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Stable Isotope Mass Balance Method to Find the Water Budget of a Lake

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Abstract

Determination of a lake water budget is essential for water resources engineers. Some of these elements, such as surface inflow, precipitation, evaporation, surface outflow, and variation of lake level, can be measured easily at the site. However, it is difficult to determine the groundwater inflow and outflow. Although these quantities could be calculated as a residual term of the water budget equation, they yield information only on absolute contributions of groundwater flow. At least one additional equation is needed to solve the unknowns separately. Stable isotopes (oxygen-18 and deuterium), exist in water naturally, provide additional equations, and simultaneous solutions of these equations make it possible to quantify the groundwater inflow into and the outflow from the lake. In this paper, the stable isotope mass balance method, which is an expensive technique, together with the conventional water budget method are applied to Mogan Lake, located south of Ankara, Turkey, to determine the groundwater contribution to the lake. It was found that the average groundwater inflow to Mogan Lake is 20.42 million $m^3 \pm$ 17.22% while the average groundwater outflow is 16.44 million $m^3 \pm$ 24.95% for the 1994 water year.

Key Words: Stable Isotope, Water Budget, Isotope Mass Balance, Mogan Lake

Göl Su Bütçesinin Kararlı İzotop Kütle Denge Yöntemi ile Belirlenmesi

Özet

Göl su bütçesinin bulunması su kaynakları mühendisleri için çok önemlidir. Yüzey girdisi, yağış, buharlaşma, yüzey çıktısı ve göl su seviyesi değişimi gibi su bütçesi elemanlarının arazide ölçülmesi kolaydır. Fakat yeraltısuyu girişi ve çıkışının bulunması zordur. Bu iki değer su bütçesi eşitliğinden hesaplanabilmesine karşılık, elde ettiğimiz bilgi sadece mutlak yeratısuyu katkısıdır. Bilinmeyenlerin ayrı ayrı bulunabilmesi için ilave olarak enaz bir eşitliğe daha ihtiyaç vardır. Bu gerekli eşitlikleri doğal olarak sularda mevcut olan kararlı izotoplar (oksijen-18 ve döteryum) sağlar ve bu eşitliklerin çözümü göle giren ve çıkan yeraltısuyu miktarının bulunmasını mümkün kılar. Bu çalışmada pahalı bir yöntem olan izotop kütle dengesi yöntemi ve klasik su bütçesi yöntemi Ankara-Türkiye'nin güneyinde bulunan Mogan Gölü'ne yeraltısuyu katkısının bulunması için uygulanmıştır. Mogan Gölü'ne 1994 su yılında ortalama yeraltısuyu girişi 20,42 milyon $m^3 \pm$ % 17,22 ve ortalama yeraltısuyu çıkışı 16,44 milyon $m^3 \pm$ % 24,95 bulunmuştur.

Anahtar Sözcükler: Kararlı İzotop, Su Bütçesi, İzotop Kütle Dengesi, Mogan Gölü

Introduction

In order to determine the water balance of a lake, one needs to consider all the hydrologic components of the lake and its surroundings. Among these components, groundwater inflow to and outflow from the lake are especially difficult to quantify due to the complicated patterns of groundwater flow around lakes. Conventional methods require measurements from extensive piezometer networks to estimate the groundwater contribution to a lake. This is rather time consuming and expensive, and yields information only on the absolute contribution of groundwater. The stable isotope mass balance method can be applied satisfactorily to determine groundwater inflow/outflow contributions separately. The most widely used stable isotopes are oxygen-18 (δ^{18}O) and deuterium (δ^{2}H), which are almost ideal tracers for investigation of lake parameters since they are part of the water and can be applied naturally to the hydrologic system. Stable isotopes have been used for the study of lakes by many investigators, including Dinçer (1968), Gat (1970), Zimmerman and Ehhalt (1970), Zuber (1983), Turner et al. (1984), Gonfiantini (1986), Krabbenhoeft et al. (1990), and Yehdegho et al. (1995).

Isotopic compositions of lake water, precipitation and groundwater are continuously modified due to natural isotopic fractionation. These modifications give useful information in determining the interaction among lakes, creeks, groundwater, and precipitation, and this enables one to separate the sources.

The monthly absolute contribution of groundwater to Mogan Lake was estimated by both the conventional water budget and the isotopic mass balance methods for the 1994 water year (Özaydın, 1997). Since the groundwater inflow and outflow elements cannot be found separately by the conventional water budget method, the contributions of groundwater inflow and outflow to the hydrological balance of a lake are quantified separately by using the stable isotope mass balance method. Monthly water samples were collected from the water budget elements at selected points and analyzed to determine their isotopic compositions. These isotopic compositions were used to write the mass balance equations. The most difficult one among the isotopic composition of water budget elements is the determination of the isotopic composition of the evaporating water body.

Theory

The main inflows to the lakes are precipitation to the lake, P; surface inflow, I_s; and groundwater inflow (subsurface), I_{ss}. The main outflows from the lakes are surface outflow, O_s; groundwater outflow (subsurface), O_{ss}; and evaporation from the lake surface, E.

The precipitation, surface inflow, evaporation, and surface outflow are measured at the site, while the groundwater inflow and outflow can be found by the residual of the water budget equation. The general water budget relationship for a lake is:

$$I_s + I_{ss} + P - E - O_s - O_{ss} = \frac{d}{dt}(V) \qquad (1)$$

where V is the volume of water in the lake and t is the time. An additional equation can be written by considering the isotope composition of the water budget elements, and since a small and shallow lake is rather well mixed, one can assume that the isotopic compositions of surface (δ_{Os}) and subsurface (δ_{Oss}) outflows are the same as those of the lake (δ_L):

$$\delta_{Os} = \delta_{Oss} = \delta_L \qquad (2)$$

where L represents lake. The isotopic mass balance equation becomes

$$\begin{aligned} &I_s\delta_s + I_{ss}\delta_{Iss} + P\delta_P - E\delta_E - O_s\delta_L - O_{ss}\delta_L \\ &\quad = \frac{d}{dt}(V\delta_L) \end{aligned} \qquad (3)$$

If the unknown terms I_{ss} and O_{ss} are kept on the left side of Equations 1 and 3:

$$I_{ss} - O_{ss} = C^* = \frac{d}{dt}(V) + E + O_s - I_s - P \qquad (4)$$

$$I_{ss}\delta_{Iss} - O_{ss}\delta_L = C^{**} = \frac{d}{dt}(V\delta_L) + E\delta_E + O_s\delta_L - I_s\delta_s - P\delta_P \tag{5}$$

The groundwater inflow rate, I_{ss}, and the outflow rate, O_{ss}, can be solved as follows:

$$I_{ss} = \frac{\delta_L C^* - C^{**}}{\delta_L - \delta_{Iss}} \tag{6}$$

$$O_{ss} = I_{ss} - C^* \tag{7}$$

Theoretically, δ_E is derived from the resistance model of the evaporation process described by Craig and Gordon (1965):

$$\delta_E = \frac{\frac{\delta_L}{\alpha} - h\delta_{atm} - \varepsilon}{1 - h + \Delta\varepsilon} \tag{8}$$

