

# Non-Linear Seismic Soil-Structure Interaction Analysis Based on the Substructure Method in the Time Domain

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## Abstract

This paper presents an idealized 2-dimensional plain strain finite element seismic soil-structure interaction (SSI) analysis based on a substructure method by using original software developed by the authors. To investigate the effects of SSI the following types of analysis were performed: linear SSI analysis and non-linear SSI analysis. For the same structure, analysis was carried out by the procedure without the consideration of soil-structure interaction. These computations were achieved for different peak accelerations: 0.15g, 0.30g and 0.45g. In another case for a different site soil with a shear wave velocity of 200, 300 and 500 m/s, a linear SSI analysis was performed. In the analysis, the radiation condition was fully accounted for, the soil plasticity was modeled with the Von Mises failure criterion, basemat uplift was not considered, and the action of gravity was not taken into consideration.

**Key Words:** Seismic soil-structure interaction, substructure formulation, finite element method

## Sismik Yükler Altında Yapı-Zemin Etkileşiminin Zaman Tanım Alanında Altsistem Yaklaşımı Çerçevesinde Analizi

### Özet

Bu çalışmada 2-boyutlu düzlem şekil değiştirme elemanı olarak sonlu elemanlarla modellenen bir sistemin altsistem yaklaşımı çerçevesinde, sismik yükler altında yapı-zemin dinamik etkileşimi incelenmiştir. Yapılan hesaplamalarda yazarlar tarafından geliştirilen program kullanılmıştır. Etkileşimi izlemek amacıyla sonsuz rijit zemin, lineer yapı-zemin etkileşimi ve lineer olmayan yapı-zemin etkileşimi ile ilgili durumlar, farklı düzeydeki ivme değerleri ve farklı zemin rijitlikleri için ( $V_s=200, 300$  ve  $500$  m/s) hesaplamalar yapılmıştır. Burada radyasyon koşulunun tam olarak sağlandığı, zemin plastik davranışının Von Mises akma kriteri ile belirlendiği, zemin ayrılma etkisinin ve yapının kendi ağırlığının ihmal edildiği bir çözümleme yapılmıştır.

**Anahtar Sözcükler:** Dinamik yapı-zemin etkileşimi, altsistem formülasyonu, sonlu elemanlar yöntemi

### Introduction

During the last quarter of the 20th century, the importance of dynamic soil-structure interaction for several structures founded on soft soils was well recognized. If not accounted for in analysis, the accu-

racy in assessing structural safety in the face of earthquakes cannot be accounted for adequately. For this reason, seismic soil-structure interaction analysis has become a major topic in earthquake engineering.

In dealing with the analysis of dynamic soil-structure interaction, one of the most difficult tasks

is the modeling of unbounded media. Many numerical methods or techniques have been developed to solve this problem, such as transmitting boundaries of different kinds, boundary elements, and infinite elements and their coupling procedures.

There are two main approaches for analyzing soil-structure interaction, namely the direct method and the substructure method. These methods are well documented in two textbooks by Wolf (1985, 1988). Both methods are still being developed to achieve the desired results in recent years. Among them, a common formulation equally applicable to both methods is presented by Aydinoglu (1993a, 1993b). This is achieved by changing the size of an irregular soil zone and the definition of dynamic boundary conditions along the interaction horizon. To determine the interaction force-displacement relationships of the degrees of freedom in the nodes along the soil-structure interface for use in the consistent formulation of direct and substructure methods, rigorous formulation is applied. It is based on the similarity and finite element method, and was originally developed by (Wolf and Song, 1996). Recent studies have proven that it is very effective and accurate.

Recent research results in the field of soil-structure interaction indicate that SSI has an important effect on the dynamic response of the structures when the soil is soft. In general, there are three major influences: (1) It will change the dynamic characteristics of the soil-structure system, such as modal frequencies and vibrating shapes. In particular, the fundamental frequency will have significant drops and the rigid body motion of the structure

will be produced or enhanced. (2) It will increase the modal damping as some vibrating energy in the structure will be transferred to the soil. This type of damping is called radiation damping. (3) It will influence free-field ground motion (Menglin and Jingning, 1998).

