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Steady Slow Flow of an Oldroyd 8-Constant Fluid in a Corner Region with a Moving Wall

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Abstract

Viscoelastic fluids have gained increasing importance recently in technological applications. They are considered more realistic when compared to Newtonian fluids in some situations where flow phenomena can only be explained by using viscoelastic fluids' models. This paper discusses problem of dealing with the steady slow flow of an Oldroyd 8-constant viscoelastic fluid in a corner region with a moving wall. The aim of this study is to examine theoretically whether or not fluid elasticity is responsible for the formation of circulating cells near the corner, which has been observed experimentally in various polymer processes. Using series expansions given by Strauss (1975) for the stream function and stress components, the governing equations of the problem are reduced to linear ordinary differential equations. These equations have been solved analytically. It is shown that streamline patterns are strongly dependent on viscoelastic parameters. There is, unlike the case of Newtonian fluid, a secondary flow near the corner point.

Key Words: Non-radial flow, Oldroyd 8-constant fluid, slow flow, circulating cells.

8 Sabitli Oldroyd Akışkanının Cidarlarından Birisi Hareketli Olan Bir Köşe içindeki Daimi ve Yavaş Akımı

Özet

Viskoelastik akışkanlar son zamanlarda teknolojik uygulamalarda giderek artan bir ölçüde önem kazanmaktadırlar. Akış olaylarının yalnızca viskoelastik akışkan modelleri yardımıyla açıklanabileceği bazı durumlarda bu türden akışkanların Newtonian akışkanlara kıyasla daha gerçekçi modeller olduğu düşünülmektedir. Bu makale, 8 sabitli viskoelastik Oldroyd akışkanının cidarlarından birisi hareketli olan bir köşe içindeki daimi ve yavaş akımıyla ilgili olan böyle bir problemi tartışmaktadır. Çalışmanın amacı; çeşitli polimer proseslerinde köşe noktası civarında oluştukları deneysel olarak gözlemlenmiş olan sirkülasyon halkalarına akışkanın elastisitesinin neden olup olmadığını teorik olarak incelemektir. Akım fonksiyonu ve gerilme bileşenleri için Strauss (1975) tarafından verilen seri açılımları kullanılarak problemi yöneten denklemler analitik olarak çözümü verilebilen lineer diferansiyel denklemlere indirgenmiştir. Akım çizgilerinin yapısının viskoelastik parametrelere kuvvetli bir şekilde bağlı olduğu gösterilmiştir. Newtonian akışkandan farklı olarak, köşe noktası civarında ikincil akımların oluştuğu görülmüştür.

Anahtar Sözcükler: Radyal olmayan akım, 8 sabitli Oldroyd akışkanı, yavaş akım, sirkülasyon halkaları.

Introduction

The creeping corner flow induced by a steady in-plane motion of one of the walls has been examined by Moffatt (1964) and Batchelor (1970), but their works are restricted to Newtonian fluids. Hancock and Lewis (1981) have investigated the effects of inertia forces, by constructing a regular perturbation series for the stream function, of which the leading term is the known similarity solution. The two-dimensional steady and slow flow of an incompressible Maxwell fluid in a corner formed by two planes, one of which is sliding past the other at a certain angle, was first investigated by Strauss (1975) using a truncated series expansion for the stream function of the form

$$\psi(r,\theta) = \sum_{n=-1}^{N} \frac{\psi_n(\theta)}{r^n} \tag{1}$$

where $\psi(r,\theta)$ denotes the stream function in a polar coordinate system. Strauss (1975) solved the problem up to three terms (N=1) in this assumed series and found circulating cells adjacent to the moving plane. Riedler and Schneider (1983) studied the non-inertial flow of a power law fluid in a corner region with a moving wall and showed that the streamline patterns near the moving wall were considerably less affected by the power law exponent than near the wall at rest. Strauss' work (1975) on the Maxwell fluid has been recently extended by Huang et al. (1993) for the exact same geometry and boundary conditions as Strauss' study (1975), to the case of an Oldroyd-B fluid. They find that an increase in the elastic parameter reduces cellular structure. Bhatnagar et al. (1996) reconsidered Strauss' problem by taking into account the next term in the series expansion (1) and demonstrated that the solution corresponding to N=2 is significantly different from that for N=1.

