Optimum Design of Space Trusses with Buckling Constraints by Means of Spreadsheets

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Abstract

In this paper, an algorithm is developed for the optimum design of space trusses with the help of spreadsheets. The algorithm depends on the interactive computing capability of spreadsheets. A general purpose optimization tool in spreadsheets is used for the optimization procedures. The analyses of space trusses are performed by the matrix displacement method. Specific macros have been developed for matrix calculations related to the truss systems. The displacement, tensile stress, buckling stress and minimum size constraints are considered in the formulation of the design problem. A number of design examples are presented to demonstrate the application of the algorithm. The optimum designs obtained using the spreadsheets are compared with those where a classical optimization method is employed.

Key Words: Space Truss; Displacement; Buckling; Optimization; Spreadsheet

Uzay Kafes Sistemlerin Burkulma Sınırlayıcıları Altında Çalışma Tablolarıyla Optimum Boyutlandırılması

Özet

Bu çalışmada, uzay kafes sistemlerin optimizasyonunu çalışma tablolarıyla gerçekleştiren bir algoritma geliştirilmiştir. Algoritmanın temeli, çalışma tablolarının etkileşimli hesaplama yeteneklerine dayanmaktadır. Kafes sistemin analizi, matris-deplasman yöntemiyle yapılmıştır. Optimizasyon işleminde, çalışma tablosu paket programları içerisinde yerleşik olarak bulunan genel amaçlı optimizasyon makrosundan yararlanılmıştır. Kafes sistemle ilgili matrislerin kurulmasında, özel olarak geliştirilen makrolar kullanılmıştır. Boyutlandırma probleminin formülasyonunda deplasman, çekme gerilmesi, burkulma gerilmesi ve minimum alan sınırlayıcıları gözönüne alınmıştır. Geliştirilen algoritmanın uygulanabilirliği, çözülen sayısal örneklerle gösterilmiştir. Çalışma tabloları kullanılarak elde edilen optimum boyutlandırma sonuçları, klasik optimizasyon yöntemlerinin sonuçlarıyla karşılaştırılmıştır.

Anahtar Sözcükler: Uzay Kafes; Deplasman; Burkulma; Optimizasyon; Çalışma Tablosu

Introduction

Structural optimization techniques are quite well adapted for structural design problems and they are commonly used at the present time. Much research has been carried out on optimum structural designs for a variety of subjects ranging from bar systems to continuum systems, and connections and effective algorithms have been developed as well (Atrek et al., 1984). However, the problems which are viable in practice with respect to code specifications are solved in only a small number of the above mentioned studies. Two different approaches are found in structural optimization when examining these algorithms. One is mathematical programming methods, which are quite general and can be used to obtain the solution of any optimum structural design problem. It is possible to obtain optimum cross sectional areas by using linear, nonlinear, geometric and dynamic programming methods under stress, displacement and frequency constraints (Haug, 1981; Kiusalaas, 1978; Saka, 1980, 1981; Yamakawa, 1981). Shape optimization can also be performed through these methods (Ding, 1986; Lin, 1982; Majid and Saka, 1977; Topping, 1983). Although mathematical programming methods are general, they cause divergence problems and become impractical when applied to large-scale systems.

The second approach is the optimality criteria method, in which the difficulties of mathematical programming methods are not encountered. A recursive relationship for the design variables is developed. This method is used for the optimum design of both linear and nonlinear structures (Fleury, 1978; Khot, 1978; Saka, 1984, 1987,1988; Khot, 1983; Zacharopoulos, 1984).

Saka (1988) has obtained optimum steel truss systems using the optimality criteria approach in accordance with AISC and DIN specifications. In Saka's paper, displacement, stress, buckling and minimum size constraints are considered. Since classical optimization techniques are utilized in the above mentioned research, lengthy and complicated algorithms have been developed. The formulation and programming of optimization algorithm have an essential role in structural optimization.

The optimization tool of Microsoft Excel 7.0 is used for the design algorithm presented in this paper. Displacement, stress, buckling and minimum size constraints are considered in the optimum design of steel truss systems. Spreadsheet programs, which are commonly used at the present time, are employed both in the optimization procedure and in the analyses for the solution of the structural optimization problem. The algorithm is based on the automatic interaction and matrix calculation abilities as well as the optimization tool of spreadsheets. The using of the general-purpose optimization tool in the optimization procedure reduces the problem to a simple form. Microsoft Excel 7.0 was chosen as the spreadsheet program and its notations together with its Excel 4.0 macros are used in the present study.