In a seismic soil-structure interaction analysis, it is necessary to consider the infinite and layer characteristics of soil strata, and the nonlinear behaviors of soft soil. The objective of this study is to perform a rigorous seismic non-linear soil-structure interaction analysis in the time domain to satisfy the above requirements while the results are compared with those of fixed based structural analysis.

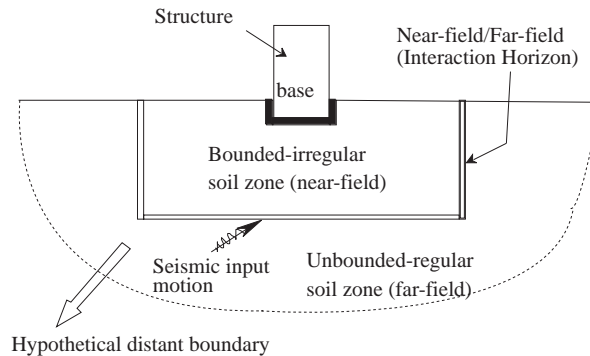
## Equations of Motion

### Basic Equations of Motion for Fixed Base Structures

If the soil-structure interaction is not considered, the equation of the motion for the structure under the seismic excitation in the time domain can be written in the well-known form as follows:

$$[M]\{\ddot{r}(t)\} + [C]\{\dot{r}(t)\} + [K]\{r(t)\} = -[M]\{\ddot{u}_g(t)\} \quad (1)$$

in which  $[M]$ ,  $[C]$  and  $[K]$  are  $n \times n$  mass, damping and stiffness matrices, respectively,  $n$  is the number of degrees of freedom of the structure,  $\{r\}$  is the total displacement vector of the system, and  $\{\ddot{u}_g\}$  is the acceleration vector of the free-field ground motion.

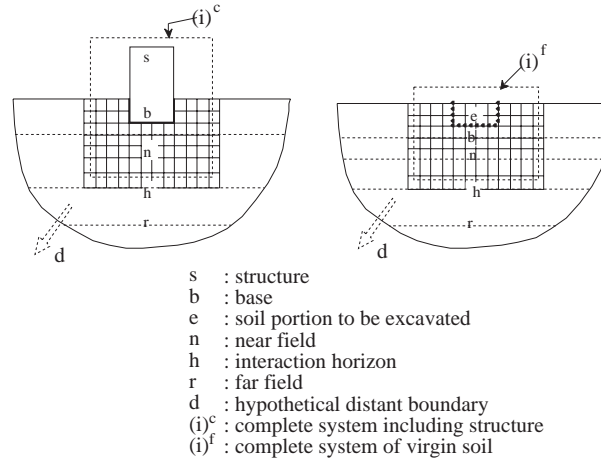


**Figure 1.** Common model for direct and generalized substructure methods (Aydinoglu, 1993a)

**Governing Equations of the Substructure Formulation**

A systematic formulation and discussion of nonlinear soil structure interaction is presented by Aydinoglu

(1993a). Referring to the soil structure model given in Figure 1 together with corresponding indices shown in Figure 2, the basic equations of the soil-structure system can be expressed in the time domain as:



**Figure 2.** Identification of (a) soil structure system, (b) unexcavated free field (Aydinoglu, 1993a)

$$\begin{bmatrix} M_{ii}^{(c)} & M_{ih} \\ M_{hi} & M_{hh}^i \end{bmatrix} \begin{Bmatrix} \dot{i}_i^t(t) \\ \dot{i}_h^t(t) \end{Bmatrix} + \begin{Bmatrix} Q_i(t) \\ Q_h^i(t) \end{Bmatrix} + \begin{Bmatrix} 0 \\ R_h^r(t) \end{Bmatrix} = \begin{Bmatrix} 0 \\ P_h^i(t) \end{Bmatrix} \quad (2)$$

where  $[M]$ ,  $\{Q\}$ ,  $\{R\}$  and  $\{P\}$  are mass matrix, nonlinear internal forces, interaction forces and effective force vector, respectively. The response vector,  $\{r\}$  of Eq. (2) is represented by total displacement indicated by superscript  $t$ . The first term on the left-hand side represents the inertial forces in respective parts of the system with the last component,  $Q_h^i(t)$ , being the nonlinear internal forces acting on the inner face of the interaction horizon.

