Turkish Journal of Electrical Engineering & Computer Sciences

http://journals.tubitak.gov.tr/elektrik/

Research Article

Turk J Elec Eng & Comp Sci
(2021) 29: 1836 – 1851

doi:10.3906/elk-2011-8

Constrained discrete-time optimal control of uncertain systems with adaptive Lyapunov redesign

Oğuz Han ALTINTAŞ[1,*], **Ali Emre TURGUT**[2]
[1]Advanced Technologies & Systems Division, Roketsan Missiles Inc., Ankara, Turkey
[2]Department of Mechanical Engineering, Faculty of Engineering, Middle East Technical University, Ankara, Turkey

Received: 02.11.2020 • **Accepted/Published Online:** 01.04.2021 • **Final Version:** 31.05.2021

Abstract: In this paper, the conventional estimation-based receding horizon control paradigm is enhanced by using functional approximation, the adaptive modifications on state estimation and convex projection notion from optimization theory. The mathematical formalism of parameter adaptation and uncertainty estimation procedure are based on the redesign of optimal state estimation in discrete-time. By using Lyapunov stability theory, it is shown that the online approximation of uncertainties acting on both physical system and state estimator can be obtained. Moreover, the convergence criteria for online parameter adaptation with fully matched and partially matched cases are presented and shown. In addition, it is shown that the uniform boundedness of tracking and adaptation errors can be maintained by projection-based parameter update laws in discrete-time with adequate sampling times. Finally, the proposed method is implemented to quadrotor case study and the gradual recovery of feasible sub-optimal solutions are presented despite actuation, modeling and measurement errors. By using the proposed method, the uncertainty estimates are successfully converged to their prescribed values and both state prediction and command tracking of model predictive controller are corrected within the convergence bounds.

Key words: Adaptive control, optimal control, kalman filtering, radial basis function networks, lyapunov stability, discrete-time systems

1. Introduction

In many practical applications physical constraints on system inputs and the states may occur and they are crucial especially for the safety of mission. Moreover, while satisfying these constraints a certain performance index might be required to be minimized or maximized in order to accomplish the mission within a predefined operation time or with a minimum control effort which fulfill the low energy requirements such as orbital approach of spacecraft, coasting phase of guided missiles or descent guidance of rocket stages [1–3]. This requirements lead designers to use celebrated optimal controllers which, in general, generate a control sequence that optimizes a mission-specific performance index while satisfying the constraints [4–7]. Model predictive control (MPC) is a receding horizon control formulation which is extensively used and developed over few decades [8, 9] for such optimal control problems. Basically, MPC uses a system model to predict or generate future state trajectories via forward sequential simulations inside the prediction horizon. Then, the future input trajectories are optimized to minimize certain cost function which usually consists of set-point tracking errors and control efforts. In linear formulation with quadratic costs, the optimization of future trajectories turns out

*Correspondence: han.altintas@roketsan.com.tr

to be quadratic programming (QP) which can be efficiently solved in both unconstrained and constrained cases [10, 11].

The effect of uncertainties in physical systems affects the performance of optimal controllers in a disruptive manner. Besides, the states fed to the controller may not be available by direct measurements all the time. In many industrial applications MPC is used along with a state estimator which is usually a variant of Kalman filter (KF). Disturbances acting on the system can be formulated as unmeasured states and are left to KF to be estimated. This approach works well especially in the input uncertainty or disturbances case. However, when the KF itself has uncertainties coming from the faulty measurements then the estimates will also be deteriorated. Consequently, this faulty estimation will lead to the loss of feasibility and optimality of the feedback control as well. Thus, the estimation of output uncertainties or measurement errors play a vital role almost all the time in constrained optimal control problems.

There are several studies that focus on the estimation of output disturbances. Chen et al. [12] showed that both system states and measurement errors can be concurrently estimated via linear parameter varying (LPV) sliding mode observers (SMO). Sufficient conditions for asymptotic stability of fault estimation were also discussed. Pyrkin and Bobtsov [13] worked on the design of an adaptive controller in the presence of input delays and output disturbances. They claimed that the frequencies of disturbance can be identified via simpler adaptive schemes such that the disturbances are assumed to be harmonic functions. Using H_∞ robust control theory, Lee et al. [14] worked on the estimation of output disturbances with SMO for singularly perturbed uncertain systems. In addition, Pannocchia and Bemporad [15] proposed a method which combines the design of both disturbance model and the observer for MPC. Their approach relied on H_∞ theory to design a dynamic observer which minimizes the effects of unmeasured disturbances and they showed that to achieve offset-free MPC, the design of observer is in fact equivalent to the design of disturbance model. A novel work on min-max type MPC for uncertain dynamical systems was done by Limon et al. [16]. Rather than estimating the uncertainties, they analyzed the stability and feasibility of min-max type MPC in the presence of input and state dependent uncertainties via Lyapunov-like approaches and practical stability notions. Similarly, Shi et al. [17] were focused on the predictive control of uncertain systems with input saturations. They proposed a new design method for robust MPC which particularly handles the input constraints by using parameter-dependent Lyapunov functions.

As a predecessor of this work, Altıntaş and Turgut [18] proposed a novel uncertainty estimation method which particularly deals with the sensor bias in the operation of KF. They used a universal approximator structure to estimate sensor bias and Lyapunov-based stability criteria to derive a parameter adaptation law. However, the convergence criteria, parameter bounds and error analysis were not studied intensively. Moreover, the cases with mismatched uncertainties were not included in the detailed analysis. Based on the corollaries of the detailed analysis in thesis work of Altıntaş [24], the challenges in MPC can be overcome by the proposed Lyapunov redesign approaches which can be extended to aerial robotics applications.

In this study, an online adaptive method which eliminates the deteriorations caused by input, output and modeling uncertainties is proposed. The proposed method uses an external reference measurement to redesign the correction step of traditional KF which is used alongside a linear MPC. Then, the input, output and modeling uncertainties are lumped into a single output uncertainty whose functional approximation is sought to be done via radial basis function networks. It is observed that both reference tracking and feasibility of MPC are recovered as the disruptive effects of uncertainties are estimated and eliminated. Moreover, the corollaries of Lyapunov-based analysis of online learning procedure are investigated and the parameter update laws are derived for the fully matched and the partially matched cases of functional approximation. The proposed method, in fact, unifies the correction dynamics of traditional KF with the notion of approximation-based discrete-time adaptive control and shows that uncertainties acting to the system on different locations can be estimated via sufficiently few regression parameters. Hence, it is shown that the challenges of constrained optimal control under uncertain or defected state estimation can be overcome by using online adaptive techniques particularly on the state estimator part.

2. Background

In this section, the general theories behind recursive state estimation and model predictive control are summarized. In addition, the reflections of both state and disturbance estimation to model predictive control are presented by giving detailed background knowledge.

2.1. State estimation with Kalman filtering

Consider a discrete-time stochastic dynamical system with additive Gaussian white noise.

$$\mathbf{x}_k = f(\mathbf{x}_{k-1}, \mathbf{u}_k) + \mathbf{w}_k, \quad \mathbf{y}_k = h(\mathbf{x}_k) + \mathbf{v}_k, \tag{1}$$

where the system states $\mathbf{x} \in \mathbb{R}^n$, the system inputs $\mathbf{u} \in \mathbb{R}^m$ and the system outputs $\mathbf{y} \in \mathbb{R}^l$. The functions $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ and $h : \mathbb{R}^n \to \mathbb{R}^l$ are assumed to be Lipschitz-continuous and solution of the dynamical system (1) exists and it is unique. The additive Gaussian white noise in the system dynamics is represented by $\mathbf{w}_k \sim \mathcal{N}(\mathbf{0}, Q_k)$ and in the measurement is represented by $\mathbf{v}_k \sim \mathcal{N}(\mathbf{0}, R_k)$. The symmetric positive semi-definite matrices $Q = Q^\top \geq 0 \in \mathbb{R}^{n\times n}$ and $R = R^\top \geq 0 \in \mathbb{R}^{l\times l}$ are the covariance matrices of process and measurements noise, respectively.

