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Binary flower pollination algorithm based user scheduling for multiuser MIMO systems

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Abstract: In this article, a multiuser (MU) multiinput multioutput (MIMO) system is considered, which is essential to support a huge number of subscribers without consuming extra bandwidth or power. Dirty paper coding (DPC) for MU MIMO channel achieves the peak sum-rate for the MU multiple antenna system at the cost of high computational complexity. Both user and antenna scheduling with a population based meta-heuristic algorithm, i.e. binary flower pollination algorithm (binary FPA) has been demonstrated in this article to achieve system sum-rate comparable to DPC with very less computational complexity and time complexity. Moreover, binary FPA shows a significant improvement in system throughput/sum-rate performance as compared to other population based meta-heuristic algorithms like binary bat algorithm (binary BA) and binary genetic algorithm (binary GA). Furthermore, the proposed binary FPA algorithm successfully achieves higher system sum-rate as compared to random search scheme and different existing suboptimal scheduling algorithms from literature as well. The binary FPA has also better convergence rate and searching ability than both binary BA and binary GA techniques. The percentage deviation achieved by the proposed binary FPA algorithm is quite less than that of binary BA, binary GA, random search method, and existing suboptimal scheduling algorithms from the literature. The efficiency of binary FPA in all these fronts is verified using exhaustive simulation studies.

Key words: Binary flower pollination algorithm, multiuser MIMO, antenna/user scheduling, dirty paper coding, computation complexity, time complexity

1. Introduction

A lot of interest has already been poured to the multiple-input multiple-output (MIMO) smart network in the recent past by different researchers [1]. This is because of its promise to achieve tremendous increase in data capacity due to spatial multiplexing benefit that can be achieved by conveying separate streams of data over each transmit-receive antenna pair at the very same time period [2, 3]. It has been observed that the channel capacity of MIMO systems increases linearly with $S = \min\{M, N\}$, where M and N are the number of base station (BS) transmitting antennas and the number of receiving antennas present at the user end to receive data respectively [4–6]. Moreover, MIMO systems deliver high reliability to the users due to spatial diversity benefit along with the tremendous increase in data capacity (which is achieved due to spatial multiplexing benefit) [7]. Furthermore, the MIMO systems delivering services to various users is referred as multi-user MIMO systems

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(MU MIMO) in the literature. These MU MIMO systems exploit a special kind of advantage due to the existence of multiple users known as multiuser diversity (MUD) [8].

In the cellular context, there can be two ways of communications possible as downlink/broadcast communications (communications from BS to users) and uplink communications (communications from users to BS). In this paper, the downlink communications of the MU MIMO systems are considered and analyzed. Multiple users are getting their services from the BS as per the MU MIMO broadcast communications. The dirty paper coding (DPC) [9] is considered to be the optimum scheme for MIMO communications. As per the DPC, the BS provides services to M number of best users simultaneously instead of providing services to all the users simultaneously [10]. Therefore, selection of M best users among all the K present users is vital for achieving the best data rate for the system. This process is popularly known as scheduling. Specifically selection of best users and antennas are termed as user scheduling and antenna scheduling respectively. First of all, user scheduling needs to be performed where the best set of users are going to be selected for data communications. After the user scheduling, the best antenna of the selected users needs to be selected as per the antenna scheduling method. The DPC is regarded as the optimum user scheduling scheme for MU MIMO downlink scenario [11]. However, the computational complexity associated with DPC is tremendously high [12]. Thus, the practical implementation of DPC is not quite feasible. Therefore, a number of suboptimal scheduling algorithms have been suggested for MU MIMO downlink transmission in [11, 13–17]. However, most of these schemes rely mainly on time division multiple access (TDMA) in which the BS only sends data to a single person in a time slot. Consequently, it was concluded that the highest possible sum-rate by making use of TDMA for the MU MIMO broadcast network is a small percentage of the total achievable throughput of the MU MIMO broadcast systems [18, 19].

The MU MIMO broadcast system's near optimum performance is accomplished by assigning several users using a process called DPC, which is an interference precancellation technique [9]. DPC has established itself as one of the best approaches to achieve MIMO broadcast channel (MIMO BC) system capacity by providing data communications to multiple users simultaneously [20, 21]. This process involves the precoding of users serially at the transmitter side and involves the process of decoding of users in the reverse manner at the receiver section. By this process the interference of the users are precancelled. Moreover, for such interference cancellation technique, the transmitter (BS) should have the full information regarding the channel states (CSI, i.e. channel state information) of all the users. This full CSI requires high feedback load in the uplink direction. This full feedback of CSI from the end users to the BS is very complex and cumbersome. Therefore, limited feedback scheduling algorithms have been considered widely in the recent past [11, 15, 17, 19, 22].

From the computational complexity perspective, the implementation of DPC is regarded as the exhaustive search algorithm (ESA), which finds the best set of users by exploring all the possible combinations of users. Therefore, the DPC process is regarded as the ESA for the user scheduling of MU MIMO BC. A total number of possible ordered selections is represented by

$$N_{OrderedUsers} = \sum_{k=1}^{M} (k!) \binom{K}{k} \tag{1}$$

It would not be feasible to run this many number of ordered selections during few coherence time period of the advanced packet data communication systems [23]. It is also not possible to search such high dimensional search space in the span of a usual scheduling interval (few coherence time periods). Recently meta-heuristic soft computing techniques are implemented for various multiantenna system models [24–26]. This motivates us to explore the possibilities of implementing an evolutionary algorithm to achieve this task.

In the search of some efficient evolutionary algorithms for solving the scheduling problem of MU MIMO broadcast scenario, bat algorithm (BA) [27], flower pollination algorithm (FPA) [28], and genetic algorithm (GA) [29, 30] are being considered in this paper due to the various advantages associated with these soft computing schemes. Futher, the binary versions of BA, FPA, and GA are considered for various comparisons. In [30], authors used GA for downlink scheduling scenario for multiuser single-carrier and multicarrier multiantenna systems. This binary GA is based on the performances of genes to find out the best solution for any objective optimization function. BA is inspired by echo locative characteristics of bats. This algorithm affects search process by the use of artificial bats as searching agents imitating the natural emission rate and pulse loudness of real bats. The binary version of BA is known as binary bat algorithm (binary BA). It uses the navigating and hunting capabilities of artificial bats in binary search spaces by alternating their positions from 0 to 1 [27]. FPA is a good candidate to solve multiobjective optimization problems [31]. FPA is inspired by the pollination process of flowering plants. FPA depends on the flowering plants' characteristics and some pollinating insects. In [32], the authors have verified that the binary FPA finds the optimal comprehensive solutions very swiftly. Moreover, an analytical study regarding the computational complexity of MIMO uplink networkers is presented in [33]. Authors in the study in [24] have used ant colony optimization (ACO) and grey wolf optimization (GWO) algorithms to solve the user and antenna scheduling jointly for multiuser multiantenna systems.