The fractionation factor, α, is temperature dependent. The equations of the water-vapor fractionation factor α (valid between 0°C and 100°C), obtained from laboratory experiments, are given for $\delta^{18}O$ and δ^2H as follows (Majoube, 1971):

$$\ln \alpha_{18_O} = (-2.0667 - \frac{415.6}{T} + \frac{1137}{T^2} {*} 10^3)/1000 \tag{9}$$

$$\ln \alpha_{2_H} = (52.612 - \frac{76278}{T} + \frac{24844}{T^2} {*} 10^3)/1000 \tag{10}$$

where T is temperature in Kelvins (0°C=273.15 K). The total enrichment factor ε is defined as

$$\varepsilon = (1 - \frac{1}{\alpha}) + \Delta\varepsilon \tag{11}$$

The kinetic enrichment fractionation factor, $\Delta\varepsilon$, is proportional to the moisture deficit, (1-h), of the atmosphere and the relative difference in the transport resistance in air between isotopic molecules. Vogt (1978) expresses it as

$$\Delta\varepsilon^{18}O = 14.2^*(1 - h)‰ \tag{12}$$

$$\Delta\varepsilon^{2}H = 12.5^*(1 - h)‰ \tag{13}$$

δ_{atm} represents the isotopic composition in the region of turbulent transport. It is practically impossible to measure this parameter in the field with adequate frequency and for sufficiently long periods. It can be usually derived from the isotopic composition of local precipitation as

$$\delta_{atm} = \frac{\delta_p}{\alpha} - (1 - \frac{1}{\alpha}) \tag{14}$$

Since the relative humidity, h, is measured near the lake, it does not reflect the relative humidity just over the lake surface. Therefore, it should be normalized to the lake surface by the following equation, which was derived from the Magnus formula and gives specific humidity (g/m^3) under saturation conditions (Moeller, 1973):

$$e_s = \frac{1321.7e^{\frac{17.27t}{237+t}}}{273 + t} \tag{15}$$

where e_s is the saturation vapor pressure and t is the temperature in degrees Celsius measured near the lake. To obtain normalized relative humidity, e_s should be calculated for air and water temperature separately by using Equation 15. Then, the relative humidity measured near the lake should be multiplied by the ratio of e_s calculated from air temperature to water temperature.

Description of the Basin

There were extensive environmental problems including pollution of the water in Mogan Lake. Therefore the amount of the groundwater inflow and outflow will help in the analysis of these problems.

Mogan Lake is located in Central Anatolia, 20 km south of Ankara, the capital of Turkey (Figure 1). The total catchment area of Mogan and Eymir Lakes is 971 km^2 (Mogan's catchment is 925 km^2). The highest and the lowest points in the basin are 1650 m and 970 m, respectively (Altınbilek et al., 1995 and 1996).

Continental climate is dominant in the study basin with cold and rainy/snowy winters, but very hot and dry summers. The basin can be identified as a semi-arid region in terms of precipitation, and as a steppe type in terms of its vegetation cover. The morphometric characteristics of Mogan Lake are given in Table 1.

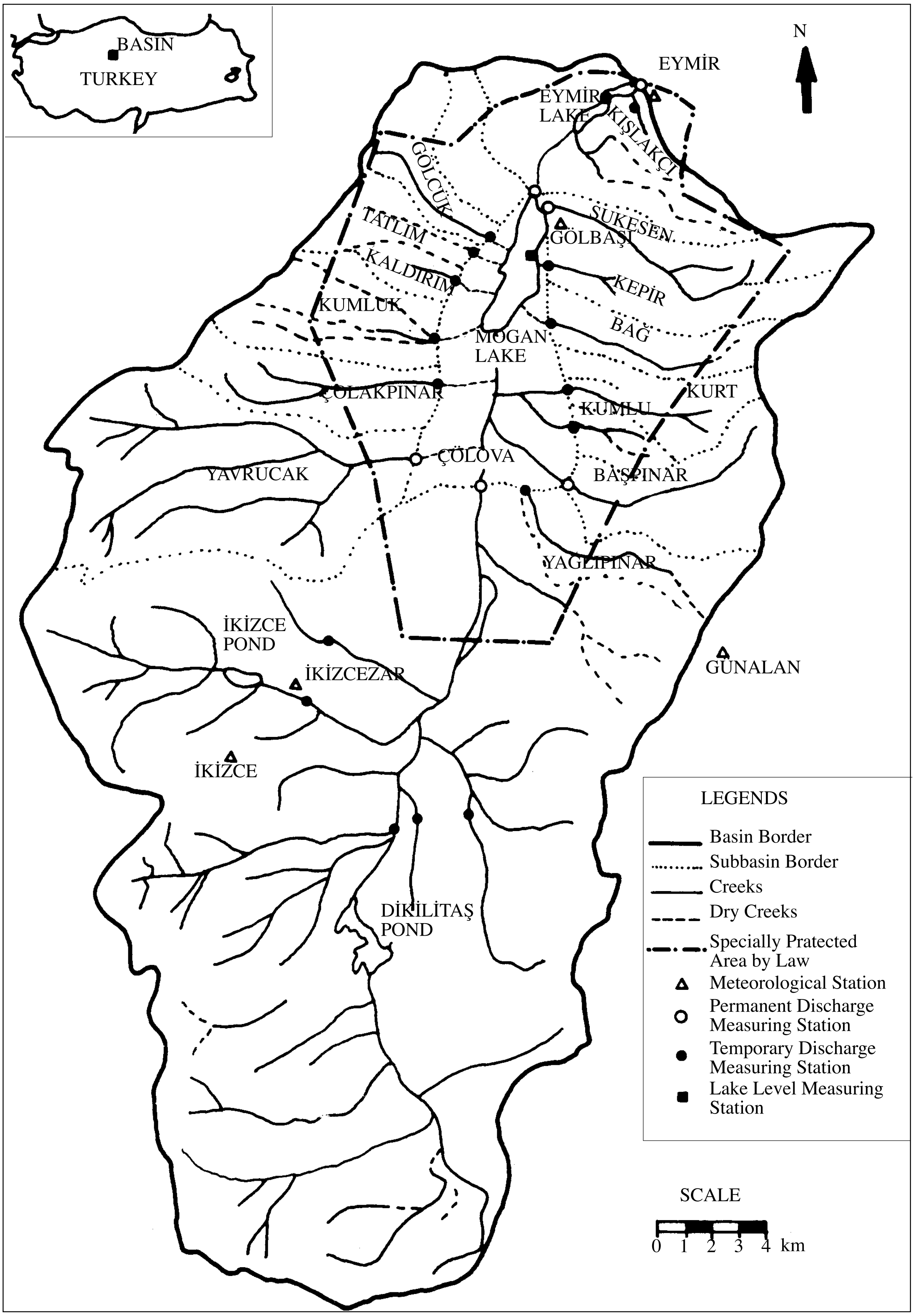

Figure 1. General layout of the basin

Table 1. Morphometric characteristics of Mogan Lake

	Minimum	Mean	Maximum
Water depth (m)	2.00	3.00	4.25
Volume (10^6 m^3)	6.20	11.63	20.24
Area (km^2)	4.77	5.43	7.70
Basin area (km^2)		925	
Length (km)		6	
Width (km)		1	

There are 16 creeks that feed Mogan Lake. The major ones are Çölova, Yavrucak, Sukesen and Başpınar. In addition to Mogan Lake, there are Eymir Lake and two artificial ponds, which are inside the Çölova Creek subbasin.