The original formulation of Kalman filtering consists of linear stochastic dynamics to iterate both mean values of states and state prediction covariance under Gaussian noise assumption. In each prediction step, the new states are computed from previous states and inputs by passing through system (i.e. process) dynamics while the prediction covariance is iterated and dispersed based on the linear process dynamics and the covariance of process noise. Moreover, in each correction (update) step, the error between predicted outputs from linear model (1) and the measured outputs from actual dynamics is decreased by a normalizing Kalman gain. The normalizing gain is computed based on whether the predictions or measurements are statistically more reliable by putting a weight on prediction and measurement covariances.

The linearity assumption of Kalman filter can be relaxed by various celebrated approaches such as extended Kalman filter (EKF), unscented Kalman filter (UKF) and multihypothesis Kalman filter (MHKF). In this study, EKF is preferred over other filtering techniques since the point-wise linearization procedure of process dynamics is highly compatible with the linear MPC formulation. The EKF formulation consists two main parts namely prediction and correction which are given below.

Here the identity matrix is $I \in \mathbb{R}^{n\times n}$, the symmetric positive definite matrix $P = P^\top > 0 \in \mathbb{R}^{n\times n}$ denotes the prediction error covariance matrix. The linearization of process dynamics gives Jacobian matrices $F_k \in \mathbb{R}^{n\times n}$ and $H_k \in \mathbb{R}^{l\times n}$ which are defined as $F_k = \partial_{\mathbf{x}} f|_{\hat{\mathbf{x}}_{k-1|k-1},\mathbf{u}_k}$ and $C_k = \partial_{\mathbf{x}} h|_{\hat{\mathbf{x}}_{k|k-1}}$, respectively. The alternative way of updating prediction covariance is given by Joseph form which restricts the positive definiteness of prediction covariance.

$$P_{k|k} = (I - K_k C_k)\, P_{k|k-1}\, (I - K_k C_k)^\top + K_k R_k K_k^\top \tag{2}$$

The Joseph form of prediction update is computationally less efficient than the standard form given in Table 1, however, it guarantees to preserve the positive definiteness (i.e. $P_{k|k} > 0\ \forall k$) in each filter recursion.

Table 1. KF prediction and correction steps.

Prediction	Correction
$\hat{\mathbf{x}}_{k|k-1} = f(\hat{\mathbf{x}}_{k-1|k-1}, \mathbf{u}_k)$	$K_k = P_{k|k-1} C_k^\top \left(C_k P_{k|k-1} C_k^\top + R_k\right)^{-1}$
$P_{k|k-1} = F_k P_{k-1|k-1} F_k^\top + Q_k$	$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + K_k \left(\mathbf{y}_k - C_k \hat{\mathbf{x}}_{k|k-1}\right)$
	$P_{k|k} = (I - K_k C_k)\, P_{k|k-1}$

2.2. Model predictive control

The MPC formulation is based on a system model which predicts the future behavior of the actual system and optimizes the future sequence of control inputs to minimize some certain performance index (i.e. cost function). The prediction of future system states is usually done by sequential simulations over the discrete-time linear system model known as the prediction model. Utilizing the linearity between the current states and future outputs, finding the optimal sequence of controls turns into a QP which needs to be solved at each step. Moreover, linear constraints on input variables and state variables may make the problem inequality-type QPs.

2.2.1. Prediction model

Consider a discrete-time linear time-invariant (LTI) system in the following form

$$\mathbf{x}_{k+1} = F\mathbf{x}_k + G\mathbf{u}_k, \quad \mathbf{y}_k = C\mathbf{x}_k + D\mathbf{u}_k, \tag{3}$$

where the system states $\mathbf{x} \in \mathbb{R}^n$, system inputs $\mathbf{u} \in \mathbb{R}^m$ and system outputs $\mathbf{y} \in \mathbb{R}^l$. In addition, the corresponding state-space matrices are $F \in \mathbb{R}^{n\times n}$, $G \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{l\times n}$ and $D \in \mathbb{R}^{l\times m}$. Then, by using the states at k^{th} sample and a sequence of control inputs, the future outputs within prediction horizon, N_p can be obtained by the following sequential simulation matrices.

$$\mathbf{Y}_k = \begin{bmatrix} \mathbf{y}_k \\ \mathbf{y}_{k+1} \\ \mathbf{y}_{k+2} \\ \vdots \\ \mathbf{y}_{k+N_p} \end{bmatrix}, \ \mathbf{U}_k = \begin{bmatrix} \mathbf{u}_k \\ \mathbf{u}_{k+1} \\ \mathbf{u}_{k+2} \\ \vdots \\ \mathbf{u}_{k+N_p} \end{bmatrix}, \ \Phi = \begin{bmatrix} C \\ CF \\ CF^2 \\ \vdots \\ CF^{N_p} \end{bmatrix}, \ \Gamma = \begin{bmatrix} D & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ CG & D & \mathbf{0} & \dots & \mathbf{0} \\ CFG & CG & D & \dots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ CF^{N_p-1}G & CF^{N_p-2}G & CF^{N_p-3}G & \dots & D \end{bmatrix} \tag{4}$$

In a compact form the prediction model is expressed as $\mathbf{Y}_k = \Phi\mathbf{x}_k + \Gamma\mathbf{U}_k$, where the input and output variables of the prediction model are $\mathbf{U}_k \in \mathbb{R}^{mN_p}$ and $\mathbf{Y}_k \in \mathbb{R}^{lN_p}$, respectively.

2.2.2. Optimal control problem

The synthesis of future input sequence in MPC is, in fact, a parametric optimization problem such that the minimum of some certain cost function is searched and the minimizer is applied to the actual system over the control horizon. Consider a linear quadratic (LQ) performance index to be minimized over the prediction horizon, then, the LQ receding horizon optimal control problem, or MPC in our case, can be presented as follows.

Theorem 1 *Consider the LTI system given in (3), then N_p-step optimal control problem with a control horizon of unity and a terminal cost of zero is*

$$\begin{aligned} \text{minimize} \quad & J_k = \sum_{i=0}^{N_p} \left\| \mathbf{y}_{k+i} - \mathbf{y}^d_{k+i} \right\|^2_{R_1} + \left\| \mathbf{u}_{k+i} \right\|^2_{R_2} \\ \text{subject to} \quad & \mathbf{x}_{k+1} = F\mathbf{x}_k + G\mathbf{u}_k,\ \mathbf{y}_k = C\mathbf{x}_k,\ D = \mathbf{0},\ \mathbf{u}_k \in \mathcal{U},\ \mathbf{x}_k \in \mathcal{X}, \end{aligned} \tag{5}$$

where $\mathbf{y}^d \in \mathbb{R}^l$ is the desired set-points on output variables, $\|\cdot\|_R$ is the standard weighted Euclidean norm with $R_1 = R_1^\top \geq 0$, $R_2 = R_2^\top \geq 0$, and the feasible sets $\mathcal{U} = \{\mathbf{u}_k \in \mathbb{R}^m : W_1\mathbf{u}_k \leq \mathbf{z}_1\}$ and $\mathcal{X} = \{\mathbf{x}_k \in \mathbb{R}^n : W_2\mathbf{x}_k \leq \mathbf{z}_2\}$. Furthermore, using the prediction model (4), the original optimal control problem (5) is converted to inequality constrained QP such as

$$\text{minimize} \quad \tilde{J}_k = \frac{1}{2}\mathbf{U}_k^\top Q \mathbf{U}_k + \mathbf{f}_k^\top \mathbf{U}_k \quad \text{subject to} \quad \mathbf{U}_k \in \Omega_k, \tag{6}$$

where $Q = 2(\Gamma^\top \tilde{R}_1 \Gamma + \tilde{R}_2)$ and $\mathbf{f}_k = 2\Gamma^\top \tilde{R}_1(\Phi\mathbf{x}_k - \mathbf{Y}^d_k)$ with $\mathbf{Y}^d_k := \mathbf{1}_{N_p} \otimes \mathbf{y}^d_k$, $\tilde{R}_1 := I_{N_p} \otimes R_1$ and $\tilde{R}_2 := I_{N_p} \otimes R_2$. Then, the control sequence at k^{th} step that solves the optimal control problem in (5) is $\mathbf{U}^*_k = \arg\min_{\mathbf{U}\in\Omega} \tilde{J}_k$.

In the operation of MPC, the QP defined in (6) is required to be solved at each sample time and the minimizing input sequence $\mathbf{U}^*_k$ is fed to the actual system, then, the whole procedure is repeated by shifting time horizon with new states.

