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Performance of Prefiltered Model-Based Frequency Estimators *

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Abstract

In this work, the performance improvement due to prefiltering of inputs in model-based frequency estimators is investigated based on simulation experiments. Initial estimates on the tone frequency locations, which are obtained via DFT peak picking type preanalysis, are used to design a prefilter to remove noise and interference. The simulations indicate that prefiltering can improve the accuracy of Pisarenko and AR frequency estimators and MUSIC and KT frequency estimators with low subspace order significantly. The SNR thresholds of model-based frequency estimators are lowered by prefiltering. Additionally, interesting trade-offs between prefiltering gain and the gain due to subspace noise filtering have been investigated.

Keywords: Frequency estimation, parametric estimation, model-based estimation.

1. Introduction

The problem of frequency estimation of tone type signals in the presence of additive noise given a short record of observations is still a current problem in the theory and application of digital signal processing. This problem has been studied extensively and its applications are found in a variety of fields such as geophysics, radar, sonar, astronomy, etc. [1, 2]. When the available data record has limited number of samples, model-based spectral estimation techniques show in general superior performance. Several such methods and their performance are well documented in the literature.

However there are applications where the approximate locations of the tone frequencies are known a priori as in the case of multifrequency signalling or can be estimated using a preliminary analysis, e.g., via locating peaks in the DFT (Discrete Fourier Transform) spectrum [3, 4]. In these instances it is conjectured that prefiltering the input signal with band-pass filters at and around the spectral peaks corresponding to the presumed tone frequencies will improve the estimation performance.

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Thus in this work, we investigate the effect of prefiltering on the tone frequency estimation performance. In this respect several estimators will be considered for the tone frequency estimation problem, namely, MUltiple SIgnal Classification (MUSIC) , Kumaresan-Tufts (KT) and Yule-Walker (YW) methods. The signal model assumed consists of multiple real sinusoids

$$s_n = \sum_{k=1}^{K} A_k sin(\omega_k n + \theta_k) \tag{1}$$

observed in additive white Gaussian noise (AWGN)

$$x_n = s_n + e_n \qquad n = 1, \cdots, N \tag{2}$$

where A_k denotes the real amplitude, ω_k is the frequency, θ_k is the phase of the k'th real sinusoid K denotes the number of sinusoids e_n is a sequence of white Gaussian random variables with zero mean and variance σ^2.

The prefilter will typically be a band-pass filter, centered on the hypothesized/estimated tone frequency locations. The band-pass filter will eliminate not only the out-of-band noise, but also mitigate the interference effects. The impulse response of the prefilter which is centered around $\hat{\omega}_1$, is even and can be given as:

$$h_n = g_{n,p} \frac{sin(0.5Bn)}{\pi n} cos(\hat{\omega}_1 n) \tag{3}$$

where $g_{n,p}$ denotes some window function which is nonzero in the interval $-p \leq n \leq p$ for some integer p, B is the bandwidth of the prefilter and $\hat{\omega}_1$ is the frequency estimate which is obtained via a preliminary analysis on the data, for one of the tones. In summary, the proposed tone frequency estimation scheme consists of the following steps:

- Obtain an initial estimate of the location of the tone frequency (or frequencies).
- Use a model-based tone frequency estimator after prefiltering the *out-of-band* components with an appropriately centered bandpass filter(s).

The performance of various frequency estimators are compared against the Cramèr-Rao Bounds (CRB). The exact CRBs for line spectra are found generating the Fisher information matrix J, whose (i,j)'th element is given by [8]:

$$J_{i,j} = \frac{1}{\sigma^2} \sum_{n=1}^{N} \left[\frac{\partial s_n}{\partial \eta_i} \frac{\partial s_n}{\partial \eta_j} \right] \tag{4}$$

where η_i denotes the i'th element of the $1 \times 3K$ dimensional parameter vector η which is constituted by the unknown parameters of the K sinusoids as:

$$\eta = [A_1, \cdots, A_K, \omega_1, \cdots, \omega_K, \theta_1, \cdots, \theta_K]^T \quad . \tag{5}$$

The CRB of the variance of the i'th element of η is given by:

$$\mathrm{var}(\eta_i) \geq J_{ii}^{-1} \tag{6}$$

where J_{ii}^{-1} is the i'th element on the diagonal of the inverse of the Fisher information matrix.

As a result of prefiltering, the contaminating noise becomes colored. The case of CRBs in the presence of colored noise has also been recently investigated [9]. However our case is not compatible with the results in [9] since our filter parameters depend upon the cisoid parameters while the analysis in [9] assumes that the filter coefficients are strictly independent of the cisoid parameters we want to estimate. Notice that the filter coefficients in Eq.(3) depend upon the tone frequency estimate $\hat{\omega}$. Furthermore the cisoid signals themselves can be affected by the prefiltering stage as a result of the filter(s) being wrongly centered in the precense of noise. Therefore the effects of prefiltering on cisoid parameter estimation will have to be investigated based on extensive simulation studies.

The outline of the paper is as follows. In Section 2. the tone frequency estimators, that is MUSIC, KT and Yule-Walker frequency estimators as used in simulations, are briefly discussed. For the case $K = 1$, the single tone versions of the mentioned estimators are utilized. Different estimates of the autocorrelation (AC) matrix used in the above algoritms are also considered. Section 3. contains details of the performed simulation experiments.

