


Adaptive switching gain sliding mode control for speed regulation in PMSMs

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Abstract: To suppress uncertainties caused by parametric variations or disturbances, an adaptive switching gain (ASG) is proposed for integral sliding mode control (ISMC) to regulate speed in permanent magnet synchronous motors (PMSMs). According to system uncertainties, the ASG is designed to adjust the switching gain to suppress the chattering. The adaptive law is a positive value resulting in an increment of the switching gain when the tracking trajectory is outside the boundary layer. Conversely, it is negative with a decrement of the gain. Further, it improves a convergent rate by the function of the reciprocal of the tracking error when the trajectory enters the sliding phase. ISMC with the ASG is applied to a PMSM experiment with the result that this method can effectively suppress high frequency chattering and shows excellent steady state performance of the current loop and speed in a servo system.

Key words: Permanent magnet synchronous motors, integral sliding mode control, switching gain, adaptive law, chattering

1. Introduction

Vector control technology is used for permanent magnet synchronous motors (PMSMs) to obtain an equivalent DC motor model, which obtains a linear relationship between stator current and torque. However, uncertainties such as external disturbance and parameter perturbation affect vector model accuracy. Robust control strategies have been introduced for PMSM systems to suppress the uncertainties. Sliding mode control (SMC) is a nonlinear control method. The main advantage is that it can effectively suppress the influence of parameter perturbation and external disturbance when the state trajectory of the system enters the sliding phase [1–3]. Integral SMC (ISMC) is often used for PMSMs to regulate speed [4,5]. Although ISMC improves dynamic system response performance compared with SMC, the discontinuous switching control law is also in ISMC, which results in the chattering phenomenon and limitation of steady-state system performance [6]. An adaptive switching gain (ASG) for SMC is a practical method to suppress the chattering. The nonlinear function is introduced to adjust the switching gain, which improves the dynamic response performance in the SMC system, but at the expense of the robustness of the system in the sliding modus [7]. A neural network is used to estimate the bounds of uncertain quantities accurately, but the computational complexity is very expensive [8]. The adaptive law is designed for SMC by the sliding surface to regulate the switching gain [9]. Although this method is simple in design, it ensures the robustness of the system. Since the adaptive law is $\dot{\rho} > 0$, the switching gain ρ monotonically increases, and the switching gain is too large in the steady state, resulting in more intense chattering and worsening system performance. The sliding modulus is redefined by introducing a dead zone for the adaptive law, which suppresses the chattering [10]. However, it still does not solve the overestimation

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of the switching gain. The concept of first-order inertial link is introduced to design the adaptive law, and the switching gain can be adjusted in both directions to vary the gain value. However, the law is relatively complicated and difficult to implement [11]. In [12], an adaptive gain law with a boundary layer is designed for SMC. The adaptive gain law is proportional to the absolute value of the sliding modulus. In order to overcome the overestimation of the switching gain, the sign of the adjustment law changes when the sliding trajectory enters the boundary layer. Therefore, the switching gain decreases and effectively weakens the chattering. However, the descending rate for the switching gain is too slow since the sliding modulus is very small when the sliding trajectory enters the boundary layer.

An ASG is proposed for ISMC (ASGISMC) to regulate speed in the PMSM servo system. When the sliding trajectory is in the reaching phase, the value of the adjustment law is proportional to the absolute value of the tracking error to ensure the robustness and rapidity of the control system. After the tracking trajectory enters the boundary layer, the value of the adjustment law is inversely proportional to the absolute value of the tracking error to accelerate the adjustment rate of the switching gain. The adaptive switching gain can suppress the chattering and improve the steady-state system performance.

2. Mathematical model

The mathematical model of PMSM with the $d - q - o$ synchronous rotating coordinate system is [13]

$$\begin{cases} U_d = R_s i_d + L_{dq} \frac{di_d}{dt} - n_p \omega L_{dq} i_q \\ U_q = R_s i_q + L_{dq} \frac{di_q}{dt} + n_p \omega L_{dq} i_d + n_p \omega \psi_f \end{cases} \tag{1}$$

Here i_d, i_q, U_d, U_q are stator current and stator voltage in the $d - q$ rotating coordinate system coordinate components, respectively; L_{dq} is stator winding in $d - q$ rotating coordinate system equivalent inductance; R_s is stator resistance; ψ_f is permanent magnet flux; n_p is pole pair of motor stator windings; w is mechanical angular velocity of the rotor.

