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Global stabilization of a class of fractional-order delayed bidirectional associative memory neural networks

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Abstract: This paper focuses on the stabilization problem of a class of fractional-order bidirectional associative memory neural networks with time delays. Based on feedback control, a sufficient condition is derived to achieve the global stabilization of systems by using the fractional inequality, the Lyapunov stability theory, and the comparison principle. In particular, this kind of control scheme is proved to be robust in the presence of external disturbances when the feedback gains are sufficiently large. In addition, a condition is obtained to achieve the global quasi-stabilization of systems with some external disturbances, and the corresponding error bound is estimated. Finally, some numerical simulations are presented to verify the effectiveness of theoretical results.

Key words: Fractional-order neural networks, stabilization, time delays, feedback control

1. Introduction

In recent decades, fractional calculus has captured considerable attention in various fields, such as physics, biology, economics, engineering, and technology. As is well known, fractional-order derivatives can provide a powerful tool for describing memory and hereditary properties of many materials and dynamical processes [1]. It has been shown that many physical systems [2–4] exhibit fractional dynamical behavior. Therefore, fractional-order models would be more appropriate to describe most systems in the real world than classical integer-order ones.

Neural networks have become an active topic due to their powerful applications in many fields. There have been all kinds of neural networks, such as Hopfield neural networks, memristor-based neural networks, bidirectional associative memory neural networks, and recurrent neural networks. For bidirectional associative memory (BAM) neural networks, it has been revealed that they can offer potential applications in pattern recognition, signal processing, and combinatorial optimization [5, 6]. A BAM neural network [7] consists of some associative neurons ordered in two layers, where every neuron in one layer is interconnected with all neurons in another layer. In particular, there are no interconnections within each layer. In order to describe the dynamical behavior of neurons in neural networks better, fractional-order neural networks have been introduced by incorporating fractional calculus into neural networks, and various types of fractional-order neural networks have been developed. Among these types, fractional-order BAM neural networks have attracted considerable attention from many researchers due to their potential applications in many fields; see [8–12] and the references therein.

Generally, the behaviors of a large number of interacting units need to be regulated in many practical

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applications. It is very desirable that the unpredicted ultimate states of systems can be controlled to the required ones. Since the pioneering work of Ott et al., the stabilization problems of dynamics systems have received close attention from many researchers. In order to meet the practical requirements, there have been all kinds of stabilization types, such as global asymptotic stabilization, exponential stabilization, guaranteed cost stabilization, sampled-data stabilization, and Mittag-Leffler stabilization. Meanwhile, various suitable stabilization control schemes have been proposed to regulate the behaviors of systems. However, to the best of the authors' knowledge, there have been few works on the stabilization control problems of fractional-order BAM neural networks. For example, Wu et al. [10] considered the global Mittag-Leffler stabilization of fractional-order BAM neural networks without time delays based on linear feedback control and partial feedback control.

Note that time delay is always unavoidable in practical dynamics systems. In this paper, we will consider the stabilization problem of fractional-order BAM neural networks with time delays. Based on feedback control, we derive a sufficient condition to realize the global stabilization of systems by using the fractional inequality, the Lyapunov stability theory, and the comparison principle. In particular, we also theoretically prove that this kind of control scheme is robust in the presence of external disturbances when the feedback gains are sufficiently large. In addition, we obtain a condition that can guarantee the global quasi-stabilization of systems with some external disturbances, and we give the corresponding estimated error bound, which can be adjusted to meet the requirement in practical applications. Finally, some numerical simulations are given to verify the effectiveness of theoretical results.

The rest of this paper is organized as follows. In Section 2, some preliminaries and network models are introduced. Section 3 focuses on the sufficient conditions that can guarantee the global stabilization of systems. In Section 4, some numerical simulations are shown to illustrate the effectiveness of the main results. Finally, some conclusions are drawn in Section 5.

2. Preliminaries and model description

In this section, we introduce some preliminaries and network models. We start by recalling some definitions and properties associated with the Caputo fractional-order derivative.

Definition 1 *[13] The Caputo derivative of fractional order q of a function $\theta(t) \in C^m([t_0, +\infty), \mathbb{R}^n)$ is defined by*

$$ {}_{t_0}^{C}D_t^q \theta(t) = \frac{1}{\Gamma(m-q)} \int_{t_0}^{t} (t-s)^{m-q-1} \theta^{(m)}(s) ds, $$

where $t \geq t_0$, $q > 0$, and m is a positive integer satisfying $m-1 < q < m$.

From this definition, we have the following immediate properties:

(i) ${}_{t_0}^{C}D_t^q(\alpha\theta_1(t) + \beta\theta_2(t)) = \alpha {}_{t_0}^{C}D_t^q \theta_1(t) + \beta {}_{t_0}^{C}D_t^q \theta_2(t)$,

(ii) ${}_{t_0}^{C}D_t^q \alpha = 0$,

where $\theta_1, \theta_2 \in C^m([t_0, +infinite), \mathbb{R}^n)$, and α and β are any constants.