2. The Tone Frequency Estimators

For the various estimates of the AC-matrix consider the following organizations of the data:

$$\begin{aligned} L_m &= \begin{bmatrix} x_1 & \cdots & 0 & 0 \\ \vdots & \ddots & \ddots & \vdots \\ x_{m-1} & \cdots & x_1 & 0 \end{bmatrix}_{(m-1)\times m} \\ T_m &= \begin{bmatrix} x_m & \cdots & \cdots & x_1 \\ x_{m+1} & \cdots & \cdots & x_2 \\ \vdots & & & \vdots \\ x_N & \cdots & \cdots & x_{N-m+1} \end{bmatrix}_{(N-m+1)\times m} \\ U_m &= \begin{bmatrix} 0 & x_N & \cdots & x_{N-m+2} \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & x_N \end{bmatrix}_{(m-1)\times m} \end{aligned} \tag{7}$$

$$X_m = \begin{bmatrix} L_m \\ T_m \\ U_m \end{bmatrix}_{(N+m-1)\times m} \tag{8}$$

The dimension m of the above matrices should be chosen as $\frac{N}{2}$ approximately, in an effort to maximize both m and $(N-m)$ at the same time where m determines the subspace order and $(N-m)$ represents the index for averaging the statistics of the data. The Least Squares Linear Prediction (LSLP) estimator is based on the equation:

$$R_m \begin{bmatrix} 1 \\ a_1 \\ \vdots \\ a_{m-1} \end{bmatrix} = \begin{bmatrix} \rho_{m-1} \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{9}$$

where $\mathbf{a}_{m-1} = [a_1 \cdots a_{m-1}]^T$ is the autoregressive (AR) parameter vector of order $(m-1)$ and ρ_{m-1} is the corresponding prediction error power. In Eq. (9) R_m is estimated as:

$$R_m = X_m^\dagger X_m = \begin{bmatrix} r_m(0,0) & \cdots & r_m(0,m-1) \\ \vdots & & \vdots \\ r_m(m-1,0) & \cdots & r_m(m-1,m-1) \end{bmatrix} \tag{10}$$

where $(\cdot)^\dagger$ denotes conjugate transpose operation. This method is referred to as the *autocorrelation* or *window* method of LSLP-analysis. It can be shown that:

$$\check{r}(i-j) = \frac{r_m(i,j)}{N} \tag{11}$$

is a biased sample estimate of the autocorrelation terms. Recall that the *autocorrelation method* of the LSLP analysis is equivalent to the Yule-Walker method in the AR-parameter estimation.

2.1. The Case of Multiple Real Tones

2.1.1. MUSIC Frequency Estimator

To obtain the MUSIC frequency estimator first the following sample estimate of the autocorrelation matrix is built:

$$\hat{R}_m = \frac{1}{N-m+1} T_m^\dagger T_m \ . \tag{12}$$

This matrix is decomposed into its eigenvectors as:

$$\hat{R}_m = \sum_{i=1}^{2K} \lambda_i \mathbf{v}_i \mathbf{v}_i^\dagger + \sum_{i=2K+1}^{m} \sigma_\omega^2 \mathbf{v}_i \mathbf{v}_i^\dagger \tag{13}$$

where σ_ω^2 is the noise variance, the set $\{\mathbf{v}_i, i = 1, 2, ..., 2K\}$ are the m-dimensional principal eigenvectors which span the signal subspace, $\{\lambda_i, i = 1, 2, ..., 2K\}$ are the corresponding ordered eigenvalues corresponding to $\mathbf{v}_i$ such that $\lambda_i > \lambda_{i+1}$ and $\{\mathbf{v}_i, i = 2K+1, ..., m\}$ are the noise subspace eigenvectors which are orthogonal to principal eigenvectors. The MUSIC power spectrum is given as:

$$P_{MUSIC}(f) = \frac{1}{\sum_{k=2K+1}^{m} |\mathbf{s}^\dagger(f)\mathbf{v}_k|^2} \tag{14}$$

where $\mathbf{s}^\dagger(f)$ is the complex sinusoidal vector:

$$\mathbf{s}^\dagger(f) = [1 \ e^{-j2\pi f} \ \cdots \ e^{-j2\pi(m-1)f}]. \tag{15}$$

The estimated tone frequency (or frequencies) is (are) found picking the K peak-pairs of this power spectrum which correspond to K real tones. The dimension m of the sample estimate of the autocorrelation matrix is used as a suffix in the name of the estimator, such that "MUSIC-m".

2.1.2. Kumaresan-Tufts (KT) Method

KT method is an extention of LSLP method. For this method the following data matrix and vector are generated:

$$T_m^{'} = \begin{bmatrix} x_m & \cdots & \cdots & x_1 \\ x_{m+1} & \cdots & \cdots & x_2 \\ \vdots & & & \vdots \\ x_{N-1} & \cdots & \cdots & x_{N-m} \end{bmatrix} \qquad \mathbf{x}_{m+1} = \begin{bmatrix} x_{m+1} \\ x_{m+2} \\ \vdots \\ x_N \end{bmatrix} \ . \tag{16}$$

Then the AR parameter vector $\mathbf{a}_m = [a_1 \cdots a_m]^T$ will be found as:

$$\mathbf{a}_m = -(T_m^{'})^{\#} \mathbf{x}_{m+1} \tag{17}$$

where

$$(T_m^{'})^{\#} = \sum_{i=1}^{2K} \frac{1}{\lambda_i} \mathbf{u}_k \mathbf{v}_k^{\dagger}$$

denotes the Moore-Penrose pseudo-inverse of $T_m^{'}$. Here $\{\mathbf{u}_k\}$ and $\{\mathbf{v}_k\}$ are the left and right singular vectors of $T_m^{'}$. The frequency estimates are found as the roots of the polynomial:

$$A(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_m z^{-m} \quad . \tag{18}$$

The number of rows of the data matrix T_m which is given by m is used as a suffix in the name of the estimator, such that "KT-m".

