Turkish Journal of Electrical Engineering & Computer Sciences

http://journals.tubitak.gov.tr/elektrik/

Research Article

Turk J Elec Eng & Comp Sci
(2020) 28: 969 – 983

doi:10.3906/elk-1906-177

Correlation coefficients of Pythagorean hesitant fuzzy sets and their application to radar LPI performance evaluation

Chengxiu YANG[1], **Qianzhe WANG**[1,*], **Weidong PENG**[1], **Shaoting PEI**[2]
[1]Air Traffic Control and Navigation College, Air Force Engineering University, Xi'an, P.R. China
[2]Aeronautics Engineering College, Air Force Engineering University, Xi'an, P.R. China

Received: 26.06.2019 • **Accepted/Published Online:** 13.11.2019 • **Final Version:** 28.03.2020

Abstract: Evaluating low probability of intercept (LPI) performance is the first step to design parameters and arrange radar resources. In the evaluation process it is hard to rely on the intercept receiver's working scenarios and operating parameters. On the other hand, indicators that affect the LPI performance of radiating side are difficult to consider comprehensively. Thus, building an effective evaluation system is crucial. This research considers the natural parameters of radar extracted from a radiating scenario. Subsequently, a number of criteria are selected, including spatial, time, frequency domain, polarization status, energy status, and waveform features. A multidomain radar LPI performance evaluation method is established, which is based on Pythagorean hesitant fuzzy sets (PHFSs). The paper is motivated by other scholars' research on fuzzy set theories and derives correlation coefficients as well as their properties for PHFSs. Concretely speaking, this study takes account of membership degree, nonmembership degree, and the hesitation of decision makers, so it integrates the benefits of correlation coefficients of hesitant fuzzy sets with Pythagorean fuzzy sets. Meanwhile, weighted correlation coefficients of PHFSs and their properties are proposed in detail. This provides a feasible approach for evaluation problems. For the sake of application, this article gives the specific LPI performance evaluation process. Finally, a novel method is presented to evaluate four fire control radars' LPI performances and is proved to be viable.

Key words: Correlation coefficients, Pythagorean hesitant fuzzy sets, low probability of interception, multidomain evaluation

1. Introduction

With the development of modern warfare, the low probability of intercept (LPI) performance of radar has become more and more important [1]. Accurately evaluating the radar stealth performance of adversaries in complex battlefields will guide real-time decision making. Therefore, establishing a reasonable LPI evaluation system is the premise of radar low intercept performance design [2]. At present, the radar LPI evaluation methods mainly concentrate on calculating interception factors and probabilities. The interception factors proposed by Schleher use distance radio as a measure of the low intercept performance of radars [3]. According to the spatial characteristics of radar signals, CEVR and SEVR used the radius of a circular area and sphere diameter as the vulnerable area, respectively [4]. Shi et al. used intercept probability to evaluate LPI performance [5]. In addition, scholars also analyzed different indicators that affect radar LPI performance. Li et al. divided the LPI evaluation index system into three aspects, which include covering, detection, and interception layers [6]. Wang et al. took advantage of mutual information to evaluate radar LPI performance [7]. However, the above

*Correspondence: 425253199@qq.com

studies still have limitations because they require accurate radar signal real-time data and do not involve an evaluation algorithm. They should be improved.

This study proposes a LPI performance evaluation system based on correlation coefficients of Pythagorean hesitant fuzzy sets (PHFSs). Considering several aspects that may pose threats to radar stealth performance, the paper uses a multidomain evaluation method, which gets rid of the "intercepting-receiving" scenario. This method has six indices, including spatial, time, frequency, polarization status, energy status, and waveform domains.

Correlation is often used in evaluation problems. According to correlation analysis, the interrelationship of alternatives will be determined; that is to say, the correlation coefficient reflects the linear relationship between two variables. However, decision makers often need to make judgments based on uncertain and imprecise information. Therefore, the correlation coefficient extends into fuzzy set theory. In the past decade, hesitant fuzzy sets (HFSs) made great progress, which were first proposed by Torra and widely applied [8]. By using HFSs, Xu and Xia initially proposed formulas for the correlation coefficients [9]. Chen et al. extended the correlation coefficients of HFSs [10]. Later, Liao et al. proposed an improved model corresponding to the above research [11]. At present, correlation analysis for HFSs is widely used in clustering analysis, multiple attribute decision making, and evaluation problems, but there are some drawbacks of those methods introduced above. HFSs only contain the information about membership degree. Therefore, Yager [12, 13] introduced the Pythagorean membership grades, based on which they proposed the Pythagorean fuzzy set (PFS) and its basic operational laws. A PFS consists of the membership degree and the nonmembership degree. Furthermore, the correlation coefficients for PFSs were defined in [14].

More recently, several authors separately derived the PHFS [15–17]. Their research combined HFS with PFS for introducing hesitation to membership degree and nonmembership degree, i.e. the subsets of PHFSs may have different crisp values. They also paid attention to the basic operation of PHFSs, such as aggregation operators and practical applications. As discussed above, this paper proposes the correlation coefficients of Pythagorean hesitant fuzzy sets and derives their properties. The method takes account of membership, nonmembership, and hesitation information, which is suitable to evaluate the radar LPI performance. In this article, four fire control radars from different countries are selected as examples. It was found that the assessment results are in line with expert expectations. We also analyzed the impact of weight vector changes on the evaluation results. The unique innovations and contributions of this article include:

· Extracting parameters from a radiating scenario and concluding the indices that influence radar LPI performance comprehensively.

· Proposing the correlation coefficients of PHFSs and proving their properties.

· Integrating a multidomain evaluation system and correlation coefficients of PHFSs to solve the radar LPI performance evaluation problem.

· The feasibility of the proposed method is demonstrated by using fire control radars as an example.

The present paper is organized as follows: Section 2 describes the multidomain radar evaluation system. In Section 3, some concepts of HFS and PFS as well as their correlation coefficients are reviewed. In Section 4, PHFSs and their basic concepts are introduced. In Section 5, correlation coefficients and weighted correlation coefficients of PHFSs with properties are proposed, respectively. The radar LPI performance evaluation specific process based on the correlation coefficients of PHFSs is presented in Section 6. The application of the proposed novel method for fire control radars' LPI performance evaluation is analyzed in Section 7. The conclusion and future work are discussed in Section 8.

2. Radar LPI performance evaluation methods

The current methods for evaluating LPI performance mainly involve calculating interception factors and probability. These methods have drawbacks as the analysis of factors affecting LPI performance is not comprehensive. Zhao et al. used the intercept receiver and gave some evaluation indices, and they were the first to introduce fuzzy set theory into LPI evaluation problems [18]. With the help of previous studies, this novel model extends the classical evaluation problems, and we should analyze the factors that influence radar LPI performance first.

This paper, from the perspective of radar radiating, proposes a multidomain evaluation method, which includes spatial, time, frequency, polarization, energy, and waveform domains. In the spatial domain, interception occurs in the passive detection equipment points in the radar beam direction, which leads to the intercept of the spatial domain, as shown in Figure 1. Hence, in order to enhance the LPI performance in the spatial domain, narrow half-power beam width and ultralow side lobe techniques are used to concentrate radiant energy in the main lobe and reduce the power of the side lobe radiation. Experts need to obtain radar beam width and side lobe level according to the design parameters in advance, or estimate them by frequency, antenna size, and structure of the enemy radar with unknown parameters.

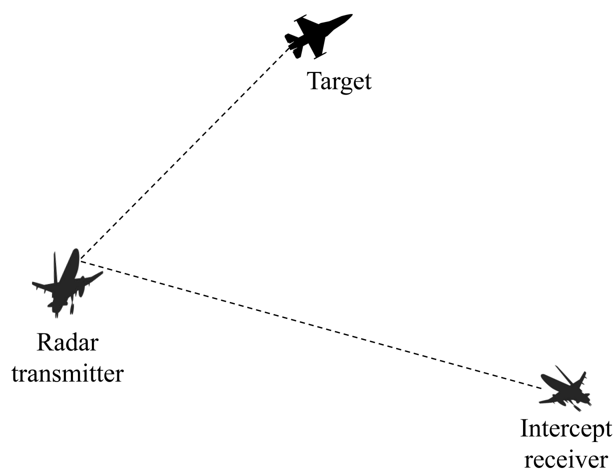

Figure 1. Spatial interception figure.

