are highlighted in Table 6. LMR-1 and LMR-2 showed better performance than WLAV but were outperformed by ULMR.

Table 6. Bad-data estimation results for the 118-bus MNI case.

Measurement types	Actual	Applied bad data	Estimated values				
			WLS	WLAV	LMR-1	LMR-2	ULMR
Vm_4	0.9980	1.0328	1.0020	1.0022	1.0125	1.0045	1.0003
Vm_5	1.0020	1.0304	1.0058	1.0062	0.9945	1.0067	1.0043
Vm_9	1.0429	1.0163	1.0507	1.0488	1.0067	1.0488	1.0451
QG_13	-0.160	0.1639	0.0290	-0.161	-0.1567	-0.1554	-0.156
PF_4-11	0.6423	-0.6419	0.3123	0.6366	0.6237	0.6305	0.6375
PF_12-14	0.1831	-0.1836	-0.1032	0.1818	0.1790	0.1867	0.1808
QF_12-14	0.0262	-0.0261	-0.0015	-0.026	0.0459	0.0345	0.0323

5.4. Comparison with postprocessed WLS (WLS-PP)

None of the five approaches of state estimation presented above has any postprocessing feature. WLS, because of not having any bad-data cleaning module within itself, fails badly in the bad-data cases.

This section gives the results of WLS with a separate module for removing bad data from the measurement set. By checking the normalized residuals of the measurements, bad-data existence is detected, eliminated, and then used for estimation in the further step [1]. Table 7 shows the comparison of the proposed method with WLS-PP in terms of CEE indicator.

Table 7. Comparison between WLS-PP and updated LMR by CEE indicator.

Cases	30-bus system		118-bus system	
	WLS-PP	Updated LMR	WLS-PP	ULMR
WN	0.64851	0.4104	5.2016	4.4702
SBD-1	0.6484	0.4104	5.2074	4.4510
SBD-2	0.65173	0.4292	5.2363	4.5232
SBD-3	0.6298	0.4104	5.2527	4.4725
MNI	0.64505	0.4195	5.3669	4.5088
MI	0.60192	0.4221	5.2301	4.4989

It is clear from the table above that the ULMR outperforms the WLS-PP in every case. Because of removal of the bad measurements in a separate module, the performance of WLS-PP does not deteriorate with bad-data presence like normal WLS. However, the proposed approach of robust estimation still performed better with a lower error indicator.

Though it is not fair to compare a single module estimator like ULMR with the double module approach of WLS-PP, it is still chosen for comparison as most of the current-day power systems are using this feature of estimation. The comparison has clearly proven the effectiveness of the ULMR, which does not have any separate bad-data processing module but still results in superior performance.

5.5. Required CPU time analysis

Computational efficiency is one of the basic concerns in the field of state estimation as the estimators are required to be suitable to work with online systems. Table 8 shows the time requirements for all the estimators for a single bad-data case.

Table 8. CPU time (seconds) comparison between the estimators.

No. of buses in the systems	Estimators					
	WLS	WLS-PP	WLAV	LMR-1	LMR-2	ULMR
30	0.621	1.027	0.367	0.423	0.465	0.479
118	2.680	4.851	1.094	1.052	1.127	1.129

WLAV shows the most efficient performance in terms of CPU time, while WLS takes the longest time. WLS with its postprocessing step has the biggest drawback of requiring a long time for estimation as it needs to check and clean the bad data present manually before doing the estimation. Moreover, in the case of multiple bad data presence, it requires a sequential approach to clean those one after another, and thus results in a very poor computational efficiency [21,22]. The proposed ULMR, along with the other two versions of LMR, showed fair performance in terms of time.

6. Conclusion

This paper presented a robust estimator capable of handling bad measurements efficiently during the estimation process. Estimation results of the proposed robust estimator are compared with those of the most frequently used conventional estimator WLS, the well-established robust estimator WLAV, and the two versions of LMR from the literature. The robustness is checked based on the criteria of error indicator, bad-data estimation accuracy, and computational efficiency. Comparison is made with the postprocessed WLS as well with different bad-data scenarios. It was found that the proposed ULMR outperforms other estimation approaches in every aspect. In selecting the tolerance values of measurements, a new approach has been established, and all the results demonstrate its efficacy. Results for IEEE 30- and 118-bus power systems confirm the effectiveness of the proposed method irrespective of the power system size. In the practical scenario of bad-data occurrence, where most of the traditional estimators fail to cope, a robust estimator like the updated LMR will be an excellent choice as it can play a major role in power system management to protect the system from any blackouts.

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