The present paper concerns the steady slow motion of an Oldroyd 8-constant fluid near a corner of plane rigid walls, one of which is stationary and the other moving parallel to itself with a steady velocity U. Our results are similar to those of Strauss (1975) and Huang et al. (1993) but differ in some details. Also, it is, as expected, possible to establish a relationship to their works.

Formulation of the problem and its solution

Consider the steady, two-dimensional, incompressible, laminar flow of the Oldroyd 8-constant fluid in a corner region bounded by two non-parallel planes, one of which is moving with constant velocity U, as is schematically illustrated in Fig. 1.

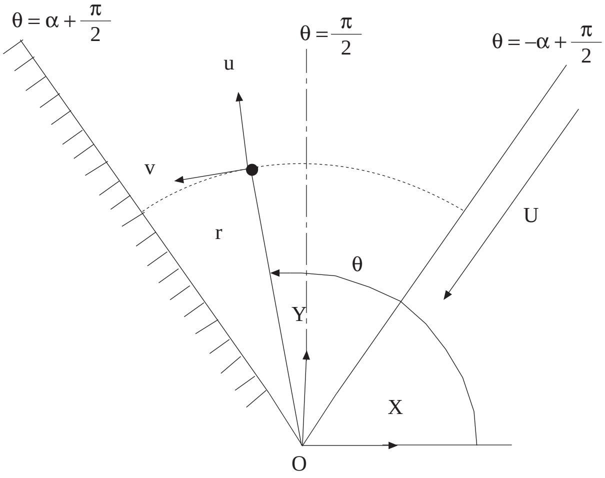

Figure 1. Basic geometry of the problem

The viscoelastic fluid model used here is the Oldroyd 8-constant model, constitutive equation of which is given as follows (Bird et al., 1987)

$$\mathbf{T} = -p\mathbf{I} + \mathbf{S} \tag{2}$$

$$\begin{aligned}&\mathbf{S} + \Lambda_1 \frac{\mathcal{D}\mathbf{S}}{\mathcal{D}t} + \Lambda_3(\mathbf{S}\cdot\mathbf{A_1} + \mathbf{A_1}\cdot\mathbf{S}) + \\ &\Lambda_5(tr\mathbf{S})\mathbf{A_1} + \Lambda_6[tr(\mathbf{S}\cdot\mathbf{A_1})]\mathbf{I} = \\ &\mu(\mathbf{A_1} + \Lambda_2\frac{\mathcal{D}\mathbf{A_1}}{\mathcal{D}t} + \Lambda_4\mathbf{A_1^2} + \Lambda_7[tr(\mathbf{A_1^2})]\mathbf{I})\end{aligned} \tag{3}$$

where $\mathbf{T}$ is the Cauchy stress tensor, p is the pressure, $\mathbf{I}$ is the identity tensor, $\mathbf{S}$ is the extra stress tensor, μ is the coefficient of viscosity, and Λ_i $(i = 1, 2, ..., 7)$ are the material constants. $\mathbf{A_1}$ is the first Rivlin-Ericksen tensor and $\mathcal{D}/\mathcal{D}t$ the contravariant convected derivative is defined as follows, respectively

$$\mathbf{A_1} = \nabla\mathbf{v} + \nabla\mathbf{v}^T \tag{4}$$

$$\frac{\mathcal{D}\mathbf{S}}{\mathcal{D}t} = \frac{\partial\mathbf{S}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{S} - \mathbf{S}\cdot\nabla\mathbf{v} - \nabla\mathbf{v}^T\cdot\mathbf{S} \tag{5}$$

where $\mathbf{v}$ is the velocity vector, ∇ is the gradient operator, while the superscript T denotes a transpose operation.

When $\Lambda_i = 0$ $(i = 1, 2, ..., 7)$ the model (3) reduces to the classical linearly viscous Navier-Stokes fluid (Newtonian fluid). Also, it should be noted that this model includes the Maxwell fluid for $\Lambda_1 \neq 0, \Lambda_i = 0$ $(i = 2, 3, ..., 7)$ and the Oldroyd-B fluid for $\Lambda_1 \neq 0, \Lambda_2 \neq 0, \Lambda_i = 0$ $(i = 3, 4, ..., 7)$

In addition to Eqs. (2) and (3), the field equations consist of the equations of motion and the continuity equation. In the case of a steady flow, the former equations in the absence of body forces take the form

$$\rho(\mathbf{v} \cdot \nabla \mathbf{v}) = \nabla \cdot \mathbf{T} \tag{6}$$

where ρ is the (constant) density. The continuity equation is

$$tr\mathbf{A_1} = 0. \tag{7}$$

We shall assume a velocity field in a plane polar coordinate system (r, θ) of the form

$$\mathbf{v}(r, \theta) = u(r, \theta)\mathbf{e_r} + v(r, \theta)\mathbf{e}_\theta \tag{8}$$

where u and v denote the velocity components in the directions of r and θ respectively.