The PMSM mechanical equations are

$$\begin{cases} J \dot{\omega} = T_e - B \omega - T_L \\ T_e = K_f i_q = \frac{3}{2} n_p \psi_f i_q \end{cases} \tag{2}$$

Here T_e is motor electromagnetic torque, J is moment of inertia, B is viscous damping coefficient, and T_L is load torque.

With the condition of ignoring the perturbation and nonlinear friction in the system, the mechanical equation is

$$\dot{\omega} = A_n \omega + B_n i_q + C_n, \tag{3}$$

where $A_n = -B_o / J_o$, $B_n = -K_f / J_o$, $C_n = -T_L / J_o$, and the subscript “o” is the parameter nominal value. Considering the parameter perturbation and the nonlinear friction force, the dynamic equation can be reexpressed as

$$\begin{aligned} \dot{\omega} &= (A_n + \Delta A) \omega + (B_n + \Delta B) i_q + (C_n + \Delta C) \\ &= A_n \omega + B_n i_q + H \end{aligned} \tag{4}$$

Here the sign “ Δ ” is the bias of J and B caused by the uncertainty. The bounded uncertainty is $H = \Delta A * \omega + \Delta B * i_q + (C_n + \Delta C)$ and H is assumed as bounded, namely,

$$|H| \leq D, \tag{5}$$

where D is the upper bound, $D > 0$.

3. ASGISMIC strategy

3.1. ISMC

To obtain an accurate tracking regulation for PMSMs with uncertainties, the tracking error for the ISMC system is

$$e = w^* - w, \quad (6)$$

where w^* is a given angular velocity and w is the velocity response.

The integral sliding surface S is defined as [2]

$$S = e + \lambda \int_0^t e d\tau \quad (7)$$

The derivative of Eq. (7) is

$$\dot{S} = \dot{e} + \lambda e = \dot{\omega} - A_n \omega - B_n i_q - H \quad (8)$$

According to the system state equation given in Eq. (4), the robust control law of ISMC is designed as Eq. (9), which includes the equivalent control law in Eq. (10) and the switching law in Eq. (11). The robust control law in Eq. (9) can ensure the stability of the proposed control system and the tracking error will converge to a zero neighborhood in a finite time.

$$i_q^* = i_{eq} + i_r \quad (9)$$

$$i_{eq} = \frac{1}{B_n} [\dot{\omega}^* - A_n \omega + \lambda e] \quad (10)$$

$$i_r = \frac{1}{B_n} \left[\rho \cdot \text{sat} \left(\frac{S}{\phi} \right) \right], \quad (11)$$

where $\text{sat}(\cdot)$ is a saturation function, ϕ is the boundary layer of the saturation function, and ρ is the switching gain.

Proof of stability: A Lyapunov function is defined as

$$V = \frac{1}{2} S^2 \quad (12)$$

With Eq. (8) and i_q as the control variable i_q^* in Eq. (9), the Lyapunov function (12) is derived:

$$\begin{aligned} \dot{V} &= S \dot{S} \\ &= S[\dot{\omega} - (A_n \omega + B_n i_q + H)] \\ &= S[\dot{\omega} - (A_n \omega + \dot{\omega}^* - A_n \omega + \lambda e + \rho \cdot \text{sat} \left(\frac{S}{\phi} \right) + H)] \\ &= S(-\dot{e} - \lambda e) - S \rho \cdot \text{sat} \left(\frac{S}{\phi} \right) - SH \\ &= -S \dot{S} - S \rho \cdot \text{sat} \left(\frac{S}{\phi} \right) - SH \\ &= \frac{1}{2} [-S \rho \cdot \text{sat} \left(\frac{S}{\phi} \right) - SH] \\ &\leq |S| |H| + S [-\rho \cdot \text{sat} \left(\frac{S}{\phi} \right)] \\ &= |S| (|H| - \rho) < 0 \end{aligned} \quad (13)$$

When the switching gain satisfies $\rho > D$, the ISMC system can maintain asymptotic stability.

