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Sliding Mode Control of a Multi-Degree-of-Freedom Structural System With Active Tuned Mass Damper

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Abstract

In this study, a sliding mode control system is designed for a multi-degree-of-freedom structure having an Active Tuned Mass Damper (ATMD) to suppress earthquake or wind induced vibration. Since the model might have uncertainties and/or parameter changes, sliding mode control is preferred because of its robust character and superior performance. In addition this control method can easily be applied to non-linear systems. The simulated system has five degrees of freedom. In this study, a linear motor is used as the control device. At the end of the study, the time history of the floor displacements, control voltage and frequency response of the both uncontrolled and sliding mode controlled structures are presented and the results are discussed.

Key Words: Sliding mode control, active tuned mass damper (ATMD), multi-degree-of-freedom structure, earthquake or wind induced vibration.

Aktif Ayarlı Kütle Sönümleyicili Çok Serbestlik Dereceli Yapısal Bir Sistemin Kayan Kipli Denetimi

Özet

Bu çalışmada, aktif ayarlı kütle sönümleyiciye (ATMD) sahip çok serbestlik dereceli bir yapının deprem veya rüzgar kaynaklı titreşimlerini bastırmak için kayan kipli bir kontrol sistemi tasarlandı. Model belirsizlikler ve/veya parametre değişikliklerine sahip olabileceği için, robust niteliğe ve üstün performansa sahip olan kayan kipli denetim tercih edildi. Bunun yanında, bu denetim yöntemi doğrusal olmayan sistemlere de kolaylıkla uygulanabilir. Benzetimi yapılan sistem beş serbestlik derecesine sahiptir. Bu çalışmada, kontrol cihazı olarak bir doğrusal motor kullanılmıştır. Çalışmanın sonunda, kontrolcüsüz ve kayan kipli kontrolcülü yapının kat hareketlerinin, denetim voltajının zaman cevapları ve frekans cevapları sunulmuş ve sonuçlar irdelenmiştir.

Anahtar Sözcükler: Kayan kipli denetim, aktif ayarlı kütle sönümleyici (ATMD), çok serbestlik dereceli yapı, deprem veya rüzgar kaynaklı titreşim.

Introduction

Structural vibration control has improved rapidly both in theory and in practice recently. Vibration isolation using rubber bearings is one of the most popular method of passive vibration control. It is known that a seismic isolation rubber bearing, consisting of rubber sheets and steel plates, is effective for an architectural structure whose base is subjected to an earthquake input. Also, semi-active vibration methods are proposed in the literature. Yoshida and Fujio (1999) applied a semi-active control method to a base in which the viscous damping coefficient is changed for vibration control. In recent years, there have been studies where active actuators are used for isolation systems in order to isolate the earthquake induced vibrations. Fukushima et al. (1996) developed an active-passive composite tuned mass damper aimed at reducing wind and earthquake induced vibrations of tall building structures. Since there are uncertainties in building structures, and system parameters are not constant, robust control methods are offered for the active control of the structures (Nishimura et al., 1996). Because the actual buildings have a non-linear character, sliding mode controls have gained more importance (Adhikari and Yamaguchi, 1997).

The aim of this study is to apply non-chattering sliding mode control to structural systems. If not prevented, chattering causes damage to mechanical components. Sabanovic (1994) proposed an effective method for chattering free sliding mode applications. The improvements in electromagnetic force sources and sensors make it possible. Dan Cho (1993) presented the application of sliding mode control to stabilize an electromagnetic suspension system with experimental results. Yagiz et al. (2000) proposed the application of sliding mode control on a vehicle. Presently this method has been applied to robot control, flight control, motor control and power systems successfully. The superiorities of this method are its applicability on nonlinear systems, simplicity, high performance and robust character.

Dynamic Model of the Structural System

The structural system has five degrees of freedom that are all in a horizontal direction. An ATMD with the active element and passive elements, which are optimally tuned for the first mode of the primary structure, is placed over the top floor. The aim of an ATMD control system is to reduce wind and earthquake induced vibrations of tall buildings. The physical system has been shown in Figure 1. The masses of each floor are m_1, m_2 ,m_3 and m_4 respectively, where m_5 is the mass of the ATMD. x_1 x_2 x_3 x_4 and x_5 are the horizontal displacements belonging to them. All springs and dampers are acting in a horizontal direction. The system parameters are presented in the Appendix.