Throughout this paper, the parameters q and t_0 satisfy $0 < q < 1$ and $t_0 = 0$, respectively. In this case, ${}_{t_0}^{C}D_t^q$ is written as D_t^q for the sake of convenience. More precisely,

$$ D_t^q \theta(t) = \frac{1}{\Gamma(1-q)} \int_{0}^{t} (t-s)^{-q} \theta'(s) ds, \quad 0 < q < 1. $$

Proposition 1 *[14] Let $0 < q \leq 1$. For $h(t) \in C^1([0,+\infty),\mathbb{R})$, the inequality*

$$D^q|h(t^+)| \leq sgn(h(t))D^q h(t)$$

holds almost everywhere, where $h(t^+) = \lim_{\tau \to t^+} h(\tau)$.

Proposition 2 *[15] Let $V(t) \in R^1$ be a continuous differentiable and nonnegative function. For $0 < q < 1$, suppose that $V(t)$ satisfies*

$$\begin{cases} D_t^q V(t) = -aV(t) + bV(t-\tau), \\ V(t) = \varphi(t) \geq 0, \quad t \in [-\tau, 0], \end{cases}$$

where $t \in [0,\infty)$. If $a > b > 0$, then $\lim_{t\to\infty} V(t) = 0$.

Proposition 3 *[16] Let $0 < q \leq 1$. Consider the following two fractional-order systems with time delay:*

$$\begin{cases} D_t^q x(t) = f_1(t, x(t)) + g_1(t, x(t-\tau)), \\ x(t) = h(t), \quad t \in [-\tau, 0], \end{cases}$$

and

$$\begin{cases} D_t^q y(t) = f_2(t, y(t)) + g_2(t, y(t-\tau)), \\ y(t) = h(t), \quad t \in [-\tau, 0], \end{cases}$$

where $f_1(t, x(t))$ and $f_2(t, y(t))$ are Lipschitz continuous in $[0,+\infty) \times G$ $(G \subset R)$, and $g_1(t, x(t-\tau))$ and $g_2(t, y(t-\tau))$ are Lipschitz continuous in $[-\tau,+\infty) \times G$ $(G \subset R)$. If

$$f_1(t, x(t)) \leq f_2(t, x(t)), \quad g_1(t, x(t-\tau)) \leq g_2(t, x(t-\tau)), \quad \forall t \in [0,+\infty),$$

then

$$x(t) \leq y(t), \qquad \forall t \in [0,+\infty).$$

Proposition 4 *[16] Let $0 < q < 1$. For a fractional-order system with delay, $D_t^q x(t) = Ax(t) + Bx(t-\tau)$, if all eigenvalues of $A + B$ satisfy $|arg(\lambda)| > \frac{\pi}{2}$ and the characteristic equation $det(\Delta(s)) = 0$ has no purely imaginary roots, then the zero solution of this system is Lyapunov asymptotically stable.*

Based on this proposition, we easily get the following result, which is crucial for our main results. For the convenience of readers, we give some key points.

Lemma 1 *Let $0 < q < 1$. For a fractional-order system with delay:*

$$D_t^q x(t) = -ax(t) + bx(t-\tau) + \rho, \tag{1}$$

where $a, b, \rho > 0$, and if $asin(q\pi/2) > b$, then the solution $x^* = \frac{\rho}{a-b}$ of the system of Eq. (1) is Lyapunov asymptotically stable.

Proof Let $\tilde{x}(t) = x(t) - x^*$. The fractional-order system of Eq. (1) is rewritten as:

$$D_t^q \tilde{x}(t) = -a\tilde{x}(t) + b\tilde{x}(t-\tau). \tag{2}$$

Taking the Laplace transformation of Eq. (2), we can get

$$(s^q + a - be^{-s\tau})X(s) = s^{q-1}X(0) + be^{-s\tau}(\int_{-\tau}^{0} e^{-st}\tilde{x}(t)dt), \tag{3}$$

where $X(s)$ denotes the Laplace transformation of $\tilde{x}(t)$. Furthermore, this gives the characteristic equation $det(\Delta(s)) = s^q + a - be^{-s\tau} = 0$. Based on the proof of contradiction, we can derive that this equation has no pure imaginary root.