3.2. ASG method

Since prior knowledge of uncertainties of system parameters or unpredictable external disturbances is difficult to obtain, it is difficult to select the switching gain ρ for ISMC. In [12], an ASG is designed for SMC to estimate the switching gain accurately. This ASG not only ensures system robustness but also prevents overestimation of switching gain. The law for the ASG is

$$\dot{\rho} = \begin{cases} \mu & , if \rho \leq \mu \\ \bar{\rho} \cdot |\sigma| \cdot sgn(|\sigma| - \varepsilon), & if \rho > \mu \end{cases} \quad (14)$$

where σ is the sliding modulus; $\bar{\rho}$ is the initial gain, $\bar{\rho} > 0$; ε is the boundary layer, $\varepsilon > 0$; μ is the lower bound for the switching gain, $\mu > 0$. This ASG does not require a priori knowledge of the system uncertainties.

The performance of an ASG in [12] is analyzed through a PMSM experiment with the same experiment condition described in Section 4. In Figure 1a, the chattering produced by ISMC will exceed the boundary layer and the system will not satisfy the conditions $|\sigma| \leq \varepsilon$ when the value of $\varepsilon = 0.02$ is too small. The gain keeps increasing monotonically. The chattering of the control amount i_q^* will continue to deteriorate and the amplitude of the chattering will continue to increase, which will seriously deteriorate system performance and eventually lead to instability of the control system. In Figure 1b with $\varepsilon = 0.05$, the gain will increase to keep a constant, and the chattering of the control variable i_q^* remains a constant. In Figure 1d with $\varepsilon = 0.2$, the gain firstly increases and then descends quickly to a small value, but the control variable i_q^* has no chattering, which means it cannot maintain system robustness and ε is too large. In Figure 1c with $\varepsilon = 0.08$, the gain firstly increases and then descends to a small value, and the chattering varies from a large value to a small value for the control variable i_q^* , which is suitable to maintain system robustness. From Figure 1a to Figure 1d, it means the parameter ε is very important for the ASG. However, the adjustment process is too long for the ASG in [12] from Figure 1c since this ASG does not consider the sliding modulus is very small in the sliding phase.

Here a novel ASG is designed for ISMC motivated by the method in [12]. The ASG is designed as

$$\dot{\rho} = \begin{cases} \mu, & if \rho \leq \mu \\ \bar{\rho} \cdot \left(\frac{|\sigma|}{\varepsilon}\right)^\Delta \cdot \Delta, & if \rho > \mu \end{cases} \quad (15)$$

Here Δ is the sign of $(|\sigma| - \varepsilon)$; $\bar{\rho}$ is the initial gain, $\bar{\rho} > 0$; ε is the boundary layer, $\varepsilon > 0$; μ is the lower bound for the switching gain, $\mu > 0$. Eq. (15) also does not require a priori knowledge of the upper bound of uncertainties. Due to the role of the sign function, the ASG can be divided into two parts: 1) $|\sigma| > \varepsilon$; 2) $|\sigma| \leq \varepsilon$ when $\rho > \mu$. At the same time, ε is guaranteed to satisfy $0 < \varepsilon < 1$. When $|\sigma| > \varepsilon$ and $\dot{\rho} = \bar{\rho} \cdot \left(\frac{|\sigma|}{\varepsilon}\right)^\Delta > \bar{\rho} \cdot |\sigma|$, the switching gain increases monotonically to estimate the uncertainties in the system. Thus, σ approaches zero rapidly. When $|\sigma| \leq \varepsilon$, the sliding trajectory enters the boundary layer, $\dot{\rho} = -\bar{\rho} \cdot \left(\frac{\varepsilon}{|\sigma|}\right) \leq -\bar{\rho} < -\bar{\rho} \cdot |\sigma|$, the switching gain begins to decrease adaptively. Since ISMC is in the sliding phase, the tracking modulus is closer to zero. The function of the reciprocal of the sliding modulus is designed for the adaptive law to speed up the descending rate of the gain. Therefore, this ASG significantly suppresses the chattering and improves steady-state system performance.