Spreadsheets

General characteristics

Lotus 1-2-3 was among the first programs to be used as spreadsheets. Nowadays, spreadsheets are quite popular computer software. Although spreadsheet programs developed by various software firms have their own special features, they are based on the same working principles. Moreover, most of them are compatible with each other. The spreadsheets developed for the graphic-based operating systems such as Microsoft Windows and OS/2 have commonly and efficiently been used in recent years. A spreadsheet comprises many 'workbooks'. A workbook has 16 sheets each of which is a group of cells and contains 256 columns and 16384 rows in the Excel 7.0 program as shown in Figure 1. The columns are called A, B, ..., Z, AA, AB, ..., IV and the rows are numbered from 1 to 16384 in general. A user can move around among cells and write information on them. The information may be numeric or alphanumeric values or formulae. Values of variables are written on the cells and cell addresses are used as variable names such as A1, M25. The cells or the group of cells can be named if required and formulations can be expressed clearly with the help of these names. All operations concerning spreadsheets are conducted by a core program. This program scans all the filled cells in the sheet and searches for logical relations and updates the operations at once when entering new information into the cells. This feature is called automatic interaction. One of the important concepts of spreadsheets is that of range. A range covers one or more rectangular cells of a sheet. The address of a range can be defined by the addresses of both ends of its diagonal, such as A2:B5. It is also possible to give names to the ranges and use these names in operations. The addresses representing the ranges or

the names of the ranges can be used as parameters. Some formulations can also be written on their defined range. These types of formulations are used particularly in matrix operations.

Macros

Simply defined, macros are small programs which can be written and executed in spreadsheets (Orwis 1991; Weisskopf, 1997; Microsoft Corp., 1994). Macros are defined in special sheets called macro sheets. They use the cells of the sheets as variables. There are two kinds: command macros and function macros. Function macros assign the values of special functions used in spreadsheets. Command macros need special commands to be executed.

Analysis of Space Trusses

Entering data into the sheets

The data concerning a truss system are written on a *template table* prepared beforehand in a sheet of a workbook. There are some ranges on the template table which have general information and others related to the joints, members and member groups of a truss as shown in Figure 1 and Figure 2. Some of these ranges are arranged for users to enter data into while the others are prepared for the information that will be obtained and transferred after calculations.

Giving names to the special ranges of the sheets

Giving names to the special ranges and then calling them by those names give the algorithm greater flexibility. The data are used in the range named 'General Information' when defining the addresses of the ranges. The naming of the ranges is performed with the help of a command macro. First the range addresses are determined and then related ranges are defined by using the DEFINE.NAME() function with this macro (Weisskopf, 1997; Microsoft Corp., 1994).

Carrying out the matrix displacement method

The matrices related to the matrix displacement method are constituted with the help of a command macro. The macro uses the information on the data range and obtains the elements of the member stiffness matrix, transformation matrix and external load vector and then it writes down this information on the relevant ranges. Matrix functions are used for obtaining the system stiffness matrix, joint displacements and axial member forces. The required automatic interaction feature is utilized by writing the matrix elements in the form of formulations. When the cross sectional area of a member is changed, the axial member forces and displacements of the system are affected by this change immediately due to the interaction feature. The MMULT(),
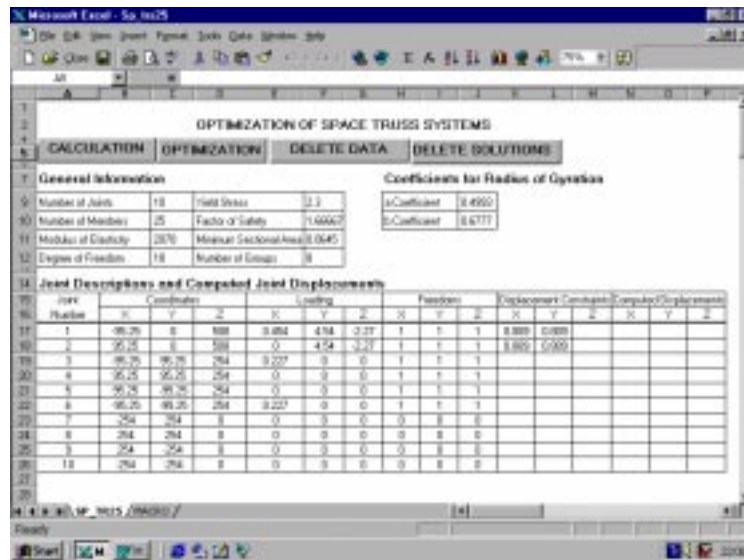

Figure 1. General information and joint descriptions

Figure 2. Member and member group descriptions

TRANSPOSE(), MINVERSE() functions are used in the matrix operations (Weisskopf, 1997; Microsoft Corp., 1994).

The calculated axial member forces and system joint displacements are placed in the relevant parts of the tables in the sheets by means of referring. The visual selection of the initial cross sectional areas becomes possible in an interactive way by monitoring the data and the results with this feature. The presence of the calculated displacements together with their limiting values in the same order of the table makes the expression of displacement constraints simple.