2.2. The Case of a Single Real Tone

The special form of the MUSIC estimator, when the *autocorrelation method* is used as in Eq. (11) and when the noise subspace is one dimensional, that is when $m - 2K = 1$, is called the Pisarenko estimator. In the case of a real single tone $K = 1$, and the 3 by 3 AC-matrix, is constructed using biased autocorrelation coefficients as:

$$\hat{R}_3 = \frac{1}{N} X_3^{\dagger} X_3 \tag{19}$$

where X_3 is defined as in Eq. (8). Then since $\hat{R}_3$ is Toeplitz, the following explicit formula can be used to obtain Pisarenko frequency estimate (PISFE) [5]:

$$\begin{aligned} \hat{\omega}_{PISFE} &= \arccos \frac{\gamma}{2} \\ \gamma &= \frac{\check{r}(2) + \sqrt{\check{r}^2(2) + 8\check{r}^2(1)}}{2\check{r}(1)}. \end{aligned} \tag{20}$$

We call the ordinary MUSIC frequency estimator when $K = 1$ and $m = 3$ as MUSIC-3. Then one uses *non-windowed* or *covariance* method for AC-matrix estimation and substituting 3 for m in Eq. (12) $\hat{R}_3$ can be found as:

$$\hat{R}_3 = \frac{1}{N - 3 + 1} T_3^{\dagger} T_3 \tag{21}$$

If an estimation formula as in Eq. (21) is used, then the AC-matrix is not any more Toeplitz and the frequency is estimated via an eigenanalysis as in Eq. (14).

In addition, for the single tone case only, we used a second estimator, namely, the autoregressive frequency estimate (ARFE), which is obtained by solving for the angle of the root of the second order AR polynomial (with coefficients a_1 and a_2) using the Yule-Walker method. The ARFE is given by [6]:

$$\hat{\omega}_{ARFE} = \arctan \frac{-\sqrt{4a_2 - a_1^2}}{a_1}. \tag{22}$$

3. Simulation Results

In the extensive simulation experiments, the sample size is taken as 30 points and 64 points, in the single and double tone cases, respectively. The variance estimates are obtained by averaging 100 realizations. The center frequencies of the prefilters are estimated via DFT spectrum peak picking. The prefilter impulse response is given by Eq. (3). In the single and double tone cases, the rectangular window and the minimum 3-term Blackman-Harris window with 67 dB sidelobe reduction are used, respectively. The effective bandwidth of the prefilter is 0.14 for the single tone case and 0.33 for the double tone case and SNR is 5 dB, unless otherwise specified. Finally the local SNR for the k'th sinusoid is defined as

$$SNR_k = 10\log_{10}\frac{A_k^2}{\sigma^2} \quad . \tag{23}$$

3.1. The Case of a Single Real Tone

3.1.1. Frequency Dependence of Prefiltering Gain

In Figure 1 the sample variances of MUSIC frequency estimator of 3rd order (MUSIC-3), KT frequency estimator of 3rd order (KT-3), PISFE and ARFE with and without prefiltering for a single tone in AWGN problem are plotted against the tone frequency. The Cramèr-Rao Bound (CRB), which is the lowest possible error for an unbiased estimator, is also shown in the subplots. It can be observed that the prefiltering makes the frequency estimator MUSIC-3 and KT-3 nearly attain CRB, with a 13 dB reduction in variance on the average (refer to Figure 2 To show the improvement of the prefiltering method the sample variance of the utilized DFT peak picking results is also shown in the first subplot. PISFE and ARFE also attain significant variance reductions when the prefiltering method is utilized as shown in Figure 1 For the simulation of PISFE and ARFE in Figure **??** no data windowing and no prefilter windowing were used as also in the cases of MUSIC and KT frequency estimators. But at high SNR's PISFE and ARFE type frequency estimators suffer from scalloping loss which is defined to be the loss in the processing gain for a tone of frequency midway between two DFT bins [10]. So data and prefilter windowing will be required. This windowing in turn degrades the performance improvement of prefiltering and makes the lowest attainable sample variance equal to the sample variance of the utilized prior frequency estimate. So prefiltering method is not advisable for PISFE and ARFE, on the other hand it is advisable for the estimators such as MUSIC or KT of low order.