3. Methodology

Standard MPC formulation requires the full state information while solving QP defined in (6). In many practical applications having a full state information is, unfortunately, not possible and needs to be estimated. There are plenty of methods which estimate the states and feed them to the MPC optimization step. Even if the states are available or they are estimated, the uncertainties or disturbances on the states degrade the performance of the controller.

There are mainly two approaches to estimate disturbances. The first approach assumes the disturbances as an augmented state and estimates it via standard state estimation procedures [19]. On the other hand the second tries to find the functional approximation of these disturbances. However, these methods are valid only for specific cases where the disturbances or uncertainties are assumed to be acting on the inputs or outputs separately. In the proposed method the disturbances acting on the different parts of the closed-loop system are estimated and eliminated concurrently in a generalized manner.

3.1. MPC with disturbance estimation

Consider the discrete-time system in (1) with additive input and output uncertainties as Lipschitz-continuous functions of available states such that $d^i : \mathbb{R}^n \to \mathbb{R}^m$ and $d^o : \mathbb{R}^n \to \mathbb{R}^l$, respectively.

$$\mathbf{x}_k = f(\mathbf{x}_{k-1}, \mathbf{u}_k + d^i(\mathbf{x}_{k-1})) + \mathbf{w}_k, \quad \mathbf{y}_k = h(\mathbf{x}_k) + d^o(\mathbf{x}_k) + \mathbf{v}_k, \tag{7}$$

where $\mathbf{w}_k$ and $\mathbf{v}_k$ are the process and measurement noise with aforementioned definitions. Now, consider a linear prediction model of the above system with Jacobian matrices similar to the LTI system in the form (3) and suppose that $\hat{\mathbf{d}}^i_k \in \mathbb{R}^m$ and $\hat{\mathbf{d}}^o_k \in \mathbb{R}^l$ are the estimated images of d^i and d^o at kth sample instance.

$$\hat{\mathbf{x}}_{k|k-1} = F\hat{\mathbf{x}}_{k-1|k-1} + G\mathbf{u}_k + G^d\hat{\mathbf{d}}^i_{k-1|k-1}, \quad \hat{\mathbf{d}}^i_{k|k-1} = \hat{\mathbf{d}}^i_{k-1|k-1}, \quad \hat{\mathbf{y}}_k = C\hat{\mathbf{x}}_{k|k-1}, \tag{8}$$

where $G^d = G$ since the input uncertainty is assumed. Here, the input uncertainty is also assumed to be constant at each prediction step. Then, the estimate of input uncertainty $\hat{\mathbf{d}}^i$ can be augmented as the unmeasured portion of state estimates and the new prediction model can be written as follows.

$$\begin{bmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{d}}^i \end{bmatrix}_{k|k-1} = \begin{bmatrix} F & G \\ \mathbf{0} & I \end{bmatrix} \begin{bmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{d}}^i \end{bmatrix}_{k-1|k-1} + \begin{bmatrix} G \\ \mathbf{0} \end{bmatrix} \mathbf{u}_k, \quad \mathbf{y}_k = \begin{bmatrix} C & \mathbf{0} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{d}}^i \end{bmatrix}_{k|k-1} \tag{9}$$

Moreover, the augmented prediction model can be also written in a compact form with $\hat{\mathbf{z}} := \begin{bmatrix} \hat{\mathbf{x}} & \hat{\mathbf{d}}^i \end{bmatrix}^\top$ such as $\hat{\mathbf{z}}_{k|k-1} = \tilde{F}\hat{\mathbf{z}}_{k-1|k-1} + \tilde{G}\mathbf{u}_k$, $\hat{\mathbf{y}}_k = \tilde{C}\hat{\mathbf{z}}_{k|k-1}$. Then, the same filter recursions can be made to estimate input uncertainties as well as the system states by using the augmented prediction error covariance matrix and the process noise covariance matrix $P, Q \in \mathbb{R}^{(n+m)\times(n+m)}$, and the estimation of uncertainty requires an initial guess on the distribution of the augmented states.

Standard MPC formulation can be unified with this augmented estimation procedure to make controller more robust to input uncertainties. This is nothing but using MPC with KF of augmented input uncertainty states. The synthesis of control sequence via (6) is unchanged except the prediction model created from (9) such that $\hat{\mathbf{Y}}_k = \tilde{\Phi}\hat{\mathbf{x}}_k + \tilde{\Gamma}\mathbf{U}_k$. Then, the optimal control sequence can be found by solving another constrained QP with the augmented prediction model matrices $\mathbf{U}^*_k = \arg\min_{\mathbf{U}\in\Omega} 1/2\ \mathbf{U}_k^\top H \mathbf{U}_k + \tilde{\mathbf{f}}_k^\top \mathbf{U}_k$, where $H := 2(\tilde{\Gamma}^\top \tilde{R}_1 \tilde{\Gamma} + \tilde{R}_2)$ and $\tilde{\mathbf{f}}_k = 2\tilde{\Gamma}^\top \tilde{R}_1(\tilde{\Phi}\hat{\mathbf{z}}_k - \mathbf{Y}^d_k)$.

The aforementioned estimation-based control procedure uses the estimation of input uncertainties and optimizing the control sequence based on the approximate knowledge of input uncertainties. A similar approach can be used to estimate the output uncertainties (e.g., augmenting $\hat{\mathbf{d}}^o_k$ to the original states), however there are several issues that make the problem to be handled more extensively which is also described in [19].

3.2. Functional approximation of disturbances

Consider the correction step of traditional Kalman filter where the probability distribution of predicted states is combinated with the probability distribution of measurements and normalized with the Kalman gain. Consequently, both the mean and variance of predicted states are updated according to information gathered from external measurements. Then, an uncompensated output uncertainty appears on the information coming from external measurements and degrade the correction performance (as well as the estimation performance) of the filter. Thus, the uncertainty in measurements which are fed to KF (i.e. output uncertainty of the system (7)) should be eliminated.

Consider the update step of KF with input disturbances as augmented states and also consider the uncertain external measurements $\mathbf{y}_k$ such that $\hat{\mathbf{z}}_{k|k} = \hat{\mathbf{z}}_{k|k-1} + K_k(\mathbf{y}_k - C\hat{\mathbf{z}}_{k|k-1})$ and convert it to a discrete-time dynamical system using an associative rule as $\hat{\mathbf{z}}_{k|k} = (I - K_k C)\hat{\mathbf{z}}_{k|k-1} + K_k\mathbf{y}_k$. Suppose that the output uncertainty is a function of augmented state estimates which is bounded above and below (i.e. $\sup_{k\in\mathbb{N}} |d^o(\hat{\mathbf{z}}_k)| < \infty$). Let $\mathbf{y}^*_k \in \mathbb{R}^l$ be the true measurements then, the correction dynamics can be written in terms of true measurements and an additive bounded uncertainty function as $\hat{\mathbf{z}}_{k|k} = (I - K_k C)\hat{\mathbf{z}}_{k|k-1} + K_k(\mathbf{y}^*_k + d^o(\hat{\mathbf{z}}_k))$. Noting that for the system defined by this approach, it is possible to create an analogy between input uncertainty case such that d^o is the additive input uncertainty to correction dynamics. Now, consider a reference model for the correction step which consists of true augmented state estimates $\hat{\mathbf{z}}^*_{k|k} \in \mathbb{R}^{n+m}$ and true measurements which is given by $\hat{\mathbf{z}}^*_{k|k} = (I - K_k C)\,\hat{\mathbf{z}}^*_{k|k-1} + K_k\mathbf{y}^*_k$. Then, the functional approximation of uncertainty function d^o needs to be obtained in order to steer the values of actual augmented state estimates to the true ones.

The idea behind approximation of d^o is, basically, expanding it on a space of known basis functions $\Phi : \mathbb{R}^{n+m} \to \mathbb{R}^N$ and superposing them via regression coefficients (or weights) $\Theta \in \mathbb{R}^{N\times l}$ which need to be found. As a universal approximator radial basis function neural networks (RBFNN) can be used to approximate the unknown disturbances or uncertainty functions [20], [21].