Optimization of Space Truss Systems

General purpose optimization tool

In the classical structural optimization methods, an essential part of computer programming consists of the optimization processes. In the present study optimization processes are performed by a general purpose optimization tool (*Solver*) that exists in the spreadsheet. The following steps are carried out when solving an optimization problem by the *Solver*:

1- The initial values for the design variables are written at random on the relevant places of the sheet.

2- The objective function and constraints are formulated and put in the appropriate places in the sheet referring to the cells where the design variables are present. The objective function is the volume of the structure.

3- The best initial values for design variables are selected by trial and error making use of the automatic interaction feature.

4- The *Solver* is run and the objective function and constraints are entered into the relevant places of its dialog box as shown in Figure 3.

5- The 'Solver Options' dialog box is activated by the *'Options'* button of the *Solver* dialog box, and information such as optimization technique, precision and number of iterations is entered into that box as shown in Figure 4.

6- The optimization process is started by the *Solve* button. The variation of the values in the tables is visually monitored during the optimization with the interaction feature. The process terminates when the adequate convergence is satisfied.

The above mentioned steps are followed in the optimum design of a structure under displacement, tensile and buckling stresses.
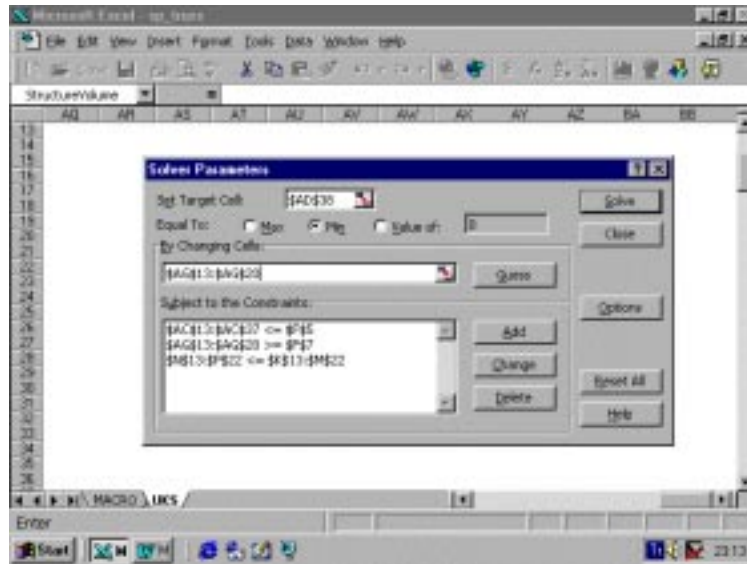
StructureVolume:	$AD38$		
GroupSectionalAreas:	$AG13:AG20$	MinSectionalArea:	$F7$
YieldStress:	$F5$	RealStresses:	$AC13:AC37$
ComptDisplacements:	$N13:P22$	DisplConstraints:	$K13:M22$

Figure 3. Solver Parameters dialog box

Figure 4. Solver options

Tensile and buckling stress constraints

The stress constraints given in Saka (1990), which are taken from AISC Spec. (1987), are considered in the present study. The tensile stress constraint can be expressed as

$$\sigma_i = \frac{F_i}{A_i} \leq \sigma_i^t \tag{1}$$

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where F_i and A_i are the axial member force and cross-sectional area of a tension member i, respectively. σ_i is the computed axial tensile stress and σ_i^t is the permitted axial tensile stress, which is given in AISC Spec. (1987) as

$$\sigma_i^t = 0.6\sigma_y \tag{2}$$

where σ_y is yield stress.

The stress constraints are considered as 'buckling stress constraints' in compression members. Buckling of an i-th compression member occurs either in elastic range or in plastic range depending on the slenderness ratio $S_i = L_i/r_i$, where L_i and r_i are the length and radius of gyration of the i-th member.

The design problem is formulated by only considering cross-sectional areas as variables. Hence, it becomes necessary to express the above radius of gyration in terms of area. This relationship is given by

$$r = aA^b \tag{3}$$

where a and b are constants whose values are obtained by applying the least square approximation to a practically available AISC standard section such as angles, pipes, tees and double angles (Saka, 1990). The values of a and b for some sections are given in Table 1.

Table 1. The constants in Eqn. (3) for some sections

Constants	Section Shapes			
	L	O	T	JL
a	0.8338	0.4993	0.2905	0.5840
b	0.5266	0.6777	0.8042	0.5240

According to AISC (Specification 1987), the computed axial compressive stress $\sigma_i = F_i/A_k$ does not exceed the permitted buckling stress:

$$\text{if} \quad S_i > C \quad \text{then} \quad \sigma_i \leq \frac{12\pi^2 E}{23S_i^2}$$

(elastic buckling) $\tag{4}$

$$\text{if} \quad S_i < C \quad \text{then} \quad \sigma_i \leq \frac{[1 - S_i^2/(2C^2)]\sigma_y}{\frac{5}{3} + \frac{3S_i}{8C} - \frac{S_i^3}{8C^3}}$$

(plastic buckling) $\tag{5}$

where A_k is the area of members belonging to group k, E is modulus of elasticity and $C = \sqrt{2\pi^2 E/\sigma_y}$. Eqns (1), (4) and (5) can be arranged in the following forms, such that each of them is constrained by σ_y.