3.1.2. SNR and Subspace Order Dependence of Prefiltering Gain

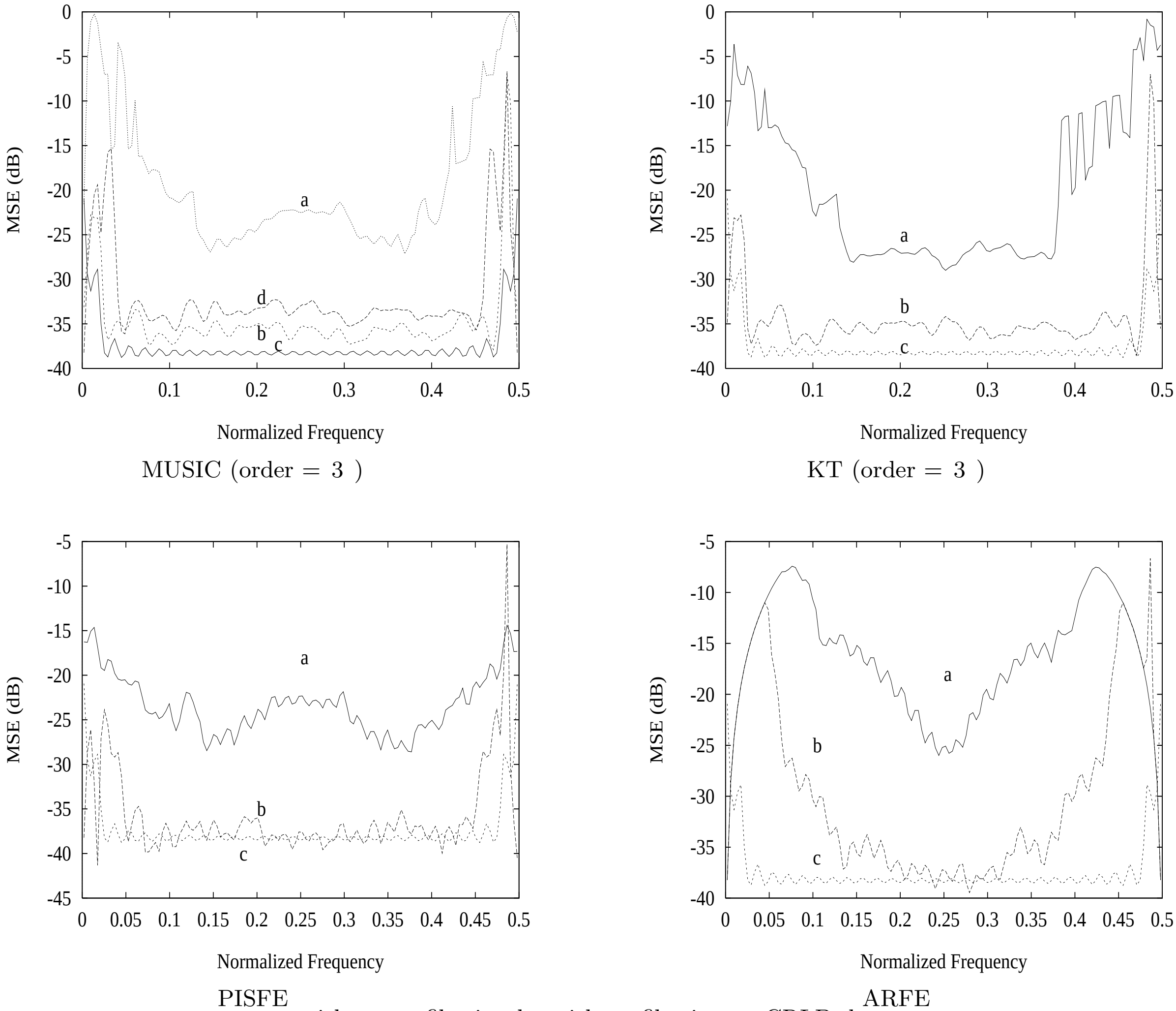
In Figure 2, the improvement in sample variance due to prefiltering is shown as a function of SNR for the estimators MUSIC and KT. The curves represent the average gain in the Mean Square Error (MSE) on the whole frequency range. It can be argued that a prefilter, determined by DFT peak picking method, becomes useful when the SNR is greater than or equal to 0 dB. The lack of gain with prefiltering below 0 dB is mostly due to uncertainties in tone location estimation, i.e., as in the DFT peak picking procedure. The improvements attain their highest value, which is approximately 14 dB both for MUSIC-3 and KT-3, when $SNR \approx 4dB$. The improvement for high SNR's is more than 6 dB for MUSIC-3, while it is 2 dB for prefiltered KT-3. Though prefiltering seems to contribute more to MUSIC-3 than to KT-3, their performances become equal only after the prefiltering, as the MUSIC-3 starts from a more disadvantaged level, in other words without prefiltering the variance of KT-3 estimate is smaller than the variance of MUSIC-3 estimate . When the order of these estimators is larger, the prefiltering does not yield any improvements. This fact is demonstrated in the prefiltering gain curves for MUSIC-6 and KT-6. Obviously an effective noise filtering has been realized via the subspace method, and there is no room left for prefiltering improvements.

3.1.3. Comparison of Subspace Filtering and Prefiltering Gains

To clarify the lack of variance improvement when the subspace order is high, it is useful to compare the variance reduction of *subspace filtering* and prefiltering. The role of prefiltering in subspace related methods is to recuperate the loss of performance when a lower subspace dimensionality is used up to the level when the optimum dimensionality is chosen. To illustrate this point, in Figure 3 two gain curves are plotted as a function of SNR. The two gain items in this figure are as follows:

1. *Subspace filtering* gain is measured by finding the difference of sample variances of the MUSIC algorithm with orders 6 and 3.