Ideally, the intercept probability of the time domain is defined as

$$P = \frac{T_{OT}}{T_{OT} + t_i + t_s}, \tag{1}$$

where T_{OT} represents the time the radar beam remains pointing at the intercept receiver, i.e. dwell time. t_i represents radar beam emission interval and t_s represents radar search time. t_s depends on practical application. Hence, by increasing T_{OT} or decreasing t_i, the time domain intercept probability will increase and radar LPI performance will degrade.

The reasons for interception in the frequency domain include: (i) frequency bandwidth of the intercept receiver is wide enough to distinguish the frequency currently used by the radar; (ii) when the radar beam is illuminating the target, the intercept receiver turns to the right frequency. The intercept receiver sometimes knows the radar parameters (e.g., pulse repetition frequency, wavelength) in advance through the intelligence system. Modern radar uses a frequency hopping mechanism, which can reduce the probability of interception by increasing the band. Another antiintercepting method is spread-spectrum. With the derivation by Rajgopal et al., spread-spectrum radar's intercept probability is related to the waveform and energy domain [19]. Hence, the multidomain evaluation indices are connected with each other and inextricably linked.

Let us suppose that a pulse radar radiation electric field detected by the intercept receiver is

$$\overrightarrow{E}(t) = \widehat{E}(t) \sum_{n=0}^{\infty} A_n rect\left(\frac{t - nT_r}{\tau}\right) \cos(2\pi f_0 t + \phi), \tag{2}$$

where $\widehat{E}(t)$ denotes the signal electric field time-variable polarization status, A_n represents the nth pulse amplitude, $rect\,(*)$ denotes rectangle function, τ denotes pulse width, and T_r denotes pulse repetition frequency (PRF). ϕ represents the radar signal phase, and f_0 denotes the carrier wave frequency.

The interception is related not only to radar signal but also to identifying and sorting. That is to say, the passive detection equipment needs to perform coherent processing. If the polarization status of a radiated signal remains unchanged during the detection process, whether it is horizontal polarization, vertical polarization, circular polarization, or elliptically polarized, this determined polarization status is most easily captured by the enemy. In contrast, complex polarization status leads to difficulty for the intercept receiver. For example, the random changing polarization status will be harder to be intercepted than constant polarization status.

In the energy domain, under the constraints of detection accuracy, lower transmit power can be difficult to detect by the intercept receiver. Therefore, modern radar uses power management technology to improve LPI performance, and the energy information is a key factor in LPI evaluation. Besides, the pulse train energy changing period also affects the intercept probability. The signal is easily detected when the power of each pulse of the radiated signal is the same. If the energy of the pulse train changes randomly, it is considered that the signal is the most difficult to be detected, and the LPI performance is the best. Methods for extracting energy domain indicators include airspace power measurement techniques and pulse energy spectrum analysis.

ϕ and f_0 in (2) are the intrapulse characteristics of the waveform. The more complicated the waveform feature is, the harder it will be for it to be identified by the intercept receiver. In addition, time-bandwidth product and duty cycle are parts of the interpulse waveform characteristics. Peak power will decrease with the reduction of time-bandwidth. Duty cycle is proportional to pulses that are emitted per unit time; therefore, by increasing the duty cycle, the LPI performance degrades.

The proposed multidomain LPI performance evaluation system includes subindicators, as shown in Figure 2. In addition to the six evaluation indices summarized above, the radar LPI performance also can be studied from the modulation domain. However, the parameters of radar modulation domain can be mapped to the six indices mentioned in this paper. For example, for analog amplitude modulated (AM) signal and analog frequency modulated (FM) signal, the main LPI performance evaluation parameters include carrier frequency and bandwidth, which can be described by waveform feature and frequency domain.

3. HFS and PFS with their correlation coefficients

3.1. HFS and its correlation coefficients of PFS

Definition 1 *Let X be a finite set. The hesitant fuzzy set (HFS) on X returns a subject of* $[0,1]$,

$$A = \{\langle x, h_A(x)\rangle \,|x \in X\},\tag{3}$$

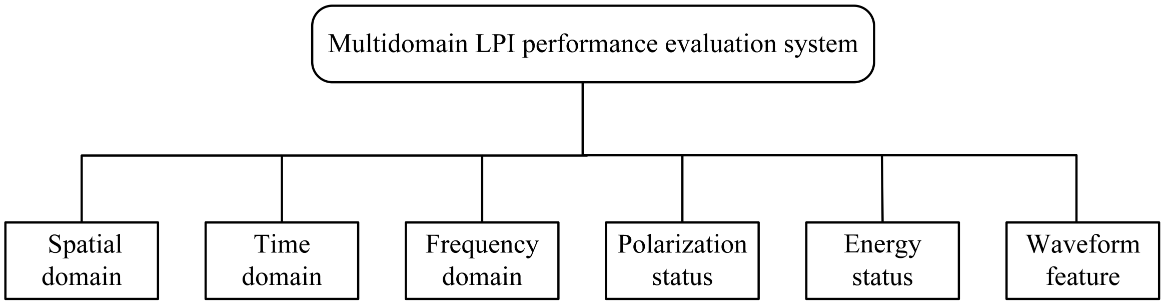

Figure 2. Multidomain evaluation system.

where $h_A(x)$ denotes the possible membership degrees of element $x \in X$ to set A by taking values in $[0,1]$. Xu and Xia called $h_A(x)$ a hesitant fuzzy element (HFE) [8].

Definition 2 *Let A and B be two hesitant fuzzy sets in a finite set $X = \{x_1, x_2, \cdots, x_n\}$ denoted as $A = \{\langle x_i, h_A(x_i)\rangle | x_i \in X, i = 1, 2, \cdots, n\}$ and $B = \{\langle x_i, h_B(x_i)\rangle | x_i \in X, i = 1, 2, \cdots, n\}$, respectively. Then two kinds of correlation coefficients of HFSs between A and B are defined as*

$$\rho_{1HFS}(A,B) = \frac{\sum_{i=1}^{n}\left(\frac{1}{l_i}\sum_{j=1}^{l_i} h_{A\sigma(j)}(x_i)\, h_{B\sigma(j)}(x_i)\right)}{\sqrt{\sum_{i=1}^{n}\left(\frac{1}{l_i}\sum_{j=1}^{l_i} h_{A\sigma(j)}^2(x_i)\right)}\sqrt{\sum_{i=1}^{n}\left(\frac{1}{l_i}\sum_{j=1}^{l_i} h_{B\sigma(j)}^2(x_i)\right)}}, \tag{4}$$

$$\rho_{2HFS}(A,B) = \frac{\sum_{i=1}^{n}\left(\frac{1}{l_i}\sum_{j=1}^{l_i} h_{A\sigma(j)}(x_i)\, h_{B\sigma(j)}(x_i)\right)}{\max\left\{\sum_{i=1}^{n}\left(\frac{1}{l_i}\sum_{j=1}^{l_i} h_{A\sigma(j)}^2(x_i)\right), \sum_{i=1}^{n}\left(\frac{1}{l_i}\sum_{j=1}^{l_i} h_{B\sigma(j)}^2(x_i)\right)\right\}}. \tag{5}$$

$l(h_A(x_i))$ and $l(h_B(x_i))$ represent the number of values in $h_A(x_i)$ and $h_B(x_i)$, respectively. Supposing $l_i = l(h_A(x_i)) = l(h_B(x_i))$, the size of values in $h(x)$ is in descending order. Therefore, $h_{A\sigma(j)}(x_i)$ and $h_{B\sigma(j)}(x_i)$ are the jth largest values in $h_A(x_i)$ and $h_B(x_i), j = 1, 2, \cdots, n-1$, respectively.