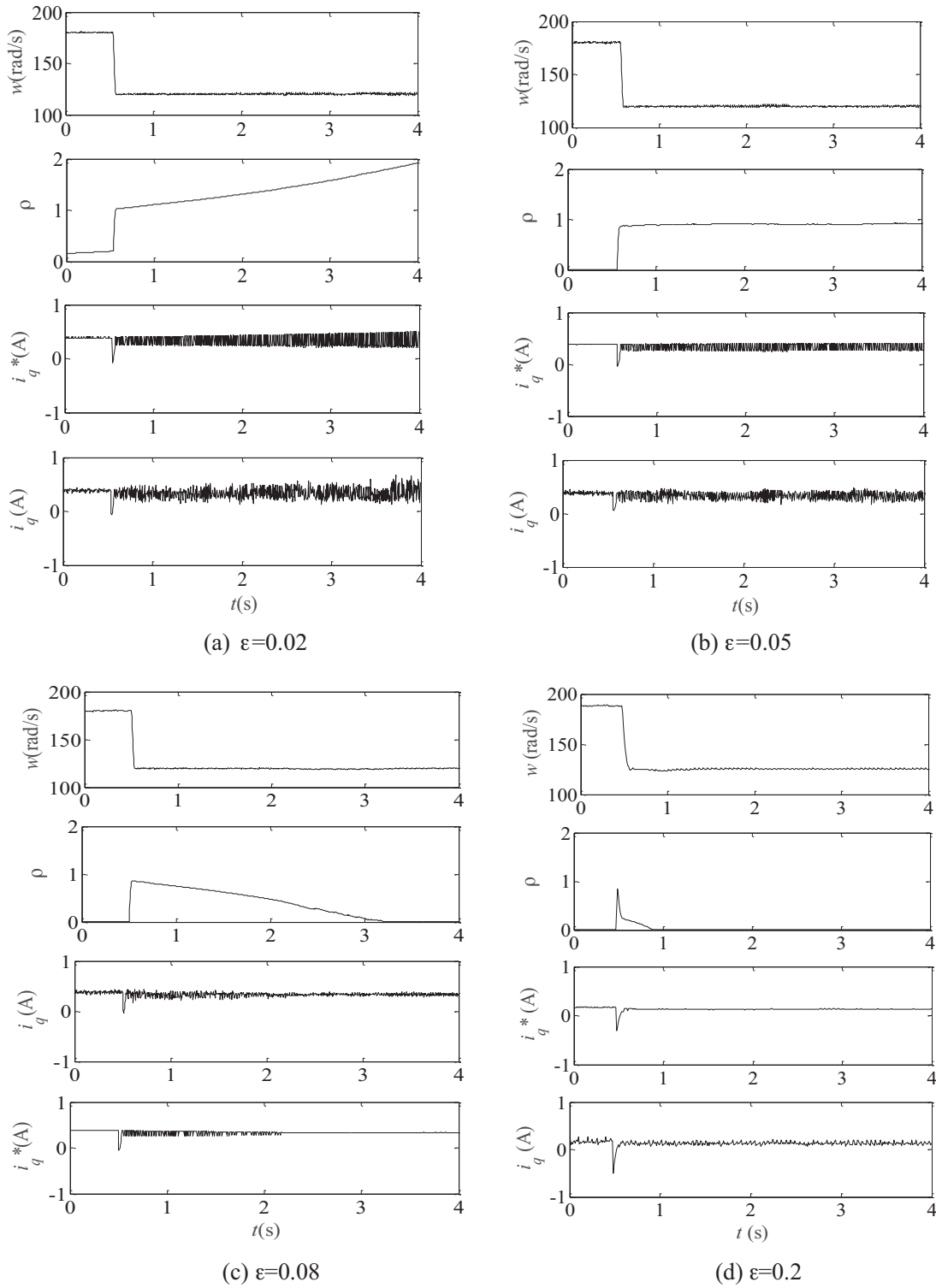


Figure 1. System responses with different ε . (a) $\varepsilon = 0.02$ (b) $\varepsilon = 0.05$ (c) $\varepsilon = 0.08$ (d) $\varepsilon = 0.2$.