Suboptimal antenna and user scheduling algorithms with less complexity and minimal feedback are suggested as a substitute for DPC in literature [20, 22, 23]. We focus on performance analysis of system sum-rate by applying binary FPA, binary BA, random search and ESA. As seen from the results, binary FPA achieves output near optimal to DPC in a very less computation time and complexity. Furthermore, two existing sub-optimal scheduling algorithms which are based on the present system model have been considered for various comparative studies in this paper. These two suboptimal scheduling algorithms are

- the full feedback scheduling scheme of [11], where all the users feedback their CSI to the BS for the completion of the scheduling process. This algorithm is referred as suboptimal algorithm 1 throughout this paper.
- the two-bit quantized scheduling scheme discussed in [4], where all the users feedback the two bits (used to quantize their CSI) to the BS for accomplishing the scheduling task. This algorithm is referred as the suboptimal algorithm 2 throughout the paper.

Moreover, it is shown in this paper that the binary FPA outperforms the techniques of binary BA, binary GA, random search, and both the suboptimal algorithms in achieving higher system sum-rate capacity.

In the recent past various works have been proposed in the area of wireless communications using different meta-heuristic soft computing techniques. Few of them are discussed here. The concentric circular array antenna produces very good directivity gain. However, there exists problem of high sidelobes levels. To address this problem of concentric circular array antenna, authors in [34], have discussed a new approach of quantum particle swarm optimization (QPSO). This new approach of QPSO minimizes the sidelobe levels of circular antenna array with concentric rings. Different multi objective problems are successfully solved by decomposition based multi objective evolutionary algorithm (DMOEA) [35]. Authors in [35] have used DMOEA to reduce the maximum sidelobe level. DMOEA method outperforms the firefly algorithm, cuckoo search, PSO, nondominated sorting genetic algorithm-II, and nondominated sorting genetic algorithm-III. A compact triband antipodal Vivaldi antenna with frequency selective surface structures are designed by using the global and local optimization processes [36]. Authors in [36] have used honey bee mating optimization technique and differential evolutionary (DE) technique as the global optimizer and local optimizer respectively. A hybrid DE technique is proposed by the authors in [37]. This hybrid DE technique combines the advantages of numerical algorithm and evolutionary algorithm. This hybrid DE technique is used for antenna array pattern synthesis [37]. Three grey wolf inspired optimization algorithms are used for designing a single transistor low noise amplifier [38]. Authors in [24] have used ACO and GWO algorithms to solve the user and antenna scheduling jointly for multiuser multiantenna uplink systems. Invasive weed optimization (IWO) technique has been used in [39]. Authors in [39] have used IWO technique to design multilayer lenses to enhance the gain of a traditional horn antenna upto 2.9 dB.

The remaining parts of this study are configured as follows, section 2 describes the system model. Section 3 includes the user scheduling in MU MIMO BC using binary FPA. Further section 4 describes the effects of the experiment and simulation results. Binary FPA's system sum-rate performance is discussed and compared with that of binary BA, random search as well as DPC (ESA). Moreover, both the computational complexity and time complexity are analyzed with that of DPC. Finally, a brief interpretation of the various simulation results is given in section 5 as the conclusion of this paper.

2. System Model

In this article, we suggest MU-MIMO networks comprising of a BS with M transmitting antennas to the K number of subscribers (users/user terminals (UTs)) on the receiving end. Each user has a N number of receive antennas. This paper considers $M > 1$, $N > 1$, $M \geq N$ and $K >> M$. The system model is depicted in Figure 1.

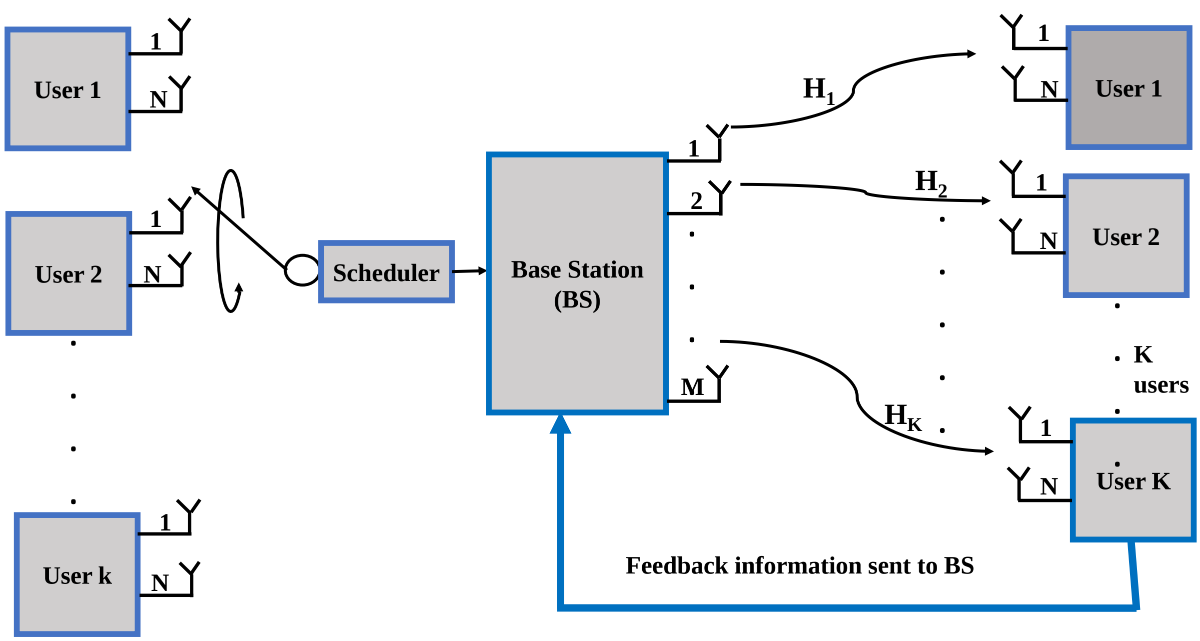


Figure 1. The MU MIMO system model for downlink transmission with M transmit antennas, K users each with N receive antennas.

The expression of the received signal of user k at time slot t is denoted as

$$\mathbf{Y}_k^t = \sqrt{\Re_k}\mathbf{H}_k^t\mathbf{X}^t + \mathbf{W}_k^t, k = 1, \ldots, K, \tag{2}$$

where Y_k^t is a received signal vector of the k^{th} user at t^{th} time slot having dimension $N \times 1$. X^t shows the transmission signal vector of dimension $M \times 1$ at time slot t and W_k^t is the $N \times 1$ additive noise vector of user k with zero mean and unit variance. H_k^t is a complex functional channel matrix of dimension $N \times M$. Each feature of this matrix is a complex channel gain coefficient from the transmit antenna m to the receive antenna n of the user k. $\sqrt{\Re_k}$ models the attenuation of power due to shadowing effect and path loss. The data transmission from all M antenna arrays of the BS is given by X^t at the time slot t which is the vector of the size $M \times 1$. The additive white Gaussian noise is w_k^t. All w_k^t components are independent and identically distributed (i.i.d) Gaussian complex with zero mean and variance N_0. The expected received signal at n^{th} antenna of the user k is denoted as:

$$y_k(n) = \sqrt{\Re_k}\sum_{m=1}^{M} h_k(n,m)x(m) + w_k(n), \tag{3}$$

where $w_k(n)$ is the additive white Gaussian noise for user k with receive antenna n. From this point on wards, the time slot parameter t is being suppressed for the simplicity of the analysis. Here $x(m)$ is assumed to be the desired signal for the k^{th} user. Then other signal $x(m')$ is the interference signal with the condition $m' \neq m$. The signal to interference noise ratio (SINR) at the k^{th} receiver for $y_k(n)$ is given by:

$$SINR_{m,n}^k = \frac{|h_k(n,m)|^2}{\left\{\left(\frac{M}{\rho_k}\right) + \sum_{m' \neq m} |h_k(n,m')|^2\right\}} \tag{4}$$

where $\rho_k = (\Re_k/N_0)$ is the average signal to noise ratio (SNR) of user k. We consider $\rho_k = \rho$, for users $k = 1, ..., K$. We consider that no interference cancellation is performed at the receiver terminal. We can define the sum-rate capacity of the system with the upper bound as,

$$\begin{aligned} &C_{sum}\left(H_1, \ldots, H_K\right) \leq \\ &\mathbb{E}\left[\sum_{m=1}^{M} \log_2\left(1 + \max_{1 \leq k \leq K, 1 \leq n \leq N} SINR_{m,n}^k\right)\right], \end{aligned} \tag{5}$$

where C_{sum} is the upper bound of the achievable system capacity and $H_1, ..., H_K$ are the channel matrices of all the users. We can alternately write the system capacity of K users having finite M value with maximum feedback of SINR as [11]

$$C_{sum}\left(H_1, \ldots, H_K\right) = \sum_{m=1}^{M} \log_2\left(1 + \max_{1 \leq k \leq K, 1 \leq n \leq N} SINR_{m,n}^k\right) \tag{6}$$

3. Scheduling using binary flower Pollination algoritm

In this section, the DPC scheduling process is elaborated in subsection 3.1, where the process of scheduling using DPC is explained. Then, the general process of FPA, binary FPA are described in subsection 3.2 and 3.3 respectively. The process of user scheduling using binary FPA is discussed and explained in subsection 3.4.

3.1. DPC scheduling

In DPC, M users are receiving service through BS simultaneously. The scheduler is responsible to select best M users in K UTs. It means that from $N \times K$ number receiving antennas, only M number of receiving antennas from different UTs should be selected. At the BS, DPC relies upon sequencing of user encoding. Selection of M number users have unique encoding order and is presented in Eq. (1), that is considered in the period of couple milliseconds. In today's world of communication system, for computing complex multiplications and additions, BS takes the help of DSP processors and if any complexity is noted, the operation is moved to the algorithm which is implemented by BS. Therefore, we put forward to assess a subset of collection of user sequences by citing below mentioned limitations:

- There is a relaxation in the systematization of user sequences, i.e. unique M users will be combined and examined at a time.
- There is no repetition in the user sequence and the user sequence has different users, i.e. data streams which are independent with regards to distinct users are sent by transmit antennas.
- Transmitting antennas should always be active and hence, every transmit antenna should assign one unique user.

Here the number of feasible unique user sequences is less than Eq. (1) and is represented by

$$N_{UniqueUserSequence} = \binom{K}{M} \tag{7}$$

It indicates that if we increase the transmitting antennas as well as users, then there will be increase in the user sequence number. Thus, this study describes the best way to decrease the computational complexity through using the procedure of binary FPA. Let us say that ϕ denotes a specific order of the number of M users out of the existing K active users. ϕ is a subset of a bigger set, i.e. the array of all possible unique user sequences as shown in Eq. (7), i.e. $\phi \in \Theta$ and $\phi_i = \left\{\phi_i^1, \phi_i^2, \phi_i^3, ..., \phi_i^M\right\}, 1 \leq i \leq |\Theta|$, where $|B|$ denotes set B set's cardinality. Then, the sum-rate capacity of the framework reached by a given selected user set is

$$C_{sum}\left(\theta, H_1, \ldots, H_K\right) = \sum_{m=1}^{M} \log_2 \left(1 + \max_{k \in \theta, 1 \leq n \leq N} SINR_{m,n}^k\right) \tag{8}$$

The set of user antennas of the current user set (θ) resulting from the highest SINR for each of the BS antennas are stored as $N_i = \{n_i^1, n_i^2, \ldots, n_i^M\}$. The current problem of scheduling receiving antennas with transmitting antennas is a combinatorial optimization problem, i.e.

$$\max_{\theta \in \Theta} \sum_{m=1}^{M} \log_2 \left(1 + \max_{1 \leq k \leq K, 1 \leq n \leq N} SINR_{m,n}^k\right) \tag{9}$$

The optimal user sequence is represented as $\theta opt = arg\max_{\theta \in \Theta} C_{sum}\left(\theta, H_1, \ldots, H_K\right)$. The set of user antennas selected for the users in θ_{opt} can be denoted as: $N_{opt} = \{\, N_{\theta_{opt}^{i=1}}^{i=1},\ \ldots,\ N_{\theta_{opt}^{i=M}}^{i=M} \,\}$,

where $N_{\theta_{opt}^i}^i = arg\max_{1 \leq n \leq N} SINR_{i,n}^{(\theta_{opt}^i)}$.

ESA evaluates all the possible combinations to maximize Eq. (9). Though scheduling is optimal, it needs to be completed within a time frame of a few ms. As such, ESA becomes computationally expensive and is inefficient in light of the requirements of the scheduling. The binary FPA has lesser computational complexity and reaches near optimum at a relatively faster rate as compared to DPC and binary BA. Thus, it becomes suitable for current scheduling problems of MU MIMO broadcast scenario.

3.2. Flower pollination algorithm (FPA)

Flower pollination algorithm is a nature motivated algorithm based on the phenomenon of flower pollination. It solves the multi objective optimization problem with multiple criteria. In consideration with the above characteristics Xin-She established a flower pollination algorithm and highlighted four basic rules for its easy utilization [28].

$(Rule1)$: The global pollination process includes cross and biotic-pollination and these pollinators carrying pollen moves such that it obeys Levy flights.

$(Rule2)$: Local pollination utilizes self and abiotic-pollination.

$(Rule3)$: Insects produce flower constancy and such pollinators are similar to the probability of reproduction, which is further proportional to the presence of two flowers.

$(Rule4)$: A switch probability would control the interaction between global and local pollination with some inclination on local pollination where $p \in [0, 1]$.

For rule 1, by converting the rule global pollination and flower constancy where the pollinators transfer the pollen gametes over a far of distance and move through Levy flight method; a mathematical expression can be represented as:

$$X_i^{(t+1)} = X_i^{(t)} + \alpha L(g_* - X_i^{(t)}) \tag{10}$$

where

$$L = \frac{\lambda.\Gamma(\lambda).\sin(\pi\lambda/2)}{\pi}.\frac{1}{S^{1+\lambda}}, \quad S >> S_0 > 0. \tag{11}$$

$X_i^{(t)}$ is the solution vector (pollen i) at iteration t and g_* indicates recent best solution amongst all solutions. The scaling factor controlling the step size is α. L denotes step size for Levy flights that signifies the strength of pollination. $\Gamma(\lambda)$ represents gamma function and the step size is denoted by S. As insects with various distance steps can fly over long distances, a Levy flight can be used to effectively copy this attribute.