Suppose that the equation $s^q + a - be^{-s\tau} = 0$ has a pure imaginary root s. Let $s = \omega i$, where ω is a real nonzero number. Notice that s can be written as $s = \omega i = |\omega|(\cos(\frac{\pi}{2}) + i\sin(\pm\frac{\pi}{2}))$. Substituting $s = |\omega|(\cos(\frac{\pi}{2}) + i\sin(\pm\frac{\pi}{2}))$ into the equation $s^q + a - be^{-s\tau} = 0$, we can derive

$$|\omega|^{2q} + 2a\cos(\frac{q\pi}{2})|\omega|^q + a^2 - b^2 = 0. \tag{4}$$

Let $|\omega|^q$ be the variable of the above equation. In view of $b < a\sin(\frac{q\pi}{2})$, we get the discriminant $\Delta = 4(b^2 - a^2 sin^2(\frac{q\pi}{2})) < 0$, which implies that Eq. (4) has no real solutions. This is in contradiction with ω being a real number. Hence, the equation $det\Delta(s) = 0$ has no pure imaginary roots.

On the other hand, it is obvious that every eigenvalue of the matrix $(-a+b)I$ satisfies $|arg(\lambda)| > \frac{\pi}{2}$ due to $b < a\sin(\frac{q\pi}{2}) < a$. Based on Proposition 4, the zero solution of the system of Eq. (2) is globally asymptotically stable, i.e. $\tilde{x}(t) \to 0 \ (t \to +\infty)$. Hence, the solution x^* of the system of Eq. (1) is Lyapunov asymptotically stable. □

In the following, we focus on a class of fractional-order bidirectional associative memory neural networks with time delays, which can be expressed as:

$$\begin{aligned} D_t^q x_i(t) &= -c_i x_i(t) + \sum_{j=1}^{n_2} a_{ij}(t) f_{1j}(y_j(t)) + \sum_{j=1}^{n_2} b_{ij}(t) g_{1j}(y_j(t-\tau)) + I_i(t), \\ D_t^q y_j(t) &= -d_j y_j(t) + \sum_{i=1}^{n_1} p_{ji}(t) f_{2i}(x_i(t)) + \sum_{i=1}^{n_1} q_{ji}(t) g_{2j}(x_i(t-\tau)) + J_j(t), \end{aligned} \tag{5}$$

where $0 < q < 1$, $i = 1, 2, \cdots, n_1$ and $j = 1, 2, \cdots, n_2$. $x_i(t), y_j(t) \in \mathbb{R}$ denote the states of a neural unit at time t. The constant $c_i > 0$ is the self-regulating parameter. The constant $\tau > 0$ stands for the time delay. $a_{ij}(t)$ and $b_{ij}(t)$ are the time-varying connections at times t and $t - \tau$, respectively. f_{1j} and g_{1j} are the activation functions such that $f_{1j}(0) = 0$ and $g_{1j}(0) = 0$. $I_i(t)$ corresponds to the time-varying external inputs. For d_j, $p_{ji}(t)$, $q_{ji}(t)$, f_{2i}, g_{2i}, and $J_j(t)$, they have the same assumptions as those in the first equation of Eq. (5), respectively.

Assume that the system of Eq. (5) satisfies the following initial conditions:

$$x_i(t) = \varphi_i(t), \quad y_j(t) = \psi_j(t), \quad t \in [-\tau, 0],$$

or

$$x(t) = \varphi(t), \quad y(t) = \psi(t), \quad t \in [-\tau, 0], \tag{6}$$

where $x(t) = (x_1(t), x_2(t), \cdots, x_{n_1}(t))^T$ and $y(t) = (y_1(t), y_2(t), \cdots, y_{n_2}(t))^T$.

For some parameters of the network model, we make the following assumptions.

Assumption 1 For $i = 1, 2, \cdots, n_1$ and $j = 1, 2, \cdots, n_2$, the connection functions $a_{ij}(t)$, $b_{ij}(t)$, $p_{ji}(t)$, and $q_{ji}(t)$ are continuous and bounded on $[0, +\infty)$.

Assumption 2 For $i = 1, 2, \cdots, n_1$ and $j = 1, 2, \cdots, n_2$, the activation functions $f_{1j}(x)$, $g_{1j}(x)$, $f_{2i}(x)$, and $g_{2i}(x)$ satisfy the following Lipschitz conditions:

$$|f_{1j}(x) - f_{1j}(y)| \le \alpha_1 |x - y|, \quad |g_{1j}(x) - g_{1j}(y)| \le \beta_1 |x - y|, \quad i = 1, 2, \cdots, n_1,$$

$$|f_{2i}(x) - f_{2j}(y)| \le \alpha_2 |x - y|, \quad |g_{2i}(x) - g_{2j}(y)| \le \beta_2 |x - y|, \quad j = 1, 2, \cdots, n_2,$$

for any $x, y \in \mathbb{R}$, where α_1, α_2, β_1, and β_2 are some positive constants.