3.3. Stability analysis of the ASGISMC system

With Eq. (4), the control law equation given in Eq. (9) with Eq. (15) is

$$i_q^* = i_{eq} + i_r \tag{16}$$

A finite positive constant γ satisfies $D = |H| + \gamma$. A Lyapunov function is

$$V = \frac{1}{2}S^2 + \frac{\alpha}{2\beta}(\rho - D)^2,$$

where $\alpha, \beta > 0$, and the time derivative of V is given as

$$\begin{aligned} \dot{V} &= S\dot{S} + \frac{\alpha}{\beta}(\rho - D)\dot{\rho} \\ &\leq |S||\dot{H}| + S \left[-\rho \cdot \text{sat}\left(\frac{S}{\phi}\right) \right] + \frac{\alpha}{\beta}(\rho - D) \cdot \bar{\rho} \left(\frac{|S|}{\varepsilon}\right)^\Delta \cdot \Delta \end{aligned}$$

The sliding trajectory S is in a different mode to discuss system stability.

- (1) When $|S| > \varepsilon$ and $|S| > \phi$, it means S is out of the boundary layer,

$$\begin{aligned} \dot{V} &\leq |S|(|H| - \rho) + \frac{\alpha}{\beta}(\rho - D) \cdot \bar{\rho} \left(\frac{|S|}{\varepsilon}\right) \\ &= |S|(|H| - D) - |S|(\rho - D) + \frac{\alpha}{\beta}(\rho - D) \cdot \bar{\rho} \left(\frac{|S|}{\varepsilon}\right) \\ &\leq -\gamma|S| - \left(1 - \frac{\alpha}{\varepsilon} \cdot \frac{\bar{\rho}}{\beta}\right) \cdot |S|(\rho - D) \end{aligned}$$

Choosing $\alpha = \varepsilon, \beta = \bar{\rho}$,

$$\dot{V} \leq -\gamma|S| < 0$$

- (2) When $\varepsilon \geq |S| \geq \phi$

$$\dot{V} \leq -\gamma|S| - \left(|S| + \frac{\varepsilon^2}{|S|}\right) \cdot (\rho - D)$$

As $\rho > D, \dot{V}(t) < 0$, which means the system is asymptotical stable. As $\rho \leq D$, ISMC cannot effectively compensate for the system uncertainties and the system is unstable, resulting in an error increment. That means the sliding modulus $|S|$ increases, until $|S| > \varepsilon, \dot{V}(t) < 0$, and the system becomes stable. Therefore, the sliding modulus will converge to $|S| \leq \varepsilon$ in a finite time, but it cannot be guaranteed that the sliding modulus converges to $|S| \leq \phi$ in a finite time.

- (3) When $\phi \geq |S| \geq \varepsilon$, due to the existence of saturated function $\text{sat}(\cdot)$ in the robust control law, $\dot{V}(t) < 0$ cannot be guaranteed. The sliding modulus will converge to $|S| \leq \phi$ for a finite time, but it is also impossible to guarantee the convergence of the gain adaptive law boundary layer $|S| \leq \varepsilon$.

Here, in order to ensure that the system converges not only to the boundary layer $|S| \leq \phi$ of the saturation function but also to the boundary layer $|S| \leq \varepsilon$ of the adaptive gain in a finite time, the boundary layer parameters are selected as equal,

$$\varepsilon = \phi \tag{17}$$

3.4. Parameter optimization

The boundary layer thickness ε is a very critical parameter in the ASG. With the stability of the control system and the precision guaranteed, a tuning of parameter ε in ISMC is discussed. The time derivative of Eq. (6) is

$$\dot{e} = \dot{\omega}^* - \dot{\omega} = H - \lambda e - \rho \text{sat}\left(\frac{Sg}{\phi}\right),$$

where $|H| \leq D$. When $\rho \geq |H|$, the result is

$$\lim_{t \rightarrow \infty} |S(t)| \leq \varepsilon,$$

where t is the time variable. T is the sampling time, assuming $t = t_1$, $|S(t_1)| \neq 0$. $S(t)$ is differential by the Euler formula

$$\begin{aligned} S(t_1 + T) &\sim S(t_1) + \dot{S}(t_1) \cdot T \\ &= S(t_1) + \left[H - \rho(t_1) \cdot \text{sat}\left(\frac{S(t_1)}{\phi}\right) \right] \cdot T \end{aligned}$$

Since $H \leq \rho(t)$, $\left| H - \rho(t_1) \cdot \text{sat}\left(\frac{S(t_1)}{\phi}\right) \right| \leq 2\rho(t_1)$ is obtained.