Rule 2 and 3 may be expressed for local pollination as

$$X_i^{(t+1)} = X_i^{(t)} + \epsilon(X_j^{(t)} - X_k^{(t)}), \tag{12}$$

where $X_j^{(t)}$ and $X_k^{(t)}$ represent the pollen from distinct flower j and k of the same species of plant. ϵ is chosen from a uniform distribution of [0,1].

3.3. Binary flower pollination algorithm (binary FPA)

In the regular FPA, the approaches are modified towards real valued locations in the problem space. However, the sear space is modeled as a d-dimensional binary lattice in the conceptual binary FPA, for which the answers are modified around the edges of a hypercube [28]. Here, the local and global pollination are done in a continuous manner. Here, a sigmoid normalization function is being used, which is expressed as:

$$S(X_i^j(t)) = \frac{1}{1+e^{-X_i^j(t)}}, \tag{13}$$

Moreover, as the solution is only binary values, the new solution is being updated by the following expression:

$$X_i^j(t) = \begin{cases} 1 & \text{if } S(X_i^j(t)) > \sigma, \\ 0 & \text{otherwise} \end{cases} \tag{14}$$

where $\sigma \sim U(0,1)$. Figure 2 shows the main framework of binary FPA.

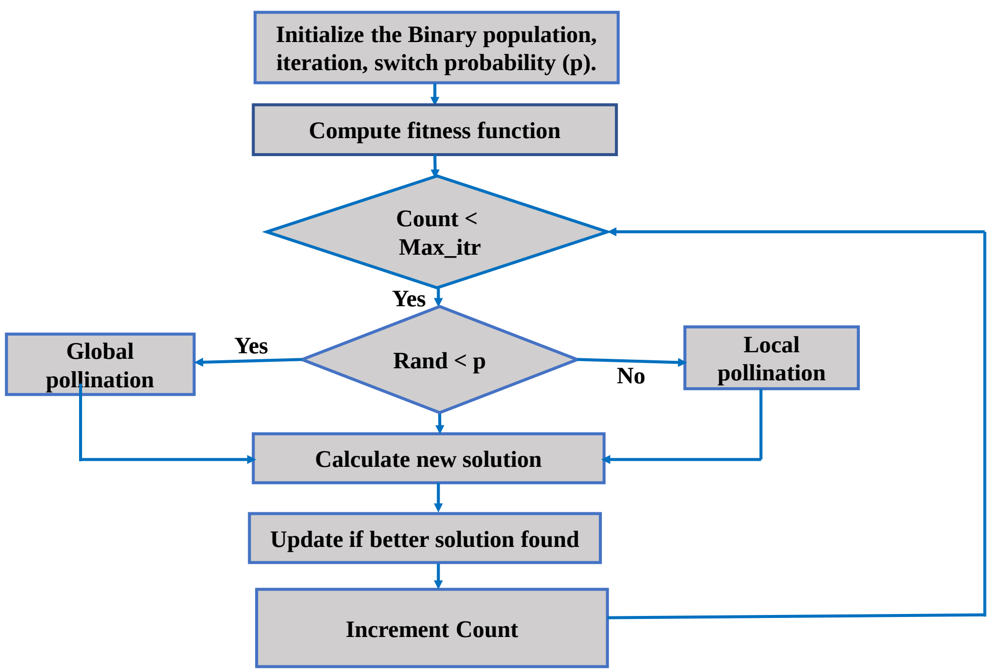


Figure 2. Framework of binary FPA.

3.4. Binary FPA for user scheduling

Various parameters and movements employed in binary FPA are shown below:

- Z is referred as the utility function which is represented in Eq. (9).
- Pop_size denotes the population size.
- P is the proximity probability in [0,1].

- D includes the binary representation of the scheduled users and of length $M \times [log_2 K]$.
- L is a parameter which denotes step size and drawn from a Levy distribution.
- Max_itr is the iteration number.
- $SelectedUT$ is a M size row vector that stores the M users to be served selected by BS.
- $Selectedreceivedantenna$ is always a size M vector that holds the receive antennas selected by the selected UTs present in $SelectedUT$.

◇ **Step 0**:

1. Initialize the population which is created randomly in D dimensional search spaces with binary values.
2. The initial solutions are generated by Eq. (14) in the proposed binary FPA.
3. A set of θ represents each row and those are represented as binary string μ.
4. The binary depiction of the user arbitrarily selected to be aided by m^{th} antenna for transmission is present in the $m^{th} log_2 K$ bits of μ.
5. For instance, a case with $K = 15$, $N = 3$ and $M = 4$. Suppose that the 6th user, 12th user,13th and 3rd user are randomly chosen to be served by four BS transmission antennas. Then, number of bits require to represent each user = $[log_2 15] = 4$. Then, the user sequence (θ) reflecting to this collection will be

$$[\overbrace{0,1,1,0,}^{6^{th}\text{ user}}\underbrace{1,1,0,0,}_{12^{th}\text{ user}}\overbrace{1,1,0,1,}^{13^{th}\text{ user}}\overbrace{0,0,1,1,}^{3^{rd}\text{ user}}].$$

◇ **Step 1**:

For better binary FPA efficiency, a set of constraints is given below:

1. The transmitting antenna must send different streams of data to various user, i.e. there should be no redundant user index in any of the population rows.
2. Each population rows should be identical, i.e. different user configurations should be tested in less time.
3. No user index should be negative or greater than the number of users by some amount, i.e. K.

If any pollen gamete breaks any constraint, so certain bits must be randomly toggled and this process can be continued until none of the above limitations are broken.

◇ **Step 2**:

1. Compute Θ with the current population number.
2. Update each pollen's fitness by assessing Eq. (9).

3. In the decreasing order of their fitness, the pollen are sorted.

4. Update $SelectedUT = \theta_{opt}$ as well as $selectedreceivedantenna = N_{opt}$.

◇ **Step 3**:

1. In this article, it is assumed that the value of p is 0.8.

2. For each iteration and each dimension, calculate a D-dimensional step vector L for all flowers in the population which obey a Levy distribution by using Eq. (11).

3. Update global best position via Eq. (10).

◇ **Step 4**:

1. If $count < max\_itr$ and random value($rand$) $< p$, find random flowers in the neighbourhood and update local best position based on Eq. (12). Go to Step 2 and follow all the steps again.