For tension members

$$\frac{1}{0.6}\frac{F_i}{A_k} \leq \sigma_y \tag{6}$$

For compression members

$$\text{if} \quad S_i > C \quad \text{then} \quad \frac{23S_i^2}{6C^2} \cdot \frac{F_i}{A_k} \leq \sigma_y \tag{7}$$

$$\text{if} \quad S_i < C \quad \text{then} \quad \frac{\left[\frac{5}{3} + \frac{3S_i}{8C} \frac{S_i^3}{8C^3}\right]}{[1 - S_i^2/(2C^2)]} \cdot \frac{F_i}{A_k} \leq \sigma_y \tag{8}$$

Eqns (6), (7) and (8) have the form

$$n_i\sigma_i \leq \sigma_y \tag{9}$$

where n_i may be perceived as a variable factor of safety and $\sigma_i^r = n_i\sigma_i$ is described as 'real stress'.

The macro for the real stress functions

A function macro called the 'real stress function' is prepared to transfer the stress constraints to the *Solver*. This macro computes σ_i^r real stress values from eqns (6), (7) and (8) depending on the signs of member axial forces and the member slenderness ratio. The definition of the macro is given in Figure 5.

The optimization macro for space truss systems

The optimization macro informs the *Solver* about the objective function, the constraints and the optimization options and it also starts the optimization process. This macro is defined in Figure 6.

Real Stress	
Member force	=ARGUMENT("Fi")
Member sectional area	=ARGUMENT("Ak")
Member length	=ARGUMENT("Li")
	=IF(A<=0)
	=RETURN(2*!YieldStress)
	=END.IF()
Tension member?	=!IF(Fi>=0)
	=RETURN(!FactorofSafety*Fi/Ak)
	=ELSE()
Member slenderness ratio (Si)	=Li/(!aCoefficient*Ak^!bCoefficient)
Critical slenderness ratio (C)	=SQRT(2*PI()^!2*!ModulusofElasticity/!YieldStress)
Elastic buckling?	=IF(Si>=C)
	=RETURN(ABS(Fi)*23*Si^2/(Ak*6*C^2))
Plastic buckling?	=ELSE()
	=RETURN(ABS(Fi)*(5/3+3* Si/(8*C)-Si^3/(8*C^3))/(Ak*(1-Si^2/(2*C^))))
	=END.IF()
	=END.IF()
	=RETURN()

Figure 5. Real stress function macro

Optimization	
Error control notice	=ERROR(TRUE;ErrorinOptimization)
	=FORMULA.GOTO("GroupSectionalAreas")
Reset the Solver	=SOLVER.RESET()
Optim. Options	=SOLVER.OPTIONS(9999;100;0.0001;FALSE;FALSE;2;1;1;0.05;FALSE)
Objective function	=SOLVER.OK(!StructureVolume;2;0;!GroupSectionalAreas)
Min. size constraint	=SOLVER.ADD(!GroupSectionalAreas;3;!MinimumSectionalArea)
Tens. & buckl. Stress const.	=SOLVER.ADD(!RealStresses;1;!YieldStress)
Displ. constraints	=SOLVER.ADD(!ComputedDisplacements;1;!DisplConstraints)
Start the optimization	=SOLVER.SOLVE(TRUE)
	=ERROR(FALSE)
	=SOLVER.FINISH(TRUE)
	=RETURN()

Figure 6. Space truss optimization macro

Design Examples

The algorithm is developed for the optimum design of both space and plane trusses by means of spreadsheets. However, the 27-bar system presented in Saka (1990) is only designed as plane truss in the present study. The optimum design of 25-, 56- and 244-bar space truss systems, which are applicable in practice, is performed by the spreadsheet afterwards. In all these examples, the yield stress, the permitted tensile stress and the factor of safety for tension members are taken as 233.3 MPa, 140 MPa and 1.6667, respectively.