Definition 1 *Let the Gaussian function $\phi : \mathbb{R}^p \to \mathbb{R}$ be in the form of $\phi(\mathbf{x}; \mathbf{c}, R) = \exp\left\{-\|\mathbf{x} - \mathbf{c}\|^2_R\right\}$ with the input argument $\mathbf{x} \in \mathbb{R}^p$, the given arguments $\mathbf{c} \in \mathbb{R}^p$, $R \in \mathbb{R}^{p\times p}$ and $R = R^\top > 0$ and the weigthed Euclidean norm $\|\cdot\|_R$. Suppose that ϕ is integrable on $\mathbb{R}^p$, then, it is called radial basis function (RBF). Consider the linear combination of RBFs, $\Theta^\top\Phi(\mathbf{x}) = \sum_{i=1}^N \theta_i\phi_i(\mathbf{x}; \mathbf{c}_i, R_i) + \mathbf{b}$, where $\Phi \in \mathbb{R}^{N+1}$ is the array of regressors, $\Theta \in \mathbb{R}^{(N+1)\times q}$ are the weights, $\mathbf{b} \in \mathbb{R}^q$ are the bias terms with the centers of reception $\mathbf{c}_i$, then, the combination is called radial basis function neural network.*

Definition 2 *Consider a Gaussian function ϕ with the weight matrix $R_i = 1/2\sigma_i^2$, where σ_i denotes the width of i^{th} basis function. Then, the RBF is called isotropic if the width of Gaussian is fixed according to the spread of centers $\mathbf{c}_i$ with $\sigma_i = d_{max}/\sqrt{2N}$ and is in the form $\phi_i(\mathbf{x}) = \exp\left\{-\frac{N}{d^2_{max}}\|\mathbf{x} - \mathbf{c_i}\|^2\right\}$, where N is the number of centers and d_{max} is the maximum distance between centers.*

Using the definitions of RBFNNs, the universal approximation theorem for RBFNNs [21] can be adapted to the functional approximation of output uncertainty with isotropic RBFNNs.

Theorem 2 *Let $\phi : \mathbb{R}^p \to \mathbb{R}$ be a bounded and integrable function such that $\int_{\mathbb{R}^n} \phi(\mathbf{x}; \mathbf{c}, R)d\mathbf{x} \neq 0$. Then, for any function $d \in \mathcal{C}^1(\mathbb{R}^p)$ which represents the uncertainty and ball with radius $\epsilon > 0$, there exists an isotropic RBFNN structure with the approximation tolerance $\varepsilon > 0$ of order $\mathcal{O}(N^{-1/2n})$ such that $\hat{d}(\mathbf{x}) = \sum_{i=1}^N \theta_i\phi_i(\mathbf{x}, \mathbf{c}_i, R_i) = \Theta^\top\Phi(\mathbf{x})$ with $R_i = d_{max}/\sqrt{2N}I_n$ for $i = 1, \ldots, N$ such that $\int_{\mathcal{B}_\epsilon} |d(\mathbf{x}) - \hat{d}(\mathbf{x})|^2 d\mathbf{x} \leq \varepsilon$.*

Returning back to the correction dynamics with output uncertainty d^o, define a Lyapunov redesign term [22] which approximates and eliminates the uncertainty such that $\hat{\mathbf{z}}_{k|k} = (I - K_k C)\,\hat{\mathbf{z}}_{k|k-1} + K_k(\mathbf{y}_k -$

$\hat{\Theta}_k^\top \Phi(\hat{\mathbf{z}}_{k|k-1}))$ where $\hat{\Theta}_k \in \mathbb{R}^{(N+1)\times l}$ denotes the estimates of the true regression coefficients which are initially unknown.

3.3. Adaptive Lyapunov redesign on state estimator

It was shown that the output uncertainty d^o can be expanded in terms of known RBFs within a sufficiently small approximation radius such that $d^o(\hat{\mathbf{z}}_k) \approx \sum_{i=1}^N \theta_i^* \phi_i(\hat{\mathbf{z}}_{k|k-1}) + \mathbf{b}_k^* = \Theta^{*\top}\Phi(\hat{\mathbf{z}}_{k|k-1})$ where $\Theta^* \in \mathbb{R}^{(N+1)\times l}$ are the true regression parameters (or weights) which approximate the uncertainty and are initially unknown. Consider the new measurement law defined in Lyapunov redesign and note that the actual measurements can be written in terms of uncertainty function and the true measurements. Thus,

$$\hat{\mathbf{z}}_{k|k} = (I - K_k C)\hat{\mathbf{z}}_{k|k-1} + K_k \left(\mathbf{y}_k^* + d^o(\hat{\mathbf{z}}_k) - \hat{\Theta}_k^\top \Phi(\hat{\mathbf{z}}_{k|k-1})\right). \tag{10}$$

Suppose now the true correction dynamics is a reference model to (10) with the tracking error $\hat{\mathbf{e}}_{k|k} := \hat{\mathbf{z}}_{k|k} - \hat{\mathbf{z}}_{k|k}^*$ and the parameter adaptation error $\tilde{\Theta}_k := \Theta^* - \hat{\Theta}_k$, and let $\Phi_{k|k-1} := \Phi(\hat{\mathbf{z}}_{k|k-1})$. Then, the error dynamics relating the uncertain augmented state estimates to the true augmented state estimates with the functional approximation of d^o can be defined as

$$\hat{\mathbf{e}}_{k|k} = (I - K_k C)\,\hat{\mathbf{e}}_{k|k-1} + K_k \tilde{\Theta}_k^\top \Phi_{k|k-1}. \tag{11}$$

In order to approximate the output uncertainty function while estimating the augmented states which also includes the input uncertainties, the stability of estimator must be preserved. Then, a proper stability and convergence analysis should be maintained.

First, the following mathematical concepts [23] which were adapted to discrete-time systems are redefined in order to create a base to prove the stability of parameter adaptation and state estimation.

Definition 3 *The equilibrium point $\mathbf{x}^* = \mathbf{0}$ of the system (1) is asymptotically stable if it is stable and the solution trajectories stay bounded within the ball $\mathcal{B}_\delta = \{\mathbf{x} \in \mathbb{R}^p : \|\mathbf{x}_0\| < \delta(k_0)\}$ such that $\mathbf{x}_k \to \mathbf{0}$ $\forall \mathbf{x}_k \in \mathcal{B}_\delta$ as $k \to \infty$ $\forall k \in \mathbb{N}$. The equilibrium point $\mathbf{x}^*$ is said uniformly asymptotically stable if it is uniformly stable and the ball radius δ is independent of k_0.*

Theorem 3 *Let $\mathbf{x}^* = \mathbf{0}$ be an equilibrium point of the discrete-time system (1) with $\mathbf{x}_0, \mathbf{x}^* \in \mathcal{D} \subset \mathbb{R}^p$ and $k_0 = 0$. Suppose that on the domain $\mathcal{D}$ there exists a continuously differentiable locally (globally) positive definite Lyapunov function $V : \mathcal{D} \to \mathbb{R}$ whose backward difference along the system trajectories is locally (globally) negative semidefinite such as $\Delta V_k := V(f(k, \mathbf{x}_{k-1})) - V(\mathbf{x}_{k-1}) \leq 0$ $\forall \mathbf{x} \in \mathcal{D}$ $\forall k \in \mathbb{N}$. Then, $\mathbf{x}^*$ is locally (globally) uniformly stable and if ΔV_k is negative definite, then, it is locally (globally) uniformly asymptotically stable.*

Now, consider the error dynamics obtained in (11), then, in order to have a stable and convergent parameter adaptation which steers the uncertain correction dynamics to the true ones the following theorem must hold. Moreover, if the theorem holds then the concurrent estimation of the sequences of output uncertainty $\{\hat{\mathbf{d}}_k^o\}_{k=0}^\infty$ and input uncertainty $\{\hat{\mathbf{d}}_k^i\}_{k=0}^\infty$ should be convergent. The theorem is restated for the sake of completeness of the study and the original proof can be found in [24].