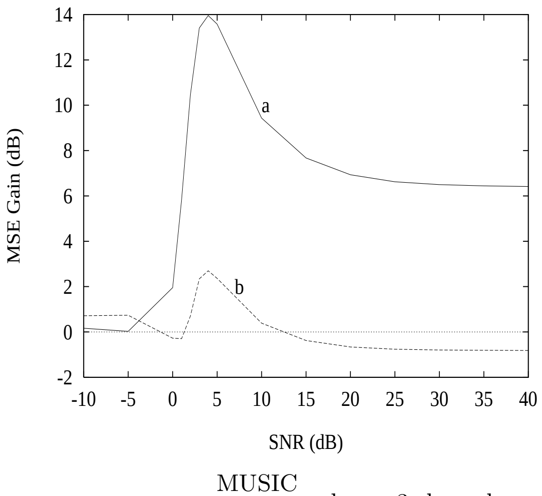
2. Prefiltering gain is measured by finding the difference of sample variances of the MUSIC-3 with prefiltering and without any prefiltering.

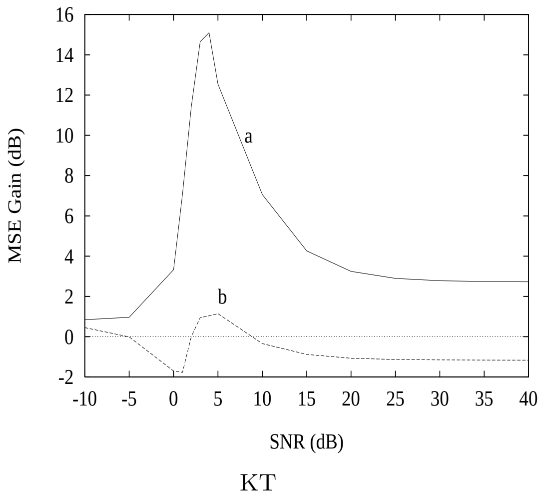


a: without prefiltering b: with prefiltering c: CRLB d: DFT analysis

Figure 1. Variances of MUSIC-3, KT-3, PISFE and ARFE frequency estimators before and after prefiltering versus normalized frequency (SNR=5 dB, sample size=30, effective bandwidth of the prefilter=0.14, average of 100 simulation runs)

A third curve representing the *Gain* of Maximum Likelihood Estimate (MLE) which is the difference of the variance of MLE and the sample variance of MUSIC-3 without prefiltering is also plotted in this figure in order to show the performance of estimators after *subspace filtering* or *prefiltering* w.r.t. CRB. According to the simulation results which are shown in Figure 3, the subspace filtering gain and prefiltering gain show the same behaviour when the SNR is increasing. Either subspace filtering or prefiltering method make the model-based estimators nearly attain the CRB. The results for KT frequency estimator are similar to the results for MUSIC frequency estimator [7].





a: order = 3, b: order = 6

Figure 2. Improvements in sample variances of MUSIC and KT frequency estimators averaged over the whole frequency range versus SNR (sample size=30, effective bandwidth of the prefilter=0.14, average of 100 simulation runs)

3.1.4. Dependence Of Prefiltering Gain Upon Prefilter Bandwidth

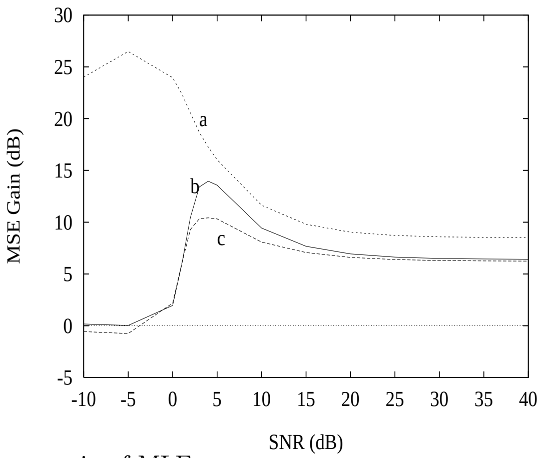
The less the uncertainty about the location of tone frequencies, the narrower a band-pass filter can be afforded. In Figure 4, the variance reduction gain of the prefiltering method is plotted as a function of the bandwidth parameter B of the prefilter. As the effective bandwidth of the prefilter increases the noise rejection capability of the prefilter decreases, which in turn results in a loss of gain. On the other hand when we continue decreasing B even after the prefilter length dependent minimum effective bandwidth is reached, the prefiltering gain does not increase any more. As can be seen in Figure 4 for the dimensions of data we analyzed the bandwidth parameter B of the prefilter need not be less than approximately 0.1.

3.2. The Case of Two Real Tones

In these simulations we kept one tone frequency fixed at $\omega_1 = \pi/2$, while the other tone frequency changed from $\omega_2 = 0$ to $\omega_2 = \pi$. The gain results are given in terms of SNR and the sample variances are plotted against the frequency difference parameter $\Delta\omega = \omega_2 - \omega_1$.