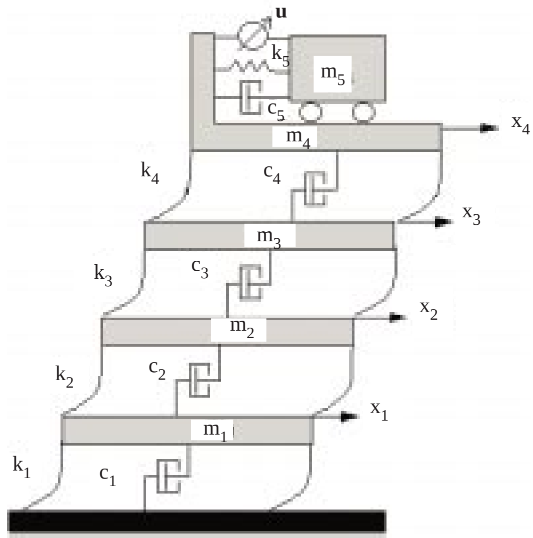

Figure 1. Physical model of structural system

The equation of motion of the system is given below:

$$[M]\underline{\ddot{X}} + [C]\underline{\dot{X}} + [K]\underline{X} = \underline{F}_u + \underline{F}_d \quad (1)$$

where $\underline{X} = [x_1\ x_2\ x_3\ x_4\ x_5]^T$ and $\underline{F}_u = [\ 0\ 0\ 0\ F_u - F_u]^T$. F_u is the control force produced by a linear motor and $\underline{F}_d$ is the disturbance vector to the structural system. $[M]$, $[C]$ and $[K]$ are mass, damping and stiffness matrices and are given in the Appendix.

In Figure 2, the working principle of the linear motor is depicted. It is comprised of two main parts:

i) a number of base-mounted permanent magnets forming the stator,

ii) a translator (as counterpart of the rotor in a rotating motor) formed by a number of iron-core coils.

By applying a three-phase current to three adjoining coils of the translator, a sequence of attracting and repelling forces between the poles and the permanent magnets will be generated. This results in a thrust force experienced by the translator. Basically, the motor is a synchronous permanent-magnet motor with electronic commutation (Otten et al., 1997), (Nasar and Boldea, 1987).

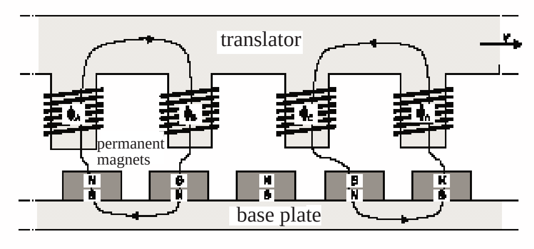

Figure 2. Working principle of a linear motor.

The equation of the linear motor is

$$R\ i + K_e(\dot{x}_5 - \dot{x}_4) = u \tag{2}$$

u and i are the voltage and current of the armature coil respectively where u is the control voltage input at the same time. R and K_e are the resistance value and induced voltage constant of the armature coil. The current of the armature coil and control force has the following relation:

$$F_u = K_f i \tag{3}$$

where K_f is the thrust constant. The inductance of the armature coil is neglected (Nishimura et al., 1996). By combining the equations (1) through (3) and arranging them, it is also possible to get the governing equations in state space form.

The Sliding Mode Controller Design

Sliding mode control theory has been applied to many nonlinear systems. The main idea is to bring and keep the error on a sliding surface such that the system is insensitive to the disturbances and parameter changes (Utkin, 1981; Yagiz et al., 1997). The nonlinear system is defined in state space form as

$$\underline{\dot{x}} = \underline{f}(x) + [B] * \underline{u} \tag{4}$$

where $\dim[B] = n_* m, \dim(\underline{f}(x)) = n_* 1$ and $\dim(\underline{u}) = m_* 1; \underline{f}(x)$ is continuous, but $\underline{u}(t)$ may be discontinuous. The aim is to hold the system motion on a sliding surface S. The surface can be expressed as

$$S = \{\underline{x} : \underline{\sigma}(x,t) = 0\} \tag{5}$$

In order to obtain a stable solution of the system, it must stay on this surface, i.e. $\underline{\sigma}(x,t) = 0$ as shown in Figure 3.

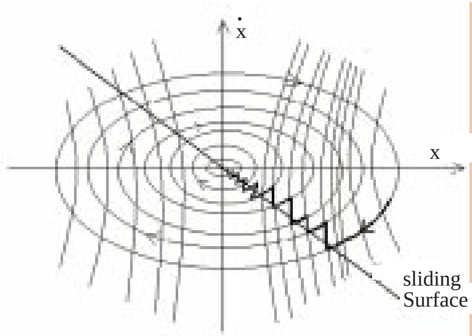

Figure 3. Phase plane diagram of the state variables.