Theorem 4 *For the correction step of the state estimator as an uncertain discrete-time dynamical system* $\hat{\mathbf{z}}_{k|k} = (I - K_kC)\hat{\mathbf{z}}_{k|k-1} + K_k\mathbf{y}_k$ *with the measurement law* $\mathbf{y}_k = \mathbf{y}_k^* + d^o(\hat{\mathbf{z}}_k) - \hat{\Theta}_k^\top \Phi_{k|k-1}$ *and the parameter adaptation law* $\hat{\Theta}_k = \hat{\Theta}_{k-1} + \Gamma_\Theta \Phi_{k|k-1}\hat{\mathbf{e}}_{k|k}^\top P_{k|k}^{-1} K_k$, *where* $\Gamma_\Theta = \Gamma_\Theta^\top > 0$ *is the diagonal learning rate matrix and* $P_{k|k}, K_k$ *are the prediction error covariance and Kalman gain matrices, the uncertain augmented state estimates* $\hat{\mathbf{z}}_{k|k}$ *asymptotically track the true augmented state estimates* $\hat{\mathbf{z}}_{k|k}^*$ *of the reference model* $\hat{\mathbf{z}}_{k|k}^* = (I - K_kC)\hat{\mathbf{z}}_{k|k-1}^* + K_k\mathbf{y}_k^*$ *driven by any bounded true measurement* $\mathbf{y}_k^*$, *while all the signals in the closed-loop system remain uniformly bounded. Moreover, the closed-loop tracking error dynamics* $\hat{\mathbf{e}}_{k|k} = (I - K_kC)\hat{\mathbf{e}}_{k|k-1} + K_k\tilde{\Theta}_k^\top \Phi_{k|k-1}$ *are globally uniformly asymptotically stable.*

3.4. Parameter adaptation within approximation tolerances

In the previous section, the output uncertainty function d^o was assumed to be matched perfectly via sufficient structure of RBFNN. On the other hand, the tolerance of approximation may cause regression coefficients to grow unbounded while the tracking errors are still bounded at all times, hence, the parameter adaptation process suffers from lack of robustness. This phenomenon is named as parameter drift and there are several well-adopted techniques to prevent drift such as σ-modification, e-modification, dead-zone modification and projection-based adaptation. These modifications, in essence, modifies the original parameter adaptation law in such a way that the regression coefficients stay uniformly bounded in the presence of bounded approximation errors (or external disturbances).

Consider the same RBFNN expansion of d^o with approximation error $\varepsilon_k \in \mathbb{R}^{n+m}$ and a constant upper bound $\varepsilon_{max} \in \mathbb{R}$ such that $\sup_{k\in\mathbb{N}} |\varepsilon_k| \leq \varepsilon_{max}$. Then, the function d^o can be represented via RBFNN structure as $d^o(\hat{\mathbf{z}}_k) = \sum_{i=1}^N \theta_i^* \phi_i(\hat{\mathbf{z}}_{k|k-1}) + \mathbf{b}_k^* + \varepsilon_k = \Theta^{*\top}\Phi(\hat{\mathbf{z}}_{k|k-1}) + \varepsilon_k$. Consequently, the error dynamics slightly differ and the bounded approximation error term occurs in such a way that $\hat{\mathbf{e}}_{k|k} = (I - K_kC)\,\hat{\mathbf{e}}_{k|k-1} + K_k\tilde{\Theta}_k^\top \Phi_{k|k-1} + K_k\varepsilon_k$. Now, focus on how to sustain the uniform boundedness of both tracking error and the parameter adaptation error via modifying Theorem 4 by using projection-based adaptive law. Similar to Theorem 4, the proof of the following theorem is given in [24] and some major points are restated.

Theorem 5 *For the correction step of the state estimator as an uncertain discrete-time dynamical system* $\hat{\mathbf{z}}_{k|k} = (I - K_kC)\,\hat{\mathbf{z}}_{k|k-1} + K_k\mathbf{y}_k$ *with the measurement law* $\mathbf{y}_k = \mathbf{y}_k^* + d^o(\hat{\mathbf{z}}_k) - \hat{\Theta}_k^\top \Phi_{k|k-1}$ *and the projection-based parameter adaptation law* $\hat{\Theta}_k = \hat{\Theta}_{k-1} + \mathrm{Proj}_{\Gamma_\Theta}(\hat{\Theta}_k, \Phi_{k|k-1}\hat{\mathbf{e}}_{k|k}^\top P_{k|k}^{-1} K_k)$, *where* $\mathrm{Proj}_\Gamma(\cdot,\cdot)$ *is the* Γ*-Projection operator and* $P_{k|k}, K_k$ *are the prediction error covariance and Kalman gain matrices, the uncertain augmented state estimates* $\hat{\mathbf{z}}_{k|k}$ *asymptotically track the true augmented state estimates* $\hat{\mathbf{z}}_{k|k}^*$ *outside the compact set* $\Omega := \{(\hat{\mathbf{e}}_{k|k}, \tilde{\Theta}_k) : \|\hat{\mathbf{e}}_{k|k-1}\| \leq c_1 \wedge \|\tilde{\Theta}_k\| \leq c_2\}$ *where the positive scalars*

$$c_1 := 2\,\frac{\lambda_{max}(P_{k|k}^{-1})}{\lambda_{min}(N_{k|k-1})}\|I - K_kC\|\|K_k\|\varepsilon_{max}, \quad c_2 := \frac{1}{\|\Phi_{k|k-1}\|}\sqrt{\kappa(K_k^\top P_{k|k}^{-1} K_k)}\varepsilon_{max}.$$

Consequently, driven by any bounded true measurement $\mathbf{y}_k^*$, *all the parameter adaptation errors remain uniformly bounded and asymptotically converge towards* Ω *within the set* $\mathcal{A} := \{\tilde{\Theta}_k : c_2 \leq \|\tilde{\Theta}_k\|_F \leq c_3\}$ *with a positive upper bound* $c_3 := \|\Theta^*\|_F + \left(\sum_{i=1}^l (\Theta_{max}^i)^2 + \epsilon\frac{(\gamma^i)^2(\Theta_{max}^i)^3}{2(1+\gamma^i)^2(\hat{\Theta}_k^i)^\top \mathbf{x}_k^i}\right)^{1/2}$, *where* $\mathbf{x}_k^i := (\Phi_{k|k-1}\hat{\mathbf{e}}_{k|k}^\top P_{k|k}^{-1} K_k)^i$,

Θ^i_{max} *is the maximum allowed parameter error,* ϵ *is the relaxation of parameter bounds due to discretization and* γ^i *is the projection tolerance for* $i = 1, 2, \cdots, l$. *Moreover, the closed-loop tracking error dynamics* $\hat{\mathbf{e}}_{k|k} = (I - K_k C)\,\hat{\mathbf{e}}_{k|k-1} + K_k(\tilde{\Theta}_k^\top \Phi_{k|k-1} + \varepsilon_k)$ *are globally uniformly asymptotically stable in the presence of bounded approximation error* ε_k.

4. Simulation and results

The proposed method is applied to a widely encountered problem in engineering practice which is a concurrent estimation of actuation and sensory uncertainties (or faults at some level). Moreover, how the capabilities and stability properties of traditional estimation methods can be enhanced are shown by simulations via universal approximators in the presence of exaggerated uncertain situations. In this section, a quadrotor motion in plane is preferred as a case study to present how sensory and actuation faults and limitations can be handled via adaptive Lyapunov redesign KF and estimation based MPC.