For the generalized substructure method, the interaction force-displacement relationships in the time domain can be expressed in terms of the relative interaction displacements calculated along the interaction horizon, namely, the difference between total and free-field displacements, which is formulated as:

$$R_h^r(t) = \int_0^t S_{hh}^r(t-\tau) r_h^t(\tau) d\tau - P_h^r(t) \quad (3)$$

with  $S_{hh}^r(t)$  representing the far-field dynamic stiffness matrix in the time domain. The second term on the right-hand side is time effective forces, and can

be expressed as:

$$P_h^r(t) = \int_0^t S_{hh}^r(t-\tau) v_h^f(\tau) d\tau \quad (4)$$

where  $v_h^f(\tau)$  is obtained from nonlinear analysis of the unexcavated free-field. The relative interaction displacements are defined as:

$$r_h^\Delta(t) = r_h^t(t) - v_h^f(t) \quad (5)$$

Thus from Eqs. (3)-(5):

$$R_h^r(t) = \int_0^t S_{hh}^r(t-\tau) r_h^\Delta(\tau) d\tau \quad (6)$$

Finally, the non-zero effective force vector component of Eq. (3) can be expressed as:

$$P_h^i(t) = [ M_{hi} \quad M_{hh}^i ] \begin{Bmatrix} \ddot{v}_i^f \\ \ddot{v}_h^f \end{Bmatrix} + \bar{P}_h^i(t) \quad (7)$$

where the second term represents the internal forces acting on the inner face of the interaction horizon as obtained from one- or two-dimensional nonlinear analysis of unexcavated free-field system (Figure 2) incident seismic waves.

To overcome the numerical difficulties and simplify the formulation and derivation, the interaction forces are expressed as a convolution integral of the accelerations (Wolf and Song, 1996), as follows:

$$R_h^r(t) = \int_0^t M^\infty(t - \tau) \ddot{r}_h^\Delta(\tau) d\tau \quad (8)$$

where  $M^\infty(t)$  is the acceleration unit impulse response matrix in the time domain. It can be determined directly with the consistent infinitesimal finite element cell method which is addressed in (Wolf and Song 1995). Therefore, it will not be repeated in this paper.

The interaction forces of the soil medium at the

$$\left[ \begin{array}{cc} M_{ii}^{(c)} & M_{ih} \\ M_{hi} & M_{hh}^i + \gamma \Delta t [M^\infty]_1 \end{array} \right]_n \left\{ \begin{array}{c} \ddot{r}_i^t(t) \\ \ddot{r}_h^t(t) \end{array} \right\}_n + \left\{ \begin{array}{c} Q_i(t) \\ Q_h^i(t) \end{array} \right\}_n + \left\{ \begin{array}{c} 0 \\ R_h^r(t) \end{array} \right\}_n = \left\{ \begin{array}{c} 0 \\ P_h^i(t) - \tilde{R}_h^r \end{array} \right\}_n \quad (11)$$

With this formulation, true nonlinearity of soil can be consistently taken into account within the near field by properly defined constitutive models (Aydınoğlu, 1993a). To solve Eq. (11), Wilson- $\theta$  method is applied to treat the time variable, and the initial stiffness method is used to treat material plasticity.

### Material Nonlinearity

With SSI analysis there are two kinds of nonlinearities. The first one which has received most attention from researchers and practicing engineers, and is associated with the nonlinear behavior of the soil. The second is associated with the partial separation (uplift) of the foundation from the soil mass, resulting from the inability of the soil to resist tension (Jianguo et al., 1998). Soil is the most complicated engineering material, especially when considering the effects of seismic and dynamic loading.