Definition 2 *Let $I_i(t) = 0$ ($i = 1, 2, \cdots, n_1$) and $J_j(t) = 0$ ($j = 1, 2, \cdots, n_2$). Suppose that $(x(t), y(t))$ is the solution of the system of Eq. (5) with the initial condition of Eq. (6). The system of Eq. (5) is said to be asymptotically stable if*

$$\|x(t)\| + \|y(t)\| \to 0, \quad t \to +\infty,$$

where $\|x(t)\| = \sum_{i=1}^{n_1} |x_i(t)|$ and $\|y(t)\| = \sum_{j=1}^{n_2} |y_j(t)|$.

Definition 3 *The system of Eq. (5) is said to achieve global stabilization if there exist suitable feedback controls $I(t)$ and $J(t)$ such that the system of Eq. (5) is asymptotically stable.*

3. Main results

In this section, we focus on the global stabilization of the system of Eq. (5). A kind of control scheme is proposed based on feedback control, and a sufficient condition is derived to achieve the global stabilization of the system. In particular, this kind of control scheme is proved to be robust in the presence of external disturbances. In addition, a sufficient condition is obtained to achieve the global quasi-stabilization of the system with external disturbances, and the corresponding error bound is estimated.

For $i = 1, 2, \cdots, n_1$ and $j = 1, 2, \cdots, n_2$, $I_i(t)$ and $J_j(t)$ are designed as follows:

$$I_i(t) = -\mu_i x_i(t) \text{ and } J_j(t) = -\nu_j y_j(t), \tag{7}$$

where μ_i and ν_j are any positive constants.

Now we state our main theorems. Let $a_{ij}^* = \sup_{t \ge 0} |a_{ij}(t)|$, $b_{ij}^* = \sup_{t \ge 0} |b_{ij}(t)|$, $p_{ij}^* = \sup_{t \ge 0} |p_{ij}(t)|$, and $q_{ij}^* = \sup_{t \ge 0} |q_{ij}(t)|$.

Theorem 1 *Suppose that Assumptions 1 and 2 hold. If $\min\{l_1, l_2\} > \max\{m_1, m_2\}$, where*

$$l_1 = \min_{1 \le i \le n_1} \{c_i + \mu_i - \alpha_2 \sum_{j=1}^{n_2} |p_{ji}^*|\}, \quad m_1 = \max_{1 \le i \le n_1} \{\beta_2 \sum_{j=1}^{n_2} q_{ji}^*\},$$

$$l_2 = \min_{1 \le j \le n_2} \{d_j + \nu_j - \alpha_1 \sum_{i=1}^{n_1} |a_{ij}^*|\}, \quad m_2 = \max_{1 \le j \le n_2} \{\beta_1 \sum_{i=1}^{n_1} b_{ij}^*\},$$

then the global stabilization of the system of Eq. (5) can be achieved with the control of Eq. (7).

Proof Let

$$V(t) = \sum_{i=1}^{n_1} |x_i(t)| + \sum_{j=1}^{n_2} |y_i(t)|.$$

In view of Proposition 1, we obtain

$$\begin{aligned} D_t^q V(t^+) \leq & \sum_{i=1}^{n_1} sgn(x_i(t)) D_t^q x_i(t) + \sum_{j=1}^{n_2} sgn(y_j(t)) D_t^q y_j(t) \\ = & \sum_{i=1}^{n_1} sgn(x_i(t))[-(c_i+\mu_i)x_i(t) + \sum_{j=1}^{n_2} a_{ij}(t) f_{1j}(y_j(t)) + \sum_{j=1}^{n_2} b_{ij}(t) g_{1j}(y_j(t-\tau))] \\ & + \sum_{j=1}^{n_2} sgn(y_j(t))[-(d_j+\nu_j)y_j(t) + \sum_{i=1}^{n_1} p_{ji}(t) f_{2i}(x_i(t)) + \sum_{i=1}^{n_1} q_{ji}(t) g_{2i}(x_i(t-\tau))]. \end{aligned}$$

From $f_{1j}(0) = 0$ and Assumption 2, it follows that $|f_{1j}(y_j(t))| \leq \alpha_1 |y_j(t)|$. Similar inequalities can also be obtained for the functions g_{1j}, f_{2i}, and g_{2i}. Consequently, we have

$$\begin{aligned} D_t^q V(t^+) \leq & \sum_{i=1}^{n_1} \big[-(c_i+\mu_i)|x_i(t)| + \alpha_1 \sum_{j=1}^{n_2} a_{ij}^* |y_j(t)| + \beta_1 \sum_{j=1}^{n_2} b_{ij}^* |y_j(t-\tau)| \big] \\ & + \sum_{j=1}^{n_2} \big[-(d_j+\nu_j)|y_j(t)| + \alpha_2 \sum_{i=1}^{n_1} p_{ji}^* |x_i(t)| + \beta_2 \sum_{i=1}^{n_1} q_{ji}^* |x_i(t-\tau)| \big]. \end{aligned}$$