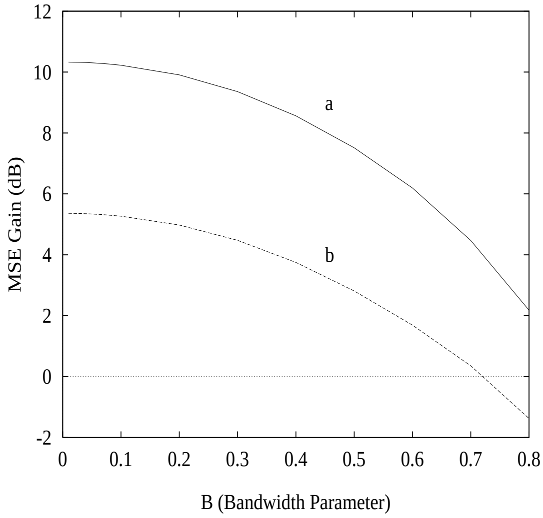
3.2.1. Frequency Dependence of Prefiltering Gain

In Figure 5, the sample variances of the fixed tone and varying tone frequency estimates with and without prefiltering are plotted against $\Delta\omega$ for the 5'th order KT and MUSIC frequency estimators. The prefilter now has two pass-bands centered around the two estimated tone frequencies. Like the single tone case, it can be seen that prefiltering method offers a gain of about 10 dB for a wide range of $\Delta\omega$'s. The gain disappears however when $\Delta\omega$ gets too small, obviously, as the two spectral peaks alias to each other due to filter main lobe. As it is shown in the figure, the results for the shifting tone are similar to the results of the fixed tone.



a: gain of MLE
b: prefiltering gain
c: subspace gain

Figure 3. Comparison of the gains achieved with subspace filtering (MUSIC-6) and with prefiltering (MUSIC-3) both w.r.t. MUSIC-3 without prefiltering (sample size=30, effective bandwidth of the prefilter=0.14, average of 100 simulation runs)



a: prefiltered MUSIC-3
b: prefiltered KT-3

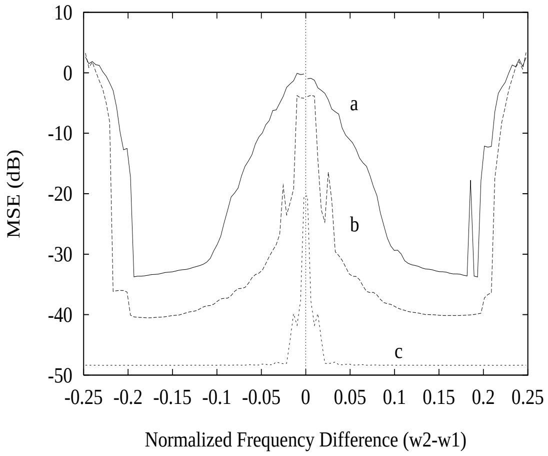
Figure 4. Drop in sample variances of MUSIC-3 and KT-3 averaged over the whole frequency range versus bandwidth parameter B of the prefilter (SNR=5dB, sample size=30, 15 tap prefilter, average of 100 simulation runs)

3.2.2. SNR and Subspace Order Dependence of Prefiltering Gain

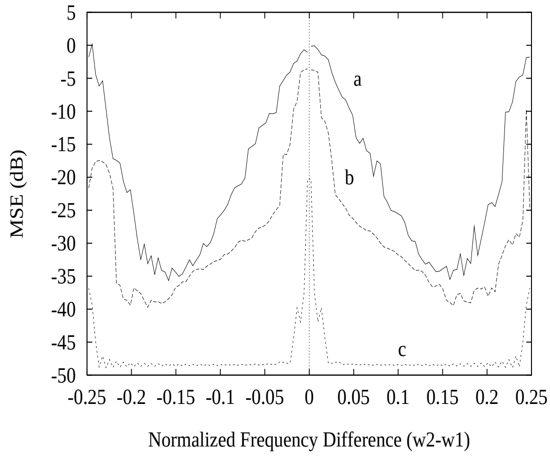
In Figure 6 the improvement in sample variance due to prefiltering is shown as a function of SNR for the estimators MUSIC and KT. The curves represent the average gain in the Mean Square Error (MSE) on the whole frequency range for the fixed tone. Again the results for the shifting tone are similar to the results of the fixed tone. The improvements attain their highest value , which is more than 20 dB both for MUSIC-5 and KT-5, when $SNR \approx 0\ dB$. The improvement for high SNR's is around 5 dB for MUSIC-5, while it disappears for prefiltered KT-5. As it is in the one tone case, the difference of these gains is deceiving since the performances of MUSIC-5 and KT-5 become equal only after the prefiltering, as MUSIC-5 starts from a more disadvantaged level. When the subspace order is 10, no prefiltering gain is observed for the estimators excluding the SNR range between -5 dB and 0 dB. The prefiltering gain peaks for the mentioned estimators at these SNR values are due to the effect of prefiltering on these estimators to lower their *SNR thresholds* at which those estimators become effective.

3.2.3. Comparison of Subspace Filtering and Prefiltering Gains

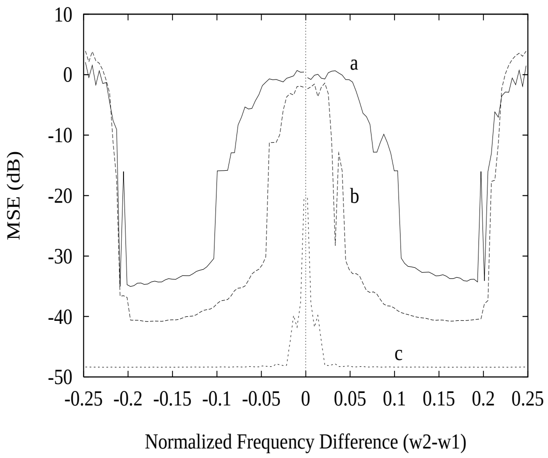
We have also compared subspace and prefiltering gains for the case of two real tones. In Figure 7 again three gain curves are plotted as a function of SNR. As in Figure 6, the curves represent the average gain in the Mean Square Error (MSE) on the whole frequency range for the fixed tone. As it is in the single tone case, the curves represent subspace, prefiltering and MLE gains for MUSIC frequency estimator for the fixed and varying frequency tones. The higher subspace order is 10 and the lower one is 5 in this case. Again the pass-bands of the prefilters are determined via DFT peak picking. According to the plotted simulation results the subspace filtering gain is now approximately 4 dB higher than the prefiltering gain. But both gains show again the same behaviour when the SNR is increasing. The results obtained with KT frequency estimator are similar to the results obtained with MUSIC.