Mogan and Eymir Lakes are natural alluvial lakes. Water released from Mogan Lake feeds Eymir Lake through an artificial canal. The water level of Mogan Lake is approximately 3 m higher than that of Eymir Lake. There are many springs inside the basin, which are the source of the creeks.

The geological map of the surroundings of both lakes together with the locations of the observation wells is given in Figure 2 (Kalkan et al., 1992). The primary structures in Gölbaşı and vicinity are the discordance, folds and faults. Schistosity and joint systems are the secondary structural features in the region. Geological studies revealed that there has not been any tectonic event since the Quaternary period.

Data Collection

Instrumentation

The existing hydrological and meteorological instrumentations and data were not sufficient for conducting water budget studies of the lake. To add to the database, a network of hydrological observation stations was established over the creeks for collecting water level data. Permanent discharge measuring stations with special weirs and automatic water level recorders were constructed at the four larger creeks (Figure 1). The existing stations at the Mogan and Eymir Lake Exits were also equipped with water level recorders. Instantaneous discharge measuring sites were selected for the other creeks. Twenty-three wells, which have a total depth of 336 m ranging in depth from 4 m to 30 m, were drilled around Mogan and Eymir Lakes (Figure 2) to collect water samples.

Meteorological data

There was a meteorological station in Gölbaşı town, but it was not operated continuously. This station was improved with equipment for measuring more meteorological data in June 1994. Measurements were conducted on climatic data including precipitation, class A pan evaporation, air temperature and wind velocity since June 1994. The missing class A pan data before this date and other climatic data such as relative humidity, sunshine hours, wind velocity and radiation flux were obtained from Ankara Meteorological Station, which is located 30 km north of the study area.

No evaporation measurements were conducted in the study area from November to March as the water freezes in Class A pan during this period. The missing evaporation data in the winter months, however, were estimated by taking the average of several methods, such as energy balance method, aerodynamic method, combined aerodynamic and energy balance method, and Penman method. Average monthly meteorological values for the 1994 water year are compiled in Table 2.

Discharge and lake level measurements

Discharges of creeks and water levels of Mogan Lake were measured by the technical staff of the General Directorate of State Hydraulic Works (DSİ) and the General Directorate of Electrical Power Resources Survey and Development Administration (EİEİ). The selected sampling points were sampled mostly once a month. However, in rainy months, more samples were taken. The average monthly surface inflow from major creeks to Mogan Lake and the average monthly lake level for the 1994 water year are given in Table 3.

Isotopic data collection

Water samples for isotopic analysis were collected once a month in the study area from creeks, lakes, lake exits and wells. Water samples from Mogan Lake were taken from twelve different locations.

Precipitation samples were collected from the meteorological station at the site, but for some months, due to lack of sufficient rainfall at the site, the isotopic composition of precipitation (oxygen-18 and deuterium) was taken from Ankara Meteorological station. The collected samples were mainly analyzed

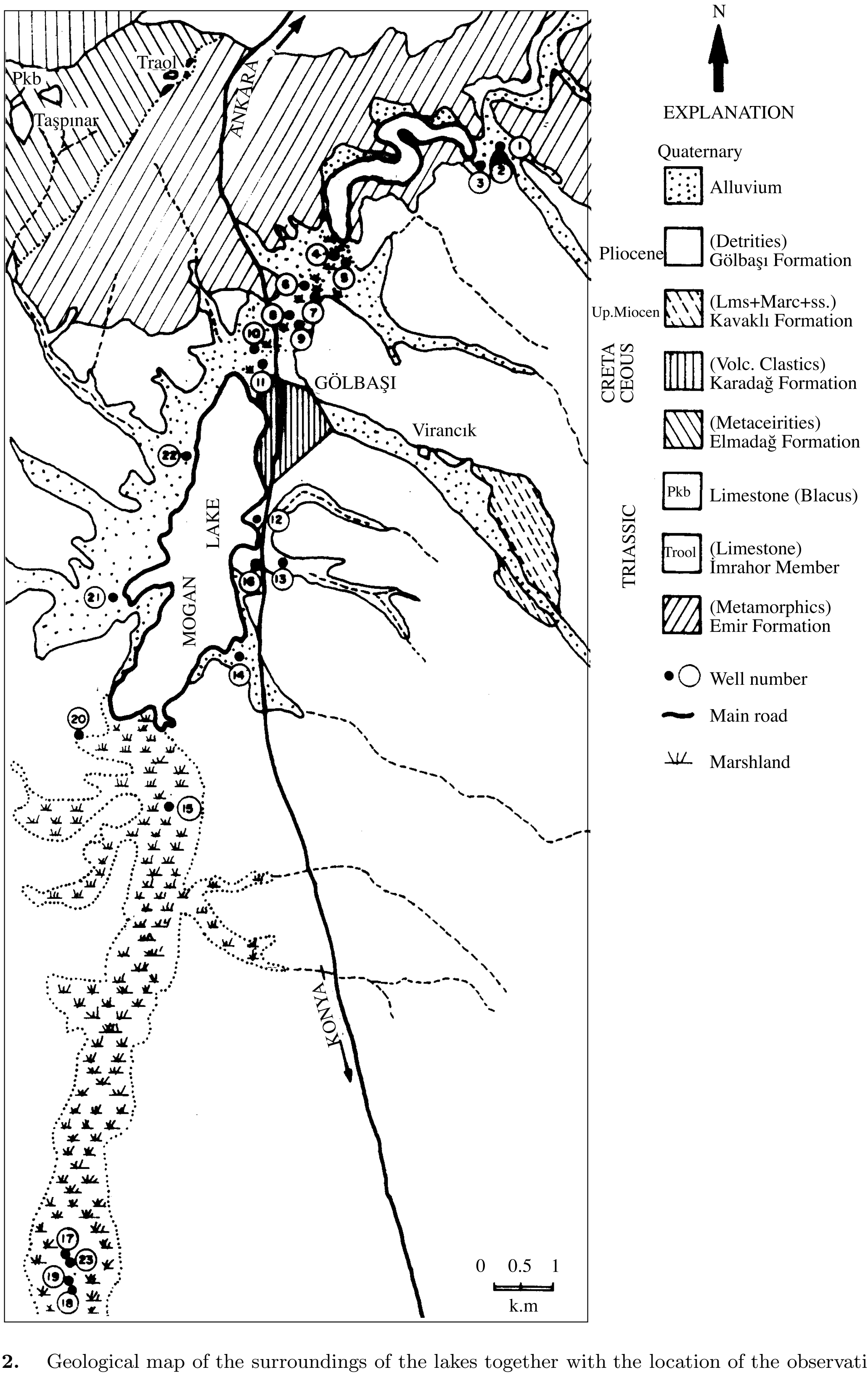

Figure 2. Geological map of the surroundings of the lakes together with the location of the observation wells