Moreover, we have

$$\begin{aligned} D_t^q V(t^+) \leq & - \sum_{i=1}^{n_1} (c_i + \mu_i - \alpha_2 \sum_{j=1}^{n_2} |p_{ji}^*|)|x_i(t)| + \beta_2 \sum_{i=1}^{n_1} \sum_{j=1}^{n_2} q_{ji}^* |x_i(t-\tau)| \\ & - \sum_{j=1}^{n_2} (d_j + \nu_j - \alpha_1 \sum_{i=1}^{n_1} |a_{ij}^*|)|y_i(t)| + \beta_1 \sum_{j=1}^{n_2} \sum_{i=1}^{n_1} b_{ij}^* |y_i(t-\tau)| \\ & \leq -l(\sum_{i=1}^{n_1} |x_i(t)| + \sum_{j=1}^{n_2} |y_i(t)|) + m(\sum_{i=1}^{n_1} |x_i(t-\tau)| + \sum_{j=1}^{n_2} |y_i(t-\tau)|) \\ & = -lV(t) + mV(t-\tau), \end{aligned}$$

where $l = \min\{l_1, l_2\}$ and $m = \max\{m_1, m_2\}$. If $l > m$, then Proposition 2 gives that $V(t) \to 0 \ (t \to \infty)$. This implies that $\|x(t)\| + \|y(t)\| \to 0 \ (t \to \infty)$. Hence, the system of Eq. (5) can achieve global stabilization with the control of Eq. (7). □

In practical applications, it is desired that the scheme with partial feedback control is always considered due to its lower complexity and less a priori information of the system. Here, we investigate the stabilization control control schema for the system of Eq. (5) with the partial feedback control.

The external inputs $I_i(t) \ (i = 1, 2, \cdots, n_1)$ and $J_j(t) \ (j = 1, 2, \cdots, n_2)$ are designed as follows:

$$I_i(t) = -\mu_i x_i(t), \quad J_j(t) = 0, \tag{8}$$

or

$$I_i(t) = 0, \quad J_j(t) = -\nu_j y_j(t), \tag{9}$$

where μ_i and ν_j are any positive constants.

Corollary 1 *Suppose that Assumptions 1 and 2 hold. Let m_1 and m_2 be defined as in Theorem 1. If $\min\{l_1, l_2\} > \max\{m_1, m_2\}$, where*

$$l_1 = \min_{1\le i\le n_1}\{c_i + \mu_i - \alpha_2 \sum_{j=1}^{n_2} |p_{ji}^*|\}, \quad l_2 = \min_{1\le j\le n_2}\{d_j - \alpha_1 \sum_{i=1}^{n_1} |a_{ij}^*|\},$$

then the global stabilization of the system of Eq. (5) can be achieved with the control of Eq. (8).

Corollary 2 *Suppose that Assumptions 1 and 2 hold. Let m_1 and m_2 be defined as in Theorem 1. If $\min\{l_1, l_2\} > \max\{m_1, m_2\}$, where*

$$l_1 = \min_{1\le i\le n_1}\{c_i - \alpha_2 \sum_{j=1}^{n_2} |p_{ji}^*|\}, \; l_2 = \min_{1\le j\le n_2}\{d_j + \nu_j - \alpha_1 \sum_{i=1}^{n_1} |a_{ij}^*|\},$$

then the global stabilization of the system of Eq. (5) can be achieved with the control of Eq. (9).

Remark 1 *For systems with constant connections, Wu et al. [10] considered the global Mittag-Leffler stabilization of the corresponding case without time delays based on the above three kinds of controllers.*

Remark 2 *In [17], the author investigated the finite-time stability of delayed fractional-order BAM neural networks with constant connection functions by using the Bellman–Gronwall inequality and other elementary inequalities.*

In practical applications, some external disturbances are always inevitable when various control are imposed on the systems. Here, we consider the global stabilization of the system of Eq. (5) under the control of Eq. (7) with some external disturbances. It is proved that the control of Eq. (7) is robust when the feedback gains are sufficiently enough. Assume that the system of Eq. (5) with some external disturbances is described as follows:

$$\begin{aligned} D_t^q x_i(t) &= -c_i x_i(t) + \sum_{j=1}^{n_2} a_{ij}(t) f_{1j}(y_j(t)) + \sum_{j=1}^{n_2} b_{ij}(t) g_{1j}(y_j(t-\tau)) - \mu_i x_i(t) + \xi_i(t), \\ D_t^q y_j(t) &= -d_j y_j(t) + \sum_{i=1}^{n_1} p_{ji}(t) f_{2i}(x_i(t)) + \sum_{i=1}^{n_1} q_{ji}(t) g_{2j}(x_i(t-\tau)) - \nu_j y_j(t) + \eta_j(t), \end{aligned} \tag{10}$$

where $\xi_i(t)$ and $\eta_j(t)$ are two external disturbances.

