Turkish Journal of Electrical Engineering & Computer Sciences

http://journals.tubitak.gov.tr/elektrik/

Research Article

Turk J Elec Eng & Comp Sci
(2020) 28: 1208 – 1216
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doi:10.3906/elk-1812-51

Lattice-reduction aided multiple-symbol differential detection in two-way relay transmission

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Received: 07.12.2018 • **Accepted/Published Online:** 06.03.2020 • **Final Version:** 08.05.2020

Abstract: Multiple-symbol differential detection (MSDD) algorithms are proposed in two-way relay transmission (TWRT). Firstly, generalized likelihood ratio test based MSDD (GLRT-MSDD) is proposed in TWRT. Unfortunately, as the number of observation windows increases, the computational complexity of GLRT-MSDD increases exponentially. Hence, this detection in TWRT constitutes a challenging problem. Moreover, we find a way to reformulate the GLRT-MSDD model and additionally propose a lattice-reduction aided MSDD (LR-MSDD) model. Performance analysis and simulations show that the proposed LR-MSDD provides bit-error rate performance close to that of GLRT-MSDD with lower complexity in TWRT.

Key words: Multiple-symbol differential detection, generalized likelihood ratio test, lattice reduction, two-way relay transmission

1. Introduction

In one-way communication, coherent detection is expensive or even infeasible because it requires accurate channel state information. Therefore, noncoherent detection algorithms, such as transmitted reference and differential detection, have become popular by avoiding channel estimation [1]. However, the transmitted reference reduces energy utilization and transmission efficiency [2], and one-symbol differential detection results in severe bit error rate (BER) performance loss due to using reference signals containing the noise [3]. Thus multiple-symbol differential detection (MSDD) has been developed since it can make full use of the correlation among the received signals [4].

From the perspective of power efficiency, two-way relay transmission (TWRT) is considered by exploiting network coding to enhance the efficiency of one-way transmission [5]. However, estimating the carrier phase brings high complexity with coherent detection at the relay. Therefore, differential detection is proposed with continuous phase frequency shift keying [6]. Furthermore, MSDD has been brought into single antenna TWRT [7].

In the present paper, differential space-time block-codes (DSTBC) based MSDD is introduced in TWRT to improve the BER performance of single antenna TWRT. Firstly, generalized likelihood ratio test based MSDD (GLRT-MSDD) is investigated in DSTBC based TWRT. Fortunately, the system performance of the proposed GLRT-MSDD is superior to that of the existing MSDDs in noncoherent TWRT [7]. However, from the perspective of computational complexity, the GLRT-MSDD has high complexity, which increases

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exponentially with the size of the observation window. Thus, GLRT-MSDD is not desirable with a large observation window. Lattice reduction (LR) represents the computationally efficient algorithms and is capable of achieving performance near the optimal. Based on the LR theory, GLRT-MSDD is transformed and LR based MSDD (LR-MSDD) is proposed, which completes the detection by reducing the search lattice points. Moreover, LR-MSDD is transformed from exhaustive search based GLRT-MSDD with an equivalent form. Performance analysis shows that, as a beneficial result, LR-MSDD achieves almost the same BER performance as that of GLRT-MSDD with a lower complexity.

The rest of the paper is organized as follows. The system model of TWRT is described in Section 2. In Section 3, GLRT-MSDD in TWRT is proposed. In Section 4, LR-MSDD is derived for reducing the computational complexity of GLRT-MSDD. BER performance is analyzed in Section 5. In Section 6, from the perspective of computational complexity, an analysis of the two MSDD algorithms is presented. Simulation results validate the performance of the proposed MSDD algorithms in Section 7. In Section 8, the conclusions are summed up.

Notations: Lower-case (upper-case) boldface symbols represent vectors (matrices); $\mathbf{I}_u$ stands for the $u \times u$ identity matrix; $(\cdot)^{\mathrm{T}}$ and $\mathrm{Tr}(\cdot)$ represent the transpose and the trace of a matrix, respectively; $*$ stands for convolution; $\mathbf{1}$ is a matrix and all its entries are 1; $|\mathbf{F}|$ denotes a matrix whose entries are the absolute value of the matrix $\mathbf{F}$; $\delta(t)$ is Dirac delta function.