Design of 27-bar plane truss

The optimum design of the 27-bar plane truss shown in Figure 7 is considered as the first example. The dimensions, the loading and member grouping are given in Figure 7. The results obtained from the present study are compared with those of Saka (1990). The pipe sections are adopted in the design. The vertical and horizontal displacements of joint 11 are restricted to 10 and 4 mm, respectively. The modulus of elasticity is taken as 210 kN/mm^2. The minimum size constraints for area variables are considered to be 400 mm^2. The optimization process

is started with 2000 mm^2 initial cross-sectional areas. The optimum designs are given comparatively in Table 2. The computed stresses remain quite below their limiting values in this example. The displacement constraints are dominant in the design.
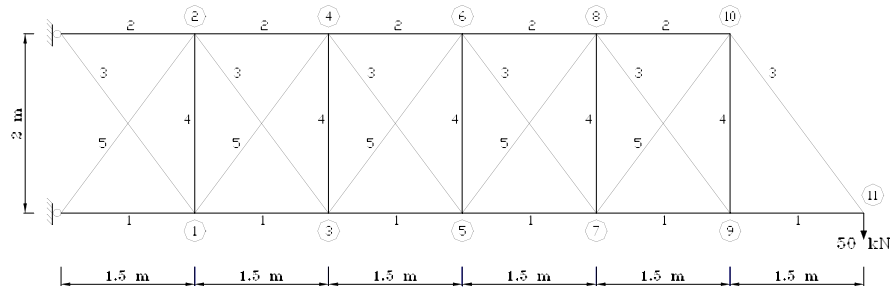

Figure 7. 27-bar plane truss

Table 2. Optimum designs for 27-bar plane truss

		Group Sectional Areas (mm^2)					Structure Volume (x 10^3 mm^3)
		A$_1$	A$_2$	A$_3$	A$_4$	A$_5$	
Initial values		2000	2000	2000	2000	2000	108000
Optimum	(Saka, 1990)	4038	4391	1174	402	1006	103484
Designs	This work	4041	4395	1175	400	1007	103554

Design of 25-bar space truss

The second example is a 25-bar space truss shown in Figure 8. This truss was designed by Venkayya et al. (1969), Adeli and Kamal (1986) and Saka (1990), with the help of different optimization techniques. The same truss is designed by spreadsheet herein. The members of the truss are collected in eight groups. The pipe sections are considered in the design. The modulus of elasticity is taken as 207 kN/mm^2. The loading of the truss and the upper bounds for the displacements of the restricted joints are given in Table 3. The minimum cross-sectional area for members is chosen as 6.45 mm^2. The optimization starts with the cross-sectional areas of 1000 mm^2 for all members. The results for the optimum design are listed and compared with those of Saka (1990) in Table 4. The joint displacements remain quite below their limiting values, and the buckling stress constraints govern the design. The reduction in the structure volume is 9.1% when compared with the design of Saka (1990).

Design of 56-bar space truss

The third example is a 56-bar space truss whose members are collected in three groups as shown in Figure 9. Angle sections are adopted for members. Joint 1 is loaded with 4 kN in the Y-direction and 30 kN in the Z-direction while the others are loaded with 4 kN in the Y-direction and 10 kN in the Z-direction. The vertical displacements of joints 4, 5, 6, 12, 13 and 14 are restricted to 40 mm while the displacement of joint 8 in the Y-direction is limited to 20 mm. The modulus of elasticity and the minimum member cross-sectional area are taken as 210 kN/mm^2 and 200 mm^2, respectively. The initial cross-sectional areas are chosen as 2000 mm^2 when starting the optimization. The results of optimum design are shown in Table 5. The obtained values of joint displacements are much smaller when compared with their upper bounds. It is found that the tensile and buckling stress constraints are dominant in the design.
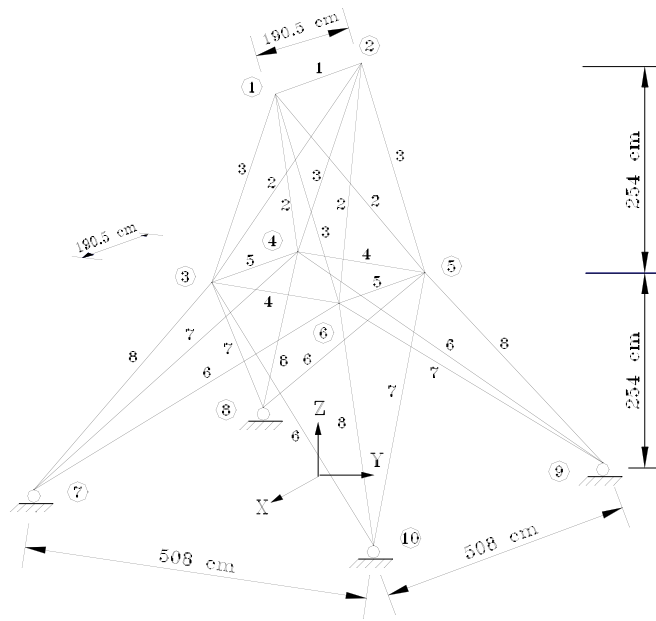
Figure 8. 25-bar space truss