Table 2. Average monthly meteorological values

	Precipitation (mm)	Evaporation[1] (mm)	Air temperature (°C)	Relative humidity (%)	Sunshine hours (hour)	Wind velocity (m/s)	Radiation flux (cal/cm^2/day)
Oct-93	1.8	132.8	15.0	46.0	8.3	1.7	303.30
Nov-93	33.2	60.1	4.1	65.0	3.9	1.8	172.30
Dec-93	33.0	40.3	4.0	77.0	2.8	1.5	127.75
Jan-94	24.2	42.2	3.8	75.5	3.2	1.9	153.18
Feb-94	33.6	63.7	1.8	73.8	3.8	2.1	198.53
Mar-94	18.4	114.3	6.8	60.1	6.0	2.3	297.60
Apr-94	30.7	144.1	14.0	55.0	8.4	2	421.20
May-94	39.0	179.8	17.0	56.5	9.0	1.6	462.83
Jun-94	6.6	267.1	17.7	47.2	12.4	1.7	559.36
Jul-94	0.8	311.9	19.9	44.4	11.3	1.7	579.45
Aug-94	1.6	278.7	20.0	46.7	11.0	1.7	505.66
Sep-94	5.1	217.8	18.7	44.0	9.7	1.3	420.12
Average	19.00	154.38	11.90	57.60	7.48	1.78	350.11
Sum	228.0	1852.6			89.80		4201.3

[1] The values for five months (Nov 93 - Mar 94) are computed by the formula and the others are uncorrected pan evaporation values.

Table 3. Catchment areas and average monthly surface inflows of major creeks, and lake level

	Çölova Creek (10^3 m^3)	Yavrucak Creek (10^3 m^3)	Sukesen Creek (10^3 m^3)	Başpınar Creek (10^3 m^3)	Monthly lake level (m)
Area (km^2)	551.0	87.3	31.6	31.0	-
Oct-93	244.2	22.4	17.2	28.5	972.11
Nov-93	232.1	202.1	30.6	27.5	972.18
Dec-93	295.1	255.2	106.1	63.3	972.27
Jan-94	369.6	192.7	153.4	160.3	972.41
Feb-94	567.9	255.8	64.1	94.9	972.57
Mar-94	524.4	415.4	267.8	133.9	972.74
Apr-94	182.1	271.5	155.6	30.4	972.86
May-94	172.2	124.0	54.4	26.4	972.82
Jun-94	49.0	2.2	4.9	1.5	972.77
Jul-94	No flow	No flow	0.1	3.9	972.56
Aug-94	No flow	No flow	No flow	1.7	972.35
Sep-94	No flow	No flow	No flow	1.3	972.15
Average					972.48
Sum	2636.6	1741.3	854.3	573.5	

in the laboratories of the International Atomic Energy Agency (IAEA), Vienna, for determining the values of $\delta^{18}O$ and $\delta^{2}H$. Some analyses were carried out at the isotope laboratory of the State Hydraulic Works. The monthly isotopic composition of the samples from major creeks, Mogan lake, and groundwater inflow into Mogan Lake for the 1994 water year are given in Table 4.

Table 4. Monthly isotopic composition of major creeks, Mogan Lake, and groundwater inflow to Mogan Lake

	Çölova Creek		Yavrucak Creek		Sukesen Creek		Başpınar Creek		Mogan Lake		Ground water Inflow	
	$\delta^{18}O$ ‰	$\delta^{2}H$ ‰	$\delta^{18}O$ ‰	$\delta^{2}H$ ‰	$\delta^{18}O$ ‰	$\delta^{2}H$ ‰	$\delta^{18}O$ ‰	$\delta^{2}H$ ‰	$\delta^{18}O$ ‰	$\delta^{2}H$ ‰	$\delta^{18}O$ ‰	$\delta^{2}H$ ‰
Oct-93	-2.12	-33.65	-10.14	-72.38	-9.72	-70.66	-9.87	-77.15	-1.70	-22.26	-8.91	-68.26
Nov-93	-3.91	-45.36	-9.73	-77.00	-9.82	-74.20	-9.97	-76.23	-0.88	-32.28	-8.81	-67.19
Dec-93	-8.77	-71.51	-10.06	-72.26	-10.46	-75.41	-10.05	-75.08	-1.19	-32.62	-8.72	-66.12
Jan-94	-8.75	-72.26	-9.90	-76.49	-10.32	-80.72	-9.97	-79.73	-1.50	-32.97	-8.62	-65.05
Feb-94	-8.96	-69.18	-9.75	-71.67	-10.33	-73.35	-9.85	-69.43	-1.82	-33.31	-8.85	-66.49
Mar-94	-7.95	-60.60	-9.47	-79.77	-9.62	-80.73	-8.74	-74.11	-2.13	-33.66	-9.07	-67.92
Apr-94	-3.68	-38.90	-9.15	-67.10	-9.59	-69.90	-8.84	-65.10	-2.44	-34.00	-9.30	-69.36
May-94	-1.16[1]	-24.60[1]	-8.82[1]	-64.90[1]	-8.94	-66.00	-9.01	-68.45	-1.09	-28.40	-9.24	-69.18
Jun-94	-1.16[1]	-24.60[1]	-8.82[1]	-64.90[1]	-8.10	-63.00	-9.17[2]	-71.80[2]	1.77	-17.10	-9.17	-68.99
Jul-94	No flow	No flow	No flow	No flow	No flow	No flow	-9.17[2]	-71.80[2]	1.00	-18.90	-9.11	-68.81
Aug-94	No flow	No flow	No flow	No flow	No flow	No flow	-9.17[2]	-71.80[2]	0.23	-20.70	-9.04	-68.63
Sep-94	No flow	No flow	No flow	No flow	No flow	No flow	-9.17[2]	-71.80[2]	0.66	-19.00	-8.98	-68.44
Average	-3.87	-36.72	-7.15	-53.87	-7.24	-54.50	-9.42	-72.71	-0.76	-27.10	-8.99	-67.87

NOTE:

[1] Since there are no measurements of $\delta^{18}O$ and $\delta^{2}H$ in Çölova and Yavrucak Creeks in June 1994, isotopic compositions are taken to be the same as those in May 1994.

