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Numerical Investigation of Subcooled Boiling in a Vertical Pipe Using a Bubble-Induced Turbulence Model

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Abstract

A bubble-induced turbulence model is applied to subcooled boiling of water in a vertical pipe. The volume fractions, velocities, temperatures of water and steam and turbulence characteristics of the flow are estimated in a range of heat fluxes, subcooling temperatures and outlet pressures. The mathematical model involves solutions of transport equations for the variables of each phase with allowance for interphase transfer of momentum and energy. The numerical results agree satisfactorily with those of experimental and numerical results in the literature.

Key words: Subcooled boiling, $k-\varepsilon$ turbulence model, bubbles

Introduction

Subcooled boiling occurs in many practical applications, such as in nuclear reactors, heat exchangers, steam generators and various power generation systems. Prediction of the void fraction profile, flow pattern and thermal field and velocity distribution is essential for design and safety analysis of such systems.

There is extensive research in the literature on the prediction of the void fraction in subcooled boiling. Most of these studies are based on empirical correlations due to the complex nature of the subcooled boiling process. Zuber *et al.* (1966) developed an expression for the axial void fraction considering the relative velocity between two phases. Using the model proposed by Zuber and Findlay (1965), Levy (1967) developed a formulation for the vapor volumetric fraction and tested the formulation with the experimental data in the literature. Kroger and Zuber (1968) developed an empirical formulation for the axial void fraction in a pipe depending on temperature, flow and local relative velocity. The formulation assumes prior knowledge of the location of incipient void formation. There are also a number of correlations available in the literature. The major drawback of these correlations is that they are valid only for the specific conditions in which they are tested. The more elaborate models consider basic transport equations governing the boiling and two-phase flow. Hu and Pan (1995) developed a mechanistic model derived from a one-dimensional two-phase model. However, the model was limited to only the axial direction and no information can be obtained in the radial direction. Zeitoun and Shoukri (1997) also developed a one-dimensional two-phase

model that accounts for interfacial mass and energy transport between two phases. However, this model also predicts the void fraction only in the axial direction and turbulence effects were not considered.

Lai and Farouk (1993) applied an advanced two-phase model to subcooled boiling flow in a pipe. This model was very useful for predicting the axial and radial void fraction profile, temperature distribution and velocity profile in the pipe. However, turbulence production due to the bubble motion was considered. It is evident from experimental measurements (Serizawa *et al.* (1986), Wang *et al.* (1987), Lopez de Bertodano *et al.* (1990)) that bubble motion affects the turbulence and therefore phase distribution. Theofanous and Sullivan (1982), and later Lance and Bataille (1991), experimentally found that at low liquid flow rates, grid generated turbulence can be linearly superimposed. However, at higher flow rates bubbles may have an adverse effect. Serizawa *et al.* (1986) and Wang *et al.* (1987) observed turbulence suppression at higher flow rates. Lopez de Bertodano *et al.* (1990) developed a $k-\varepsilon$ turbulence model that accounts for turbulence production due to bubble motion. They applied this model to predict phase distribution in turbulent upward flow of bubble-water flow in a vertical pipe. They considered an isothermal problem with a prescribed void fraction at the inlet. Bubble production due to heating of a wall was not considered. The purpose of this study is to apply Lopez de Bertodano *et al*'s (1994) model to estimate the void fraction, heat transfer and flow characteristic of subcooled boiling in a vertical pipe.

This paper is divided into four sections of which this introduction is the first. Section 2 contains the mathematical formulations and a summary of the numerical method. The computed results are presented in section 3. Section 4 contains the concluding remarks and summarizes the major findings of the study.

Mathematical Modeling

Consideration is given to upward flow of water in a vertical pipe. The pipe height and diameter are 2 m and 0.024 m, respectively. The subcooled water enters the system at the bottom as shown in Fig. 1 and subsequently boils due to the constant heat flux supplied from the pipe walls. This system mirrors the experiment of Bartolemei and Chanturiya (1967) and the numerical study of Lai and Farouk (1993) for direct comparison of the results. Only half of the pipe is considered due to the symmetry at the pipe axis.

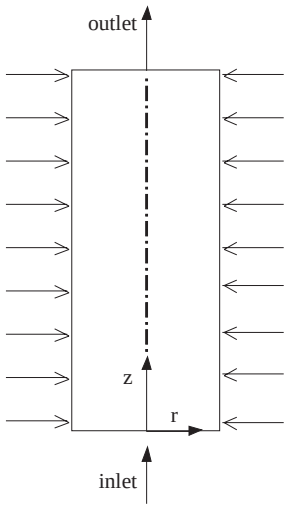

Figure 1. Schematic sketch of the system considered.