Table 3. The loading and displacement bounds for 25-bar space truss

Joint Number	Loading (kN)			Displacement Limitations (mm)	
	x	y	z	x	y
1	4.54	45.4	-22.7	8.89	8.89
2	0.0	45.4	-22.7	8.89	8.89
3	2.27	0.0	0.0	-	-
6	2.27	0.0	0.0	-	-

Table 4. Optimum designs for 25-bar space truss

Design Variables	Members	(Saka,1990) mm^2	This work (mm^2)
A1	1	6.45	6.45
A2	2,3,4,5	1327.7	1266
A3	6,7,8,9	1927.7	1708
A4	10,11	6.45	6.45
A5	12,13	6.45	6.45
A6	14,15,16,17	449	496
A7	18,19,20,21	1077.4	913
A8	22,23,24,25	1672.3	1470
Structure Volume(x10^3 mm^3)		89351	81246
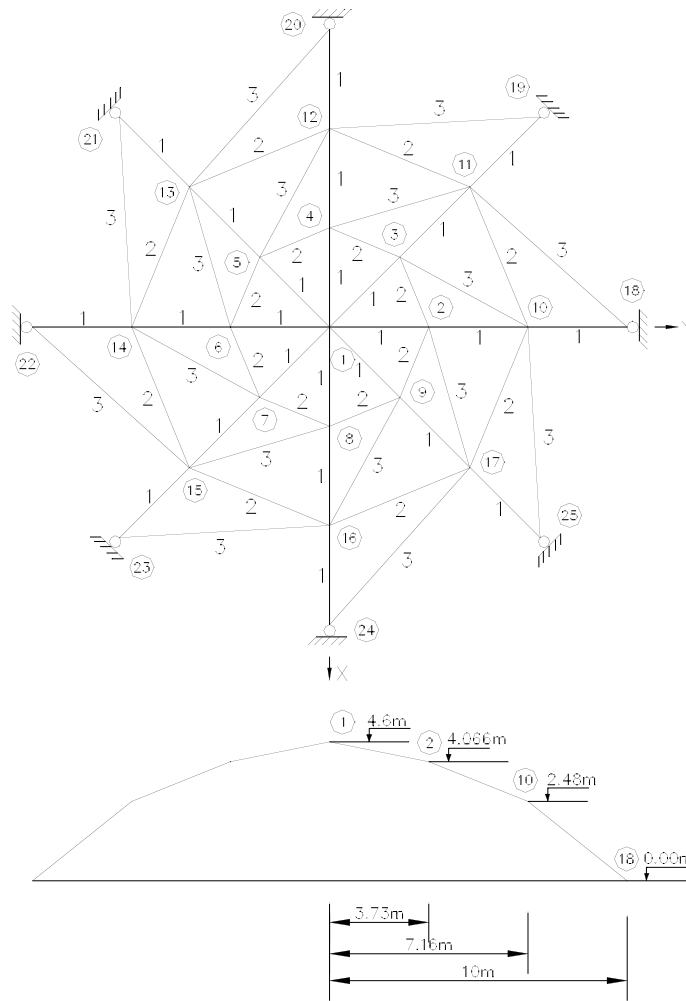
Figure 9. 56-bar space truss

Table 5. Optimum designs for 56-bar space truss

	Group Sectional Areas (mm^2)			Structure volume
	A$_1$	A$_2$	A$_3$	(x 10^3 mm^3)
Initial values	2000	2000	2000	521810
Optimum designs	773	477	832	187996

Design of 244-bar transmission tower

The design of a 244-bar transmission tower, shown in Figure 10, is considered as the last example. The members of this space truss are combined in 32 groups. The modulus of elasticity is considered to be 206 kN/mm^2. The loading and the bounds imposed on the displacements are given in Table 6. Angle sections are adopted for the members. The minimum cross-sectional areas are taken as 200 mm^2 and the initial values for the areas are selected as 1000 mm^2.