Furthermore, the \dot{S} sign is the opposite of the S sign when $\rho \geq |H|$. It means that the value of $|S(t)|$ is always decreasing. Thus, when $|S(t_1)| > \varepsilon$, the value of $|S(t_1 + T)|$ approaches ε .

When $|S(t_1)| \leq \varepsilon$, the value of $|S(t_1 + T)|$ approaches $|\sigma| = 0$. However, the value of $|S(t_1 + T)|$ is likely to decrease, or it may cross the sliding surface due to the inertia and cause the value to increase. In order to ensure that the value of $|S(t_1 + T)|$ does not exceed ε for both cases, the case of $|S(t_1)| = 0$ is discussed; it has

$$|S(t_1 + T)| \leq 2\rho(t_1)T$$

Let $|S(t_1 + T)| < \varepsilon$, and to ensure the accuracy of the system to the maximum extent, the design of the boundary layer thickness parameter ε is the time-varying function $\varepsilon(t) = 2\rho(t)T$.

3.5. ASG with optimal parameter

In order to guarantee the acceleration adjustment in Eq. (15), the boundary layer thickness meets $0 < \varepsilon(t) < 1$. Thus, an adjustment upper bound for ASG is

$$\rho(t) < \frac{1}{2T} \tag{18}$$

With Eq. (18), Eq. (15) is expressed as

$$\dot{\rho} = \begin{cases} \mu & , \text{if } \rho < \mu \\ \bar{\rho} \cdot \left(\frac{|\sigma|}{\phi}\right)^\Delta \cdot \Delta & , \text{if } \mu \leq \rho \leq \frac{1}{2T} \\ 0 & , \text{if } \rho > \frac{1}{2T} \end{cases} \tag{19}$$

4. Experimental verification

Figure 2 is a PMSM experimental servo platform, mainly composed of four major modules: PMSM with a magnetic powder brake, DSP (TMS320F2812) control subsystem, power inverter, and computer. The model

In the experiment, the speed reference input w^* is $w^* = 40\pi$ rad/s when $t < 0.1$ s, $w^* = 60\pi$ rad/s when $0.1 \text{ s} < t < 2.8$ s, and $w^* = 40\pi$ rad/s when $t > 2.8$ s. The magnetic powder brake input current is 0 A, that is, the load torque $T_L = 0$ Nm (no load). Figures 4 and 5 show the system velocity response w , the switching gain ρ , the tracking error e , and the sliding modulus S by ASGISM and by ISMC with the adaptive gain proposed in [12]. Figures 6 and 7 show the speed controller output i_q^* and the cross-axis current response i_q . Experimental results show that ASGISM only needs about 1.0 s to adjust the gain ρ , while the adaptive gain law proposed in [12] needs about 4.0 s to adjust the gain ρ . In particular, the descending rate in ASGISM is much larger than that in [12]. Thus, the gain adjustment time for ASGISM is significantly shorter than that in [12]. When the system enters the steady state, the q-axis control current i_q^* chattering amplitude obtained by ASGISM is greatly reduced compared with the method proposed in [12], which effectively suppresses the system chattering and improves the steady-state accuracy of i_q .

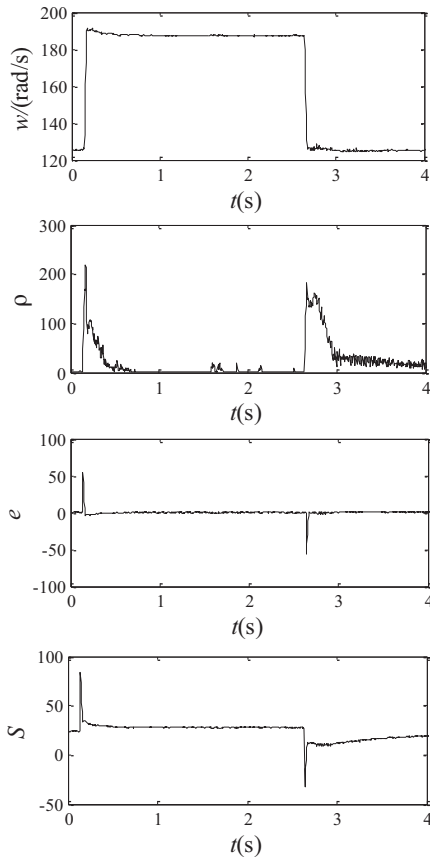


Figure 4. w , ρ , e , and S response by ASGISM.