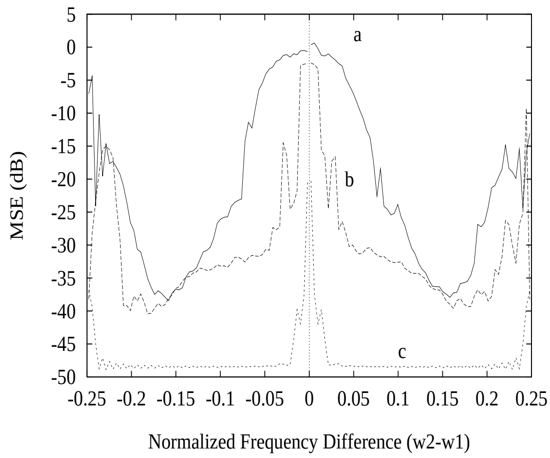
MUSIC-5 fixed frequency tone ($\omega_1 = \frac{\pi}{2}$)



MUSIC-5 varying frequency tone



KT-5 fixed frequency tone ($\omega_1 = \frac{\pi}{2}$)



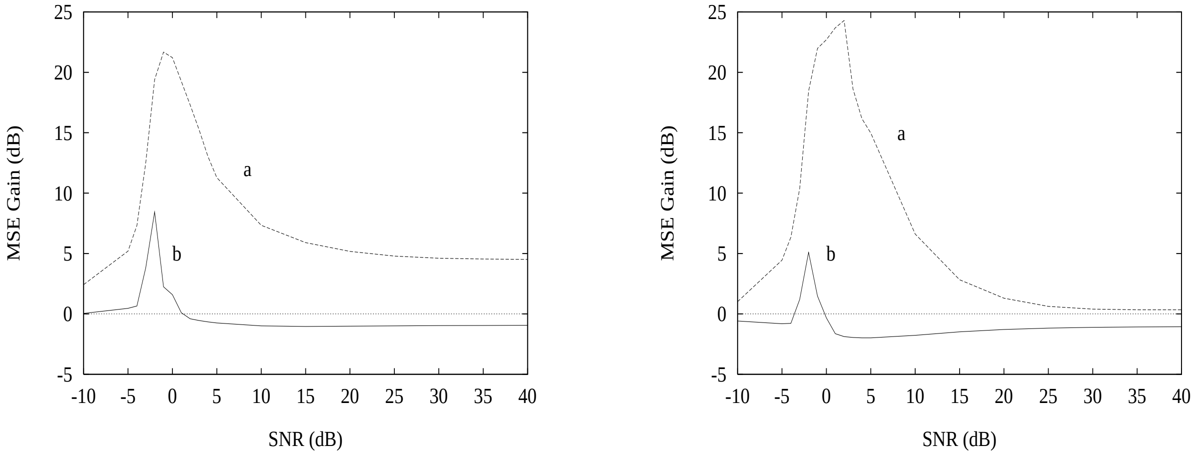
KT-5 varying frequency tone

a: without prefiltering

b: with prefiltering

c: CRLB

Figure 5. Variances of the two tone frequencies for MUSIC-5 and KT-5 before and after prefiltering versus normalized frequency difference (SNR=5 dB, sample size=64, effective bandwidth of each pass-band of the prefilter=0.33, average of 100 simulation runs)



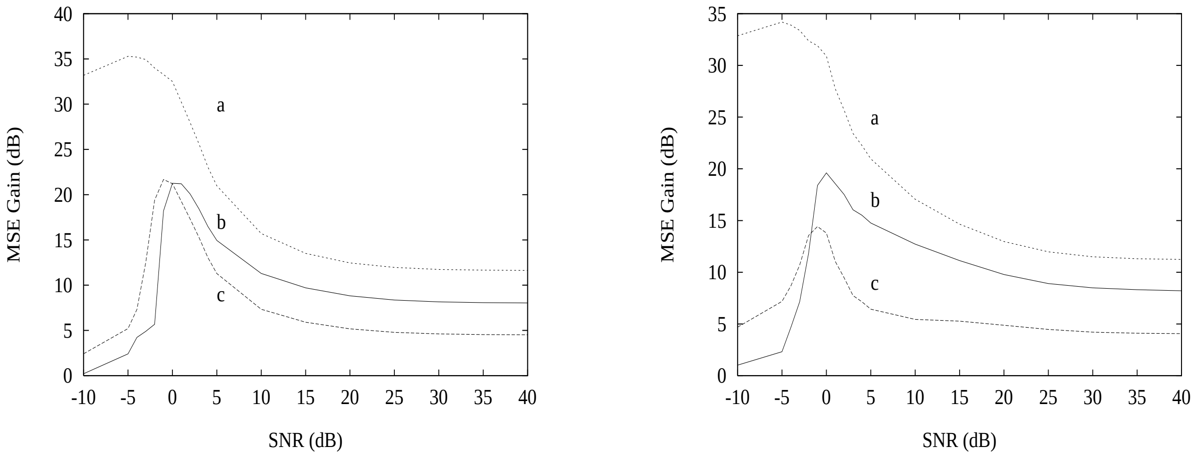
MUSIC fixed frequency tone ($\omega_1 = \frac{\pi}{2}$)

a: order = 5

b: order = 10

KT fixed frequency tone ($\omega_1 = \frac{\pi}{2}$)