For $i = 1, 2, \cdots, n_1$ and $j = 1, 2, \cdots, n_2$, assume that $\xi_i(t)$ and $\eta_j(t)$ are continuous and bounded on $[0, +\infty)$, i.e. there exist two constants ρ_1 and ρ_2 such that

$$\|\xi(t)\| \le \rho_1 \text{ and } \|\eta(t)\| \le \rho_2$$

for any $t \in [0, +\infty)$.

Theorem 2 *Suppose that Assumptions 1 and 2 hold. Let l_1 and l_2 be defined as in Theorem 1. If l_1 and l_2 are sufficiently large, then the global stabilization of the system of Eq. (10) can be achieved with the control of Eq. (7).*

Proof Let

$$V(t)=\sum_{i=1}^{n_1}|x_i(t)|+\sum_{j=1}^{n_2}|y_i(t)|.$$

According to the calculation method of $D_t^qV(t)$ in the proof of Theorem 1, we can deduce

$$D_t^qV(t)\leq -lV(t)+mV(t-\tau)+\rho_1+\rho_2.$$

Here, $l=\min\{l_1,l_2\}$, $m=\max\{m_1,m_2\}$, $m_1=\max_{1\leq i\leq n_1}\{\beta_2\sum\limits_{j=1}^{n_2}q_{ji}^*\}$, and $m_2=\max_{1\leq j\leq n_2}\{\beta_1\sum\limits_{i=1}^{n_1}b_{ij}^*\}$.

Consider the following fractional-order system:

$$D_t^qW(t)=-lW(t)+mW(t-\tau)+\rho_1+\rho_2,$$

where $W(t)\geq 0$ and the initial value of $W(t)$ is the same as that of $V(t)$. If l_1 and l_2 are sufficiently large, then we have $lsin(q\pi/2)>m$. From Lemma 1, we obtain $W(t)\to\frac{\rho_1+\rho_2}{l-m}$ $(t\to+\infty)$. Together with Proposition 3, it follows that $V(t)=\|x(t)\|+\|y(t)\|\leq W(t)\to\frac{\rho_1+\rho_2}{l-m}$ $(t\to+\infty)$. This implies that the system of Eq. (10) can achieve global stabilization when l_1 and l_2 are sufficiently large. □

Based on the proof of Theorem 2, we can easily obtain the following global quasi-stabilization result.

Theorem 3 *Suppose that Assumptions 1 and 2 hold. Let l_1, l_2, m_1, and m_2 be defined as in Theorem 1. Let $l=\min\{l_1,l_2\}$ and $m=\max\{m_1,m_2\}$. If $lsin(q\pi/2)>m$, then we have*

$$\|x(t)\|+\|y(t)\|\leq\frac{\rho_1+\rho_2}{l-m},\quad t\to+\infty,$$

where $\delta\triangleq\frac{\rho_1+\rho_2}{l-m}$ is the estimated error bound, i.e. the global quasi-stabilization of the system of Eq. (10) can be realized with the control of Eq. (7).

4. Numerical simulations

In this section, some numerical examples are presented to illustrate the effectiveness of our results. A predictor-corrector scheme is used to obtain the numerical solutions of fractional-order neural networks with step-length $\theta=0.1$.

Consider the fractional-order BAM neural networks with time delays, which are written in the following form:

$$\begin{aligned}D_t^qx(t)&=-Cx(t)+A(t)f_1(y(t))+B(t)g_1(y(t-\tau))+I(t),\\ D_t^qy(t)&=-Dy(t)+P(t)f_2(x(t))+Q(t)g_2(x(t-\tau))+J(t),\end{aligned}\tag{11}$$

where $q=0.85$, $C=diag(0.07,0.08)$, $D=diag(0.05,0.06)$, $\tau=0.1$, $f_1(y(t))=(tanh(y_1(t)),\ tanh(y_2(t)))^T$, $g_1(y(t-\tau))=(sin(y_1(t-\tau)),\ sin(y_2(t-\tau)))^T$, $f_2(x(t))=(tanh(x_1(t)),\ tanh(x_2(t)))^T$, $g_2(x(t-\tau))=$

$(sin(x_1(t-\tau)),\ sin(x_2(t-\tau)))^T$, and

$$A(t)=\begin{pmatrix}0.1 & 0.2sin(t)\\ -0.3sin(t) & -0.2exp(-t)\end{pmatrix}, B(t)=\begin{pmatrix}0.25cos(t) & 0.35cos(t)\\ 0.32cos(t) & 0.2exp(-t)\end{pmatrix},$$

$$P(t)=\begin{pmatrix}0.4exp(-t) & 0.2cos(t)\\ 0.3exp(-t) & 0.3\end{pmatrix},\ Q(t)=\begin{pmatrix}0.3*exp(-t) & -0.2*cos(t)\\ 0.2*exp(-t) & 0.3*cos(t)\end{pmatrix}.$$

Obviously, the activation vector functions satisfy Assumption 2 with $\alpha_1=\alpha_2=\beta_1=\beta_2=1$.