We shall now write the field equations in terms of a set of dimensionless variables and, for this purpose, we shall choose Λ_1, μ and U as characteristic units. If $\overline{f}$ is used to denote the dimensionless form of a quantity f, it follows that

$$\begin{aligned} \overline{r} &= \frac{r}{U\Lambda_1}, \ \overline{u} = \frac{u}{U}, \quad \overline{v} = \frac{v}{U}, \\ \overline{p} &= \frac{\Lambda_1}{\mu}p, \quad \overline{S^{ij}} = \frac{\Lambda_1}{\mu}S^{ij}. \end{aligned} \tag{9}$$

Thus the Eqs. (2), (3), (6), and (7) in non-dimensional form become

$$\overline{\mathbf{T}} = -\overline{p}\mathbf{I} + \overline{\mathbf{S}} \tag{10}$$

$$\begin{aligned} &\overline{\mathbf{S}} + \frac{\mathcal{D}\overline{\mathbf{S}}}{\mathcal{D}t} + \tau_3(\overline{\mathbf{S}} \cdot \overline{\mathbf{A_1}} + \overline{\mathbf{A_1}} \cdot \overline{\mathbf{S}}) + \\ &\tau_5(tr\overline{\mathbf{S}})\overline{\mathbf{A_1}} + \tau_6[tr(\overline{\mathbf{S}} \cdot \overline{\mathbf{A_1}})]\mathbf{I} = \\ &\overline{\mathbf{A_1}} + \tau_2\frac{\mathcal{D}\overline{\mathbf{A_1}}}{\mathcal{D}t} + \tau_4\overline{\mathbf{A_1^2}} + \tau_7[tr(\overline{\mathbf{A_1^2}})]\mathbf{I} \end{aligned} \tag{11}$$

$$Re(\overline{\mathbf{v}} \cdot \nabla\overline{\mathbf{v}}) = \nabla \cdot \overline{\mathbf{T}} \tag{12}$$

$$tr\overline{\mathbf{A_1}} = 0 \tag{13}$$

where

$$Re = \frac{\rho U^2 \Lambda_1}{\mu}, \quad \tau_i = \frac{\Lambda_i}{\Lambda_1} \quad (i = 2, 3, ..., 7), \tag{14}$$

where $\overline{\mathbf{A_1}}$ satisfies the above dimensionless equations obtained from Eq. (4) by replacing $\mathbf{A_1}$ by $\overline{\mathbf{A_1}}$.

By defining a dimensionless stream function $\overline{\psi}(r, \theta)$, such that

$$\overline{u} = \frac{1}{\overline{r}}\frac{\partial\overline{\psi}}{\partial\theta}, \ \overline{v} = -\frac{\partial\overline{\psi}}{\partial\overline{r}}, \ \overline{\psi} = \frac{\psi}{U^2\Lambda_1} \tag{15}$$

the continuity equation is satisfied automatically. We also introduce the dimensionless volumetric flow rate

$$\overline{Q} = \frac{Q}{U^2\Lambda_1} = \int_{-\alpha}^{+\alpha} \overline{u}\overline{r}d\theta = \overline{\psi}(\overline{r}, +\alpha) - \overline{\psi}(\overline{r}, -\alpha) \tag{16}$$

then we can use

$$\overline{\psi}(\overline{r}, \pm\alpha) = \pm\frac{\overline{Q}}{2}. \tag{17}$$

We shall use the truncated series expansion (1) for the first three dimensionless stream function components defined as follows:

$$\overline{\psi}_{(-1)} = \frac{\psi_{(-1)}}{U}, \ \overline{\psi}_{(0)} = \frac{\psi_{(0)}}{U^2\Lambda_1}, \ \overline{\psi}_{(1)} = \frac{\psi_{(1)}}{U^3\Lambda_1^2}. \tag{18}$$

In this section, henceforth for convenience, we shall drop the bars that appear over the dimensionless quantities.