In this study the authors will follow the material nonlinearity of both soil and structure within the framework of plasticity theory. For this purpose, the well established Von Mises model is employed

soil-structure interface given by Eq. (8) are discretized at time station  $n$  for a piecewise constant acceleration unit impulse response matrix (Wolf and Song 1995) as:

$$\begin{aligned} \{R_h^r\}_n &= \gamma \Delta t [M^\infty]_1 \{\dot{r}_h^t\}_n - \gamma \Delta t [M^\infty]_1 \{\ddot{v}_h^f\}_n \\ &+ (1 - \gamma) \Delta t [M^\infty]_1 \{\ddot{r}_h^\Delta\}_{n-1} \\ &+ \sum_{j=1}^{n-1} [M^\infty]_{n-j+1} \left( \{\dot{r}_h^\Delta\}_j - \{\dot{r}_h^\Delta\}_{j-1} \right) \end{aligned} \quad (9)$$

rewriting the Eq. (9) as:

$$\{R_h^r\}_n = \gamma \Delta t [M^\infty]_1 \{\ddot{r}_h^t\}_n + \{\tilde{R}_h^r\}_n \quad (10)$$

Substituting Eq. (9) in Eq. (2) leads to the nonlinear seismic soil-structure interaction formulation in the time domain which is expressed as:

to model the failure of the materials. For the plain strain application the failure surface of the model is expressed as:

$$F = \bar{\sigma} - \sigma_Y = 0 \quad (12)$$

where  $\sigma_Y$  is yield stress and  $\bar{\sigma}$  is deviator stress which is obtained as:

$$\begin{aligned} \bar{\sigma} &= \frac{1}{\sqrt{2}} \left[ (\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 \right. \\ &\left. + (\sigma_z - \sigma_x)^2 + 6\tau_{xy}^2 \right]^{1/2} \end{aligned} \quad (13)$$

in which  $\{\sigma_x \sigma_y \sigma_z \tau_{xy}\}$  is the Cartesian stress tensor.

An outline of the elastic-plastic algorithm, which comes after the stiffness matrix formation, is given in the flowchart represented in Figure 7.

### Numerical Application and Discussions

The proposed analysis model is applied to study the dynamic responses of structures to earthquake excitation in the time domain. The computational model employed in this section is shown in Figure 3. The

parameters of the model are given in Figure 3 and Table 1. The accelerogram (E-W component) for the Erzincan earthquake of 1992 (Figure 5) is employed as the horizontal ground motion applied to the anal-

ysis model and scaled to have different peak accelerations: 0.15g, 0.3g and 0.45g. To investigate the effects of soil-structure interaction with each input motion level, the following cases are studied:

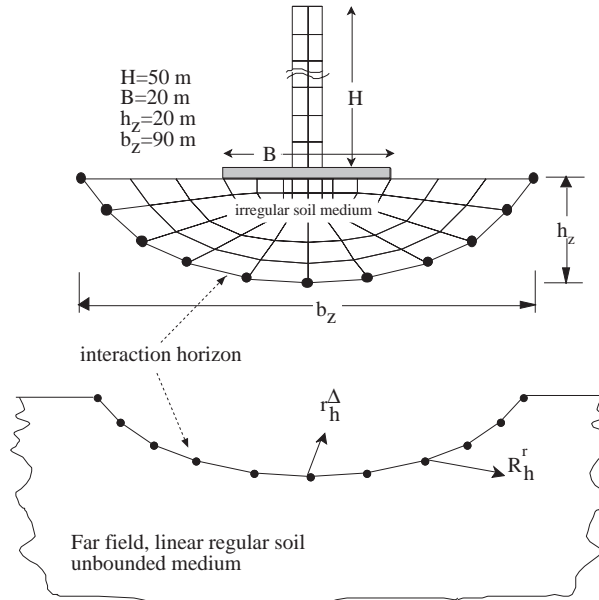


Figure 3. Geometry and discretization of the SSI system

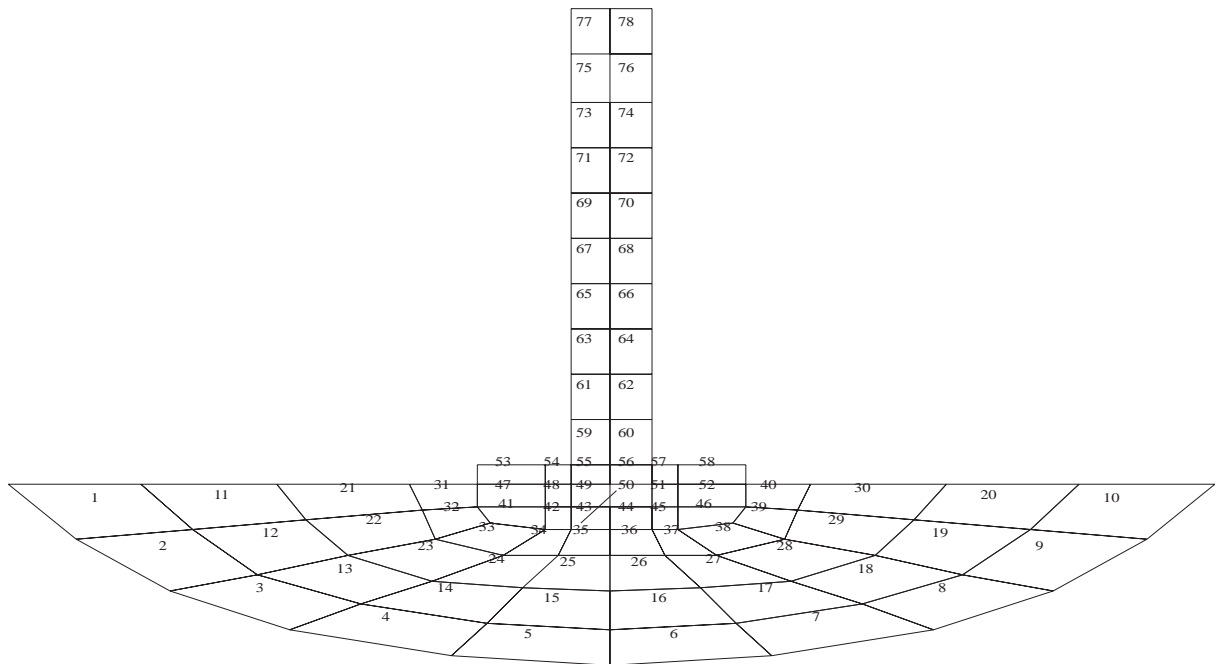
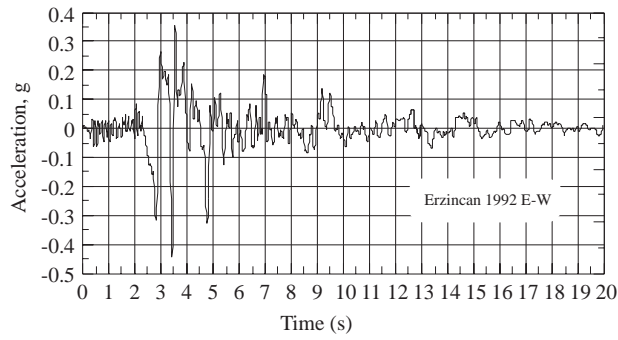


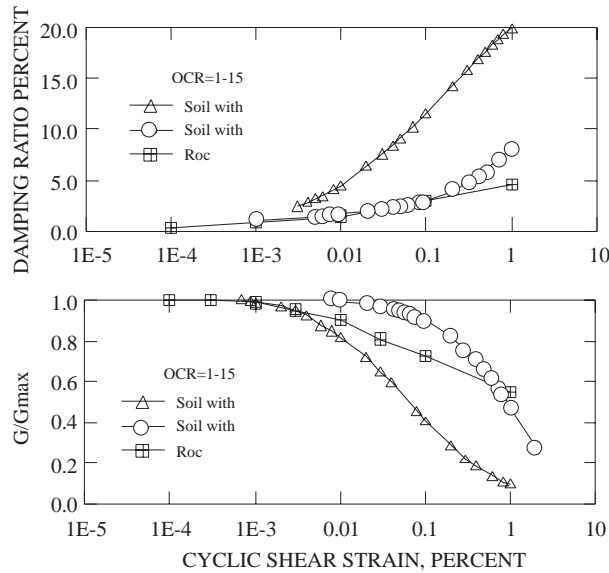
Figure 4. Element number of the SSI system