In the next part, GLRT-MSDD and LR-MSDD are investigated in DSTBC aided ultra-wideband impulse (UWB) TWRT. It is noted that the proposed MSDD algorithms can be expanded into many other types of noncoherent TWRT by taking a reasonable conversion.

2. System description

A network model with three nodes is analyzed, which is shown in Figure 1. The user nodes A_1 and A_2 are configured with T $(T > 1)$ transmit antennas. The relay node B is configured with Z $(Z \geq 1)$ receive antennas. For illustrative purposes, the transmit antenna $T = 2$ is considered in the following. At some time, the node A_1 or A_2 transmits information to the relay in the uplink phase of DSTBC aided UWB TWRT. The transmitted information bits and the DSTBC symbol correspond to each other [8]. Each DSTBC symbol belongs to the set $\Omega = \{\mathbf{C}^0, \mathbf{C}^1, \mathbf{C}^2, \mathbf{C}^3\}$.

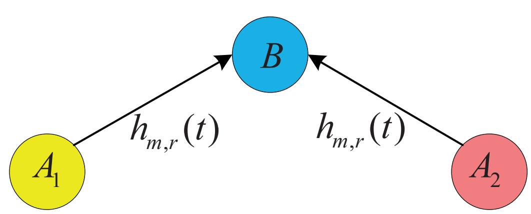

Figure 1. The uplink phase of UWB TWRT; the user nodes A_1 or A_2 transmit information to the relay B.

The corresponding rules are presented as follows:

$$00 \rightarrow \mathbf{C}^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},\ 01 \rightarrow \mathbf{C}^1 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix},\ 10 \rightarrow \mathbf{C}^2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},\ 11 \rightarrow \mathbf{C}^3 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}.$$

It is noted that each codeword symbol employed has to be a unitary matrix. Moreover, the proposed MSDD algorithms can be branched out to the TWRT where the transmit antennas satisfy $T > 2$ when the unitary matrices are regulated based on the DSTBC design [9,10]. Furthermore, the transmission symbol can be obtained with differential encoding as

$$\mathbf{G}_{m+1} = \mathbf{G}_m \mathbf{C}_{m+1}, \tag{1}$$

where m is an integer between $0 \sim M-1$, and M denotes the number of the transmitted symbol. $\mathbf{C}_m \in \Omega$ is the DSTBC information symbol and $\mathbf{G}_m$ is the transmission symbol. The reference symbol is $\mathbf{G}_0 = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$. The entry in the u-th row and the n-th column of the matrix $\mathbf{G}_m$ is represented as $g_{u,2m+n-1}$, where $u = 1, 2$ and $n = 1, 2$. For the u-th antenna, the signal transmitted is derived as

$$s_u(t) = \sum_{m=0}^{M-1} \sum_{n=1}^{2} g_{u,2m+n-1} \omega \left(t - (n-1)T_{\mathrm{f}} - mT_{\mathrm{s}} \right), \tag{2}$$

where $\omega(t)$ denotes the monocycle pulse with duration T_ω; T_f represents the frame duration; a symbol is transmitted in $T_s = 2T_f$. To facilitate the expression, $j = 2m + n - 1$ is introduced; then $g_{u,2m+n-1}$ will be represented as $g_{u,j}$. Correspondingly, (2) will be reformulated as

$$\begin{aligned} s_u(t) &= \sum_{m=0}^{M-1} \sum_{n=1}^{2} g_{u,2m+n-1} \omega(t - (2m+n-1)T_{\mathrm{f}}) \\ &= \sum_{j=0}^{2M-1} g_{u,j} \omega(t - jT_{\mathrm{f}}). \end{aligned} \tag{3}$$

A quasi-static fading channel is considered [11]. From the u-th transmit antenna to the z-th ($1 \le z \le Z$) receive antenna, the channel impulse response is

$$h_{u,z}(t) = \sum_{l=1}^{L_{m,z}} \alpha_l^{u,z} \delta(t - \tau_l^{u,z}), \tag{4}$$

where $L_{u,z}$ denotes the total number of propagation paths; $\alpha_l^{u,z}$ and $\tau_l^{u,z}$ denote the path-gain coefficient and delay of the l-th path, respectively. Then the overall channel response is formulated as

$$\rho_{u,z}(t) = \omega(t) * h_{u,z}(t) = \sum_{l=1}^{L_{u,z}} \alpha_l^{u,z} \omega(t - \tau_l^{u,z}). \tag{5}$$