The adherence boundary conditions of the problem are as follows:

$$u(r, -\alpha) = -1, \quad u(r, +\alpha) = 0, \quad v(r, \pm\alpha) = 0 \tag{19}$$

which by virtue of Eqs. (1) and (15) implies that

$$\begin{aligned} &\psi'_{(-1)}(-\alpha) = -1 \\ &\psi'_n(-\alpha) = 0, \quad (n = 0, 1, ...) \\ &\psi'_n(+\alpha) = 0, \quad (n = -1, 0, 1, ...) \\ &\psi_n(\pm\alpha) = 0, \quad (n = -1, 1, ...). \end{aligned} \tag{20}$$

Also, using Eqs. (1) and (17) we have

$$\psi_{(0)}(\pm\alpha) = \pm\frac{Q}{2}. \tag{21}$$

We now turn our attention to the equations of motion (12). For Re << 1, neglecting inertial terms compared with the viscous forces and eliminating the pressure by cross-differentiating Eqs. (12), one obtains the following governing equation:

$$\frac{1}{r}\frac{\partial^2}{\partial r\partial\theta}\left\{r\left(S^{rr}-S^{\theta\theta}\right)\right\}-\frac{\partial}{\partial r}\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r^2S^{r\theta}\right)\right\}+\frac{1}{r}\frac{\partial^2 S^{r\theta}}{\partial\theta^2}=0. \tag{22}$$

We shall express $S^{rr}, S^{r\theta}$ and $S^{\theta\theta}$ as a truncated series expansion of the form (Strauss, 1975)

$$S^{rr}(r,\theta) = \sum_{n=1}^{N}\frac{a_n(\theta)}{r^n},$$
$$S^{r\theta}(r,\theta) = \sum_{n=1}^{N}\frac{b_n(\theta)}{r^n}, \quad S^{\theta\theta}(r,\theta)=\sum_{n=1}^{N}\frac{c_n(\theta)}{r^n}. \tag{23}$$

The functions $a_n(\theta), b_n(\theta)$, and $c_n(\theta)$ may be expressed in terms of the $\psi_n(\theta)$ functions in the series expansion (1) by substituting Eqs. (4) and (23) into Eqs. (11). Next, inserting a_n 's, b_n 's, and c_n 's (up to $n=3$) into Eq. (23) and substituting these expressions for $S^{rr}, S^{r\theta}$ and $S^{\theta\theta}$ into Eq. (22), a very long and tedious calculation yields the equations at various orders of r^{-n}. Here, we carry out our analysis up to order n=4.

Equating the coefficients of r^{-2}, r^{-3} and r^{-4} to zero, we get the following differential equations, respectively

$$\psi_{(-1)}^{IV}+2\psi''_{(-1)}+\psi_{(-1)}=0 \tag{24}$$

$$\psi_{(0)}^{IV}+4\psi''_{(0)}=0 \tag{25}$$

$$\begin{aligned}&\psi_{(1)}^{IV}+10\psi''_{(1)}+9\psi_{(1)}+\delta_1\big(30\psi_{(-1)}\psi'^2_{(-1)}+6\psi^2_{(-1)}\psi''_{(-1)}+32\psi''_{(-1)}\psi'^2_{(-1)}+5\psi_{(-1)}\psi''^2_{(-1)}\\&-3\psi''^3_{(-1)}+28\psi_{(-1)}\psi'_{(-1)}\psi'''_{(-1)}+8\psi'_{(-1)}\psi''_{(-1)}\psi'''_{(-1)}+2\psi_{(-1)}\psi'''^2_{(-1)}+7\psi^2_{(-1)}\psi^{IV}_{(-1)}\\&+12\psi'^2_{(-1)}\psi^{IV}_{(-1)}+8\psi_{(-1)}\psi''_{(-1)}\psi^{IV}_{(-1)}+8\psi_{(-1)}\psi'_{(-1)}\psi^{V}_{(-1)}+\psi^2_{(-1)}\psi^{VI}_{(-1)}\Big)+\delta_2\Big(-6\psi^3_{(-1)}\\&+12\psi_{(-1)}\psi'^2_{(-1)}-12\psi^2_{(-1)}\psi''_{(-1)}+12\psi'^2_{(-1)}\psi''_{(-1)}-6\psi_{(-1)}\psi''^2_{(-1)}+24\psi_{(-1)}\psi'_{(-1)}\psi'''_{(-1)}\\&+24\psi'_{(-1)}\psi''_{(-1)}\psi'''_{(-1)}+12\psi_{(-1)}\psi'''^2_{(-1)}+12\psi''_{(-1)}\psi'''^2_{(-1)}+6\psi^2_{(-1)}\psi^{IV}_{(-1)}+12\psi_{(-1)}\psi''_{(-1)}\psi^{IV}_{(-1)}\\&+6\psi''^2_{(-1)}\psi^{IV}_{(-1)}\Big)=0\end{aligned} \tag{26}$$