2. Increase the count by 1.

3. Evaluate Θ with the new population.

4. The throughput of the system will be the main objective of fitness which is calculated by Eq. (9).

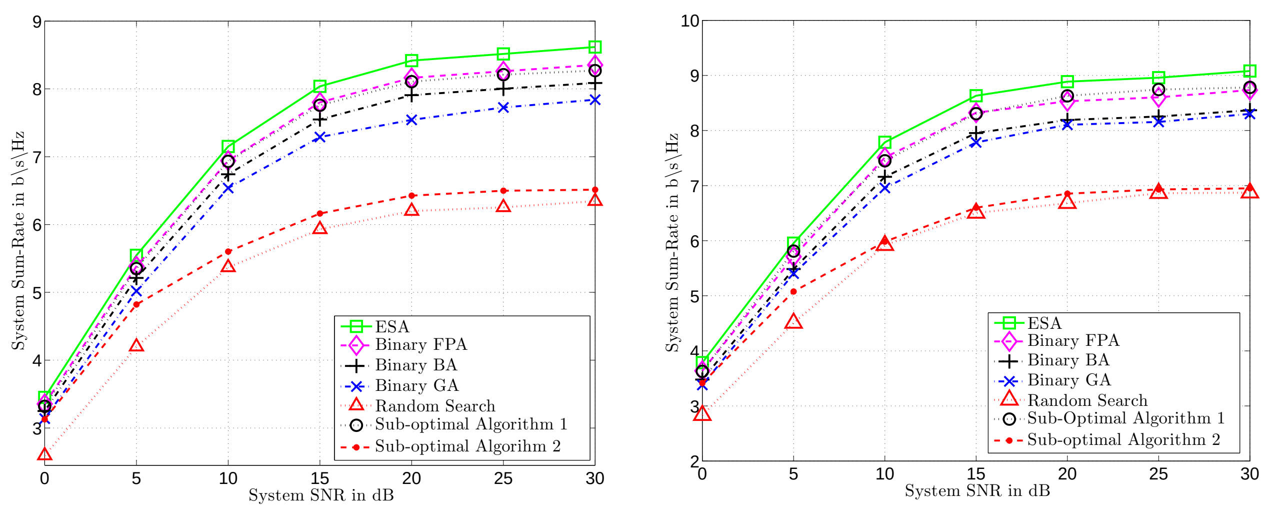
5. Update $SelectedUT = \theta_{opt}$ as well as $selectedreceivedantenna = N_{opt}$.

6. Based on the above, calculate user index, best user, and best receiving antenna index.

4. Results and Discussions

In this section, we analyze the simulation results which are obtained for different conditions and performance comparison of binary FPA, binary BA, binary GA, ESA (DPC), two existing suboptimal scheduling algorithms, and random search method. We have presented a comparison of system sum-rate for binary FPA along with binary BA, binary GA, suboptimal algorithm 1 and 2, random search, and ESA to emphasize the benefits of these methods. As per the presented simulation results in this section (afterwards) the binary FPA outperforms the binary BA, binary GA, both the suboptimal algorithms, and random search method in ascertaining better achievable system sum-rate/throughput for the MU MIMO BC systems. Further, the meta-heuristic performances of binary FPA are also presented and discussed in this section. For showcasing the meta-heuristic nature of the binary FPA, the simulation results have been presented later which depicts that the achievable system sum-rate of different MU MIMO BC systems increase with an increase of population size (i.e. Pop_size) and number of generations/iterations (i.e. Max_itr). These results are as per the meta-heuristic approach. In this section, the deviation achieved by different algorithms (viz. binary GA/BA/FPA, suboptimal algorithm 1 and 2, random search) as compared to the ESA (DPC) is also discussed and presented for various system scenarios. Moreover, the per-generation performance of binary FPA, binary BA, and binary GA are also studied to showcase the convergence ability of these three meta-heuristic algorithms/approaches for MU MIMO broadcast scenario. This has been showcased in the later part of this section that the convergence performance of binary FPA is better than that of binary BA and binary GA. Futhermore, in subsection 4.1, the complexity analysis has been performed for various algorithms. This complexity analysis highlights the advantages of binary FPA as compared to ESA (DPC) with respect to both computational and timing complexity. Also, the computational complexity has also been analyzed in the context of number of evaluations of the objective/cost function expressed in Eq. (9).

To correlate the findings, we picked two examples such as (K, N, M, Pop_size, Max_itr) = (20,4,6,20,30), and (25,5,7,25,30), which are demonstrated in Figures 3a and 3b, respectively. In a random search process, the user series is chosen randomly. From the graphs, it can be seen that binary FPA shows a better and near optimal performance as compared to binary BA, binary GA, suboptimal algorithms 1 and 2, and random search method. The probability of crossover and probability of mutation used in this paper for the binary GA are 1.0 and 0.1, respectively [30]. Therefore, the results obtained by the binary FPA scheme is compared thoroughly with that of ESA (DPC) for the rest of the results presented in this paper. Performance of random search scheme is much less than the other scheduling schemes. It is clearly observed from the Figures 3a and 3b that for the same number of Pop_size and Max_itr, the binary FPA achieves higher system throughput than binary BA and binary GA. This emphasizes that the binary FPA is a better searching algorithm than the binary BA and binary GA. However, the binary BA outperforms the binary GA in achieving higher system sum-rate.Therefore, for the scheduling process of MU MIMO broadcast scenarios, the binary FPA is a better meta-heuristic approach than the binary BA and binary GA.



(a) $K = 20$, $N = 4$, $M = 6$, $Pop\_size = 20$, and $Max\_itr = 30$.

(b) $K = 25$, $N = 5$, $M = 7$, $Pop\_size = 25$, and $Max\_itr = 30$.

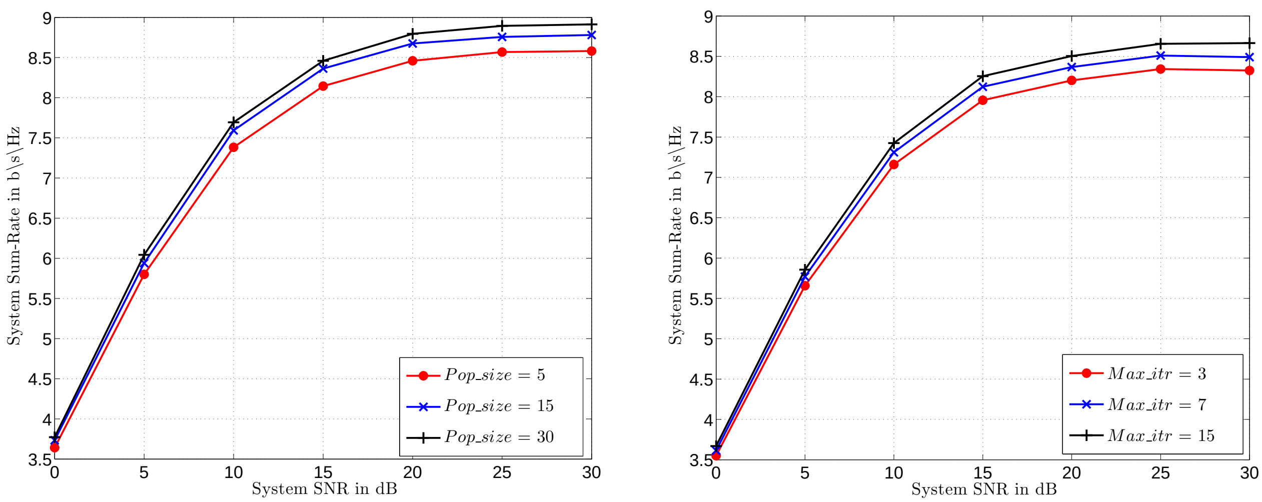
Figure 3. Comparative analysis of the sum-rate among ESA(DPC), suboptimal algorithm 1 and 2, random search, binary BA, binar BA, binary FPA for MU MIMO devices with different values of K, N, M, Pop_size and Max_itr. Every point means an average of over 1000 runs of independent simulation process.

It is ascertained that binary FPA accomplishes optimal system capacity as a DPC for a diverse variety of system SNR values. Every point in these estimates is an average of 1000 separate runs. Each run channel matrices of all users (i.e. $H_1, \ldots, H_k$) is independently produced.