Figure 6. Improvements in sample variances of the fixed frequency tone in the two tones case for MUSIC and KT frequency estimators averaged over the whole frequency range versus SNR (sample size=64, effective bandwidth of each pass-band of the prefilter=0.33, average of 100 simulation runs)



fixed frequency tone ($\omega_1 = \frac{\pi}{2}$)

a: gain of MLE

b: subspace gain

c: prefiltering gain

varying frequency tone

Figure 7. Comparison of the gains achieved with subspace filtering (MUSIC-10) and with prefiltering (MUSIC-5) both w.r.t. MUSIC-5 without prefiltering in the two tones case (sample size=64, effective bandwidth of each pass-band of the prefilter=0.33, average of 100 simulation runs)

4. Conclusions

In view of the above simulation results one can conclude that:

- The fact that with prefiltering sample variances which are smaller than the sample variances of utilized prior frequency estimates can be obtained, validates the usefulness of prefiltering (refer to Figure 1).
- Prefiltering can improve the performances of model-based frequency estimators. This improvement is higher for the initially disadvantaged estimators such as MUSIC-3 and KT-3 in the single tone case or MUSIC-5 and KT-5 in the two tones case.
- For PISFE and ARFE type frequency estimators which use autocorrelation lag estimates in the frequency estimation, prefiltering method is not advisable. The scalloping loss which becomes more effective as the SNR increases, necessitates data and prefilter windowing for these estimators. So the sample variance of these estimators do not become lower than the variance of prior frequency estimates.
- Using a coarse (low resolution) frequency estimator such as DFT peak picking procedure, the SNR thresholds of model-based frequency estimators can be lowered since DFT peak picking frequency estimation has lower SNR threshold than the model-based frequency estimators.
- Prefiltering gain and Subspace gain exhibit similar SNR trends and these gains can be improved nearly until the CRB is reached.
- Since prefiltering method with low subspace order can be used as a substitute for high subspace order for model-based frequency estimators, prefiltering method may be preferred where increasing the subspace order may not be wanted as in the case of array processing where it necessitates the number of transducers to be increased.

References

[1] S.M. Kay, and S.L. Marple, "Spectrum Analysis: A Modern Perspective", Proc. IEEE, Vol.69, November 1981, pp. 1380-1419.

[2] D.W.Tufts, R. Kumerasan, "Estimation Of Frequencies Of Multiple Sinusoids: Making Linear Prediction Perform Like Maximum Likelihood", Proc. IEEE, Vol. 70, September 1982, pp.975-989.

Algorithm For Frequency Estimation Of A Sinusoid Based On Newton's Method", IEEE Trans. on Acoustic Speech and Signal Processing, Vol. 33, February 1985, pp. 77-89.

[3] R. Kumaresan, Y. Feng, "FIR Prefiltering Improves Prony's Method", IEEE Trans. on Acoustic Speech and Signal Processing, Vol. 39, March 1991, pp. 736-741.

[4] F. Hu, T. K. Sarkar, Y. Hua, "Utilization of Bandpass Filtering for the Matrix Pencil Method", IEEE Trans. on Signal Processing, Vol. 41, January 1993, pp. 442-446.

From A Covariance Function", Geophys. J. Royal Astron. Soc., Vol. 33, September 1973, pp. 347-366.

[5] E.Anarım, Y. Istefanopulos, "Statistical Analysis Of The Pisarenko Type Tone Frequency Estimator ", Signal Processing , Vol. 24, 1991, pp. 291-298.

[6] E. Anarım, B. Sankur, "Statistical Analysis Of Tone Frequency Estimators Based On Autoregressive Models", The Fourth Inter. Conf. on Computer and Inf. Sciences (ISCIS IV), İzmir, Turkey, October 1989, pp. 1321-1329.

[7] M. A. Altınkaya, E. Anarım, B. Sankur, "Effects of Prefiltering on Model-Based Tone Frequency Estimators", MELECON, IEEE 7. Mediterranean Electrotechnical Conference, 12-14 April 1994, pp. 92-95, Antalya.

Parametric Estimation with Constraints", IEEE Trans. on Information Theory, Vol. 26, November 1990, pp. 1285-1301.

Analysis, Prentice Hall, 1997.

[8] D. C. Rife and R. R. Boorstyn, Multiple Tone Parameter Estimation from Discrete-Time Observations, The Bell System Technical Journal, Vol. 55, No. 9, pp. 1389-1410 November 1976.

[9] P. Stoica, A. Jacobsson, J. Li, "Cisoid Parameter Estimation in the Colored Noise Case: Asymptotic Cramèr-Rao Bound, Maximum Likelihood, and Nonlinear Least-Squares", IEEE Trans. on Signal Processing, Vol. 45, No. 8, pp. 2048-2059, August 1997.

[10] F. J. Harris, On the Use of Windows for Harmonic Analysis with the Discrete Fourier Transform, Proceedings of the IEEE, Vol. 66, No. 1, pp. 51-83 January 1978.