[2] Since there are no measurements of 18O and 2H in Başpınar Creek in July, August and September 1994, isotopic compositions are taken to be the same as those in June 1994.

Error Analysis

Input data are not known with precision in the field situations and assumptions behind the theory are not always satisfied. The theory, therefore, should be expected to provide precise estimates of calculated values. To assess the sensitivity of these parameters, they were examined by assuming various levels of percent errors to the water budget elements as follows:

$$w_R = \left[\sum_{i=1}^{n} \left(\frac{\partial R}{\partial X_i} \right)^2 \right]^{1/2} \tag{16}$$

where w_R is the error, R the calculated amount ($\Delta(I_{ss} - O_{ss})$ or I_{ss} or O_{ss}), and X_i the elements of the corresponding equations. Three different cases of error analyses are carried out. As can be seen from Table 5, in the first case a 5% error is assumed for I_s, P, O_s, and $\Delta V/\Delta t$ and 10% for E. In the second case a 5% error is assumed for I_s, P, E and O_s, and 10% for $\Delta V/\Delta t$. Errors of $\Delta(I_{ss} - O_{ss})$ are expressed by taking the average of case 1 and case 2. An error of 0.01‰ for $\delta^{18}O$ and 0.1‰ for δ^2H is assumed for the isotopic composition of water budget elements. Because δ_E is calculated by equations, errors 10 times larger are assumed, which are 0.1‰ for $\delta^{18}O$ and 1.0‰ for δ^2H. In the third case, exaggerated errors are assumed in the isotopic composition of water budget elements while errors in the water budget elements remain the same as those in case 1.

Table 5. Errors considered in the sensitivity analyses

	%					‰			
	I_s	P	E	O$_s$	$\Delta V/\Delta t$	$\delta^{18}O$	δ^2H	δ_E for $\delta^{18}O$	δ_E for δ^2H
Case 1	5	5	10	5	5	0.01	0.1	0.1	1.0
Case 2	5	5	5	5	10	0.01	0.1	0.1	1.0
Case 3	5	5	10	5	5	0.25	2.5	0.5	5

Discussion of the Results

Conventional water budget calculations

In the calculations, the water year is used instead of the calendar year, because it is desirable to treat annual streamflow data so that the flood season is not divided between successive years. The monthly results of computing the unknown term $\Delta(I_{ss} - O_{ss})$, which is a residual term of Equation 4, are given in the ninth column of Table 6, together with the water budget elements. The positive values of $\Delta(I_{ss} - O_{ss})$ indicate that there is a net inflow to Mogan Lake every month of the year.

As shown in Table 6, the monthly absolute groundwater volume, $\Delta(I_{ss} - O_{ss})$, to Mogan Lake in the 1994 water year obtained from conventional water budget method varies between $94.2{*}10^3 m^3 \pm$ 0.65% in December 1993 and $904.7{*}10^3 m^3 \pm$ 0.61% in October 1993. The dominant water budget element in the study area is evaporation. The summation of monthly volume in the 1994 water year is $4370{*}10^3 m^3 \pm$ 9.67% whereas, the annual conventional water budget method in 1994 water year is $3590{*}10^3 m^3 \pm$ 6.33%, i.e., there is a 21.7% difference between the values obtained from the two methods. The error obtained for monthly sum is ± 10.11% and for annual balance is ± 8.10% for Case 1. The same values are ± 9.22% and ± 4.55% for Case 2, respectively. Monthly errors vary between ± 0.40% and ± 1.43% for both cases.

Monthly calculations are more reliable since the increase or decrease in volume at the end of each month is considered. On the other hand, in annual calculations, the change in volume is obtained by taking the difference in volume between the first year's first month and the following year's first month.

If measuring errors (random or systematic) occur in any of the measured elements of Equation 1, it will be reflected in the $\Delta(I_{ss} - O_{ss})$ term as well.

Isotopic water budget calculations

All water budget elements have to be multiplied by its isotopic composition to obtain the isotopic mass balance equation. The given monthly isotopic values of the creeks, lake, and groundwater inflow in Table 4 are used in the monthly isotopic mass balance method. Because of the different air and water temperatures, relative humidity is normalized by using Equation 15 before it is used in the calculations (Özaydın, 1997; and Özaydın et al., 1998).

In the annual isotope balance method, the weighted values of δ_P and δ_E are used. Monthly

Table 6. Conventional water budget of Mogan Lake

	Surface Inflow (I_s) (10^3 m^3)	Surface Outflow (O_s) (10^3 m^3)	Average Lake Surface (km^2)	Lake Storage (S_i) (10^3 m^3)	Storage Difference ($S_2 - S_1$) (10^3 m^3)	Precipitation (P) (10^3 m^3)	Evaporation (E) (10^3 m^3)	$\Delta(I_{ss} - O_{ss})$ (10^3 m^3)	Error Case 1 (%)	Error Case 2 (%)	Error Average (%)
Oct-93	354.7	2.06	6.255	12303	438	11.3	830.7	904.7	0.71	0.51	0.61
Nov-93	607.0	51.0	6.355	12741	572	211.0	381.9	187.0	0.45	0.54	0.50
Dec-93	876.8	16.9	6.490	13313	908	214.2	260.2	94.2	0.52	0.77	0.65
Jan-94	1056.7	92.5	6.675	14221	1068	161.5	281.7	223.9	0.57	0.85	0.71
Feb-94	1199.4	104.1	6.885	15289	1172	231.3	438.6	284.0	0.62	0.88	0.75
Mar-94	1512.6	140.1	7.065	16461	848	130.0	807.5	153.0	0.72	0.73	0.73
Apr-94	678.7	269.3	7.115	17309	-285	218.4	1025.3	112.4	0.64	0.40	0.52
May-94	421.7	67.6	7.060	17024	-353	275.3	1269.4	286.9	0.77	0.45	0.61
Jun-94	63.7	0	6.900	16671	-1448	45.5	1843.0	285.8	1.19	1.03	1.11
Jul-94	5.8	62.7	6.635	15223	-1394	5.3	2069.5	727.0	1.43	1.14	1.29
Aug-94	7.1	54.8	6.380	13829	-1276	10.2	1778.1	539.5	1.37	1.12	1.25
Sep-94	5.2	4.1	6.185	12553	-743	31.5	1347.1	571.5	1.11	0.80	0.96
Average	565.77	72.1	6.667	14745	-41.08	128.80	1027.85	364.16			
Sum	6789.3	864.9				1545.7	12334.2	4369.9	10.11	9.22	9.67
Annual	6789.3	864.9	6.160		-493	1404.5	11413.3	3590.2	8.10	4.55	6.33

δ_P and δ_E values are multiplied by monthly percent precipitation and monthly percent evaporation, and then they are summed. Monthly δ_E values calculated by using Equation 8 are given in Table 7, and monthly δ_P values are given in Table 8. The weighted $\delta_E(E_i/\Sigma E)$ values for $\delta^{18}O$ and δ^2H are -19.76‰ and -105.62‰ and the weighted $\delta_P(P_i/\Sigma P)$ values are -7.76‰ and -49.35‰, respectively.