Now, the meta-heuristic nature of binary FPA is being evaluated for the below mentioned two scenarios as:

- For increasing the population size (i.e. Pop_size)
- For increasing the number of generations (i.e. Max_itr)

The performance/behavior of the binary FPA is presented in Figures 4a and 4b, where the system sum-rate is presented for different values of system SNR (i.e. 0 dB, 5 dB, 10 dB, 15 dB, 20 dB, 25 dB, and 30 dB). In Figure 4a, the other system parameters considered are $K = 30$, $N = 6$, $M = 8$, $Max\_itr = 10$, and $Pop\_size = 5$, 15, and 30. Three population size ($Pop\_size = 5$, 15, and 30) have been observed. It is observed from Figure 4a that the system sum-rate capacity of the MU MIMO broadcast system obtained by binary FPA increases with an increase of the population size. This is as per the meta-heuristic characteristics. Similarly, in Figure 4b the performance of the binary FPA is evaluated for an increase in the number of generations/iterations (Max_itr). The various system parameters taken into account for the Figure 4b are $K = 30$, $N = 6$, $M = 8$, $Pop\_size = 5$, and $Max\_itr = 3$, 7, and 15. Three generation numbers ($Max\_itr = 3$, 7, and 15) have been inspected. The attainable system sum-rate capacity by the binary FPA for the MU MIMO broadcast system increases with an increase of generation/iteration numbers as shown in Figure 4b. This observation/behavior depicted in Figure 4b also complies the meta-heuristic algorithms' characteristics.

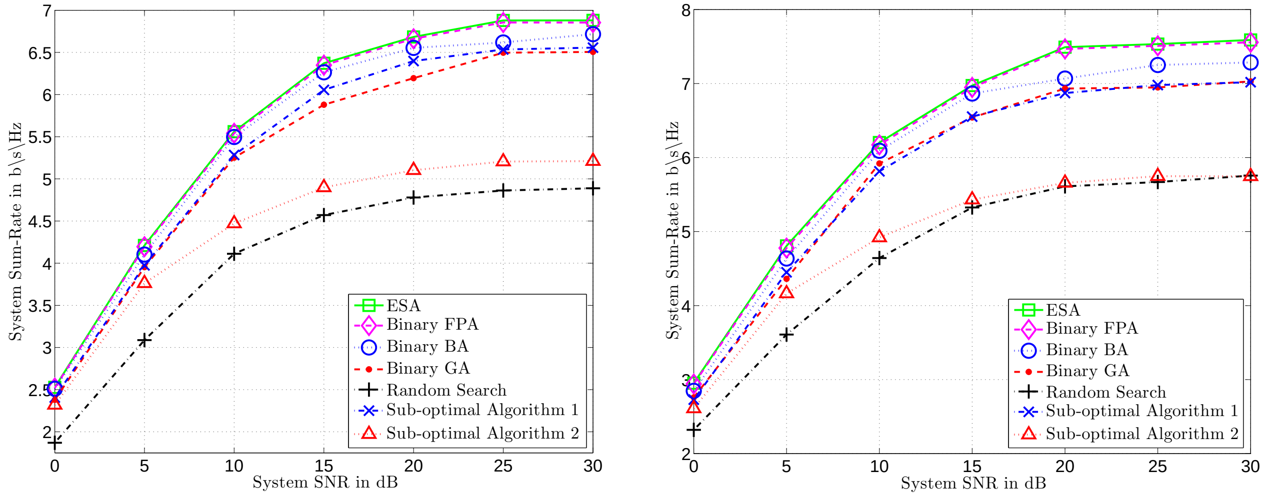


(a) $K = 30$, $N = 6$, $M = 8$, $Max\_itr = 10$, and $Pop\_size = 5$, 15, and 30.

(b) $K = 30$, $N = 6$, $M = 8$, $Pop\_size = 5$, and $Max\_itr = 3$, 7, and 15.

Figure 4. Characteristics of meta-heuristic algorithms are verified for binary FPA algorithms for MU MIMO broadcast scenarios. Every point means an average of over 1000 runs of independent simulation process.

Figures 5a and 5b show two cases where (K, N, M, Pop_size, and Max_itr) values are (10, 2, 4, 10, 30) and (12, 3, 5, 12, 30), respectively. Here, we find that binary FPA has a performance which is much similar to that of ESA, but in a very less amount of time (discussed in subsection 4.1). The binary FPA attains system sum-rate which is quite higher than that of random search algorithm, binary GA, binary BA, and both the suboptimal algorithms 1 and 2. The suboptimal algorithm 1 and 2 performs better than the random search algorithm. Binary GA performs better than suboptimal algorithm 2 and suboptimal algorithm 1 performs better than binary GA. Binary BA attains better system sum-rate than binary GA for the current system model.



(a) $K = 10$, $N = 2$, $M = 4$, $Pop\_size = 10$, and $Max\_itr = 30$.

(b) $K = 12$, $N = 3$, $M = 5$, $Pop\_size = 12$, and $Max\_itr = 30$.

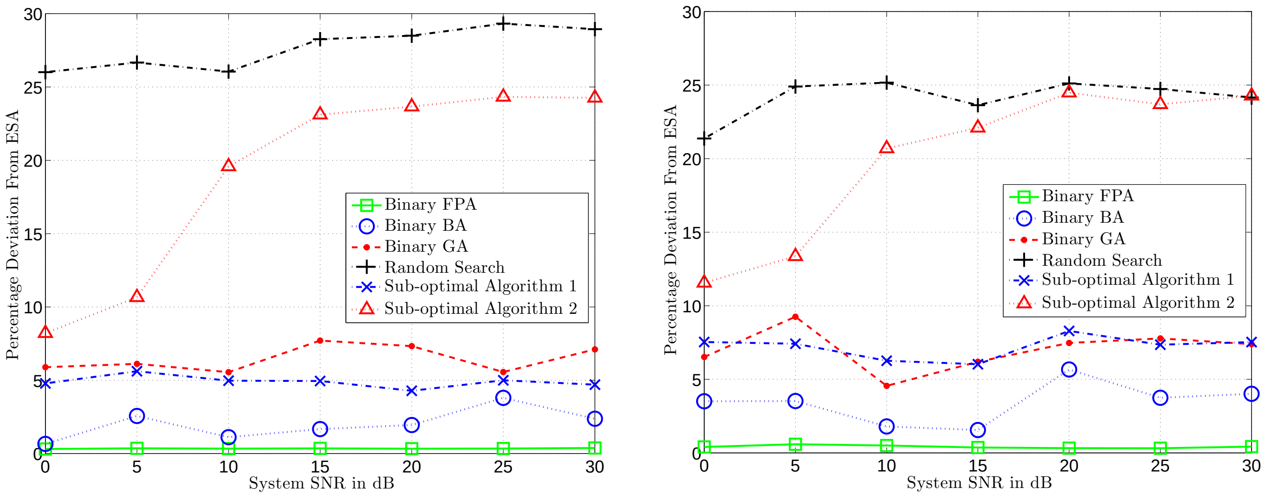
Figure 5. Comparative analysis of the sum-rate among ESA(DPC), suboptimal algorithm 1 and 2, random search, binary BA, binary GA, and binary FPA for MU MIMO devices with different values of K, N, M, Pop_size and Max_itr. Every point means an average of over 1000 runs of independent simulation process.