where δ_1 and δ_2 being the viscoelastic parameters given by

$$\delta_1=1-\tau_2, 2\delta_2=2(\tau_3+\tau_5)(\tau_2+2\tau_3+2\tau_6-\tau_4-2\tau_7)-4\tau_3+\tau_4+2(\tau_7-\tau_6-\tau_5). \tag{27}$$

These equations (24)-(26) have to be solved subject to the following boundary conditions (see Eqs. (20) and (21)):

$$\psi_{(-1)}(\pm\alpha)=0, \psi'_{(-1)}(-\alpha)=-1, \psi'_{(-1)}(+\alpha)=0 \tag{28}$$

$$\psi_{(0)}(\pm\alpha)=\pm\frac{Q}{2}, \psi'_{(0)}(\pm\alpha)=0 \tag{29}$$

$$\psi_{(1)}(\pm\alpha)=0, \psi'_{(1)}(\pm\alpha)=0. \tag{30}$$

Eq. (24) represents the two-dimensional flow of Newtonian fluid in a corner due to one rigid plane sliding on another (Batchelor, 1970). The solution of this differential equation satisfying the boundary conditions (28) is

$$\psi_{(-1)}(\theta)=C_1\sin\theta+C_2\cos\theta+C_3\theta\sin\theta+C_4\theta\cos\theta \tag{31}$$

where

$$C_1=-\frac{\alpha\cos\alpha}{2\alpha-\sin2\alpha}, C_2=-\frac{\alpha\sin\alpha}{2\alpha+\sin2\alpha},$$
$$C_3=\frac{\cos\alpha}{2\alpha+\sin2\alpha}, C_4=\frac{\sin\alpha}{2\alpha-\sin2\alpha}. \tag{32}$$

It is readily shown that the solution of Eq. (25) which satisfies (29) is

$$\psi_{(0)}(\theta)=D_1\theta+D_2\sin2\theta \tag{33}$$

where

$$D_1 = -\frac{Q\cos 2\alpha}{\sin 2\alpha - 2\alpha\cos 2\alpha},$$

$$D_2 = \frac{Q}{2\left(\sin 2\alpha - 2\alpha\cos 2\alpha\right)}. \tag{34}$$

Using the solution given above for $\psi_{(-1)}(\theta)$ in Eq. (26), it is simplified to yield

$$\psi^{IV}_{(1)} + 10\psi''_{(1)} + 9\psi_{(1)} = A_1\sin\theta + A_2\cos\theta + A_3\sin 3\theta + A_4\cos 3\theta + A_5\theta\sin\theta + A_6\theta\cos\theta + A_7\theta\sin 3\theta + A_8\theta\cos 3\theta. \tag{35}$$

The general solution of Eq. (35) is of the form

$$\psi_{(1)}(\theta) = K_1\sin\theta + K_2\cos\theta + K_3\sin 3\theta + K_4\cos 3\theta + B_1\theta\sin\theta + B_2\theta\cos\theta + B_3\theta\sin 3\theta + B_4\theta\cos 3\theta + B_5\theta^2\sin\theta + B_6\theta^2\cos\theta + B_7\theta^2\sin 3\theta + B_8\theta^2\cos 3\theta. \tag{36}$$

The constants $B_1, B_2, B_3, \cdots, B_8$ can be expressed in terms of the constants $A_1, A_2, A_3, \cdots, A_8$. Also, the constants K_1, K_2, K_3 and K_4 can be obtained with the aid of boundary conditions (30). The expressions for these constants are lengthy, and are not presented here in order to conserve space. Readers interested in these coefficients may write to the author.