3.2. PFS and its correlation coefficients

Definition 3 *Let X be a finite set. A Pythagorean fuzzy set (PFS) P in X is defined as*

$$P = \{\langle x, \mu_P(x), v_P(x)\rangle | x \in X\}, \tag{6}$$

where μ_P denotes the degree of membership and v_P denotes the degree of nonmembership of element $x \in X$ to the set P by taking values in $[0,1]$, respectively, with the condition that $0 \le (\mu_P(x))^2 + (v_P(x))^2 \le 1$. The degree of indeterminacy $\pi_P(x) = \sqrt{1 - (\mu_P(x))^2 - (\nu_P(x))^2}$.

Definition 4 *Let A and B be two Pythagorean fuzzy sets in a finite set $X = \{x_1, x_2, \cdots, x_n\}$ denoted as $A = \{\langle x_i, \mu_A(x_i), v_A(x_i)\rangle | x_i \in X, i = 1, 2, \cdots, n\}$ and $B = \{\langle x_i, \mu_B(x_i), v_B(x_i)\rangle | x_i \in X, i = 1, 2, \cdots, n\}$, respectively, where $\mu_A(x_i), v_A(x_i), \mu_B(x_i), v_B(x_i) \in [0,1]$ for every $x_i \in X$. Then the two kinds of correlation* *coefficients of PFSs between A and B are defined as*

$$\rho_{1PFS}(A,B)=\frac{\sum_{i=1}^{n}\left(\mu_A^2(x_i)\cdot\mu_B^2(x_i)+v_A^2(x_i)\cdot v_B^2(x_i)+\pi_A^2(x_i)\cdot\pi_B^2(x_i)\right)}{\sqrt{\sum_{i=1}^{n}\left(\mu_A^4(x_i)+v_A^4(x_i)+\pi_A^4(x_i)\right)}\sqrt{\sum_{i=1}^{n}\left(\mu_B^4(x_i)+v_B^4(x_i)+\pi_B^4(x_i)\right)}}\quad , \tag{7}$$

$$\rho_{2PFS}(A,B)=\frac{\sum_{i=1}^{n}\left(\mu_A^2(x_i)\cdot\mu_B^2(x_i)+v_A^2(x_i)\cdot v_B^2(x_i)+\pi_A^2(x_i)\cdot\pi_B^2(x_i)\right)}{\max\left\{\sum_{i=1}^{n}\left(\mu_A^4(x_i)+\nu_A^4(x_i)+\pi_A^4(x_i)\right),\sum_{i=1}^{n}\left(\mu_B^4(x_i)+v_B^4(x_i)+\pi_B^4(x_i)\right)\right\}}\quad . \tag{8}$$

4. Pythagorean hesitant fuzzy sets

Definition 5 *Let X be a finite set. A Pythagorean hesitant fuzzy set (PHFS) F in X is described by*

$$F=\left\{\langle x,h_F(x),g_F(x)\rangle\,|x\in X\right\}, \tag{9}$$

where $h_F(x)$ *and* $g_F(x)$ *are two sets denoting the possible degree of membership and nonmembership of element* $x\in X$ *in F by taking values in* $[0,1]$*, respectively. For convenience, the pair* $(h_F(x),g_F(x))$ *is called a Pythagorean hesitant fuzzy element (PHFE), with the condition that*

$$0\le\mu_F(x),v_F(x)\le 1, 0\le\left(\mu_F^+(x)\right)^2+\left(\nu_F^+(x)\right)^2)\le 1, \tag{10}$$

where $\mu_F(x)\in h_F(x)$ *and* $v_F(x)\in g_F(x)$*,* $\mu_F^+(x)\in h_F^+(x)=\bigcup_{\mu_F(x)\in h_F(x)}\max\{\mu_F(x)\}$ *and* $v_F^+(x)\in g_F^+(x)=\bigcup_{v_F(x)\in g_F(x)}\max\{v_F(x)\}$*. For instance, when the decision makers assign the membership degree possible values are 0.8, 0.7, and 0.5 and the nonmembership degree possible values are 0.3 and 0.1. We can write it as* $F=\{(0.8,0.7,0.5),(0.3,0.1)\}$*,* $\mu_F^+(x)=0.8$*,* $v_F^+(x)=0.3$*, and* $\left(\mu_F^+(x)\right)^2+\left(v_F^+(x)\right)^2\le 1$*.*

Definition 6 *For any PHFS* $F=\{\langle x,h_F(x),g_F(x)\rangle\,|x\in X\}$*, the degree of indeterminacy is defined as*

$$\pi_F(x)=\sqrt{1-\left(\frac{1}{l}\sum_{\mu_F(x)\in h_F(x)}(\mu_F(x))^2+\frac{1}{k}\sum_{v_F(x)\in g_F(x)}(\nu_F(x))^2\right)}, \tag{11}$$

where l *and* k *denote the number of values in* $h_F(x)$ *and* $g_F(x)$*, respectively.*

5. Correlation coefficients of Pythagorean hesitant fuzzy sets

The values of a PHFE are usually in disorder. Motivated by [10], we arrange them in a descending order. Let $\sigma:(1,2,\cdots,n)\to(1,2,\cdots,n)$ be a permutation satisfying $\mu_{F\sigma(j)}\ge\mu_{F\sigma(j+1)}, j=1,2,\cdots,n-1$, and let $\mu_{F\sigma(j)}$ be the jth largest value in h_F; let $\sigma:(1,2,\cdots,m)\to(1,2,\cdots,m)$ be a permutation satisfying $v_{F\sigma(k)}\ge v_{F\sigma(k+1)}, k=1,2,\cdots,m-1$, and $v_{F\sigma(k)}$ be the kth largest value in g_F.

Definition 7 *Let A be a PHFS in a finite set* $X=\{x_1,x_2,\cdots,x_n\}$ *denoted as* $A=\{\langle x_i,h_A(x_i),g_A(x_i)\rangle\,|x_i\in X,i=1,2,\cdots,n\}$*. Then the information energy of A is defined as*

$$E(A)=\sum_{i=1}^{n}\left[\frac{1}{l_i}\sum_{j=1}^{l_i}\left(\mu_{A\sigma(j)}(x_i)\right)^4+\frac{1}{k_i}\sum_{j=1}^{k_i}\left(v_{A\sigma(j)}(x_i)\right)^4+\pi_A^4(x_i)\right], \tag{12}$$

where l_i *and* k_i *represent the number of values in* $h_A(x_i)$ *and* $g_A(x_i)$*, respectively.*

Definition 8 *Let A and B be two PHFSs in a finite set $X = \{x_1, x_2, \cdots, x_n\}$ denoted as $A = \{\langle x_i, h_A(x_i), g_A(x_i)\rangle | x_i \in X, i = 1, 2, \cdots, n\}$ and $B = \{\langle x_i, h_B(x_i), g_B(x_i)\rangle | x_i \in X, i = 1, 2, \cdots, n\}$. Then the correlation between A and B is written as*

$$C(A,B) = \sum_{i=1}^{n} \left[\frac{1}{l_i} \sum_{j=1}^{l_i} \left(\mu_{A\sigma(j)}(x_i)\right)^2 \cdot \left(\mu_{B\sigma(j)}(x_i)\right)^2 + \frac{1}{k_i} \sum_{j=1}^{k_i} \left(v_{A\sigma(j)}(x_i)\right)^2 \cdot \left(\nu_{B\sigma(j)}(x_i)\right)^2 + \pi_A^2(x_i) \cdot \pi_B^2(x_i) \right], \tag{13}$$

where $l_i = \max\{l(h_A(x_i)), l(h_B(x_i))\}$ and $k_i = \max\{k(g_A(x_i)), k(g_B(x_i))\}$ for each x_i in X, where $l(h_A(x_i))$ and $l(h_B(x_i))$ denote the number of membership values in $h_A(x_i)$ and $h_B(x_i)$, respectively. $k(g_A(x_i))$ and $k(g_B(x_i))$ are the number of nonmembership values in $g_A(x_i)$ and $g_B(x_i)$. If $l(h_A(x_i)) \neq l(h_B(x_i))$ or $k(g_A(x_i)) \neq k(g_B(x_i))$, we can add the minimum value of the element to the PHFE with less value until the number of values in two PHFEs is equal. We call the above process a pessimistic principle [20]. For instance, if $l(h_A(x_i)) < l(h_B(x_i))$ or $k(g_A(x_i)) < k(g_B(x_i))$, $h_A(x_i)$ or $g_A(x_i)$ should be extended by adding the minimum value in it until it has the same length as $h_B(x_i)$ or $g_B(x_i)$.