4.1. Modeling of case study

The modeling of quadrotor in planar motion can be obtained by using Euler–Lagrange equations. Supposing that the reference frame attached to the center of gravity of quadrotor's body is denoted by $\mathcal{F}_b$ and the inertial frame of reference is set to $\mathcal{F}_e$, also supposing that both reference frames are right-handed and orthogonal. Body-fixed frame $\mathcal{F}_b$ is placed so that the unit vector $\mathbf{b}_2$ and $\mathbf{b}_3$ point quadrotor's right and upward directions, respectively. Now, considering the generalized coordinates placed on $\mathcal{F}_e$, then the configuration space becomes $Q = \mathbb{R}^2 \times SO(2)$ with coordinates $\mathbf{q} = \begin{bmatrix} y & z & \phi \end{bmatrix}^\top$. Then, the Lagrangian function $L : TQ \to \mathbb{R}$ to planar quadrotor can be written as

$$L(\mathbf{q}, \dot{\mathbf{q}}) = \frac{1}{2} m(\dot{y}^2 + \dot{z}^2) + \frac{1}{2} I_{xx}\dot{\phi}^2 - mgz, \tag{12}$$

where $m, I_{xx}, g \in \mathbb{R}_+$ are the mass and moment of inertia about $\mathbf{b}_1$ of quadrotor, and gravitational acceleration, respectively. Then, the Euler-Lagrange equations with generalized forces acting on the quadrotor are

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{\mathbf{q}}_i} - \frac{\partial L}{\partial \mathbf{q}_i} = \mathbf{f}_i^e, \quad \text{for } i = 1, 2, 3. \tag{13}$$

By using (12) and (13), the equations of motion for a planar quadrotor can be obtained. In addition, it is more convenient to write forces acting on the quadrotor with the basis vectors of $\mathcal{F}_b$ rather than the ones of $\mathcal{F}_e$. By using the quadrotor geometry, the forces and moments can be rotated from $\mathcal{F}_e$ to $\mathcal{F}_b$, i.e. transformed via matrix $R_{be} \in SO(2)$.

$$m\ddot{y} = -F_2^b \sin\phi, \quad m\ddot{z} + mg = F_2^b \cos\phi, \quad I_{xx}\ddot{\phi} = M_3^b \tag{14}$$

Thus, the complete nonlinear state equations $\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u})$ to the planar quadrotor with states $\mathbf{x} \in \mathbb{R}^6$ and inputs $\mathbf{u} \in \mathbb{R}^2$ are obtained as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{bmatrix} = \begin{bmatrix} x_4 \\ x_5 \\ x_6 \\ 0 \\ -g \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -\frac{1}{m}\sin x_3 & 0 \\ \frac{1}{m}\cos x_3 & 0 \\ 0 & \frac{1}{I_{xx}} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -g & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{m} & 0 \\ 0 & \frac{1}{I_{xx}} \end{bmatrix}, \tag{15}$$

where the states $x_1 = y,\ x_2 = z,\ x_3 = \phi,\ x_4 = \dot{y},\ x_5 = \dot{z},\ x_6 = \dot{\phi}$ and the inputs $u_1 = F_2^b$, $u_2 = M_3^b$. Then, considering the hovering flight condition as the equilibrium condition of the system (15) and linearizing around $\mathbf{x}_0 = \mathbf{0}$ with $u_{1,0} = mg$ and $u_{2,0} = 0$. The linearization of state equations can be done by $\dot{\mathbf{x}} = \partial_{\mathbf{x}} f|_{\mathbf{x}=\mathbf{x}_0}(\mathbf{x} - \mathbf{x}_0) + \partial_{\mathbf{u}} f|_{\mathbf{u}=\mathbf{u}_0}(\mathbf{u} - \mathbf{u}_0)$. Thus, the linearized system $\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u}$ is obtained for hovering flight with the Jacobian matrices given in (15). Assuming that all the states of quadrotor are available and there is no direct feedthrough (i.e $C = I$ and $D = \mathbf{0}$). Finally, in order to obtain the discrete-time LTI system in the form of (3), matrices in (15) can be discretized via series expansions up to desired terms as $F = I + \sum_{k=1}^{\infty} \frac{A^k \Delta t^k}{k!}$ and $G = \left(\sum_{k=1}^{\infty} \frac{A^{k-1} \Delta t^k}{k!}\right) B$, where Δt is the sampling time of the equivalent discrete-time system.

The effects of uncertainties in a planar quadrotor are investigated under the following input, output and modeling uncertainty definitions: i) the absence of feedforward control which holds the quadrotor in hover (i.e. $u_{1,0} \neq mg$ and $\mathbf{u}_0 = \mathbf{0}$) as additive input uncertainty, ii) exaggerated bias and noise in both positional and rate sensors as additive output uncertainty, iii) imperfect knowledge about quadrotor mass and moment of inertia as modeling uncertainty, and the discretization of system.

The proposed method estimates these mentioned input and output uncertainties based on bounded external reference measurements (i.e. $\mathbf{y}_k^*$). The correcting external reference measurements can be considered as either the equivalent readings from another sensor or states of reference model. For example, suppose that output uncertainties on Euler angles need to be estimated, then at some steady portion of movement (e.g., level flight) the accelerometers or gravity sensors can partially provide true Euler angle values. Consider another illustrative example, suppose that the premade calibration of accelerometers shifts due to drastic environmental changes (e.g., temperature, altitude) and consequently the attitude estimation from gyro-accelerometer leveling will deteriorate. Then, suppose that there exists a sufficiently reliable external positional measurement such as GPS or some broadcast signal, then, the deterioration on attitude estimation due to accelerometer uncertainties can be eliminated via proposed concurrent uncertainty estimation solution which is a kind of composite sensor fusion technique. Furthermore, suppose the jammed readings from magnetometers, then, the uncertainty on magnetometer readings can be estimated and eliminated by using the computational results of a local (or global) mathematical magnetic model as external reference measurements.

In this quadrotor case study, translational position and velocity states $y, z, \dot{y}, \dot{z}$ are supposed to be obtained from the integration of uncertain accelerometer measurements and rotational states $\phi, \dot{\phi}$ are supposed to be obtained from the integration of uncertain gyro measurements. Then, the uncertain measurements are corrected with the external reference measurements directly coming from the ideal measurement counterparts for simplicity.

4.2. Simulation of case study

The planar quadrotor is simulated under additive input, output and modeling uncertainties. During simulations planar quadrotor is equipped with a linear MPC to control its position while the states are estimated via KF. In the MPC formulation input and state constraints are involved and both input magnitudes and input rates are penalized to attain smooth transitions. The optimal control problem setup for the case study is given in (16). The geometric properties of quadrotor are chosen as $m = 0.5$ $[kg]$ and $I_{xx} = 5 \times 10^{-3}$ $[kgm^2]$ with the standard gravitational acceleration of $g = 9.81$ $[m/s^2]$. In addition the output tracking weights and input magnitude weights are defined as $R_1 = I_6$ and $R_2 = 0.1I_2$, respectively. The linear representation of quadrotor dynamics around hovering equilibrium is discretized with $\Delta t_c = 0.02$ $[s]$ and the simulation step size is set to $\Delta t_s = 0.01$ $[s]$.

$$\begin{aligned}
\min_{\mathbf{u}} \quad & \sum_{i=0}^{100} \left\|\mathbf{y}_{k+i} - \mathbf{y}^d_{k+i}\right\|^2_{R_1} + \left\|\mathbf{u}_{k+i}\right\|^2_{R_2} + 10\left\|\mathbf{u}_{k+1+i} - \mathbf{u}_{k+i}\right\|^2 \\
\text{s.t.} \quad & \mathbf{x}_{k+1} = F\mathbf{x}_k + G\mathbf{u}_k,\ \mathbf{y}_k = C\mathbf{x}_k,\ |u_1| \le 1\ [N],\ |u_2| \le 0.5\ [Nm], \\
& |y| \le 100\ [m],\ |\dot{y}| \le 1\ [m/s], |z| \le 100\ [m],\ |\dot{z}| \le 1\ [m/s],\ |\phi| \le 20\ [^\circ],\ |\dot{\phi}| \le 90\ [^\circ/s].
\end{aligned} \tag{16}$$

During the simulations MPC is fed by the state and input disturbance estimates of conventional KF design with Joseph form of correction step. The KF assumes augmented states $\hat{\mathbf{z}} := [\hat{\mathbf{x}}\ \hat{\mathbf{d}}^i]^\top \in \mathbb{R}^8$ and the input disturbance states are unmeasured. The process noise and measurement noise covariance matrices are chosen constant as $Q = I_8$ and $R = 100I_6$, respectively. The KF is initialized with $\mathbf{z}_0 = \mathbf{0}$ and $P_{0|0} = \text{diag}(10I_6, 100I_2)$ which represents more initial uncertainty in disturbance states. Next, 5 neurons (i.e. basis functions) are used for each state along with 1 additional bias term. The isotropic Gaussian basis functions are spread homogeneously over the state space between $[-50, 50]$ $[m]$ for y, z, between $[-50, 50]$ $[^\circ]$ for ϕ, between $[-10, 10]$ $[m/s]$ for $\dot{y}, \dot{z}$ and between $[-50, 50]$ $[^\circ/s]$ for $\dot{\phi}$, respectively. The learning rate is adjusted to $\Gamma = \text{diag}(0.4, 0.4, 0.8, 1.2, 1.2, 1.6)$ and $\Theta^i_{max} = 0.5$ with the projection tolerance of $\gamma^i = 0.01$ for $i = 1, 2, \cdots, 6$.