The optimum designs are given in Table 7. The optimum sectional areas of Saka given in the third column of the table were obtained by using Saka's computer program (1990). The buckling stress and displacement constraints are dominant in the design problem. The reduction in the structure volume is 6% when compared with Saka's design (1990).
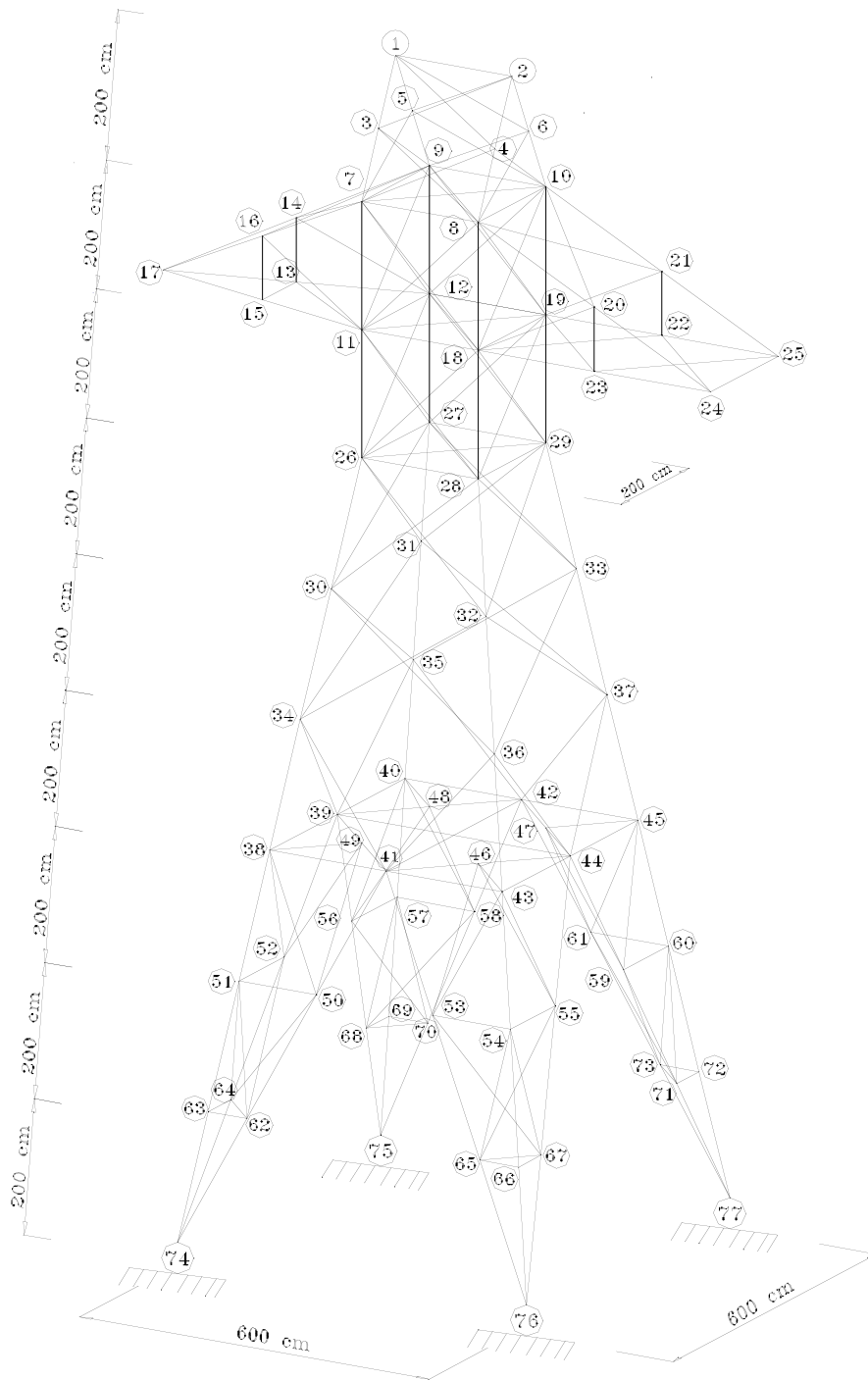

Figure 10. 244-bar transmission tower

Table 6. The loading and displacement bounds of transmission tower

Joint Number	Loading (kN)		Displacement Limitations (mm)	
	x	z	x	z
1	-10	-30	45	15
2	10	-30	45	15
17	35	-90	30	15
24	175	-45	30	15
25	175	-45	30	15

Table 7. Optimum Designs for 244-bar transmission tower

Design Variables	(Saka,1990) (mm^2)	This work (mm^2)
A1	263	267
A2	253	243
A3	200	200
A4	200	200
A5	2709	2557
A6	780	770
A7	308	291
A8	3820	3665
A9	1681	1680
A10	200	200
A11	3551	3390
A12	853	864
A13	200	200
A14	4027	3872
A15	200	200
A16	200	200
A17	200	200
A18	597	504
A19	200	200
A20	4941	4811
A21	1133	1100
A22	829	767
A23	200	200
A24	200	200
A25	200	200
A26	200	200
A27	200	200
A28	200	200
A29	200	200
A30	200	200
A31	200	200
A32	200	200
Structure Volume($\times 10^8$ mm^3)	9.21	8.64

Conclusions and Suggestions

In this work, an optimum design algorithm for the space and plane trusses with the help of spreadsheets considering displacement, stress and buckling constraints is presented. The algorithm depends on the use of the optimization tool of spreadsheets. It is also demonstrated that the algorithm can be applied effectively to the practical large systems.

It is also found from the numerical solutions that the results obtained from the present work coincide with those of previous work. Sometimes the algorithm gives even better results in comparison with those of the previous ones. The optimum design of space and plane trusses can be performed by the present algorithm without the need for any optimization methods or techniques, making use of spreadsheets which are used extensively in microcomputers.