The correlation of PHFSs has the following properties:

$$(1) C(A,B) = C(B,A).$$
$$(2) C(A,A) = E(A).$$

Definition 9 *Let A and B be two PHFSs in a finite set $X = \{x_1, x_2, \cdots, x_n\}$ denoted as $A = \{\langle x_i, h_A(x_i), g_A(x_i)\rangle | x_i \in X, i = 1, 2, \cdots, n\}$ and $B = \{\langle x_i, h_B(x_i), g_B(x_i)\rangle | x_i \in X, i = 1, 2, \cdots, n\}$. Then the correlation coefficient between A and B is defined as*

$$\begin{aligned}\rho_1(A,B) &= \frac{C(A,B)}{(C(A,A))^{1/2}(C(B,B))^{1/2}} \\ &= \frac{\sum_{i=1}^{n}\left[\frac{1}{l_i}\sum_{j=1}^{l_i}\left(\mu_{A\sigma(j)}(x_i)\right)^2\cdot\left(\mu_{B\sigma(j)}(x_i)\right)^2+\frac{1}{k_i}\sum_{j=1}^{k_i}\left(\nu_{A\sigma(j)}(x_i)\right)^2\cdot\left(\nu_{B\sigma(j)}(x_i)\right)^2+\pi_A{}^2(x_i)\cdot\pi_B{}^2(x_i)\right]}{\left\{\sum_{i=1}^{n}\left[\frac{\sum_{j=1}^{l_i}\left(\mu_{A\sigma(j)}(x_i)\right)^4}{l_i}+\frac{\sum_{j=1}^{k_i}\left(\nu_{A\sigma(j)}(x_i)\right)^4}{k_i}+\pi_A{}^4(x_i)\right]\right\}^{1/2}\left\{\sum_{i=1}^{n}\left[\frac{\sum_{j=1}^{l_i}\left(\mu_{B\sigma(j)}(x_i)\right)^4}{l_i}+\frac{\sum_{j=1}^{k_i}\left(\nu_{B\sigma(j)}(x_i)\right)^4}{k_i}+\pi_B{}^4(x_i)\right]\right\}^{1/2}}.\end{aligned} \tag{14}$$

Theorem 1 *The correlation coefficient between two PHFSs A and B satisfies the following properties:*

$$(1) \rho_1(A,B) = \rho_1(B,A).$$
$$(2) 0 \leq \rho_1(A,B) \leq 1.$$
$$(3) \rho_1(A,B) = 1, if A = B.$$

Proof (1) It is straightforward. (2) We will prove $\rho_1(A,B) \leq 1$.

$$
\begin{aligned}
&C(A,B)\\
&=\sum_{i=1}^{n}\left[\frac{1}{l_i}\sum_{j=1}^{l_i}\left(\mu_{A\sigma(j)}(x_i)\right)^2\cdot\left(\mu_{B\sigma(j)}(x_i)\right)^2+\frac{1}{k_i}\sum_{j=1}^{k_i}\left(\nu_{A\sigma(j)}(x_i)\right)^2\cdot\left(\nu_{B\sigma(j)}(x_i)\right)^2+{\pi_A}^2(x_i)\cdot{\pi_B}^2(x_i)\right]\\
&=\left[\frac{1}{l_1}\sum_{j=1}^{l_1}\left(\mu_{A\sigma(j)}(x_1)\right)^2\cdot\left(\mu_{B\sigma(j)}(x_1)\right)^2+\frac{1}{k_1}\sum_{j=1}^{k_1}\left(\nu_{A\sigma(j)}(x_1)\right)^2\cdot\left(\nu_{B\sigma(j)}(x_1)\right)^2+\cdots+{\pi_A}^2(x_1)\cdot{\pi_B}^2(x_1)\right]\\
&+\left[\frac{1}{l_2}\sum_{j=1}^{l_2}\left(\mu_{A\sigma(j)}(x_2)\right)^2\cdot\left(\mu_{B\sigma(j)}(x_2)\right)^2+\frac{1}{k_2}\sum_{j=1}^{k_2}\left(\nu_{A\sigma(j)}(x_2)\right)^2\cdot\left(\nu_{B\sigma(j)}(x_2)\right)^2+\cdots+{\pi_A}^2(x_2)\cdot{\pi_B}^2(x_2)\right]+\cdots\\
&+\left[\frac{1}{l_n}\sum_{j=1}^{l_n}\left(\mu_{A\sigma(j)}(x_n)\right)^2\cdot\left(\mu_{B\sigma(j)}(x_n)\right)^2+\frac{1}{k_n}\sum_{j=1}^{k_n}\left(\nu_{A\sigma(j)}(x_n)\right)^2\cdot\left(\nu_{B\sigma(j)}(x_n)\right)^2+\cdots+{\pi_A}^2(x_n)\cdot{\pi_B}^2(x_n)\right]\\
&=\left[\sum_{j=1}^{l_1}\frac{\left(\mu_{A\sigma(j)}(x_1)\right)^2}{(l_1)^{1/2}}\cdot\frac{\left(\mu_{B\sigma(j)}(x_1)\right)^2}{(l_1)^{1/2}}+\sum_{j=1}^{k_1}\frac{\left(\nu_{A\sigma(j)}(x_1)\right)^2}{(k_1)^{1/2}}\cdot\frac{\left(\nu_{B\sigma(j)}(x_1)\right)^2}{(k_1)^{1/2}}+{\pi_A}^2(x_1)\cdot{\pi_B}^2(x_1)\right]\\
&+\left[\sum_{j=1}^{l_2}\frac{\left(\mu_{A\sigma(j)}(x_2)\right)^2}{(l_2)^{1/2}}\cdot\frac{\left(\mu_{B\sigma(j)}(x_2)\right)^2}{(l_2)^{1/2}}+\sum_{j=1}^{k_2}\frac{\left(\nu_{A\sigma(j)}(x_2)\right)^2}{(k_2)^{1/2}}\cdot\frac{\left(\nu_{B\sigma(j)}(x_2)\right)^2}{(k_2)^{1/2}}+{\pi_A}^2(x_2)\cdot{\pi_B}^2(x_2)\right]+\cdots\\
&+\left[\sum_{j=1}^{l_n}\frac{\left(\mu_{A\sigma(j)}(x_n)\right)^2}{(l_n)^{1/2}}\cdot\frac{\left(\mu_{B\sigma(j)}(x_n)\right)^2}{(l_n)^{1/2}}+\sum_{j=1}^{k_n}\frac{\left(\nu_{A\sigma(j)}(x_n)\right)^2}{(k_n)^{1/2}}\cdot\frac{\left(\nu_{B\sigma(j)}(x_n)\right)^2}{(k_n)^{1/2}}+{\pi_A}^2(x_n)\cdot{\pi_B}^2(x_n)\right].
\end{aligned}
$$