The received signal at the z-th antenna is expressed as

$$\begin{aligned} y_z(t) &= \sum_{u=1}^{2} s_u(t) * h_{u,z}(t) + n_z(t) \\ &= \sum_{u=1}^{2} \sum_{j=0}^{2M-1} g_{u,j} \rho_{u,z} \left(t - jT_{\mathrm{f}} \right) + n_z(t), \end{aligned} \tag{6}$$

where $n_z(t)$ denotes the additive white Gaussian noise. In the form of a matrix, the received signal from Z antennas is expressed as

$$\mathbf{y}_m(t) = \begin{pmatrix} y_1(t + 2mT) & y_1(t + 2mT + T) \\ \vdots & \vdots \\ y_Z(t + 2mT) & y_Z(t + 2mT + T) \end{pmatrix}. \tag{7}$$

According to the received signal, MSDD algorithms will be proposed in the UWB TWRT as follows.

3. GLRT-MSDD in UWB TWRT

Consider that there are N DSTBC symbols in an observation window. Our particular attention is focused on detecting $N-1$ information symbols $\mathbf{C}_m$ jointly from the observation window. Thus, the relation is investigated between $\mathbf{C}_m$ and the received symbols as follows. $N-1$ information symbols are written as $\mathbb{C} = [\mathbf{C}_1, \mathbf{C}_2, \cdots, \mathbf{C}_{N-1}]$. $\mathbb{C}$ will be judged by using the received signal $\{y_z(t)\}$ in a observation window $0 < t \leq NT_s$. Assume that $\tilde{x}_z(t)$ stands for the candidate received signal, $\tilde{g}_{u,j}$ is the candidate information signal, and $\tilde{\rho}_{u,z}(t)$ is the optimum template. Based on the GLRT criterion, $\mathbb{C}$ is decided as

$$\Lambda\left(y_z(t)|\tilde{\mathbb{C}}, \tilde{\rho}_{u,z}(t)\right) = 2\int_0^{(N-1)T_s} y_z(t)\tilde{x}_z(t)dt - \int_0^{(N-1)T_s} (\tilde{x}_z(t))^2 dt, \tag{8}$$

and $\tilde{x}_z(t)$ is written as

$$\tilde{x}_z(t) = \sqrt{\frac{E_b}{2}} \sum_{u=1}^{2}\sum_{j=1}^{2N} \tilde{g}_{u,j}\tilde{\rho}_{u,z}\left(t-(j-1)T_f\right) + n_z(t), \tag{9}$$

and $\tilde{\rho}_{u,z}(t)$ is given by

$$\tilde{\rho}_{u,z}(t) = \frac{1}{M\sqrt{2E_b}} \sum_{j=1}^{2N} \tilde{g}_{u,j} y_z\left(t+(j-1)T_f\right). \tag{10}$$

According to (8), (9), and (10), the metric for GLRT-MSDD is obtained as

$$\Lambda\left(y_z(t)|\tilde{\mathbb{C}}, \tilde{\rho}_{u,z}(t)\right) = \sum_{u=1}^{2} \Lambda\left(y_z(t)|\{\tilde{g}_{u,j}\}, \tilde{\rho}_{u,z}(t)\right). \tag{11}$$

It can be observed that the GLRT-MSDD in UWB TWRT can be decomposed into the addition of that in the single antenna system [12]. Thus, $\hat{\mathbb{C}}$ can be rewritten as

$$\begin{aligned}\hat{\mathbb{C}} &= \arg\max_{\tilde{\mathbb{C}}\in\Omega^{N-1}} \left\{\max_{\tilde{\rho}_{u,z}(t)} \Lambda\left(y_z(t)|\tilde{\mathbb{C}}, \{\tilde{\rho}_{u,z}(t)\}_{u=1}^{2}\right)\right\} \\ &= \sum_{u=1}^{2} \arg\max_{\tilde{\mathbb{C}}\in\Omega^{N-1}} \left\{\max_{\tilde{\rho}_{u,z}(t)} \Lambda\left(y_z(t)|\{\tilde{g}_{u,j}(t)\}_{j=1}^{2M}, \tilde{\rho}_{u,z}(t)\right)\right\},\end{aligned} \tag{12}$$