Results and discussion

The same problem as that investigated in the present paper has been solved previously by Strauss (1975) for the Maxwell fluid, and Huang et. al (1993) for the Oldroyd-B fluid. In the special cases corresponding to Maxwell fluid ($\tau_2 = \delta_2 = 0$) and Oldroyd-B fluid ($\tau_2 \neq 0, \delta_2 = 0$), there is, as expected, an overlap between their governing equations and ours (see Eqs. (24) - (26)). This gives us confidence regarding the analytical work.

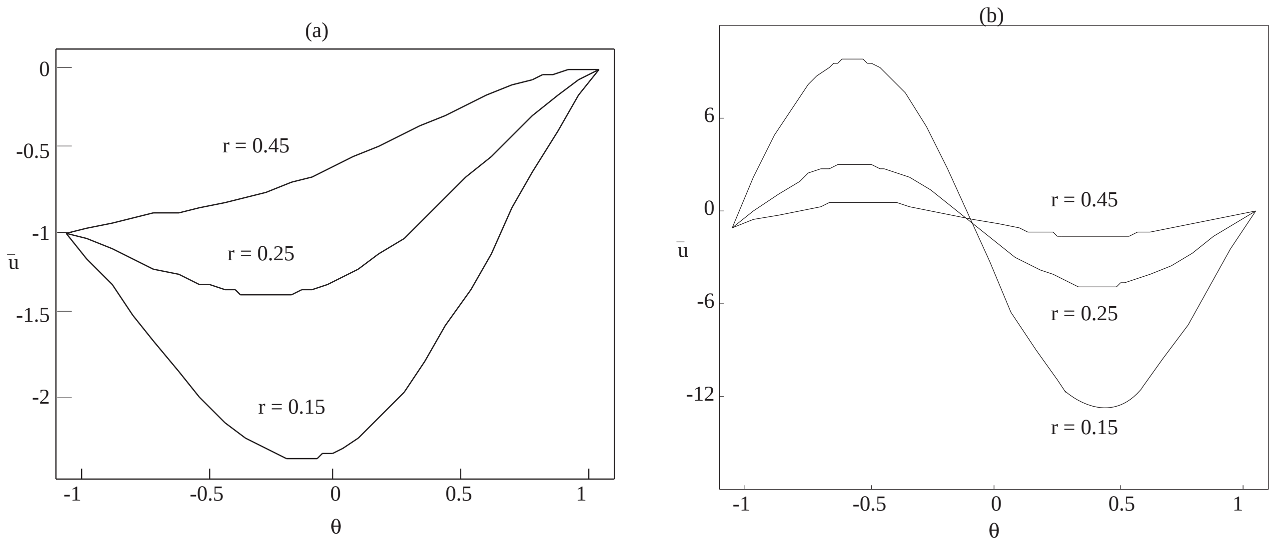

Figure 2. Radial velocity $\overline{u}$ as a function of θ for fixed radial positions for $\alpha = 60^0$ and $\overline{Q} = -0.5$, **(a)** Newtonian fluid, **(b)** Oldroyd 8-constant fluid for $\delta_1 = 0.75$ and $\delta_2 = 0.02$.

Figs. 2a and 2b show the radial component of the velocity vector, as a function of θ for Newtonian and Oldroyd 8-constant fluid respectively. The velocity profiles are plotted at the fixed radial positions $\overline{r} = 0.15, 0.25, 0.45$. It is seen that the sign of radial velocity is always negative for Newtonian fluid, while the non-Newtonian parameters δ_1 and δ_2 change its sign from negative to positive in the region between $\theta = 0$ and $\theta = -\alpha$. This change in the radial velocity is more pronounced near the corner and causes the formation of circulating cells adjacent to the moving plane (see Fig. 4). It is to be noted that such circulating cells have not been observed in Newtonian fluid (see Fig. 3).

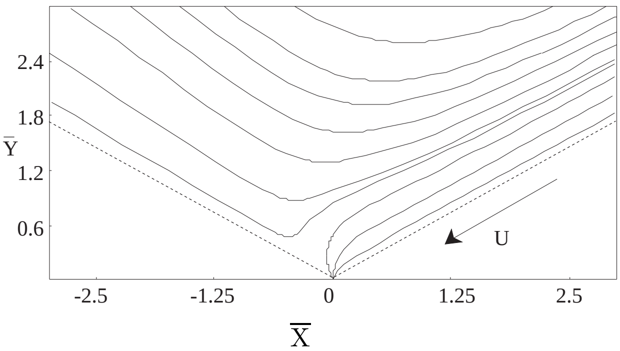

Figure 3. Streamline patterns for $\alpha = 60^0$ and $\overline{Q} = -0.5$ (Newtonian fluid).