According to the Cauchy–-Schwarz inequality,

$$(x_1y_1+x_2y_2+\cdots+x_ny_n)^2\leq\left(x_1^2+x_2^2+\cdots+x_n^2\right)\cdot\left(y_1^2+y_2^2+\cdots+y_n^2\right),$$

where $(x_1,x_2,\cdots,x_n)\in R^n$ and $(y_1,y_2,\cdots,y_n)\in R^n$, and we can get

$$
\begin{aligned}
&C^2(A,B)\\
&\leq\left[\left(\sum_{j=1}^{l_1}\frac{\left(\mu_{A\sigma(j)}(x_1)\right)^4}{l_1}+\sum_{j=1}^{k_1}\frac{\left(\nu_{A\sigma(j)}(x_1)\right)^4}{k_1}+{\pi_A}^4(x_1)\right)+\left(\sum_{j=1}^{l_2}\frac{\left(\mu_{A\sigma(j)}(x_2)\right)^4}{l_2}+\sum_{j=1}^{k_2}\frac{\left(\nu_{A\sigma(j)}(x_2)\right)^4}{k_2}+{\pi_A}^4(x_2)\right)+\cdots\right.\\
&\left.+\left(\sum_{j=1}^{l_n}\frac{\left(\mu_{A\sigma(j)}(x_n)\right)^4}{l_n}+\sum_{j=1}^{k_n}\frac{\left(\nu_{A\sigma(j)}(x_n)\right)^4}{k_n}+{\pi_A}^4(x_n)\right)\right]\cdot\left[\left(\sum_{j=1}^{l_1}\frac{\left(\mu_{B\sigma(j)}(x_1)\right)^4}{l_1}+\sum_{j=1}^{k_1}\frac{\left(\nu_{B\sigma(j)}(x_1)\right)^4}{k_1}+{\pi_B}^4(x_1)\right)\right.\\
&\left.+\left(\sum_{j=1}^{l_2}\frac{\left(\mu_{B\sigma(j)}(x_2)\right)^4}{l_2}+\sum_{j=1}^{k_2}\frac{\left(\nu_{B\sigma(j)}(x_2)\right)^4}{k_2}+{\pi_B}^4(x_2)\right)+\cdots+\left(\sum_{j=1}^{l_n}\frac{\left(\mu_{B\sigma(j)}(x_n)\right)^4}{l_n}+\sum_{j=1}^{k_n}\frac{\left(\nu_{B\sigma(j)}(x_n)\right)^4}{k_n}+{\pi_B}^4(x_n)\right)\right]\\
&=C(A,A)\cdot C(B,B).
\end{aligned}
$$

Therefore, $C^2(A,B)\leq C(A,A)\cdot C(B,B)$. Thus, $\rho_1(A,B)\leq 1$, and the inequality $\rho_1(A,B)\geq 0$ is obvious.
(3) If $A=B\Rightarrow h_{A\sigma(j)}(x_i)=h_{B\sigma(j)}(x_i)$ and $g_{A\sigma(j)}(x_i)=g_{B\sigma(j)}(x_i)\Rightarrow\rho_1(A,B)=1$. □

Definition 10 *Let A and B be two PHFSs in a finite set $X=\{x_1,x_2,\cdots,x_n\}$ denoted as $A=\{\langle x_i,h_A(x_i),g_A(x_i)\rangle|x_i\in X, i=1,2,\cdots,n\}$ and $B=\{\langle x_i,h_B(x_i),g_B(x_i)\rangle|x_i\in X, i=1,2,\cdots,n\}$. Then the correlation coefficient between A and B is defined as*

$$\rho_2(A,B)=\frac{C(A,B)}{\max\{(C(A,A)),(C(B,B))\}}$$
$$=\frac{\sum_{i=1}^{n}\left[\frac{1}{l_i}\sum_{j=1}^{l_i}\left(\mu_{A\sigma(j)}(x_i)\right)^2\cdot\left(\mu_{B\sigma(j)}(x_i)\right)^2+\frac{1}{k_i}\sum_{j=1}^{k_i}\left(\nu_{A\sigma(j)}(x_i)\right)^2\cdot\left(\nu_{B\sigma(j)}(x_i)\right)^2+{\pi_A}^2(x_i)\cdot{\pi_B}^2(x_i)\right]}{\max\left\{\sum_{i=1}^{n}\left[\frac{\sum_{j=1}^{l_i}\left(\mu_{A\sigma(j)}(x_i)\right)^4}{l_i}+\frac{\sum_{j=1}^{k_i}\left(\nu_{A\sigma(j)}(x_i)\right)^4}{k_i}+{\pi_A}^4(x_i)\right],\sum_{i=1}^{n}\left[\frac{\sum_{j=1}^{l_i}\left(\mu_{B\sigma(j)}(x_i)\right)^4}{l_i}+\frac{\sum_{j=1}^{k_i}\left(\nu_{B\sigma(j)}(x_i)\right)^4}{k_i}+{\pi_B}^4(x_i)\right]\right\}}. \quad (15)$$

Theorem 2 *The correlation coefficient between two PHFSs A and B satisfies the following properties:*

$$(1)\rho_2(A,B)=\rho_2(B,A).$$
$$(2)0\leq\rho_2(A,B)\leq 1.$$
$$(3)\rho_2(A,B)=1, if\, A=B.$$

Proof We can prove the properties (1)–(3) according to the same proof method as in Theorem 1. □

Example 1 *Assume A and B are two PHFEs in $X=\{x_1,x_2\}$, and*

$$A=\{\langle x_1,\{0.7,0.6,0.5\},\{0.4,0.3\}\rangle,\langle x_2,\{0.9,0.6\},\{0.3,0.1\}\rangle\},$$
$$B=\{\langle x_1,\{0.7,0.5\},\{0.5,0.4\}\rangle,\langle x_2,\{0.8,0.7,0.6\},\{0.4,0.2\}\rangle\}.$$

First, we normalize A and B by pessimistic principle. In order to simplify the calculation process, we calculate $\pi_A(x_i)$ and $\pi_B(x_i)$ as follows:

$$\pi_A(x_1)=\sqrt{1-(\frac{0.7^2+0.6^2+0.5^2}{3}+\frac{0.4^2+0.3^2}{2})}=0.713,$$
$$\pi_A(x_2)=\sqrt{1-(\frac{0.9^2+0.6^2+0.6^2}{3}+\frac{0.3^2+0.1^2}{2})}=0.663,$$
$$\pi_B(x_1)=\sqrt{1-(\frac{0.7^2+0.5^2+0.5^2}{3}+\frac{0.5^2+0.4^2}{2})}=0.682,$$
$$\pi_B(x_2)=\sqrt{1-(\frac{0.8^2+0.7^2+0.6^2}{3}+\frac{0.4^2+0.2^2}{2})}=0.635.$$

Then, by employing (12), the informational energy of A and B can be obtained as

$$E(A)=C(A,A)=\frac{0.7^4+0.6^4+0.5^4}{3}+\frac{0.4^4+0.3^4}{2}+{\pi_A}^4(x_1)+\frac{0.9^4+0.6^4+0.6^4}{3}+\frac{0.3^4+0.1^4}{2}+{\pi_A}^4(x_2)=0.9218,$$
$$E(B)=C(B,B)=\frac{0.7^4+0.5^4+0.5^4}{3}+\frac{0.5^4+0.4^4}{2}+{\pi_B}^4(x_1)+\frac{0.8^4+0.7^4+0.6^4}{3}+\frac{0.4^4+0.2^4}{2}+{\pi_B}^4(x_2)=0.8181.$$

The correlation between A and B can be computed as

$$C(A,B)=\frac{0.7^2\cdot0.7^2+0.6^2\cdot0.5^2+0.5^2\cdot0.5^2}{3}+\frac{0.4^2\cdot0.5^2+0.3^2\cdot0.4^2}{2}+{\pi_A}^2(x_1)\cdot{\pi_B}^2(x_1)$$
$$+\frac{0.9^2\cdot0.8^2+0.6^2\cdot0.7^2+0.6^2\cdot0.6^2}{3}+\frac{0.3^2\cdot0.4^2+0.1^2\cdot0.2^2}{2}+{\pi_A}^2(x_2)\cdot{\pi_B}^2(x_2)=0.8540.$$