Table 7. Isotopic composition of evaporation (δ_E) of Mogan Lake (‰)

	Temperature Kelvin	α		Δ_ε		ε		δ_{atm}		δ_E	
		$\delta^{18}O$	δ^2H	$\delta^{18}O$	δ^2H	$\delta^{18}O$	δ^2H	$\delta^{18}O$	δ^2H	$\delta^{18}O$	δ^2H
Oct-93	288.15	1.0102	1.0911	8.81	7.64	18.94	91.16	-21.47	-152.26	-19.82	-85.14
Nov-93	277.25	1.0113	1.1061	6.67	5.79	17.84	101.67	-18.40	-132.50	-18.79	-127.40
Dec-93	277.15	1.0113	1.1062	5.27	4.57	16.44	100.58	-17.79	-124.21	-17.08	-138.38
Jan-94	276.95	1.0113	1.1065	5.45	4.72	16.64	100.97	-22.27	-162.95	-11.22	-77.50
Feb-94	274.95	1.0115	1.1095	5.68	4.92	17.07	103.63	-22.55	-171.36	-13.09	-75.42
Mar-94	279.95	1.0110	1.1021	7.22	6.26	18.11	98.93	-21.30	-166.07	-18.89	-92.40
Apr-94	287.15	1.0103	1.0924	7.75	6.72	17.97	91.30	-13.91	-101.34	-25.50	-138.56
May-94	290.15	1.0101	1.0886	7.54	6.54	17.49	87.96	-16.50	-122.48	-20.15	-105.17
Jun-94	290.85	1.0100	1.0878	8.64	7.50	18.54	88.20	-17.43	-125.93	-16.14	-88.42
Jul-94	293.05	1.0098	1.0851	8.96	7.77	18.67	86.24	-12.68	-91.55	-20.37	-109.51
Aug-94	293.15	1.0098	1.0850	8.69	7.53	18.38	85.90	-11.68	-89.98	-22.03	-113.27
Sep-94	291.85	1.0099	1.0866	9.02	7.82	18.83	87.50	-12.28	-90.35	-21.32	-112.16
Average	285.1	1.0105	1.0956	7.47	6.48	17.91	93.67	-17.35	-127.58	-18.70	-105.28
Weighted $\Sigma(E_i/\Sigma E)$										-19.76	-105.62

Table 8. Monthly and weighted δ_P values (‰)

	δ_P	
	$\delta^{18}O$	δ^2H
Oct-93	-11.45	-75.0
Nov-93	-7.32	-40.5
Dec-93	-6.69	-31.2
Jan-94	-11.21	-73.8
Feb-94	-11.29	-80.6
Mar-94	-10.52	-80.9
Apr-94	-3.73	-18.3
May-94	-6.61	-44.7
Jun-94	-7.61	-49.2
Jul-94	-3.00	-14.2
Aug-94	-2.00	-12.6
Sep-94	-2.50	-11.6
Average	-6.99	-44.38
Weighted $\Sigma(P_i/\Sigma P)$	-7.76	-49.35

The separated quantities of groundwater inflow, I_{ss}, and outflow, O_{ss}, are calculated by the isotopic approach using Equations 6 and 7 for $\delta^{18}O$ and δ^2H and given in Table 9 (subsurface inflow and subsurface outflow for $\delta^{18}O$ and δ^2H), together with an error analysis.

Monthly errors of I_{ss} and O_{ss} are within the acceptable limits. For three different error analyses, only 8 months' percent errors of I_{ss} are greater than 30% out of 72 months' (11.1%) and only 16 months' percent errors of O_{ss} are greater than 30% out of 72 months' (22.2%). Errors of I_{ss} and O_{ss} given in Table 9 are the averaged values of three different error analyses. However, if the monthly percent error is greater than 130%, it is ignored and average of the remaining months is calculated.

For the 1994 water year, using the $\delta^{18}O$ mass balance approach, the sum of monthly I_{ss} is $25.59*10^6 m^3 \pm$ 15.50% while the sum of monthly O_{ss} is $21.22*10^6 m^3 \pm$ 23.46%. The annual volumes of I_{ss} and O_{ss} are obtained as $19.10*10^6 m^3 \pm$ 12.11% and $15.51*10^6 m^3 \pm$ 16.30%, respectively. Oxygen-18 balance yields differences between the monthly sum method and annual method of 34.0% and 36.8% in I_{ss} and O_{ss}, respectively.

If δ^2H is used instead of $\delta^{18}O$, the summation of monthly I_{ss} and O_{ss} volumes are $22.39*10^6 m^3 \pm$ 13.61% and $18.02*10^6 m^3 \pm$ 21.66%. Using δ^2H, the annual I_{ss} and O_{ss} are obtained as $14.57*10^6 m^3 \pm$ 27.64% and $10.98*10^6 m^3 \pm$ 38.37%, respectively. Deuterium balance yields differences between the monthly sum method and annual method of 53.7% and 64.1% in I_{ss} and O_{ss} respectively.

Table 9. I_{ss} and O_{ss} of Mogan Lake and their errors

a) Oxygen -18 Balance

	Mass balance	I_{ss}					O_{ss}				
			Error %					Error %			
	$\Delta(I_{ss} - O_{ss})$	10^3 m^3	Case 1	Case 2	Case 3	Average	10^3 m^3	Case 1	Case 2	Case 3	Average
Oct-93	904.7	1939.3	10.79	5.43	12.29	9.50	1034.5	21.93	11.86	24.55	19.45
Nov-93	187.0	195.8	46.10	25.90	50.93	40.98	8.9	1207.6[3]	960.1[3]	1298.8[3]	1155.5[3]
Dec-93	95.5	666.0	11.05	8.43	12.66	10.71	571.8	17.66	20.40	19.06	19.04
Jan-94	223.9	1230.8	5.77	5.10	7.31	6.06	1006.8	10.68	13.48	12.01	12.06
Feb-94	284.0	1162.3	8.28	6.41	9.84	8.91	878.3	15.43	17.48	16.96	16.62
Mar-94	153.0	339.7	61.40	36.04	68.07	55.17	186.7	128.60	92.19	139.26	120.02
Apr-94	112.4	2881.8	12.01	6.08	13.74	10.61	2769.4	13.11	6.81	14.85	11.59
May-94	286.9	2544.8	11.70	5.90	12.92	10.17	2257.9	14.40	7.47	15.67	12.51
Jun-94	285.8	2950.5	10.24	5.15	10.94	8.78	2664.8	13.56	8.60	14.21	12.12
Jul-94	727.0	4366.0	10.03	5.03	10.74	8.60	3639.0	13.45	7.70	14.22	11.79
Aug-94	539.5	4259.2	10.04	5.03	10.86	8.64	3719.7	12.56	7.12	13.43	11.04
Sep-94	571.5	3055.9	10.06	5.05	10.83	8.65	2484.5	13.60	7.41	14.47	11.83
Average	364.3	2132.7	17.29	9.96	19.26	15.50	1768.5	25.00[1]	18.23[1]	27.15[1]	23.46[1]
Sum	4369.9	25592.3					21222.3				
Annual	3590.2	19102.5	13.85	7.09	15.40	12.11	15512.3	18.71	9.74	20.46	16.30