where $\tilde{\mathbb{C}} = [\tilde{\mathbf{C}}_1, \tilde{\mathbf{C}}_2, \cdots, \tilde{\mathbf{C}}_{N-1}]$ denotes the candidate vector of $\mathbb{C}$. With the aid of the regulation in [13], (12) is reexpressed as

$$\hat{\mathbb{C}} = \arg\max_{\tilde{\mathbb{C}}\in\Omega^{N-1}} \left\{\sum_{\beta=1}^{N-1}\sum_{\gamma=0}^{\beta-1} \mathrm{Tr}\left((\prod_{i=\gamma+1}^{\beta} \tilde{\mathbf{C}}_i)\mathbf{Q}_{\beta,\gamma}\right)\right\}, \tag{13}$$

where $\mathbf{Q}_{\beta,\gamma}$ is the correlation matrix, whose entries are the correlation operation of $\mathbf{y}_\beta(t)$ and $\mathbf{y}_\gamma(t)$ given by

$$\mathbf{Q}_{\beta,\gamma} = \int_0^{T_i} \mathbf{y}_\beta^{\mathrm{T}}(t)\mathbf{y}_\gamma(t)dt, \tag{14}$$

where T_i is the integration interval. It can be observed from (13) that GLRT-MSDD is an exhaustive search algorithm, and its computational complexity increases exponentially with the number N in TWRT. In the following, we propose a model transformation and develop LR-MSDD in TWRT.

4. LR-MSDD in DSTBC aided UWB TWRT

By performing the LR algorithm on the GLRT-MSDD, a fast MSDD will be proposed, which is much easier to solve than GLRT-MSDD. The existing LR algorithm [14] and sphere decoding [15] cannot be applied in the scenes of the DSTBC aided UWB TWRT directly. Thus, some equivalent conversion has to be performed to formulate the LR-MSDD. To cast the objective function of (13) into a lattice search form, a redundant scalar is introduced and the detector is constructed as

$$\Psi(\tilde{\mathbb{C}}) = \sum_{\beta=1}^{N-1}\sum_{\gamma=0}^{\beta-1} \mathrm{Tr}\left(\mathbf{1}|\mathbf{Q}_{\beta,\gamma}| - (\prod_{m=\gamma+1}^{\beta}\tilde{\mathbf{C}}_m)\mathbf{Q}_{\beta,\gamma}\right). \tag{15}$$

Then the LR algorithm detects the points located in a sphere with the radius of D, and the LR-MSDD associated with (15) is formulated as

$$\Psi(\tilde{\mathbb{C}}) \leq D. \tag{16}$$

When $k < N-1$, in order to detect the first information symbols $\mathbf{C}_1, \mathbf{C}_2, \cdots, \mathbf{C}_k$, we define

$$\eta_k \leq D. \tag{17}$$

In addition, the recursive is given as

$$\eta_k = \eta_{k-1} + \mu_k, \tag{18}$$

where

$$\mu_k = \sum_{\gamma=0}^{k-1} \mathrm{Tr}\left(\mathbf{1}|\mathbf{Q}_{\beta,\gamma}| - (\prod_{m=\gamma+1}^{\beta}\tilde{\mathbf{C}}_m)\mathbf{Q}_{\beta,\gamma}\right), \tag{19}$$

and the reference parameter $\eta_0 = 0$. Based on (17), (18), and (19), LR-MSDD will be completed in UWB TWRT.

Remark 1: In the multiple-access phase of TWRT, we can chose GLRT-MSDD or LR-MSDD for detecting information, where the relay deals with the information with the logical exclusion or (XOR) network coding. In the broadcasting phase, the encoded information is transmitted from the relay to the users. Using the XOR operation again for self-information and information detected, the user obtains the information from the other end.