Therefore, the correlation coefficients $\rho_1(A,B)$ *and* $\rho_2(A,B)$ *are*

$$\rho_1(A,B) = \frac{0.8540}{\sqrt{0.9218 \times 0.8181}} = 0.9834,$$

$$\rho_2(A,B) = \frac{0.8540}{\max\{0.9218, 0.8181\}} = 0.9264.$$

In many practical applications, we should think about the influence of weights of different attributes. Let $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$ be the weights vector of $x_i (i = 1, 2, \cdots, n)$ with $\omega_i \geq 0$ and $\sum_{i=1}^{n} \omega_i = 1$. We can extend correlation coefficients in (14) and (15) as

$$\rho_{1\omega}(A,B) = \frac{C_\omega(A,B)}{(C_\omega(A,A))^{1/2}(C_\omega(B,B))^{1/2}} = \frac{\sum\limits_{i=1}^{n} \omega_i \left[\frac{1}{l_i} \sum\limits_{j=1}^{l_i} \left(\mu_{A\sigma(j)}(x_i)\right)^2 \cdot \left(\mu_{B\sigma(j)}(x_i)\right)^2 + \frac{1}{k_i} \sum\limits_{j=1}^{k_i} \left(\nu_{A\sigma(j)}(x_i)\right)^2 \cdot \left(\nu_{B\sigma(j)}(x_i)\right)^2 + {\pi_A}^2(x_i) \cdot {\pi_B}^2(x_i) \right]}{\left\{ \sum\limits_{i=1}^{n} \omega_i \left[\frac{\sum\limits_{j=1}^{l_i} \left(\mu_{A\sigma(j)}(x_i)\right)^4}{l_i} + \frac{\sum\limits_{j=1}^{k_i} \left(\nu_{A\sigma(j)}(x_i)\right)^4}{k_i} + {\pi_A}^4(x_i) \right] \right\}^{1/2} \left\{ \sum\limits_{i=1}^{n} \omega_i \left[\frac{\sum\limits_{j=1}^{l_i} \left(\mu_{B\sigma(j)}(x_i)\right)^4}{l_i} + \frac{\sum\limits_{j=1}^{k_i} \left(\nu_{B\sigma(j)}(x_i)\right)^4}{k_i} + {\pi_B}^4(x_i) \right] \right\}^{1/2}}, \tag{16}$$

$$\rho_{2\omega}(A,B) = \frac{C_\omega(A,B)}{\max\{(C_\omega(A,A)),(C_\omega(B,B))\}} = \frac{\sum\limits_{i=1}^{n} \omega_i \left[\frac{1}{l_i} \sum\limits_{j=1}^{l_i} \left(\mu_{A\sigma(j)}(x_i)\right)^2 \cdot \left(\mu_{B\sigma(j)}(x_i)\right)^2 + \frac{1}{k_i} \sum\limits_{j=1}^{k_i} \left(\nu_{A\sigma(j)}(x_i)\right)^2 \cdot \left(\nu_{B\sigma(j)}(x_i)\right)^2 + {\pi_A}^2(x_i) \cdot {\pi_B}^2(x_i) \right]}{\max\left\{ \sum\limits_{i=1}^{n} \omega_i \left[\frac{\sum\limits_{j=1}^{l_i} \left(\mu_{A\sigma(j)}(x_i)\right)^4}{l_i} + \frac{\sum\limits_{j=1}^{k_i} \left(\nu_{A\sigma(j)}(x_i)\right)^4}{k_i} + {\pi_A}^4(x_i) \right], \sum\limits_{i=1}^{n} \omega_i \left[\frac{\sum\limits_{j=1}^{l_i} \left(\mu_{B\sigma(j)}(x_i)\right)^4}{l_i} + \frac{\sum\limits_{j=1}^{k_i} \left(\nu_{B\sigma(j)}(x_i)\right)^4}{k_i} + {\pi_B}^4(x_i) \right] \right\}}. \tag{17}$$

It is obvious that when $\omega_i = \frac{1}{n}(i = 1, 2, \cdots, n)$, (16) and (17) reduce to (14) and (15), respectively, and the weighted correlation coefficients $\rho_{1\omega}(A,B)$ and $\rho_{2\omega}(A,B)$ satisfy the following properties:

$$(1) \rho_\omega(A,B) = \rho_\omega(B,A).$$

$$(2) 0 \leq \rho_\omega(A,B) \leq 1.$$

$$(3) \rho_\omega(A,B) = 1, if A = B.$$

We can prove the properties (1)–(3) according to the same proof method as in Theorem 1.

Example 2 *Let us suppose the weight is* $\omega = (0.4, 0.6)^T$ *and the two PHFEs are as in Example 1. Then we can get the weighted informational energy as follows:*

$$E_\omega(A) = C_\omega(A,A) = \left(\frac{0.7^4 + 0.6^4 + 0.5^4}{3} + \frac{0.4^4 + 0.3^4}{2} + \pi_A^4(x_1) \right) \times 0.4 + \left(\frac{0.9^4 + 0.6^4 + 0.6^4}{3} + \frac{0.3^4 + 0.1^4}{2} + \pi_A^4(x_2) \right) \times 0.6 = 0.4692,$$

$$E_\omega(B) = C_\omega(B,B) = \left(\frac{0.7^4 + 0.5^4 + 0.5^4}{3} + \frac{0.5^4 + 0.4^4}{2} + \pi_B^4(x_1) \right) \times 0.4 + \left(\frac{0.8^4 + 0.7^4 + 0.6^4}{3} + \frac{0.4^4 + 0.2^4}{2} + \pi_B^4(x_2) \right) \times 0.6 = 0.4144.$$

The correlation between A and B can be computed as

$$C_\omega(A,B) = \left(\frac{0.7^2 \cdot 0.7^2 + 0.5^2 \cdot 0.5^2 + 0.5^2 \cdot 0.5^2}{3} + \frac{0.4^2 \cdot 0.5^2 + 0.3^2 \cdot 0.4^2}{2} + \pi_A^2(x_1) \cdot \pi_B^2(x_1)\right) \times 0.4$$
$$+ \left(\frac{0.9^2 \cdot 0.8^2 + 0.6^2 \cdot 0.7^2 + 0.6^2 \cdot 0.6^2}{3} + \frac{0.3^2 \cdot 0.4^2 + 0.1^2 \cdot 0.2^2}{2} + \pi_A^2(x_2) \cdot \pi_B^2(x_2)\right) \times 0.6 = 0.4335.$$

Correlation coefficients $\rho_{1\omega}(A,B)$ *and* $\rho_{2\omega}(A,B)$ *are as follows:*

$$\rho_{1\omega}(A,B) = \frac{0.4335}{\sqrt{0.4692 \times 0.4144}} = 0.9831,$$
$$\rho_{2\omega}(A,B) = \frac{0.4335}{\max\{0.4692, 0.4144\}} = 0.9239.$$

6. Proposed method

This paper is aimed at solving the radar LPI performance evaluation problem by using expert judgment. According to the multidomain evaluation system, we propose a method using the correlation coefficient of Pythagorean hesitant fuzzy sets to rank different radar LPI performances as follows:

Step 1: Decision makers need to know the LPI multidomain performance parameters of the radars to be tested. The main criteria for judging include beam width and side lobe level in the spatial domain, dwell time and radar beam emission interval in the time domain, frequency agility in the frequency domain, polarization status changing mode in the polarization domain, transmit power and pulse train energy changing period in the energy domain, and intrapulse and interpulse characteristics in the waveform domain.

Step 2: According to the multidomain evaluation system, experts give the estimated value of the radar LPI performance to be tested. Then a Pythagorean hesitant fuzzy set decision matrix is formed.

Step 3: The decision matrix is normalized according to the pessimistic principle, and the values in the PHFE are arranged in order. Afterwards, we can obtain the normalized decision matrix.

Step 4: Experts consider the importance of indicators in this assessment comprehensively based on the professional knowledge and application background, then assign the attribute weights.

Step 5: Calculate the correlation and correlation coefficient between radars to be evaluated and the positive ideal alternative according to (14) and (15), respectively.

Step 6: On the basis of (16) and (17), we further compute the weighted correlation coefficient between radars to be evaluated and the positive ideal alternative.

Step 7: Compare the calculation results of the above two steps and analyze the similarities and differences. Then rank the radar LPI performance to be tested by the values of correlation coefficient and weighted correlation coefficient. Finally, a reasonable evaluation result is obtained.