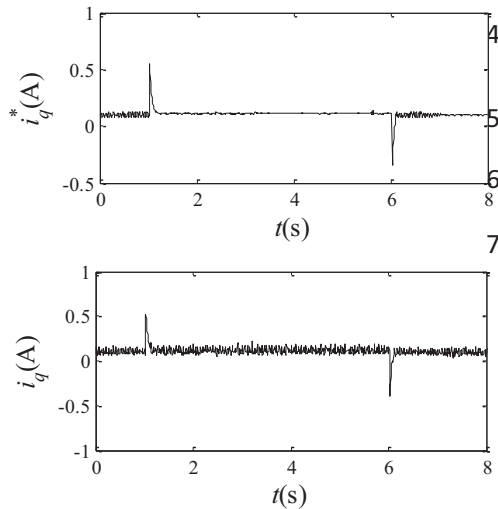


Figure 5. i_q^* and i_q response by ASGISM.

5. Conclusions

A new ASG is proposed for ISMC to suppress the systems uncertainties in PMSM systems. The adaptive law is a function of the sliding surface to accelerate the descending rate with a consideration that the sliding modulus is very small when the control system enters the sliding phase. Moreover, parameters for the adaptive law are analyzed to be equal to the boundary layer. Experiments show that (1) the adjustment process for the switching gain by ASG is much shorter than that by the method in [12], especially in the descending process; (2)

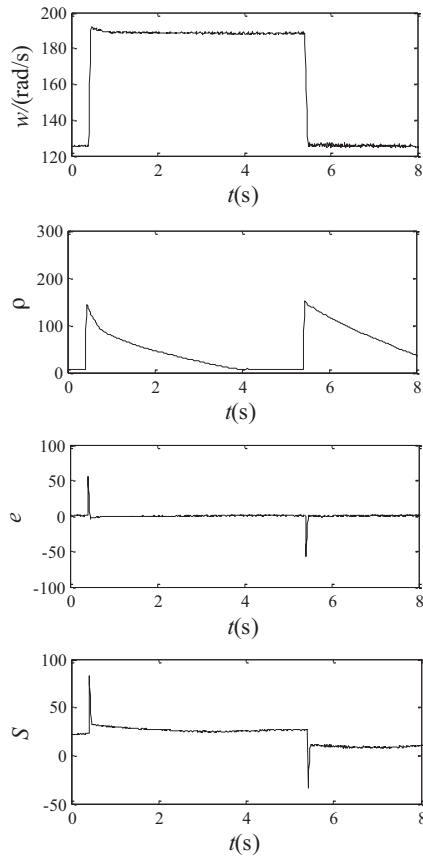


Figure 6. w , ρ , e , and S response in [12].

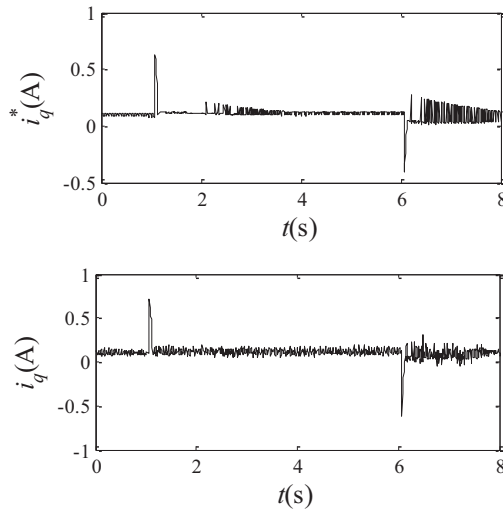


Figure 7. i_q^* and i_q response in [12].

ASGISMIC reduces the chattering for the input i_q^* of the current loop compared with ISMC with the adaptive gain in [12]; (3) ASGISMIC also maintains excellent system robustness; (4) experimental results indicate that ASG is suitable for ISMC to adjust the switching gain and has excellent system performance.

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