b) Deuterium Balance

	Mass balance	I_{ss}					O_{ss}				
			Error %					Error %			
	$\Delta(I_{ss} - O_{ss})$	10^3 m^3	Case 1	Case 2	Case 3	Average	10^3 m^3	Case 1	Case 2	Case 3	Average
Oct-93	904.7	941.4	12.25	6.41	16.95	11.87	36.6	395.6[3]	237.9[3]	497.1[3]	376.9[3]
Nov-93	187.0	783.2	13.45	6.96	23.70	14.70	596.3	20.13	14.67	32.59	22.46
Dec-93	95.5	297.3	32.78	22.40	47.67	34.28	203.1	58.83	60.08	77.64	65.52
Jan-94	223.9	1428.2	5.67	5.15	11.07	7.30	1204.3	9.50	11.71	14.74	11.98
Feb-94	284.0	1417.2	6.39	5.41	11.88	7.89	1133.3	11.61	13.63	17.08	14.11
Mar-94	153.0	651.1	25.28	17.31	36.08	26.22	498.1	40.76	33.20	52.86	42.27
Apr-94	112.4	2619.7	11.68	5.99	17.60	11.76	2507.3	12.97	6.85	18.90	12.91
May-94	286.9	2057.8	11.73	6.04	16.47	11.41	1770.9	15.50	8.26	20.52	14.76
Jun-94	285.8	2483.3	10.30	5.30	13.51	9.70	2197.5	14.72	9.84	17.73	14.1
Jul-94	727.0	3751.3	10.08	5.13	12.89	9.37	3024.3	14.44	8.57	17.54	13.52
Aug-94	539.5	3426.7	10.08	5.13	13.03	9.41	2887.1	13.64	8.13	16.79	12.85
Sep-94	571.5	2536.7	10.07	5.12	12.90	9.36	1965.2	14.81	8.35	18.11	13.76
Average	364.3	1866.2	13.31	8.03	19.48	13.61	1502.0	20.63[2]	16.66[2]	27.68[2]	21.66[2]
Sum	4369.9	22393.9					18024.0				
Annual	3590.2	14566.6	15.30	8.28	59.35	27.64	10976.4	23.04	12.57	79.51	38.37

NOTE:

[1] Error values of O_{ss} in November 1993 are ignored in oxygen-18 balance

[2] Error values of O_{ss} in October 1993 are ignored in deuterium balance

[3] High values of error terms depend on lower value of O_{ss}

Volumes of I_{ss} and O_{ss} found from the oxygen-18 balance is more than volume of I_{ss} and O_{ss} found from the deuterium mass balance. Deuterium balance yields a large difference between the monthly sum method and annual method. In the monthly sum method, the difference in I_{ss} is 14.3% and in O_{ss} 17.8%. The corresponding results in the annual method are 31.1% and 41.4%, respectively.

It would be appropriate to take the average of the results from the monthly sum and annual balance methods and give a single inflow and a single outflow value for the 1994 water year (Table 10). The average groundwater inflow to Mogan Lake is calculated to be 20.42 million m^3± 17.22% while the average groundwater outflow from Mogan Lake is calculated to be 16.44 million m^3± 24.95%.

Table 10. Averaged I_{ss} and O_{ss} of Mogan Lake for 1994 water year ($^*10^6 m^3$)

Mass Balance	Oxygen -18		Deuterium		Average	
$\Delta(I_{ss} - O_{ss})$	I_{ss}	O_{ss}	I_{ss}	O_{ss}	I_{ss}	O_{ss}
3.98 ± 8.0%	22.35 ± 13.81%	18.37 ± 19.88%	18.48 ± 20.63%	14.50 ± 30.02%	20.42 ± 17.22%	16.44 ± 24.95%

Sensitivity analysis of the hydrological and meteorological elements

Water budget elements, namely surface inflow, precipitation, evaporation, and surface outflow, and meteorological elements such as relative humidity, air temperature, water temperature, and isotopic composition of precipitation are multiplied respectively by -30%, -20%, -10%, 10%, 20% and 30%, while keeping the rest constant, in order to see their effect on: i) isotopic composition of atmosphere, δ_{atm}, ii) isotopic composition of evaporating water surfaces, δ_E, and iii) subsurface inflow, I_{ss}, and subsurface outflow, O_{ss}. Computations for monthly oxygen-18 and monthly deuterium balances are made for the 1994 water year but, due to space limitations, graphical results are given only for oxygen-18 balance. Findings from the sensitivity analysis are summarized as follows:

i) δ_{atm} depends on air temperature and the isotopic composition of precipitation. As monthly air temperature increases, δ_{atm} becomes enriched in terms of heavy isotopes, i.e., becomes slightly positive (Figure 3). However, as the isotopic composition of precipitation becomes depleted, δ_{atm} becomes depleted in terms of heavy isotopes, i.e., becomes sharply negative.

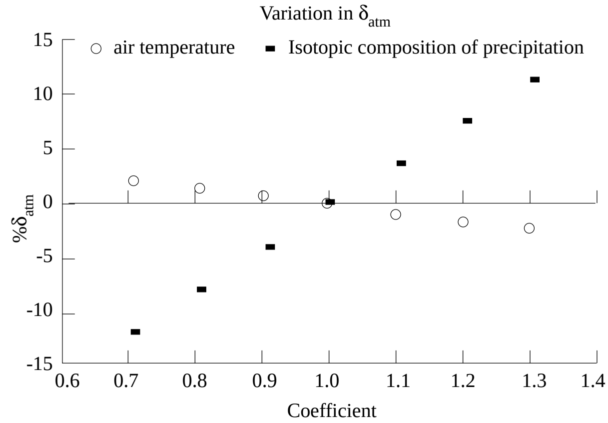

Figure 3. Effect of air temperature and isotopic composition of precipitation on δ_{atm}

ii) As seen in Equation 8, δ_E depends on hydrological and meteorological factors. Their effects are shown in Figure 4. As monthly relative humidity and air temperature increase, and the isotopic composition of the atmosphere becomes depleted, δ_E becomes enriched. However, as water temperature increases, δ_E becomes depleted.