Remark 2: In the following two scenarios, we can choose GLRT-MSDD or LR-MSDD. (1) There are two phases in TWRT. In the first phase, two users transmit information to the relay simultaneously. Then GLRT-MSDD/LR-MSDD and the XOR operation are applied at the relay. In the second phase, the relay broadcasts information operated. (2) There are three phases in TWRT. In the first and the second phases, two users transmit information, and the relay deals with information from users with the XOR operation. In the third phase, the relay broadcasts the information encoded.

5. BER performance analysis

LR-MSDD is an efficient detection algorithm and is one of the effective ways to solve the problem of the high complexity in MSDD. In LR-MSDD, the points are detected in a sphere with the radius of D. Its initial search parameter D affects the complexity of LR-MSDD. The larger the search parameter D, the lower the BER and the higher the computational complexity. When the search parameter approaches infinity, the LR-MSDD algorithm can achieve the same BER performance as GLRT-MSDD. In order to further reduce the complexity of LR-MSDD, the threshold of termination search can be set. When the Euclidean distance between the received vector and the mapping of the transmitted vector in the received signal space is less than the threshold, the transmitted vector can be considered to have been detected without further search. The setting of the threshold can be based on the literature [16].

6. Computational complexity analysis

In the present paper, GLRT-MSDD is developed in DSTBC based UWB TWRT, and LR-MSDD is proposed to simplify GLRT-MSDD. When detecting the DSTBC information symbols, the computational complexity can be measured using the constellation points visited by GLRT-MSDD or LR-MSDD. Based on the decision metric of GLRT-MSDD in (13), for each possible combination of $\hat{\mathbb{C}}$, the correlation matrix has to be computed according to (14). Specifically, GLRT-MSDD entails a computational complexity of

$$O_{\mathrm{GLRT}} = \frac{P^{N-1}}{N-1}, \tag{20}$$

where P is a parameter. According to the theory proposed by Xu et al. [16], the complexity of LR-MSDD is lower bounded by

$$O_{\mathrm{SD}} \geq \frac{P(N-1)}{N-1} = P. \tag{21}$$

Moreover, it can achieve the lower bound either $N = 2$ or $N > 2$ at high signal-to-noise ratio (SNR). LR-MSDD can reduce the search radius soon at high SNRs [17], and thus its complexity is much lower than that of GLRT-MSDD. At low SNRs, the complexity of LR-MSDD is almost the same as that of GLRT-MSDD.

Without loss of generality, LR-MSDD can be computed recursively with (17) and (18), where the computation only involves the first k symbols. Thus, the potentially unnecessary computations are avoided.

7. Simulation results and discussion

The advantages of the proposed noncoherent MSDD algorithms are validated by the Monte-Carlo simulation. For illustrative purposes, the broadcasting phase of UWB TWRT is investigated. The channel model is decided as in Zhang and Ng [8]. The monocycle waveform is $\omega(t) = [1-4\pi(t/T_\omega)^2]\exp[-2\pi(t/T_\omega)^2]$ with pulse duration $T_\omega = 0.287$ ns. The frame duration is $T_{\mathrm{f}} = 80$ ns, the maximum excess delay of channel is $T_{\mathrm{n}} = 40$ ns, and the requirement $T_{\mathrm{f}} > T_{\mathrm{n}}$ avoids intersymbol interference.

Test 1: When the receive antennas is fixed to $Z = 1$ in UWB TWRT, the BER performance of GLRT-MSDD and LR-MSDD is compared with $M = 2, 3, 5$, respectively. We can see from Figure 2 that LR-MSDD and GLRT-MSDD yield almost the same performance.

Test 2: Furthermore, we contrast the BER performance of the proposed MSDDs with $Z = 1$, 2, and 3, respectively. It is observed from Figure 3 that LR-MSDD yields detection performance close to that of GLRT-MSDD with different receive antennas Z.