**Figure 5.** Recorded acceleration time history at the ground surface (max. 0.44g)

**Table 1.** Material properties of the system considered

	Young Modulus (kgf/m <sup>2</sup> )	Shear Wave Velocity (m/s)	Mass Density (kg/m <sup>3</sup> )	Yield Stress (kgf/m <sup>2</sup> )	Poisson's Ratio
Superstructure	2.75 E+09	-	2400	1.0E+08	0.25
Near Field	-	150	1700	2.0E+05	0.35
Far Field	-	200	1800	-	0.35



**Figure 6.** Modulus reduction curves and damping ratio with cyclic shear strain (Vucetic & Dobry, 1991)

- Neglecting the effect of soil-structure interaction, i.e. assuming the structure being fixed at its base, the soil is assumed to be completely rigid and only the superstructure is considered for analysis.
- Linear soil-structure interaction analysis.
- Nonlinear soil-structure interaction analysis.

For this purpose, at first, seismic free-field input motion along the interaction horizon is determined.

This is achieved by the analysis of unexcavated virgin soil in the absence of the structure. The free field motion is calculated by assuming an upwards propagating shear wave. To carry out this step, using the data given in Table 2 and Figure 6, a well-known computer program, SHAKE, is employed. Then, assuming the far-field to be linear, dynamic boundary conditions along the interaction horizon are defined by calculating the unit-impulse response matrix of the far-field in the time domain. In the third step,

the analysis of the soil structure system under the action of free-field input motion determined in the first step, subject to the dynamic boundary conditions determined in the second step, is carried out by using the finite element software developed by the

authors. The flowchart of the computational procedure is given in Figure 7. In this analysis, basemat uplift is not considered and the action of gravity is neglected.

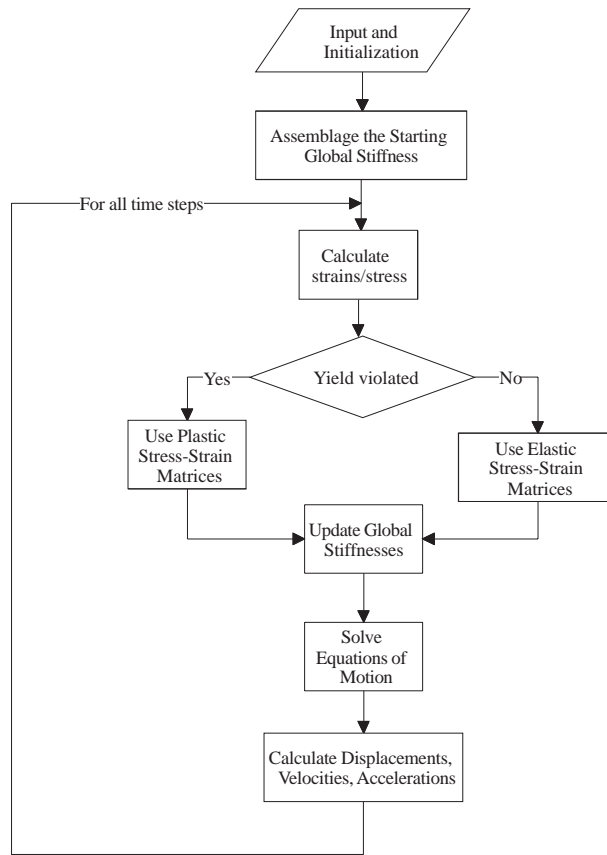


Figure 7. Flowchart for proposed procedure

Table 2. Soil profile data for free field response analysis

Layer No	Soil Type	Thickness (m)	Damping (%)	Unit Weight (kgf/m <sup>3</sup> )	Shear Wave Velocity (m/s)
1	PI=15	3	0.05	1600	150
2	PI=200	4	0.05	1700	180
3	PI=200	6	0.05	1700	180
4	PI=200	4	0.05	1700	180
5	PI=200	3	0.05	1800	200
6	PI=200	40	0.05	1900	300
7	Rock	-	0.01	2400	1600

The calculated soil-structure interaction responses for different cases are shown in Figures 8, 9, 10 and 11. As shown in Figure 8, at the 0.15g acceleration, the level linear and nonlinear response are co-

incident, but, as the acceleration level increases non-linear response becomes significant (Figures 9 and 10). From these figures, it is seen that fixed base analysis gives somewhat greater displacements. As

some of the elements have exceeded the defined yield stress at the 0.45g acceleration level, the others remain within the elastic limit (Figure 12). In Figure 11, the linear response that is calculated for the dif-

ferent soil properties is shown. From the numerical results, it is clear that as the shear wave velocity of the soil increases the response decreases.