7. Application of proposed method for radar LPI performance evaluation

In this section, we choose four kinds of airborne fire control radars as the numerical example. The AN/APG-69 and AN/APG-66 are produced by the United States, which began serving in 1983 and 1978, respectively. Furthermore, the AN/APG-69 is a modular airborne coherent multifunction digital fire control radar using a PD system. Fighters equipped with AN/APG-69 mainly perform tasks such as short-range air support and identification. AN/APG-66 is a multifunction all-digital coherent PD radar designed by Westinghouse Electric. The whole machine adopts component design, which has the characteristics of small size, light weight, high reliability, and convenient maintenance [21, 22]. EL/M-2021 is an Israeli-made radar that began service in 1977, which uses digital technology for coherent processing. During air-to-air combat, the radar performs search and automatic tracking tasks. Meanwhile, it provides ranging information to the bombing computer during air-to-ground combat. RDM was made in France and served in 1983; it is a multifunction digital airborne fire control radar developed by Thomson-CSF with more than 20 modes of operation. It can complete air defense, air-to-ground attack, and air-to-sea attack missions. These four fire control radars have performed well in war, but no one has compared their LPI performance before. The main parameters of these radars to be tested are shown in Table 1.

Table 1. Radar parameters to be evaluated.

Radar	Frequency working system	Pulse width (μs)	Average power (W)	Beam width ($^\circ$)	The aircraft used in
AN/APG-69	Frequency agility	0.25-13	160	azimuth:4.5 pitch:7.7	F-5E
AN/APG-66	Frequency agility and fast phase shift	0.81-4	250	azimuth:1.8 pitch:3	F-16
EL/M-2021	Spread Spectrum	0.95-3	200	azimuth:1.4 pitch:5	F-4
RDM	Frequency agility	0.5,0.65,4.5	360	azimuth:3 pitch:4	Mirage 2000

For simplification we define the LPI evaluation indices as follows: C_1 (spatial domain), C_2 (time domain), C_3 (frequency domain), C_4 (polarization status), C_5 (energy status), and C_6 (waveform domain). A group of experts in the field of radar and electronic countermeasures was invited to select the best LPI performance radar among them. They compared the radars' search and automatic tracking statuses according to the multidomain evaluation indices. Next, the attribute weight vector is obtained as $\omega = [0.13, 0.2, 0.14, 0.18, 0.16, 0.19]^T$. We replace the four fire control radars with $R_i (i = 1, 2, 3, 4)$, i.e. R_1 (AN/APG-69), R_2 (AN/APG-66), R_3 (EL/M-2021), R_4 (RDM). The results evaluated by the experts are contained in a Pythagorean hesitant fuzzy decision matrix, as shown in Table 2. Then we normalize the decision matrix by pessimistic principle. The normalized decision matrix is shown in Table 3.

We define the positive ideal alternative as follows:

$$R^+ = (F(\{1,1,1\},\{0,0,0\}), F(\{1,1,1\},\{0,0,0\}), F(\{1,1,1\},\{0,0,0\}),$$
$$F(\{1,1,1\},\{0,0,0\}), F(\{1,1,1\},\{0,0,0\}), F(\{1,1,1\},\{0,0,0\})).$$

By employing (14) and (15), we compute the correlation coefficients ρ_1 and ρ_2 as follows, respectively:

$$\rho_1\left(R_1, R^+\right) = 0.5642, \rho_1\left(R_2, R^+\right) = 0.4100, \rho_1\left(R_3, R^+\right) = 0.1420, \rho_1\left(R_4, R^+\right) = 0.2515;$$
$$\rho_2\left(R_1, R^+\right) = 0.4611, \rho_2\left(R_2, R^+\right) = 0.3211, \rho_2\left(R_3, R^+\right) = 0.1117, \rho_2\left(R_4, R^+\right) = 0.1872.$$

On the basis of ρ_1 and ρ_2, the four fire control radars' LPI performance sequencing is $R_1 \succ R_2 \succ R_4 \succ$

Table 2. Decision matrix.

	C_1	C_2
R_1	({0.7, 0.6}, {0.4, 0.3, 0.1})	({0.6,0.5},{0.3,0.1})
R_2	({0.7,0.6,0.4},{0.5,0.3,0.2})	({0.6,0.4,0.3},{0.4,0.2})
R_3	({0.2,0.1},{0.9,0.7})	({0.3,0.2},{0.9,0.8})
R_4	({0.5,0.3},{0.8,0.6})	({0.6,0.4},{0.7,0.5})
	C_3	C_4
R_1	({0.9,0.7},{0.2,0.1})	({0.8,0.7},{0.3,0.2})
R_2	({0.8,0.7},{0.4,0.3,0.2})	({0.6,0.5,0.3},{0.7,0.4})
R_3	({0.3,0.2,0.1},{0.8,0.7})	({0.5,0.4,0.2},{0.6,0.4})
R_4	({0.5,0.3},{0.6,0.4})	({0.6,0.5,0.3},{0.6,0.5,0.4})
	C_5	C_6
R_1	({0.9,0.8,0.5},{0.2,0.1})	({0.8,0.6},{0.4,0.3,0.2})
R_2	({0.7,0.6},{0.4,0.3})	({0.6,0.4},{0.7,0.4})
R_3	({0.4,0.2,0.1},{0.9,0.8,0.7})	({0.5,0.3,0.2},{0.5,0.4})
R_4	({0.5,0.4,0.3},{0.6,0.5})	({0.6,0.3},{0.6,0.5})

Table 3. Normalized decision matrix.

	C_1	C_2
R_1	({0.7, 0.6, 0.6}, {0.4, 0.3, 0.1})	({0.6, 0.5, 0.5}, {0.3, 0.1, 0.1})
R_2	({0.7, 0.6, 0.4}, {0.5, 0.3, 0.2})	({0.6, 0.4, 0.3}, {0.4, 0.2, 0.2})
R_3	({0.2, 0.1, 0.1}, {0.9, 0.7, 0.7})	({0.3, 0.2, 0.2}, {0.9, 0.8, 0.8})
R_4	({0.5, 0.3, 0.3}, {0.8, 0.6, 0.6})	({0.6, 0.4, 0.4}, {0.7, 0.5, 0.5})
	C_3	C_4
R_1	({0.9, 0.7, 0.7}, {0.2, 0.1, 0.1})	({0.8, 0.7, 0.7}, {0.3, 0.2, 0.2})
R_2	({0.8, 0.7, 0.7}, {0.4, 0.3, 0.2})	({0.6, 0.5, 0.3}, {0.7, 0.4, 0.4})
R_3	({0.3, 0.2, 0.1}, {0.8, 0.7, 0.7})	({0.5, 0.4, 0.2}, {0.6, 0.4, 0.4})
R_4	({0.5, 0.3, 0.3}, {0.6, 0.4, 0.4})	({0.6, 0.5, 0.3}, {0.6, 0.5, 0.4})
	C_5	C_6
R_1	({0.9, 0.8, 0.5}, {0.2, 0.1, 0.1})	({0.8, 0.6, 0.6}, {0.4, 0.3, 0.2})
R_2	({0.7, 0.6, 0.6}, {0.4, 0.3, 0.3})	({0.6, 0.4, 0.4}, {0.7, 0.4, 0.4})
R_3	({0.4, 0.2, 0.1}, {0.9, 0.8, 0.7})	({0.5, 0.3, 0.2}, {0.5, 0.4, 0.4})
R_4	({0.5, 0.4, 0.3}, {0.6, 0.5, 0.5})	({0.6, 0.3, 0.3}, {0.6, 0.5, 0.5})

R_3. The ranking results obtained by ρ_1 and ρ_2 are the same, indicating that ρ_1 and ρ_2 have consistency in this example.