The sliding surface equation for control of the system can be selected as follows:

$$\underline{\sigma}(x,t) = [G]_* \underline{e} = [G]_* (\underline{x}_{ref} - \underline{x}) \tag{6}$$

In this equation, $\underline{x}_{ref}$ represents the state vector of the reference, and the constant $[G]$ matrix represents the slope of the sliding surface (Utkin, 1977). The same equation also can be written as

$$\underline{\sigma}(x,t) = \underline{\Phi}(t) - [G]_* \underline{x} \tag{7}$$

where

$$\underline{\Phi}(t) = [G]_* \underline{x}_{ref}(t) \tag{8}$$

The first step in design is to select a Lyapunov function $\underline{\nu}$. According to the Lyapunov Stability Criteria, the Lyapunov function must have a value greater than zero, whereas its derivative should be smaller than zero. Selecting the function as in equation (9) makes its value greater than zero

$$\underline{\nu} = \underline{\sigma}^T(x,t)_* \underline{\sigma}(x,t)/2 > 0 \tag{9}$$

To have the value of the derivative of the Lyapunov Function smaller than zero:

$$\frac{d\underline{\nu}}{dt} = -\sigma^T(x,t)_* \Gamma_* \underline{\sigma}(x,t) < 0 \tag{10}$$

where Γ has a positive value. Through this, the Lyapunov Stability Criteria have been satisfied. By equating equation (10) to the derivative of (9)

$$d\underline{\sigma}(x,t)/dt = -\Gamma_* \underline{\sigma}(x,t) \text{ or } d\underline{\sigma}(x,t)/dt + \Gamma_* \underline{\sigma}(x,t) = 0 \quad (11)$$

As it is seen in equation (11), the sliding function becomes zero at infinity. But the goal is to make it very close to zero. If equation (7) is differentiated and (4) is used, the derivative of the sliding surface is obtained as

$$\begin{aligned} d\underline{\sigma}(x,t)/dt &= d\underline{\Phi}(t)/dt - [G]_* d\underline{x}/dt \\ &= d\underline{\Phi}(t)/dt - [G]_*(\underline{f}(x) + [B]_* \underline{u}(t)) \end{aligned} \quad (12)$$

The controller is designed as below by inserting (12) into (11)

$$\underline{u}(t) = \underline{u}_{eq}(t) + [GB]_*^{-1} \Gamma_* \underline{\sigma}(x,t) \quad (13)$$

where

$$\underline{u}_{eq}(t) = [GB]_*^{-1}(d\underline{\Phi}(t)/dt - [G]_* \underline{f}(x)) \quad (14)$$

If the knowledge of $\underline{f}(x)$ and $[B]$ matrices are poor, then the equivalent calculated control inputs will be far from the actual equivalent control inputs. In the literature, a number of approaches are proposed for the estimation of $\underline{u}_{eq}$, rather than calculating it. In this study, it is suggested that the equivalent control is the average of the total control. The design of an averaging filter for calculation of the equivalent control can be as below (Sabanovic, 1994):

$$\underline{\hat{u}}_{eq} = \frac{1}{\tau s + 1} \underline{u} \quad (15)$$

This is actually a low-pass filter. The value of $1/\tau$ gives the cut-off frequency. The logic behind designing a low pass filter is that low frequencies determine the characteristics of the signal and high frequencies come from unmodeled dynamics. Then

$$\underline{u}(t) = \underline{\hat{u}}_{eq} + ([G].[B])_*^{-1} \Gamma_* \underline{\sigma}(x,t) \quad (16)$$

Simulation

A structural system was simulated against 0.010 m. of initial displacement of the first floor. Figures 4.a and 4.b show the controlled and uncontrolled time responses of the top and first floors. It is observed that there is an important improvement when the horizontal displacements of the structure are considered. Figure 5.a demonstrates the change in voltage input. The motion of ATMD mass is shown in Figure 5.b.

Figure 6 shows the frequency responses of the top floor displacements and accelerations respectively for both controlled and uncontrolled cases. The natural frequencies of the structure without ATMD are at 2.1, 6.2, 9.7 and 12 Hz. The natural frequency of ATMD was tuned for the first mode.

As expected, the upper curves belong to the uncontrolled system. A significant improvement in terms of magnitudes have been witnessed again particularly at the resonance value of 6.2 Hz. on the top floor, as anticipated.

Conclusion

An ATMD with sliding mode controller has been designed for the multi-degree-of-freedom structural system. Since the destructive effects of earthquakes and wind disturbances are sourced as a result of horizontal vibrations, in this study, the degrees of freedom have been assumed only in this direction. The system is modeled to include the dynamics of a linear motor used as the control device. Since structural systems and buildings have uncertainties, and their parameters are subject to changes, a sliding mode control preferred because of its robust character, applicability to nonlinear systems and superior performance. Against disturbances, it is shown that a designed sliding mode controller has brought satisfactory seismic isolation performance.