In the following numerical simulations, the initial values of system are taken as $x(t)=(1.2,1.5)^T$ and $y(t)=(2.6,1.3)^T$ for any $t\in[-0.1,0]$.

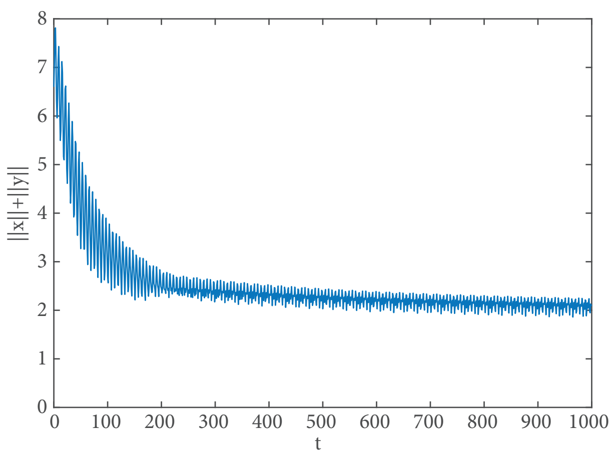

Figure 1. Evolution of the system of Eq. (11) without control.

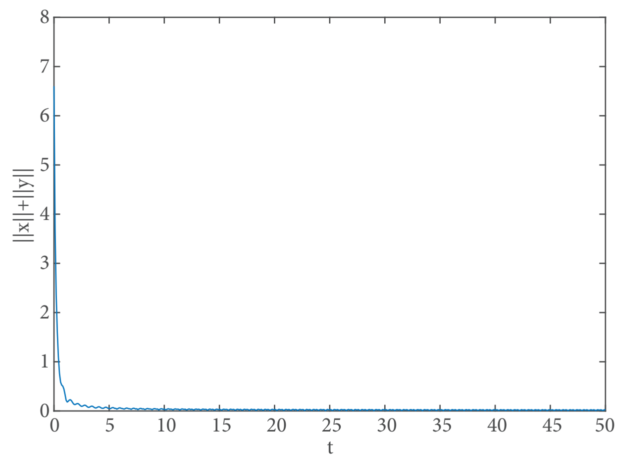

Figure 2. Evolution of the system of Eq. (11) with the control of Eq. (12).

Without external control, the time evolution of the system of Eq. (11) is shown in Figure 1. Let $\mu_1=5.3$, $\mu_2=4.7$, $\nu_1=5.5$, and $\nu_2=5.2$, i.e.

$$I(t)=(5.3x_1(t),4.7x_2(t))^T,\quad J(t)=(5.5y_1(t),5.2y_2(t))^T. \tag{12}$$

It is easily verified that the feedback gains μ_1,μ_2,ν_1,ν_2 satisfy the condition of Theorem 1. With the control of Eq. (12), the time evolution of the system of Eq. (11) is presented in Figure 2. This indicates that the global stabilization of the system of Eq. (11) can be achieved with the control of Eq. (12).

In the following, the external disturbances are considered. Let $\xi(t)$ and $\eta(t)$ be given as $\xi(t)=(-0.2*cos(t),0.1*exp(-t))^T$ and $\eta(t)=(0.1,0.1*sin(t))^T$, respectively. Assume that the feedback gains are much larger, i.e. $\mu_1=10.3$, $\mu_2=9.7$, $\nu_1=10.5$, and $\nu_2=10.2$. The corresponding control is as follows:

$$I(t)=(11.7x_1(t),11.9x_2(t))^T,\quad J(t)=(11.5y_1(t),11.2y_2(t))^T. \tag{13}$$

With this kind of control, the time evolution of the system of Eq. (11) with external disturbances is shown in Figure 3. This indicates that this kind of control scheme is robust with some external disturbances when the feedback gains are much larger.

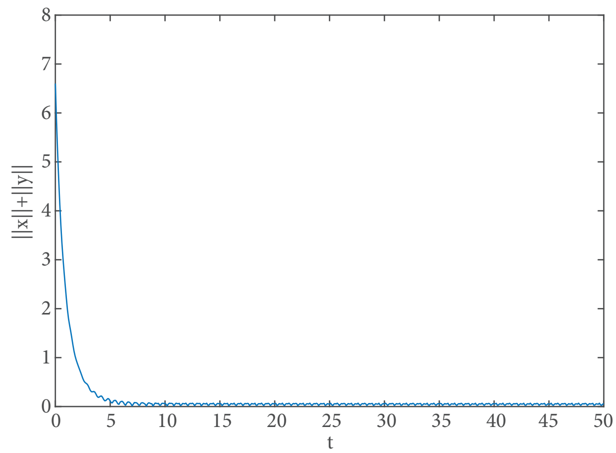

Figure 3. Evolution of the system of Eq. (11) with external disturbances under the control of Eq. (13).