The outcome of the ESA (DPC) is considered to be the optimum value for the calculation of the output variable of the various approaches [25]. We determined the percentage deviation from ESA (PDESA) for random search, binary BA and binary FPA. The percentage deviation from ESA is formulated as:

$$PDESA(\phi) = \left(\frac{(C_{sum}^{ESA}) - C_{sum}^{\phi}}{C_{sum}^{ESA}} \right) \times 100, \tag{15}$$

where C_{sum}^{ESA} is the ESA's sum-rate capacity and C_{sum}^{ϕ} is optimal algorithm's sum-rate capacity. (ϕ can be binary FPA, binary BA, binary GA, suboptimal algorithm 1 and 2, or random search).The PDESA is seen in Figures 6a to 6b for two scenarios. It is fair to conclude that binary FPA contains the lowest PDESA (very near to 0) and is better than random search, binary BA, binary GA, and suboptimal algorithm 1 and 2 across a large variety of system SNR values. However, the random search method has very high PDESA values. Each point in these figures is also an estimate of 1000 independent runs. From these figures, it is observed that the PDESA achieved by binary FPA is in the interval of 0% to 1%, the PDESA achieved by binary BA and binary GA lies in the interval of 2% to 4% and 4% to 9%, respectively. The PDESA achieved by the random search algorithm lies in the range of 21% to 30%. The PDESA achieved by the suboptimal algorithms 1 and 2 lie in the range of 5% to 8% and 9% to 25%, respectively.Therefore, among all different schemes, the PDESA attained by binary FPA is very close to 0%, which is preferable for MIMO systems.

Figures 7a to 7b demonstrate the per-generation performance (i.e. the convergence ability) of binary FPA, binary GA, and binary BA for various MU MIMO broadcast scenarios. Performance improvement is observed in these two scenarios (K, N, M, Pop_size, Max_itr) = (10, 3, 4, 10, 10) and (15, 3, 5, 15, 10) with two different SNR values 10 dB and 15 dB respectively. These results also indicate that the binary FPA has a better system sum-rate values for different generations as compared to binary BA and binary GA. The per-generation performance of binary FPA is quite better than that of binary BA and binary GA. Therefore, the binary



(a) $K = 10$, $N = 2$, $M = 4$, $Pop\_size = 10$, and Max_itr=30.

(b) $K = 12$, $N = 3$, $M = 5$, $Pop\_size = 12$, and $Max\_itr = 30$.

Figure 6. Comparative evaluation of PDESA among random search, binary BA, binary GA, suboptimal algorithm 1 and 2, and binary FPA for MU MIMO system with different values of K, N, M, Pop_size, and Max_itr. Growing level displays the average output of 1000 individual simulation tests.

FPA outperforms the binary BA and binary GA with respect to the convergence ability parameter. In all the scenarios presented in Figures 7a to 7b, the binary FPA always attains higher system sum-rate capacity during the initial generations/iterations as compared to the both binary BA and GA algorithms. Moreover, the binary FPA sustains/withstands this increment in the achievable system sum-rate for rest of the generations/iterations. Therefore, the binary FPA has higher hand over the binary BA and binary GA in the context of per-generation performance for MU MIMO broadcast scenarios. Moreover, binary BA performs better than binary GA in the context of per-generation performance/convergence performance.

The generation wise performance of binary FPA, binary BA, and binary GA is also presented in a tabular format in Tables 1 and 2 for the better understanding of the readers. By referring both the tables, we can observe that the rate of increase of the achievable system sum-rate by binary FPA is greater than binary BA and binary GA. Moreover, it can be observed that the rate of increase of the achievable system sum-rate by binary BA is greater than binary GA. Furthermore, the rate of increase of the achievable system sum-rate by binary GA is the slowest one. Therefore, either more number of generations or larger population size is required by the binary GA to achieve the optimal system sum-rate value. The binary FPA is clearly observed to be better than both the binary BA and binary GA methods.

4.1. Complexity analysis

This subsection presents a comparison of the computational complexity between binary FPA and DPC. By the number of times the utility function is calculated, the computational complexity is measured. The total number of complex additions and multiplications (CAM) accomplished by the various algorithms is regarded as a measure of computational complexity which has been discussed in [25, 40]. It may be seen from Eq. (4) that the number of CAMs required for calculating SINR is $2M$. Total SINR terms existing with the user is $N \times M$. Hence for the one user $2M^2N$ number of CAMs are required for Eq. (6). Each unique sequence of M users

Table 1. Generation wise sum-rate performance comparison of binary FPA, binary BA, and binary GA. The values of the parameters are $K = 10$, $N = 3$, $M = 4$, $Pop\_size = 10$, $Max\_itr = 10$, and $SNR = 10$ dB.

Generation Number	**Binary FPA (b\s\Hz)**	**Binary BA (b\s\Hz)**	**Binary GA (b\s\Hz)**
1^{st}	5.8209	5.7726	5.7409
2^{nd}	5.8943	5.8526	5.7409
3^{rd}	5.9362	5.9001	5.7437
4^{th}	5.9649	5.9285	5.7445
5^{th}	5.9879	5.9484	5.7445
6^{th}	6.0082	5.9615	5.7467
7^{th}	6.0221	5.9739	5.7578
8^{th}	6.0352	5.9833	5.7599
9^{th}	6.0449	5.9907	5.7618
10^{th}	6.0515	5.9977	5.7626

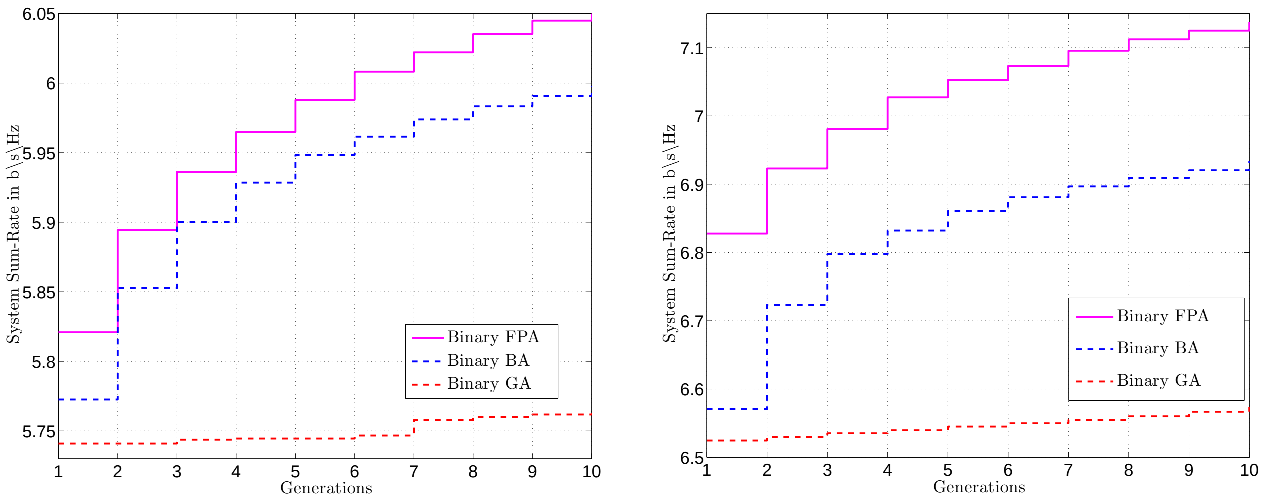
Table 2. Generation wise sum-rate performance comparison of binary FPA, binary BA, and binary GA. The values of the parameters are $K = 15$, $N = 3$, $M = 5$, $Pop\_size = 15$, $Max\_itr = 10$, and $SNR = 15$ dB.