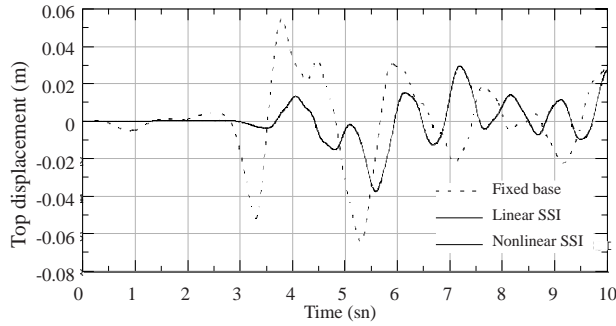


Figure 8. At 0.15g acceleration level top displacement of the structure vs time

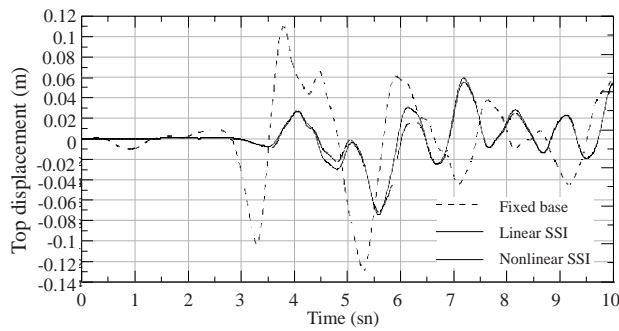


Figure 9. At 0.30g acceleration level top displacement of the structure vs time

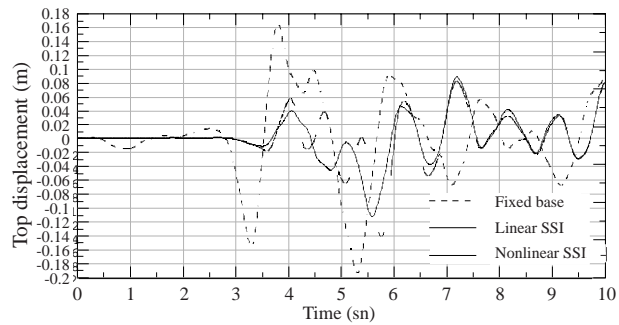


Figure 10. At 0.45g acceleration level top displacement of the structure vs time



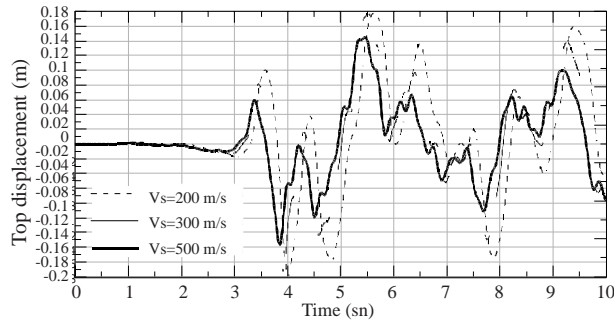


Figure 11. At 0.45g acceleration level the investigation of the effect of different soil properties, Linear SSI analysis