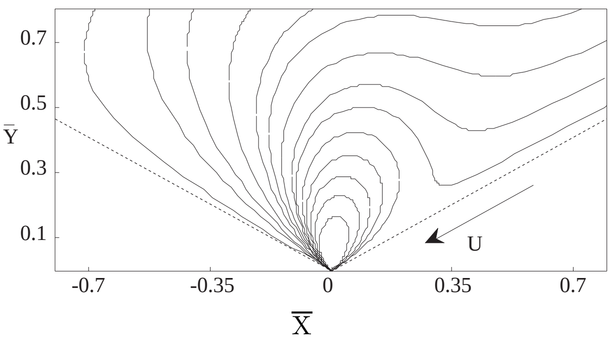

Figure 4. Streamline patterns for $\alpha = 60^0, \overline{Q} = -0.5$, $\delta_1 = 0.75$ and $\delta_2 = 0.02$.

To draw the streamlines presented in Figs. 3 to 8, the first thing to do is to give a constant value to the dimensionless stream function of the form

$$\overline{\psi}(\overline{r}, \theta) = \overline{r}\,\overline{\psi}_{(-1)}(\theta) + \overline{\psi}_{(0)}(\theta) + \frac{\overline{\psi}_{(1)}(\theta)}{\overline{r}}. \quad (37)$$

For this constant value, the proper values of $\overline{r}$ are calculated from Eq. (37) for various values of θ in the interval $-\alpha + \pi/2 \leq \theta \leq \alpha + \pi/2$. After this, the non-dimensional cartesian coordinates $(\overline{X}, \overline{Y})$ can be found from the non-dimensional polar coordinates $(\overline{r}, \theta)$ by using the relations $\overline{X} = \overline{r}\cos\theta$ and $\overline{Y} = \overline{r}\sin\theta$. If this process is repeated for different values of constants given the dimensionless stream function, the streamlines in Figs. 3 to 8 are obtained. We would prefer $(\overline{X}, \overline{Y})$ coordinates to $(\overline{r}, \theta)$ coordinates in order to depict the streamline patterns more easily.

Figs. 4 to 6 provide the streamline patterns for $\alpha = 60^0$ and various values of δ_1 and δ_2 with $\overline{Q} = -0.5$. It is clear from these figures that the size of the circulating cells decreases with increases in δ_2 and keeping δ_1 fixed, whereas it increases with increasing δ_1 and while keeping δ_2 fixed.

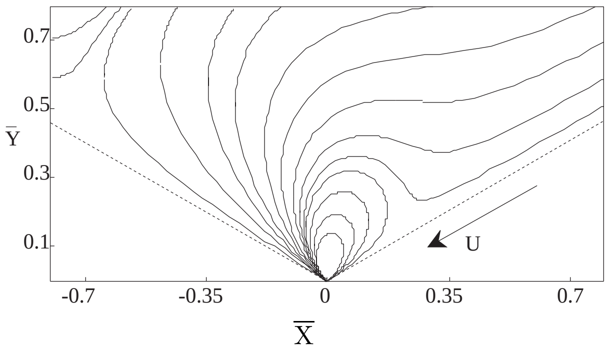

Figure 5. Streamline patterns for $\alpha = 60^0, \overline{Q} = -0.5$, $\delta_1 = 0.75$ and $\delta_2 = 0.16$.

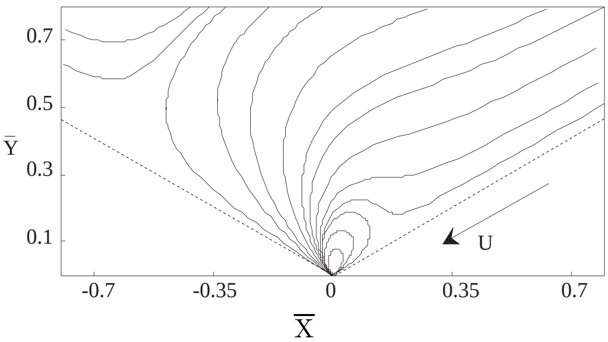

Figure 6. Streamline patterns for $\alpha = 60^0, \overline{Q} = -0.5$, $\delta_1 = 0.25$ and $\delta_2 = 0.02$