Generation Number	**Binary FPA (b\s\Hz)**	**Binary BA (b\s\Hz)**	**Binary GA (b\s\Hz)**
1^{st}	6.8277	6.5707	6.5246
2^{nd}	6.9231	6.7233	6.5296
3^{rd}	6.9808	6.7976	6.5352
4^{th}	7.0272	6.8322	6.5396
5^{th}	7.0525	6.8606	6.5451
6^{th}	7.0733	6.8809	6.5498
7^{th}	7.0955	6.8968	6.5548
8^{th}	7.1122	6.9092	6.5600
9^{th}	7.1249	6.9204	6.5668
10^{th}	7.1375	6.9340	6.5740

requires $2M^3N$ number of CAM. The possible number of user sequences for DPC is indicated in Eq. (7).

Thus, ESA requires $\left[2M^3N\binom{K}{M}\right]$ number of CAM. Binary FPA requires $\left[2M^3N \times Pop\_size \times Max\_itr\right]$ number of CAM. To achieve this, special multicore DSP processor, e.g., Texas instruments DSP processor 66AK2Ex is taken into account by modern day data communication as discussed in [23, 25]. This has been discussed in Table 3. From the table we can conclude that, the computational time required by binary FPA is very much less as compared to ESA. However, as mentioned in previous graphs/plots, binary FPA has almost similar achievable system sum-rate capacity performance as that of ESA (DPC). The time frame for binary FPA and binary BA is almost the same. However, the achievable system sum-rate capacity performance showcased by binary FPA is quite better than that of binary BA.

The time complexity of these algorithms, i.e the binary FPA and ESA (DPC) is also showcased in Table



(a) $K = 10$, $N = 3$, $M = 4$, $Pop\_size = 10$, $Max\_itr = 10$, and $SNR = 10$ dB.

(b) $K = 15$, $N = 3$, $M = 5$, $Pop\_size = 15$, $Max\_itr = 10$, and $SNR = 15$ dB.

Figure 7. Per generation efficiency analysis of binary FPA, binary GA, and binary BA for MU MIMO network with different values of K, N, M, Pop_size, Max_itr and SNR versus iteration number. Growing level displays the average output of 1000 individual simulation tests.

3. It is clearly observed that the time required for accomplishing the number of CAMs required by ESA (DPC) for higher number of users is spanning multiple coherence time period (a coherence time period is of few ms). However, the time required for accomplishing the number of CAMs required by binary FPA for higher number of users is spanning fraction of ms time. This is well within a coherence time period. This observation is crucial for wireless communication applications since the channel parameters (channel statistics/CSI) between the BS transmit antennas and the receive antennas of all users are assumed to be block fading and quasistatic. As per this assumption the CSI is valid for only a few ms. After a few ms, the CSI of the system changes. Therefore, the user and antenna scheduling done for one block time (the fading coefficients are assumed to be constant) will not be valid for the next block time. Hence, the binary FPA is preferable over ESA (DPC) for having very less time complexity.

Table 3. Computational complexity comparison between ESA (DPC) and the proposed Binary FPA for MU-MIMO BC.

Parameters	**ESA (DPC)**		**Binary FPA**	
$[K, N, M$ $Pop\_size, Max\_itr]$	**CAM**	**Time (ms)**	**CAM**	**Time (ms)**
[10, 2, 4, 10, 30]	76800	0.2482	53760	0.0271
[15, 3, 5, 18, 30]	2252250	1.5008	405000	0.0539
[20, 4, 6, 22, 30]	11628000	49.6833	1140480	0.2680
[25, 5, 7, 25, 30]	16488010000	931.3815	2572500	0.3143

The Texas instruments 66AK2Ex DSP processor can perform up to 44.8 Giga multiply-accumulate per s (GMACS). The time complexity computed is concerning this capacity of the DSP processor. The computational complexity can also be expressed in terms of the number of evaluation of the objective/cost function. The objective/cost function for the current system model is expressed in Eq. (9). The number of the objective function computation for the ESA scheme (DPC) is expressed in Eq. (1). Moreover, the number of the objective function computation for any population based meta-heuristic algorithm depends on the population size and number of generations required. Therefore, the number of the objective function evaluation/computation for any population based meta-heuristic algorithms like binary GA, binary BA, and binary FPA is expressed as:

$$N_{Meta-heuristicAlgorithm} = Pop\_size \times Max\_itr, \tag{16}$$

where the Pop_size is the population size and Max_itr is the number of generations or iterations. The computational complexity (expressed as the number of evaluation of the objective function) associated with the different population based meta-heuristic algorithms considered in this paper and ESA (DPC) is presented in Table 4.

Table 4. Computational complexity comparison in terms of the number of evaluation of the objective function expressed in Eq. (9).

Parameters $[K, N, M$ $Pop\_size, Max\_itr]$	**ESA (DPC) Eq. (1), i.e.,** $\left[\sum_{k=1}^{M}(k!)\binom{K}{k}\right]$	**Binary FPA/BA/GA Eq.** $[Pop\_size \times Max\_itr]$
[10, 2, 4, 10, 30]	5860	300
[15, 3, 5, 18, 30]	396075	540
[20, 4, 6, 22, 30]	29891200	660
[25, 5, 7, 25, 30]	2556933625	750

5. Conclusion

In this paper, we have explored the implementation of binary FPA for efficient antenna and user scheduling for MU MIMO broadcast channel. It has been found that binary FPA achieves a significant higher sum-rate which is quite close to that obtained by ESA (DPC) with quite low time complexity as well as computational complexity. Also, when we compared the binary FPA with binary BA, binary GA, the binary FPA showed significant higher system throughput than the other two meta-heuristic schemes. Morevoer, the proposed binary FPA algorithm also attains higher system throughput than some of the existing counterpart suboptimal scheduling algorithms. Simulation results verify these findings. The proposed binary FPA algorithm attains a solution very near to the optimal value quite fast and much within the coherence period for modern wireless data communications as compared to binary BA, binary GA, random search, and some of the existing suboptimal scheduling algorithms from literature. The number of evaluation/computation of the objective/cost function for the binary FPA is quite less than that of ESA (DPC). Both the computational complexity and time complexity of the binary FPA is quite less than that of ESA (DPC). Furthermore, the generation wise performance of the proposed binary FPA technique is also quite better than binary BA and binary GA. Moreover, the binary FPA also depicts the behavior of meta-heuristic algorithms. As per the various findings of this paper, binary FPA can be a prospect candidate for implementing efficient user and antenna scheduling for MU MIMO broadcast scenarios.

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