The algorithm developed for truss systems can be applied quite simply to framed structures by making small changes. Moreover, a more effective and flexible algorithm can be developed by writing macros in the Visual BASIC programming language which exists in Microsoft EXCEL.

Notation

A_i = area of member i;
A_k = area of members in group k;
a,b = constants used in relating cross-sectional area to radius of gyration;
C = critical value of slenderness ratio;
E = modulus of elasticity;
F_i = force in member i;

L_i = length of member i;
n_i = variable factor of safety for member i;
r_i = radius of gyration for member i;
S_i = slenderness ratio for member i;
σ_i = stress in member i;
σ_y = yield stress;
σ_i^r = real stress in member i;
σ_i^t = allowable tensile stress for σ_i

References

Adeli H., and Kamal O. "Efficient Optimization of Space Trusses." Comput. Struct., 24(3), 501-511, 1986.

Atrek, E., et al., "New Directions in Optimum Structural Design." John Wiley and Sons, Inc., New York, 1984.

Ding, Y. (1986). "Shape Optimization of Structures: A LiteratureSurvey." Comput. Struct., 24(6), 985-1004

Fleury, C., and Geradin, M., "Optimality Criteria and Mathematical Programming in Structural Weight Optimization." Comput. Struct., 19(8), 7-17, 1978.

Haug, E. J., and Arora, J. S., "Applied Optimal Design." John Wiley and Sons, Inc., New York, 1981.

Khot, N. S., "Nonlinear Analysis of optimized Structures with Constraints on System Stability." AIAA J., 21(8), 1181-1186, 1983.

Khot, N. S., Berke, L., and Venkayya, V. B., "Comparison of Optimality Criteria Algorithms for Minimum Weight Design of Structures." AIAA J., 19(2), 182-190, 1978.

Kiusalaas, J., and Shaw, R., "An Algorithm for Optimal Design with Frequency Constraints." Int. J. Numer. Methods Engrg., 13(2) 1089.

Lin, J. H., Che, Y. W., and Yu, Y. S., "Structural Optimization on Geometrical Configuration and Element Sizing with Statical and Dynamical Constraints." Comput. Struct., 15(5), 507-515, 1982.

Majid, K. I., and Saka, M. P., "Optimum Shape Design of Rigidly Jointed Frames." Proc. Symp. Application of Computational Methods in Engineering, Univ. of Southern California, 520-532, 1997.

Microsoft, Corp. "Microsoft Excel Function Reference.", 1994

Microsoft, Corp. "Microsoft Excel User's Guide.", 1994.

Orvis W. J., "1-2-3 for Scientists and Engineers." 2nd edition, SYBEX Inc., Alemeda, 1991.

Saka, M. P., "Optimum Design of Rigidly Jointed Frames." Comput. Struct., 11(5), 411-419.

Saka, M. P. (1981). "Optimum Design of Grillages Including Warping." Proc. Symp. Optimum Structural Design, Univ. of Arizona, 9.13-9.20, 1981.

Saka, M. P., "Optimum Design of Space Trusses with Buckling Constraints." Proc. of the 3rd Int. Conf. on Space Structures, Univ. of Surrey, Guildford, U.K, 1984.

Saka, M. P., "Optimum Design of Steel Grillage Systems." Proc. 3rd Int. Conf. Steel Structures, Singapore Steel Society, 273-290, 1987.

Saka, M. P., "Optimum Design of Nonlinear Space Trusses." Comput. Struct., 30(3), 545-551, 1988.

Saka, M. P., "Optimum Design of Pin-Jointed Steel Structures with Practical Applications." J.Struct.Div., ASCE, 116(10), 2599-2620, 1990.

"Specifications for the Design, Fabrications and Erection of Structural Steel for Buildings." American Inst. of Steel Construction, Chicago, IL, 1987.

Topping, B. H. V., "Shape Optimization of Skeleted Structures: A review." J. Struct. Div., ASCE, 109, 1933-1952, 1983.

Venkayya, V. B., Khot, N. S., and Reddy, V. S., "Energy Distribution in an Optimal Structural Design." AFFDL-TR-68-156, Flight Dynamics Laboratory, Wright Patterson AFB, Ohio, 2, 1969.

Weisskopf, G. "The ABCs of Excel 97." SYBEX, Inc., Alameda, CA, 1997.

Yamakawa, H., "Optimum Structural Design in Dynamic Response." Proc. Int. Symp. Optimum Structural Design, Univ. of Arizona, 1981.

Zacharopoulos, A., Willmert, K. D., and Khan, M. R., "An Optimality Criterion Method for Structures with Stress, Displacement and Frequency Constraints." Comput. Struct., 19(4), 621-629, 1984.