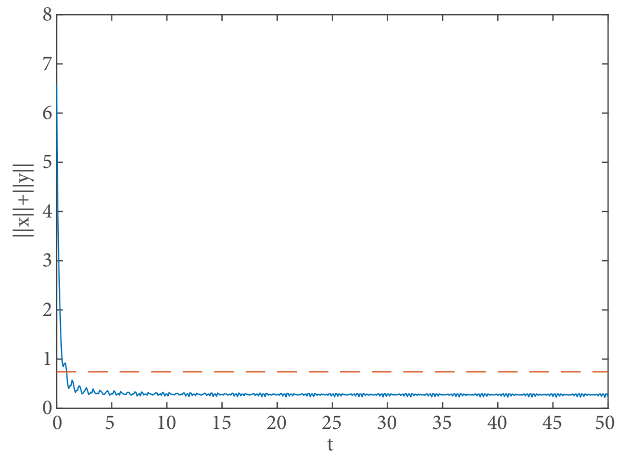

Figure 4. Evolution of the system of Eq. (11) with the estimated error bound $\delta = 0.742$.

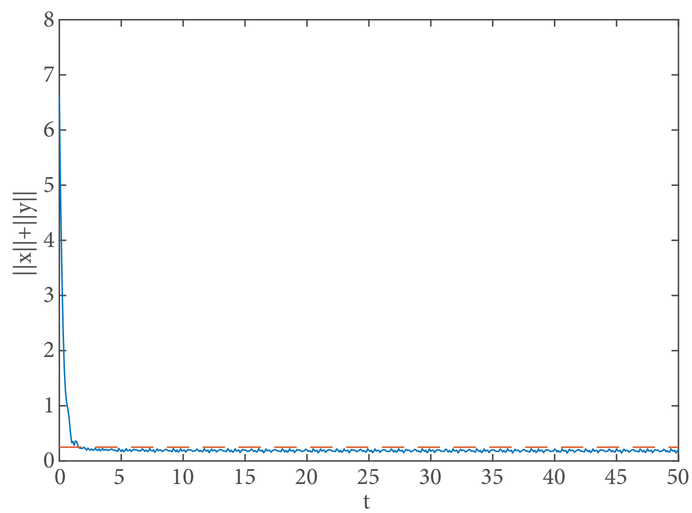

Figure 5. Evolution of the system of Eq. (11) with the desired error bound $\delta = 0.25$.

Now the global quasi-stabilization of the system of Eq. (11) is discussed in the presence of external disturbances. Assume that $\xi(t)$ and $\eta(t)$ are given as $\xi(t) = (1.2 * cos(t), -0.8 * exp(-t))^T$ and $\eta(t) = (0.6, -0.9 * sin(t))^T$.

Let $\mu_1 = 4.3$, $\mu_2 = 3.7$, $\nu_1 = 4.5$, and $\nu_2 = 4.2$. By a simple calculation, we get $lsin(q\pi/2) > m(= 0.55)$, which satisfies the condition in Theorem 3. The time evolution of the system of Eq. (11) with external disturbances is presented in Figure 4. This indicates that the global quasi-stabilization of the system can be achieved with the estimated error bound $\delta \approx 0.742$.

In order to make the error bound be controlled to $\delta = 0.25$, the value of l can be taken as $l = 10.38$, which satisfies $lsin(q\pi/2) > m$. Furthermore, in view of the condition in Theorem 3, the values of feedback gains can be chosen as $\mu_1 = 11.3$, $\mu_2 = 10.7$, $\nu_1 = 11.5$, and $\nu_2 = 11.2$. Under this control, the time evolution of the system of Eq. (11) with external disturbances is presented in Figure 4, which implies that the global stabilization of system can be achieved with the desired error bound of $\delta = 0.25$.

5. Conclusions

This paper investigated the stabilization problem of fractional-order delayed BAM neural networks. Based on feedback control, we derived a sufficient condition to realize the global stabilization of systems by using the fractional inequality, the Lyapunov stability theory, and the comparison principle. In particular, we also proved that this kind of control scheme is robust with external disturbances when the feedback gains are sufficiently large. In addition, we obtained a condition that can guarantee the global quasi-stabilization of systems with some external disturbances and we gave the corresponding estimated error bound, which can be adjusted in practical applications. Finally, we provided some numerical simulations to verify the effectiveness of the theoretical results.

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Author contributions: Z.Y. Yang gave the idea and wrote the theoretical results; X.Y. Tang and J. Zhang checked the theoretical results and did the numerical simulations.

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