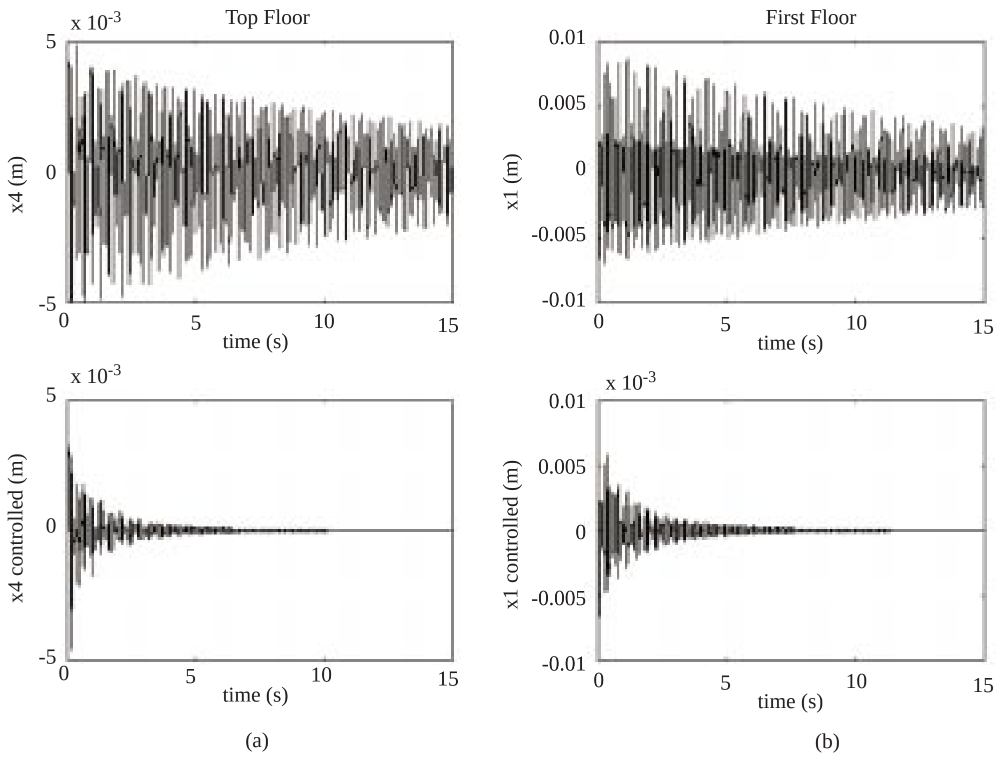

Figure 4. Controlled and uncontrolled time responses of the first and top floors.

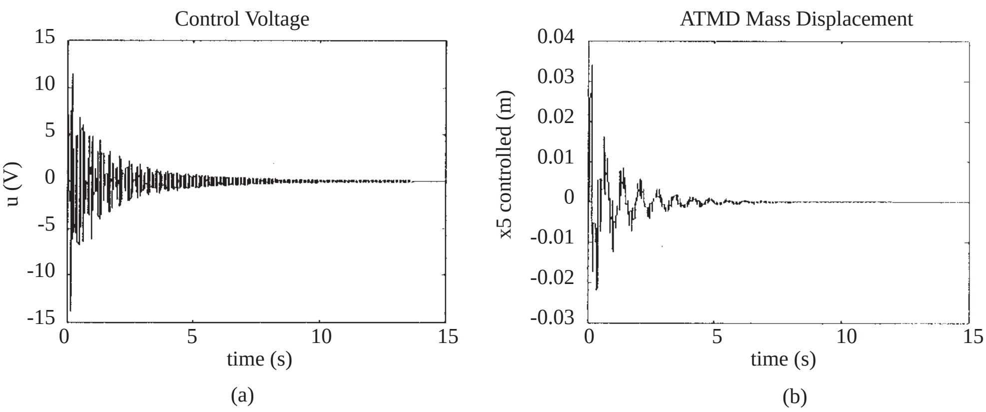

Figure 5. Time history of the control voltage and ATMD mass displacement.