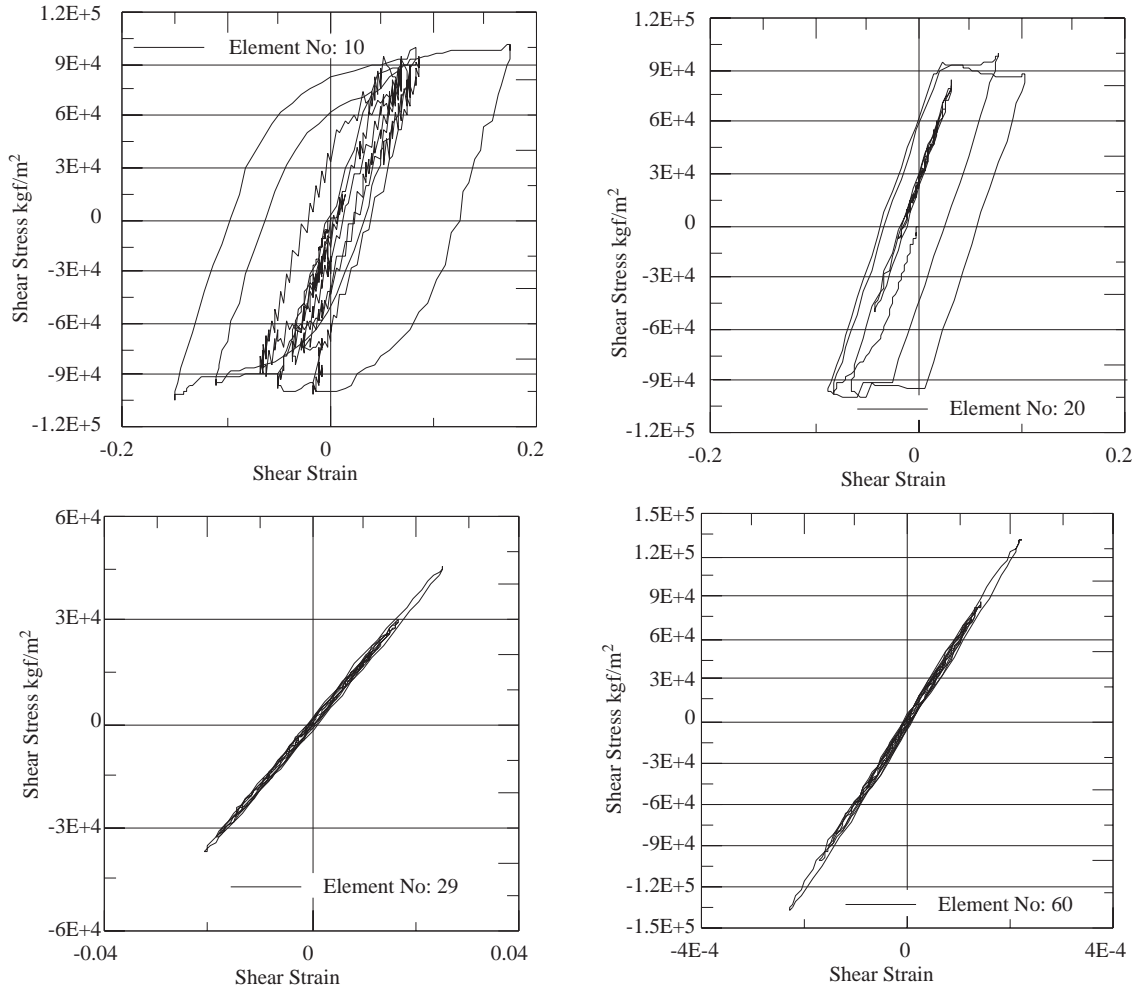


Figure 12. Calculated shear stress-strain response obtained by elastic-plastic procedure at different elements

**Conclusions**

The authors performed a calculation for the linear and nonlinear soil-structure interaction analysis based on the substructure method. It has been

shown that the procedure works well and it can be employed for the analysis of important structures. It is desirable to expand it to a three-dimensional case. Research on this subject is continuing. Much has to be done in investigating the performance of the

model, and the numerical procedures, as well as the various influence factors on the response of a soil-structure system. Moreover, the material damping effect of foundation media in the time domain needs to be improved.

### Notation

$[C]$	damping matrix
$G$	shear modulus
$g$	gravitational acceleration
$[K]$	stiffness matrix
$[M]$	mass matrix
$n$	time station

OCR	over consolidation ratio
$\{P\}$	effective force vector
PI	plasticity index
$\{Q\}$	nonlinear internal forces
$\{R\}$	interaction forces
$\{r\}$	displacement vector
$[S]$	far field dynamic stiffness matrix
$\Delta t$	time step
$\{\ddot{u}_g\}$	acceleration vector of free field
$V_s$	shear wave velocity
$\{v\}$	free field displacement vector
$\gamma$	Newmark's method constant
$\bar{\sigma}$	deviator stress
$\sigma_Y$	yield stress

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