Considering weight, we can obtain the weighted correlation coefficients $\rho_{1\omega}$ and $\rho_{2\omega}$ by employing (16) and (17) as follows, respectively:

$$\rho_{1\omega}\left(R_1, R^+\right) = 0.5380, \rho_{1\omega}\left(R_2, R^+\right) = 0.3800, \rho_{1\omega}\left(R_3, R^+\right) = 0.1533, \rho_{1\omega}\left(R_4, R^+\right) = 0.2537;$$

$$\rho_{2\omega}\left(R_1, R^+\right) = 0.4513, \rho_{2\omega}\left(R_2, R^+\right) = 0.3063, \rho_{2\omega}\left(R_3, R^+\right) = 0.1227, \rho_{2\omega}\left(R_4, R^+\right) = 0.1929.$$

According to (16), the ranking order of these four fire control radars is $R_1 \succ R_2 \succ R_4 \succ R_3$, which is the same as the result calculated by (17). Therefore, the radar LPI performance rankings obtained by the four correlation coefficients are consistent. On the other hand, this case demonstrates that the method mentioned above is effective.

There is no doubt that the weight vector plays a significant role in determining the ranking result. It is inevitable that there may exist possible differences of opinions between experts, which may lead to minor changes in weight vector. For instance, the experts diverged in determining the last three indices, and the weight vector was modified to $\omega = [0.13, 0.2, 0.14, 0.15, 0.17, 0.21]^T$. The weighted correlation coefficients $\rho_{1\omega}$ and $\rho_{2\omega}$ under the new weight vector can be obtained as follows:

$$\rho_{1\omega}\left(R_1, R^+\right) = 0.5361, \rho_{1\omega}\left(R_2, R^+\right) = 0.3819, \rho_{1\omega}\left(R_3, R^+\right) = 0.1515, \rho_{1\omega}\left(R_4, R^+\right) = 0.2513;$$
$$\rho_{2\omega}\left(R_1, R^+\right) = 0.0750, \rho_{2\omega}\left(R_2, R^+\right) = 0.0513, \rho_{2\omega}\left(R_3, R^+\right) = 0.0202, \rho_{2\omega}\left(R_4, R^+\right) = 0.0319.$$

The radar LPI performance ranking remains $R_1 \succ R_2 \succ R_4 \succ R_3$ even if the weight vector has changed slightly. Therefore, in this case, although there are possible differences in the opinions of experts, it will not affect the final evaluation results.

8. Conclusion and future work

In this study, we present a novel method to evaluate radar LPI performance. By analyzing radar parameters characteristics, a multidomain evaluation system is built. The system is in terms of characteristics of signal in space, beam illumination form, and time-varying electric field equation. This article considers all possible scenarios for radiators. It is apparent that there are no studies involving the use of PHFSs to estimate radar LPI performance before. We develop the theory of correlation coefficients and combine it with PHFSs. Benefiting from the work of previous scholars, we prove some properties of correlation coefficients and weighted correlation coefficients for PHFSs. We apply our proposed methodology to evaluate fire control radars on different aircrafts, and the ranking of selected radars' LPI performance is given. Thus, estimating LPI performance by correlation coefficients of PHFSs is feasible and effective. Future researchers may develop more PHFSs correlation coefficients and propose a novel method based on incomplete weights information.

References

[1] She J, Wang F, Zhou J. A novel sensor selection and power allocation algorithm for multiple-target tracking in an LPI radar network. Sensors 2016; 16(12): 2193. doi: 10.3390/s16122193

[2] Stove AG, Hume AL, Baker CJ. Low probability of intercept radar strategies. IEE Proceedings-Radar, Sonar and Navigation 2004; 151 (5): 249-260. doi: 10.1049/ip-rsn:20041056

[3] Schleher DC. LPI radar: fact or fiction. IEEE Aerospace and Electronic Systems Magazine 2006; 21(5): 3-6. doi: 10.1109/MAES.2006.1635166

[4] Wu PH. On sensitivity analysis of low probability of intercept (LPI) capability. In: IEEE Military Communications Conference; Atlantic City, NJ, USA; 2005. pp. 2889-2895.

[5] Shi C, Zhou J, Wang F. Adaptive resource management algorithm for target tracking in radar network based on low probability of intercept. Multidimensional Systems and Signal Processing 2017; 29 (4): 1203-1226. doi: 10.1007/s11045-017-0494-8

[6] Li B, Lei J, Wang S, Lai X. Performance evaluation of LPI communication system: a novel index system. In: 7th International Conference on Communications and Networking; Kun Ming, China; 2012. pp. 368-373.

[7] Wang F, Sellathurai M, Liu W, Zhou J. Security information factor based airborne radar RF stealth. Journal of Systems Engineering and Electronics 2015; 26 (2): 258-266. doi: 10.1109/JSEE.2015.00031

[8] Torra V. Hesitant fuzzy sets. International Journal of Intelligent Systems 2010; 25 (6): 529-539. doi: doi.org/10.1002/int.20418

[9] Xu Z, Xia M. On distance and correlation measures of hesitant fuzzy information. International Journal of Intelligent Systems 2011; 26 (5): 410-425. doi: 10.1002/int.20474

[10] Chen N, Xu Z, Xia M. Correlation coefficients of hesitant fuzzy sets and their applications to clustering analysis. Applied Mathematical Modelling 2013; 37 (4): 2197-2211. doi: 10.1016/j.apm.2012.04.031

[11] Liao H, Xu Z, Zeng XJ. Novel correlation coefficients between hesitant fuzzy sets and their application in decision making. Knowledge-Based Systems 2015; 82: 115-127. doi: 10.1016/j.knosys.2015.02.020

[12] Yager RR. Pythagorean fuzzy subsets. In: IEEE 2013 Joint IFSA World Congress and NAFIPS Annual Meeting (IFSA/NAFIPS); Edmonton, Canada; 2013. pp. 57-61.

[13] Yager RR. Pythagorean membership grades in multicriteria decision making. IEEE Transactions on Fuzzy Systems 2014; 22 (4): 958-965. doi: 10.1109/tfuzz.2013.2278989

[14] Garg H. A novel correlation coefficients between Pythagorean fuzzy sets and its applications to decision-making processes. International Journal of Intelligent Systems 2016; 31 (12): 1234-1252. doi: 10.1002/int.21827

[15] Liang D, Xu Z. The new extension of TOPSIS method for multiple criteria decision making with hesitant Pythagorean fuzzy sets. Applied Soft Computing 2017; 60: 167-179. doi: 10.1016/j.asoc.2017.06.034

[16] Lu M, Wei G, Alsaadi FE, Hayat T, Alsaedi A. Hesitant Pythagorean fuzzy Hamacher aggregation operators and their application to multiple attribute decision making. Journal of Intelligent & Fuzzy Systems 2017; 33 (2): 1105-1117. doi: 10.3233/JIFS-16554

[17] Garg H. Hesitant Pythagorean fuzzy Maclaurin symmetric mean operators and its applications to multiattribute decision-making process. International Journal of Intelligent Systems 2019; 34 (4): 601-626. doi: 10.1002/int.22067

[18] Zhao Y, Shang C, Han Z, Wang P. A new evaluation model of comprehensive radio frequency stealth performance of radar. International Journal of Modeling, Simulation, and Scientific Computing 2016; 7 (3): 1650033. doi: 10.1142/S1793962316500331

[19] Rajgopal S, Sahu OP. Metrics to define probability of interception of conventional, frequency-hopping, spread spectrum radars. IETE Journal of Research 2009; 55 (6): 304-308. doi: 10.4103/0377-2063.59171

[20] Ye J. Correlation coefficient of dual hesitant fuzzy sets and its application to multiple attribute decision making. Applied Mathematical Modelling 2014; 38 (2): 659-666. doi: 10.1016/j.apm.2013.07.010

[21] Simpson J. PRF set selection for pulse-Doppler radars. In: IEEE Region 5 Conference 1988 'Spanning the Peaks of Electrotechnolog'; Colorado Springs, CO, USA; 1988. pp. 38-44.

[22] Long WH, Harriger KA. Medium PRF for the AN/APG-66 radar. Proceedings of the IEEE 1985; 73 (2): 301-311. doi: 10.1109/proc.1985.13141

[23] Başar A. Hesitant fuzzy pairwise comparison for software cost estimation: a case study in Turkey. Turkish Journal of Electrical Engineering & Computer Sciences 2017; 25(4): 2897-2909. doi: 10.3906/elk-1604-45