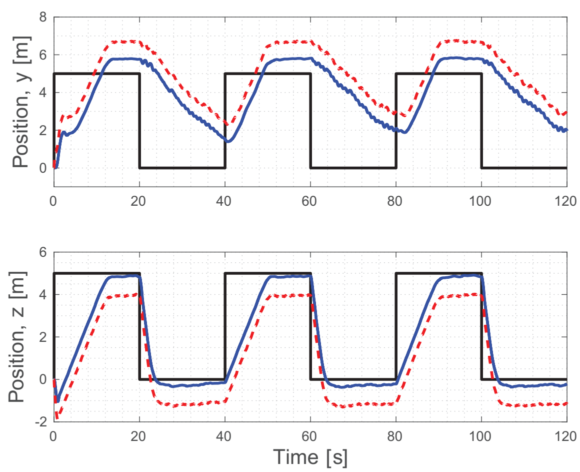
Finally, the additive input uncertainty is defined as $\mathbf{d}^i_k = [-mg\ 0]^\top$ such that the feedforward control to hold quadrotor in hover is, in fact, absent. The additive output uncertainties are normally distributed random variables and given in the following table.

Furthermore, the modeling uncertainties are defined as percentage deviation of actual values from the design values and assumed as $+20\%$ on the mass and $+30\%$ on the moment of inertia of quadrotor.

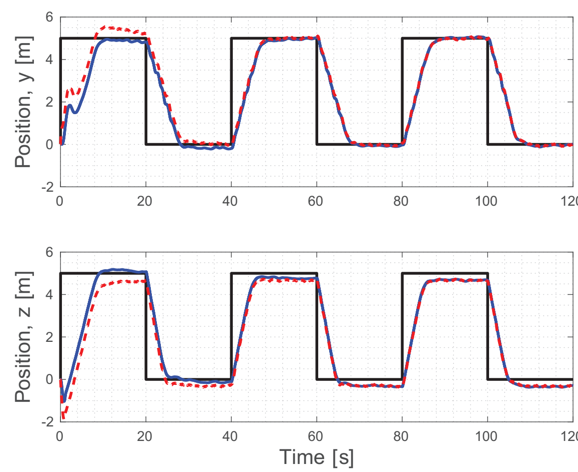
According to the simulation results, the estimation of uncertainties were converged to their prescribed values given in Table 2. In Figures 1a and 1c the disruptive effects of uncertainties on command tracking and constraint satisfaction can be seen when the proposed method was not used. After switching on the uncertainty estimation, the command tracking was gradually recovered within 20 s (based on current learning rates) and stayed within uniform bounds defined in Theorem 5. Moreover, in Fig. 1d, the velocity constraints were satisfied as the uncertainties converged toward their true values as shown in Figures 2a and 2b. On the other hand, in Figure 1c it is clearly seen that the constraints are violated due to uncertain measurements that cause biased state estimates. It can be thought that the estimator bias shifts the actual constraint boundaries and it creates a fallacy at the nominal operation of MPC. Observing that the initial violation of contraints is directly related to the transient behaviour of both KF and RBFNN dynamics. In this manner, the initialization of KF prediction covariance matrix and the distribution of initial RBFNN weights gain a particular importance for the strictly feasible real-life examples. In addition, the RBFNN weights also stayed bounded in Figures 2c and 2d despite the mismatched portion of uncertainties thanks to the projection based learning.

Table 2. Output uncertainty definitions.

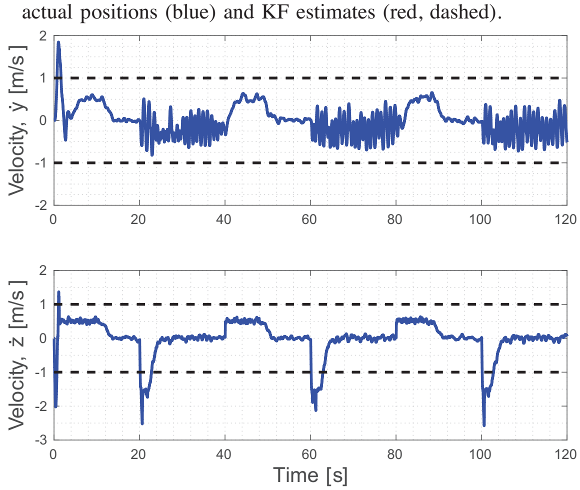
Measurement	Mean, μ	Deviation, 3σ	Measurement	Mean, μ	Deviation, 3σ
Position, y $[m]$	1.0	0.2	Velocity, $\dot{y}$ $[m/s]$	–0.5	0.5
Position, z $[m]$	–1.0	0.2	Velocity, $\dot{z}$ $[m/s]$	0.5	0.5
Roll angle, ϕ $[°]$	7.5	3.0	Roll rate, $\dot{\phi}$ $[°/s]$	5.0	10.0



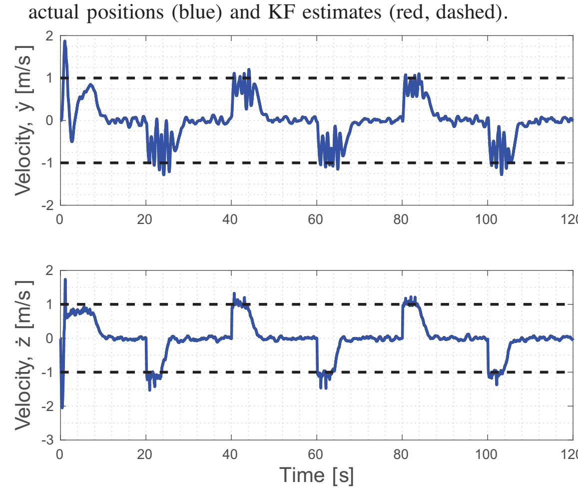
(a) Learning switched off, degenerate position command tracking due to uncertainties, reference command (black), actual positions (blue) and KF estimates (red, dashed).



(b) Learning switched on, recovering position command tracking in spite of uncertainties, reference command (black), actual positions (blue) and KF estimates (red, dashed).

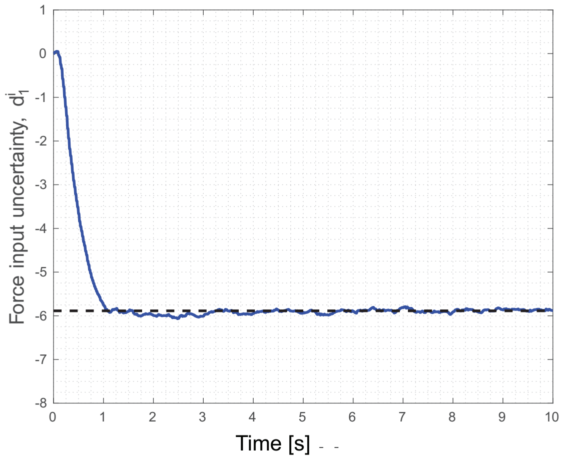


(c) Learning switched off velocity states (blue) shifted from the actual state constraints (black, dashed) due to uncertainties.

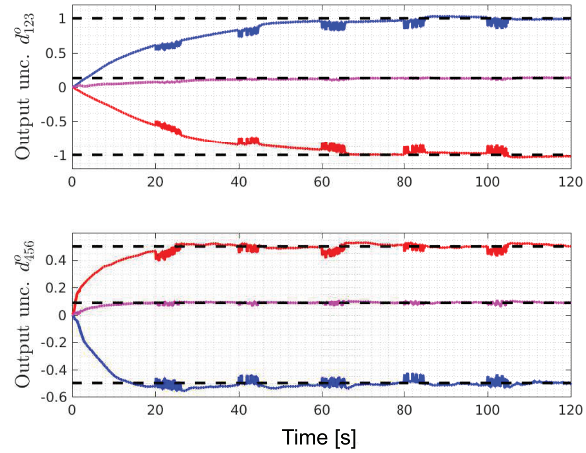


(d) Learning switched on, velocity states (blue) satisfy the state constraints (black, dashed) in spite of uncertainties.