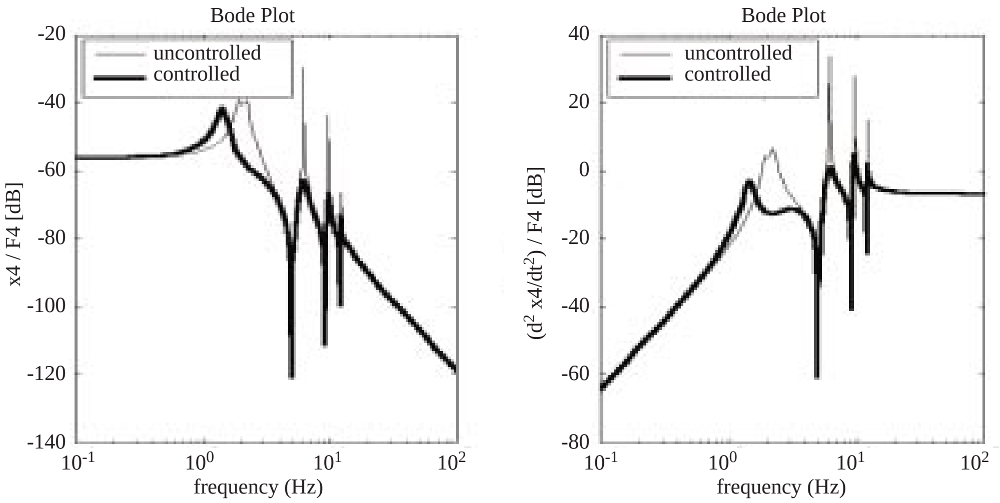

Figure 6. Controlled and uncontrolled frequency responses of the top floor.

Nomenclature

$m_1, m_2, m_3,$ m_4 and m_5	Mass of the related floors and ATMD [kg]
$k_1, k_2, k_3,$ k_4 and k_5	Stiffness of the related floors and ATMD [N/m]
$c_1, c_2,$ c_3, c_4 and c_5	Damping value of the related floors and ATMD [Ns/m]
$x_1, x_2, x_3,$ x_4 and x_5	Displacement of the related floors and ATMD [m]
K_f	Thrust constant of the armature coil [N/A]
K_e	Induced voltage constant of the armature coil [Vs/m]
R	Resistance of the armature coil [Ω]
u	Control voltage input [V]
$\underline{f}(x)$	Vector of non-linear state space equation
$\underline{\sigma}$	Vector of sliding surfaces
$[B]$	Control input matrice
$[G]$	Matrice of slopes of sliding surfaces
$\underline{\nu}$	Vector of Lyapunov functions
$\underline{u}$	Vector of control inputs
Γ	Positive term
$\underline{u}_{eq}$	Equivalent control input
τ	Time constant of the low-pass filter [s]

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Appendix

Parameters of the structural system:

$\mathbf{m}_1$	1.6 kg	$\mathbf{m}_2$=$\mathbf{m}_3$	1.5 kg
$\mathbf{m}_4$	2.2 kg	$\mathbf{m}_5$	0.135 kg
$\mathbf{k}_1$=$\mathbf{k}_2$=$\mathbf{k}_3$=$\mathbf{k}_4$	2600 N/m	$\mathbf{k}_5$	22 N/m
$\mathbf{c}_1$= $\mathbf{c}_2$= $\mathbf{c}_3$	0.08 N.s/m	$\mathbf{c}_5$	0.38 N.s/m
$\mathbf{K}_f$	2 N/A	$\mathbf{K}_e$	2 V.s/m
R	4.2 Ω	$\mathbf{x}_{10}$	0.01 m.

Mass, damping and stiffness matrices:

Mass matrix,

$$[M] = \begin{bmatrix} m1 & 0 & 0 & 0 & 0 \\ 0 & m2 & 0 & 0 & 0 \\ 0 & 0 & m3 & 0 & 0 \\ 0 & 0 & 0 & m4 & 0 \\ 0 & 0 & 0 & 0 & m5 \end{bmatrix}$$

Damping matrix,

$$[C] = \begin{bmatrix} (c1+c2) & -c2 & 0 & 0 & 0 \\ -c2 & (c2+c3) & -c3 & 0 & 0 \\ 0 & -c3 & (c3+c4) & -c4 & 0 \\ 0 & 0 & -c4 & (c4+c5) & -c5 \\ 0 & 0 & 0 & -c5 & c5 \end{bmatrix}$$

Stiffness matrix,

$$[K] = \begin{bmatrix} (k1+k2) & -k2 & 0 & 0 & 0 \\ -k2 & (k2+k3) & -k3 & 0 & 0 \\ 0 & -k3 & (k3+k4) & -k4 & 0 \\ 0 & 0 & -k4 & (k4+k5) & -k5 \\ 0 & 0 & 0 & -k5 & k5 \end{bmatrix}$$