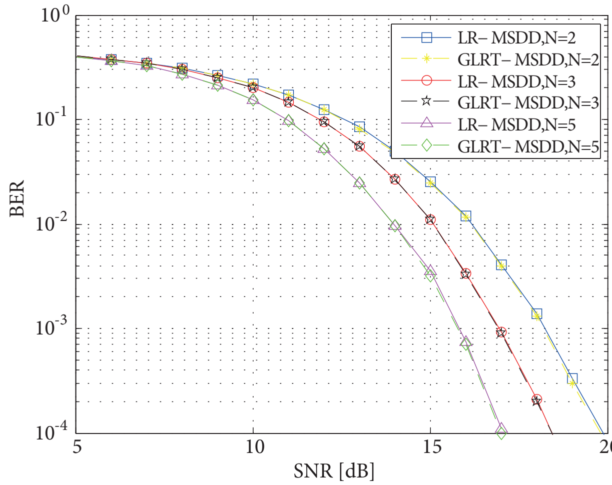

Figure 2. BER performance comparisons of GLRT-MSDD and LR-MSDD in UWB TWRT with $Z = 1$, $N = 2$, 3, and 5, respectively.

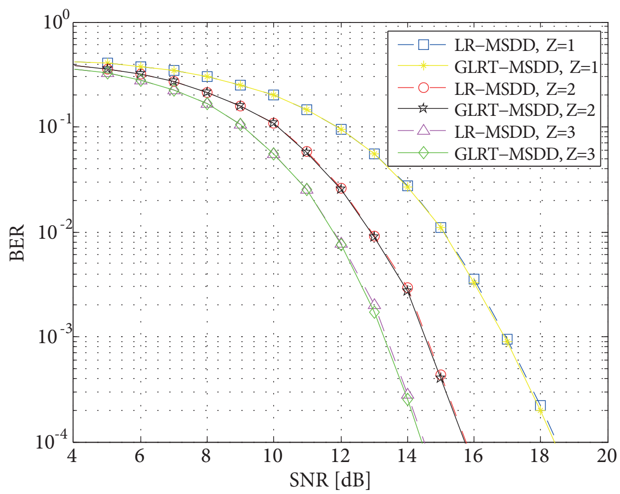

Figure 3. BER performance comparisons of GLRT-MSDD and LR-MSDD in UWB TWRT with $N = 3$, $Z = 1$, 2, and 3, respectively.

Test 3: In Figure 4, the computational complexity of the two MSDD algorithms is compared in terms of multiplication operation. It can be seen that LR-MSDD imposes lower complexity than GLRT-MSDD. Therefore, it can be concluded that in the multiantenna UWB TWRT when $T > 2$, it is more practical to use LR-MSDD than GLRT-MSDD.

8. Conclusions

According to the lattice decoding theory, GLRT-MSDD is transformed and LR-MSDD is derived by reducing the search lattice points. Performance analysis shows that, when the search parameter approaches infinity, LR-MSDD can achieve the same BER performance as GLRT-MSDD. Moreover, Monte-Carlo simulations validate that LR-MSDD provides detection performance close to that of GLRT-MSDD, but with a smaller complexity in noncoherent UWB TWRT.

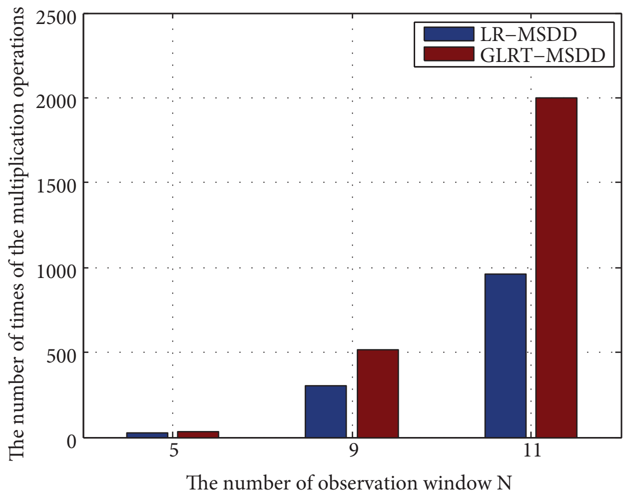

Figure 4. Complexity comparisons of GLRT-MSDD and LR-MSDD against different N in UWB TWRT.

Acknowledgments

This work was financially supported by Lanzhou University of Technology Hongliu Excellent Youth Talent Support Program and the National Natural Science Foundation of China (NSFC) under grant numbers 61875080 and 61841107.

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