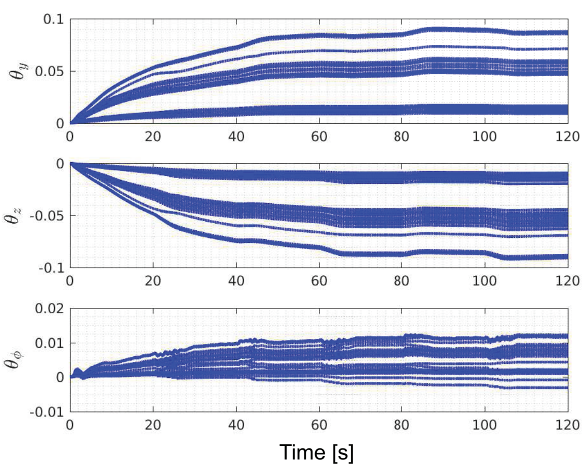
Figure 1. Command tracking and constraint satisfaction with and without uncertainty estimation.



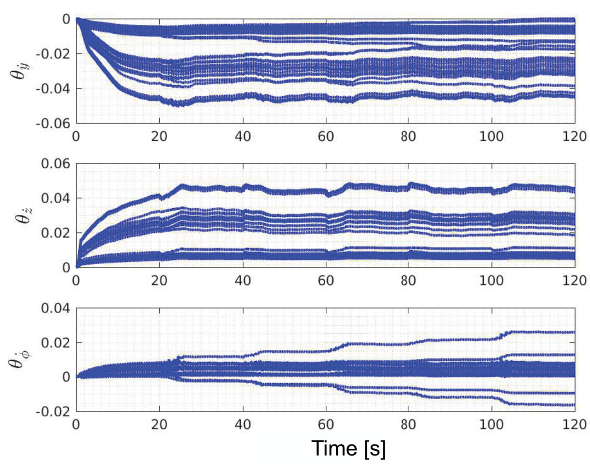
(a) Learning switched on, estimation of input uncertainty (blue), i.e. gradual recovery of the missing feedforward action ($\approx -1.2\text{mg}$), with actual feedforward value (black, dashed).



(b) Learning switched on, estimation of output uncertainties, with actual values (black, dashed), i.e. bias valus given in Table 2.



(c) Learning switched on, RBFNN parameters of positional (y, z, ϕ) states.



(d) Learning switched on, RBFNN parameters of rate $(\dot{y}, \dot{z}, \dot{\phi})$ states.

Figure 2. Estimation of prescribed uncertainties and boundedness of regression coefficients.

5. Conclusion

In this paper, the state estimation used in MPC framework was enhanced by adaptive techniques and universal approximators to recover both the tracking performance and the feasibility of MPC. By using Lyapunov-based arguments stability, convergence and boundedness properties of proposed adaptive state and uncertainty estimation method were studied in details.

It was shown that with an aid of reference measurement, it is possible to recover the correct state estimates while rejecting the disruptive effects of uncertainties. The uniform bounds on adaptive parameters were obtained and the effect of approximation tolerances on the stability and boundedness of regression coefficients is shown. Finally, the method was implemented on a planar quadrotor case and its position tracking performance with

MPC under the state and input constraints was observed. As a result both reference tracking and constraint satisfaction were attained in the presence of stringent actuation, modeling and measurement errors.

Acknowledgment

This research was supported in all aspects by Roketsan Missiles Inc. and the author O. H. Altıntaş wants to thank his colleagues in Advanced Technologies & Systems Division in Roketsan Missiles Inc.

References

[1] Lee U, Mesbahi M. Constrained autonomous precision landing via dual quaternions and model predictive control. Journal of Guidance, Control and Dynamics 2017; 40 (2): 292-308.

[2] Ma L, Wang K, Xu Z, Shao Z, Song Z. Optimal feedback guidance for powered landing of reusable rockets. In: IEEE 2019 Chinese Control And Decision Conference (CCDC); Nanchang, China; 2019. pp. 915-920.

[3] Hong H, Maity A, Holzapfel F, Tang S. Model predictive convex programming for constrained vehicle guidance. IEEE Transactions on Aerospace and Electronic Systems 2019; 55 (5): 2487-2500.

[4] Liberzon D. Calculus of Variations and Optimal Control Theory: A Concise Introduction. Princeton, NJ, USA: Princeton University Press, 2011.

[5] Diehl M, Gros S. Numerical Optimal Control [online]. Website https://www.syscop.de/files/2017ss/NOC/script/book-NOCSE.pdf [accessed 02 November 2020].

[6] Blackmore L, Açıkmeşe B, Scharf DP. Minimum-landing-error powered-descent guidance for mars landing using convex optimization. Journal of Guidance, Control and Dynamics 2010; 33 (4): 1161-1171.

[7] Carson JM, Açıkmeşe B, Blackmore L, Wolf AA. Capabilities of convex powered-descent guidance algorithms for pinpoint and precision landing. In: 2011 IEEE Aerospace Conference; Big Sky, USA; 2011. pp. 1-8.

[8] Mayne DQ, Rawlings JB, Rao CV, Scokaert POM. Constrained model predictive control: stability and optimality. Automatica 2000; 36 (6): 789-814.

[9] Camacho EF, Alba CB. Model Predictive Control. Berlin, Germany: Springer Science & Business Media, 2013.

[10] Nocedal J, Wright SJ. Numerical Optimization. Berlin, Germany: Springer Science & Business Media, 2006.

[11] Iusem AN, De Pierro AR. On the convergence properties of hildreth's quadratic programming algorithm. Mathematical Programming 1990; 47 (1-3): 37-51.

[12] Chen L, Edwards C, Alwi H. Sensor fault estimation using lpv sliding mode observers with erroneous scheduling parameters. Automatica 2019; 101: 66-77.

[13] Pyrkin AA, Bobtsov AA. Adaptive controller for linear system with input delay and output disturbance. IEEE Transactions on Automatic Control 2015; 61 (12): 4229-4234.

[14] Lee DJ, Park Y, Park YS. Robust h_∞ sliding mode descriptor observer for fault and output disturbance estimation of uncertain systems. IEEE Transactions on Automatic Control 2012; 57 (11): 2928-2934.

[15] Pannocchia G, Bemporad A. Combined design of disturbance model and observer for offset-free model predictive control. IEEE Transactions on Automatic Control 2007; 52 (6): 1048-1053.

[16] Limón D, Alamo T, Salas F, Camacho EF. Input to state stability of min-max mpc controllers for nonlinear systems with bounded uncertainties. Automatica 2006; 42 (5): 797-803.

[17] Shi T, Su H, Chu J. An improved model predictive control for uncertain systems with input saturation. Journal of the Franklin Institute 2013; 350 (9): 2757-2768.

[18] Altıntaş OH, Turgut AE. Online sensor bias estimation & calibration by kalman filtering with adaptive lyapunov redesign method. In: IEEE 2019 9th International Conference on Recent Advances in Space Technologies (RAST); İstanbul, Turkey; 2019. pp. 145-152.

[19] Ricker NL. Model predictive control with state estimation. Industrial & Engineering Chemistry Research 1990; 29 (3): 374-382.

[20] Micchelli CA. Interpolation of scattered data: Distance matrices and conditionally positive definite functions. In: Approximation Theory and Spline Functions. Berlin, Germany: Springer, 1984, pp. 143-145.

[21] Park J, Sandberg IW. Universal approximation using radial-basis-function networks. Neural Computation 1991; 3 (2): 246-257.

[22] Ge S. Lyapunov design. In: Control Systems, Robotics and Automation - Volume XIII: Nonlinear, Distributed, and Time Delay Systems-II. Paris, France: EOLSS Publications, 2009, pp. 90-130.

[23] Lavretsky E, Wise K. Lyapunov stability of motion. In: Robust and Adaptive Control. Berlin, Germany: Springer, 2013, pp. 225-261.

[24] Altıntaş OH. Adaptive lyapunov redesign of model predictive powered descent guidance. MSc, Middle East Technical University, Ankara, Turkey, 2020.

[25] Lavretsky E, Gibson TE. Projection operator in adaptive systems. arXiv Preprint 2011; arXiv:1112.4232.