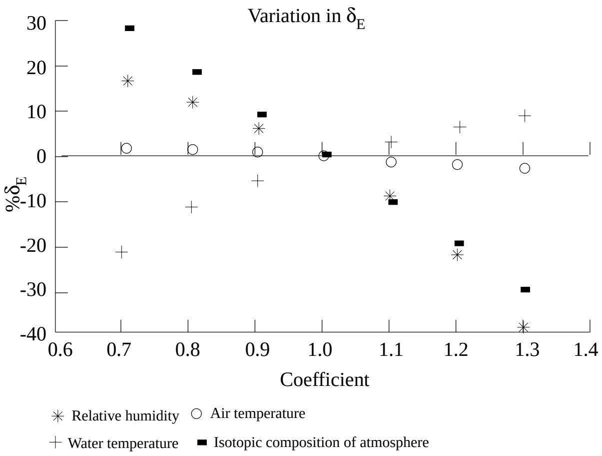

Figure 4. Effect of relative humidity, air temperature, water temperature, and isotopic composition of precipitation on δ_E

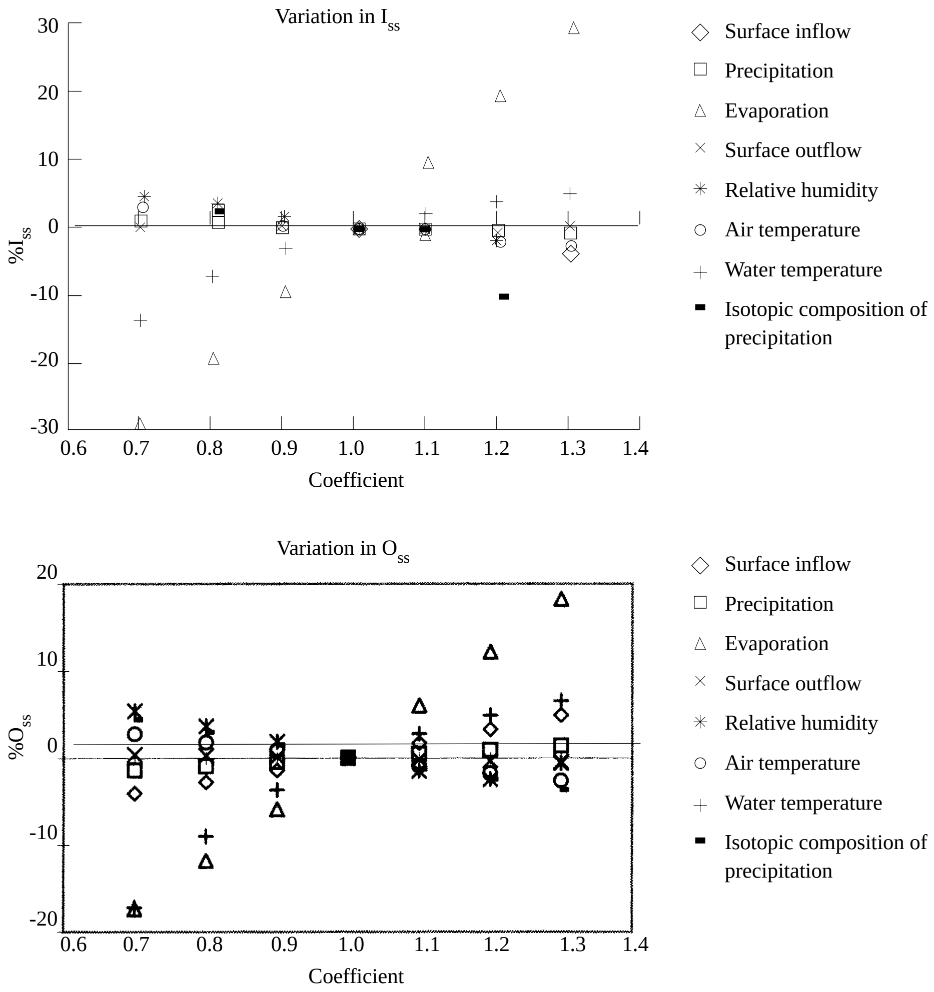

Figure 5. Effect of surface flow, precipitation, evaporation, surface outflow, relative humidity, air temperature, water temperature, and isotopic composition of precipitation on subsurface inflow, I_{ss}, and subsurface outflow, O_{ss}

iii) By increasing the monthly evaporation and water temperature, subsurface inflow, I_{ss}, and subsurface outflow, O_{ss}, will increase (Figure 5). However, increases in relative humidity and in air temperature with depletion in the isotopic composition of precipitation, decrease the subsurface inflow and subsurface outflow. Increases in surface inflow and precipitation cause a decrease in I_{ss} but cause an increase in O_{ss}. The most dominant factor on I_{ss} and O_{ss} is evaporation.

Conclusions

The aim of this study was to determine the separated quantities of subsurface inflows, I_{ss}, and subsurface outflow, O_{ss}, by using environmental isotopes. The stable isotope method is an efficient tool used in water budget analysis. The outcomes of the present study for the 1994 water year are as follows:

- Annual weighted isotopic composition of evaporating water body δ_E is expressed by meteorological and hydrological factors and was calculated to be -19.76‰ and -105.62‰ for $\delta^{18}O$ and δ^2H, respectively.
- Annual weighted δ_P was calculated to be -7.76‰ and -49.35‰ for $\delta^{18}O$ and δ^2H, respectively.
- The average groundwater inflow to and groundwater outflow from Mogan Lake, calculated by isotopic mass balance, are $20.42{*}10^6 m^3 \pm$ 17.22% and $16.44{*}10^6 m^3 \pm$ 24.95%, respectively.
- Deuterium mass balance gives lower I_{ss} and O_{ss} values than oxygen-18 mass balance and it also gives large difference between monthly sums and annual values.
- The most dominant parameter in the balance equation is evaporation. Therefore, more research is needed for the determination of evaporation.
- It would be useful to confirm the results of this study by carrying out another study by using a solute transport model not only for the 1994 water year but also for other water years to make more solid conclusions. However, the detailed information needed for such a model including the number, depth, border and the transmibilities of the existing aquifers is not known.
- The results of the conventional water budget method can be improved if more piezometers are drilled around and within the lake.

List of Symbols

atm = atmosphere
dt = time interval
E = evaporation
e_s = saturation vapour pressure
H = hydrogen
^{2}H = deuterium, isotope of hydrogen
h = relative humidity
I_s = surface flow
I_{ss} = subsurface flow
L = lake
O = oxygen
^{18}O = oxygen-18, isotope of oxygen
O_s = surface outflow
O_{ss} = subsurface outflow
P = precipitation
T = temperature in Kelvins
t = temperature in Celsius
V = volume
α = water-vapour fractionation factor
ε = total enrichment factor
$\Delta\varepsilon$ = kinetic enrichment factor
δ = isotopic composition of related water balance elements

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