Finally, we shall discuss the reliability of solutions near the apex of the wedge. The differential constitutive equations used in this paper are not limited to small, slowly changing deformation rates as in the Rivlin-Ericksen fluids, a subclass of differential type fluids. Note that the flow in a corner formed by two planes, one of which is moving, is considered to be in rapid motion and the gradients of velocity become very large near the corner. It should be pointed out clearly that the sole purpose of using Eq. (3) is to examine, qualitatively at least, whether or not the fluid elasticity (via the material constants) is responsible for the formation of circulating cells near the corner. However, the truncated series expansion (37) is not appropriate to a perturbation, for $\overline{r} < 1$. This is why the solutions based upon series expansion (37) cannot be reliable when $\overline{r} < 1$. Of course, this also depends on the nature of the functions $\overline{\psi}_n(\theta)$, that is, if $\overline{\psi}_n(\theta)$ are not identically zero for large n, the solution cannot be trusted as being meaningful for $\overline{r} < 1$.

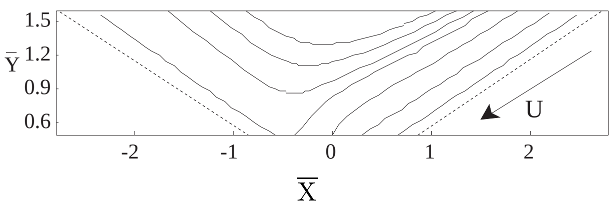

Figure 7. Streamline patterns $\alpha = 60^0, \overline{Q} = -0.5$ (Newtonianfluid) for $0.5 < \overline{Y} < 1.6$.

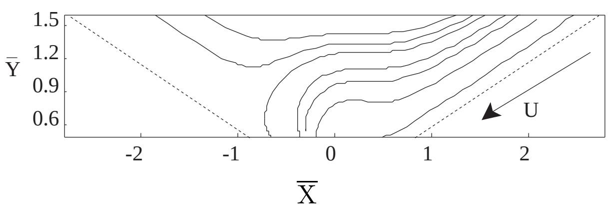

Figure 8. Streamline patterns $\alpha = 60^0, \overline{Q} = -0.5, \delta_1 = 0.75$ and $\delta_2 = 0.02$ for $0.5 < \overline{Y} < 1.6$.

On the other hand, for $\overline{r} \geq 1$, since the effects of successive terms are less significant, the solution is probably quite reliable as n increases. This gives us adequate information in the flow domain $\overline{r} < 1$ from the tendencies suggested by the results at $\overline{r} \geq 1$. For instance, the streamlines depicted in Fig. 8 indicate the presence of circulating cells in Fig. 4, whereas Fig. 7 suggests the flow without circulating cells in Fig. 3. Of course, the streamlines of the secondary flow illustrated in Figs. 4 to 6 for $\overline{r} < 1$ may not have a precise structure.

Nomenclature

$\mathbf{A_1}$	Rivlin-Ericksen tensor of rank one, T^{-1}
$\boldsymbol{I}$	Identity tensor, dimensionless
p	Pressure, $ML^{-1}T^{-2}$
Q	Volumetric flow rate, L^2T^{-1}
$\overline{r}, \theta$	Polar coordinates, dimensionless
Re	Reynolds number, dimensionless
$\mathbf{S}$	Extra stress tensor, $ML^{-1}T^{-2}$
$\mathbf{T}$	Cauchy stress tensor, $ML^{-1}T^{-2}$
U	Velocity of moving plane, LT^{-1}
u, v	Components of the velocity vector, LT^{-1}
$\mathbf{v}$	Velocity vector, LT^{-1}
$\overline{X}, \overline{Y}$	Cartesian coordinates, dimensionless
α	Half angle of corner, dimensionless
δ_1, δ_2	Viscoelastic parameters, dimensionless
μ	Coefficient of viscosity, $ML^{-1}T^{-1}$
Λ_i	Material constants, T
ρ	Density, ML^{-3}
τ_i	Ratio of two material constants, dimensionless
ψ	Stream function, L^2T^{-1}
$\overline{\psi}_{(-1)}, \overline{\psi}_{(0)}, \overline{\psi}_{(1)}$	Stream function components, dimensionless

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