To represent the flow behavior and heat transfer in the system, a two-phase mixture of liquid and gas is considered. The phases are assumed to share space in proportion to their existence probabilities such that their volume fractions sum to unity in the flow field. This can be expressed mathematically as

$$\alpha_{L+}\alpha_G = 1 \tag{1}$$

where α_L and α_G are the volume fraction of liquid and gas respectively. The zone averaged quantities are obtained through the solution of separate transport equations for each phase.

Angular variation of the variables is assumed to be negligible in this problem and only radial and axial coordinates are considered. Within this framework, the governing equations for boiling two-phase flow can be expressed in cylindrical coordinates as follows:

Mass conservation

$$\frac{\partial}{\partial z}\left(\rho_i \alpha_i w_i\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(r\rho_i \alpha_i v_i\right) = M_{i-int} \tag{2}$$

where subscripts i and j represent the phases and take the value of L and G in this problem. Subscripts L and G refer to liquid and gas phases, respectively, in this and subsequent formulations. The term on the right of the equation represents mass diffusion between two phases at the water-steam interface.

Axial momentum (z direction)

$$\frac{\partial}{\partial z}\left(\rho_i\alpha_i w_i^2\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(r\rho_i\alpha_i w_i v_i\right)=-\alpha_i\frac{\partial p}{\partial z}+ \\ +F\left(w_j-w_i\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(r\alpha_i\mu_{eff}\frac{\partial w_i}{\partial r}\right)+ \\ +\frac{\partial}{\partial z}\left(\alpha_i\mu_{eff}\frac{\partial w_i}{\partial z}\right)+F_b \tag{3}$$

Radial momentum (r direction)

$$\frac{\partial}{\partial z}\left(\rho_i\alpha_i w_i v_i\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(r\rho_i\alpha_i v_i^2\right)=-\alpha_i\frac{\partial p}{\partial r}+ \\ +F\left(v_j-v_i\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(r\alpha_i\mu_{eff}\frac{\partial v_i}{\partial r}\right)+ \\ +\frac{\partial}{\partial z}\left(\alpha_i\mu_{eff}\frac{\partial v_i}{\partial z}\right) \tag{4}$$

F in both momentum equations is the interface friction term and represents momentum exchange between the phases per unit volume, and $F_b=\rho rg$ is the buoyancy force g being the gravity vector.

Energy equation

$$\frac{\partial}{\partial z}\left(\rho_i\alpha_i w_i h_i\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(r_i\rho_i\alpha_i v_i h_i\right)= \\ \frac{1}{r}\frac{\partial}{\partial r}\left(r\alpha_i\frac{\mu_{eff}}{\mathrm{Pr}_{eff}}\frac{\partial h_i}{\partial r}\right)+\frac{\partial}{\partial z}\left(\alpha_i\frac{\mu_{eff}}{\mathrm{Pr}_{eff}}\frac{\partial h_i}{\partial z}\right)+S_{i-int} \tag{5}$$

where Pr_{eff} is the effective Prandtl number, which includes laminar and turbulent contributions, and S_{i-int} represents energy exchange between two phases at the interface.

Auxiliary equations

M_{i-int} in equation (1) represents the mass transfer between two phases at the steam-water interface. M_{i-int} is computed from the heat transfer balance since evaporation and condensation occur at the interphase, thus

$$M_{G-int}=M_{L-int}= \\ \frac{\lambda_G A_{int}(T_G-T_{sat})-\lambda_L A_{int}(T_L-T_{sat})}{\Delta H} \tag{6}$$

where λ_L and λ_G are the heat transfer coefficients at the steam-water interphase. λ_L and λ_G are calculated from experimental data on heat transfer from spheres are valid over the entire Reynolds and Prandtl number range (Rosten and Spalding, 1986). ΔH is the latent heat of vaporization or condensation at a given pressure. A_{int} is the interfacial area for unit volume and the calculated as

$$A_{int}=\frac{6\alpha}{d_p} \tag{7}$$

were d_b is the bubble diameter. A constant value of d_b=1mm is chosen (Lai and Farouk, 1993) in all the calculations presented in this study.

The interphase friction term, F, in momentum equations can be expressed as

$$F=0.75\frac{c_d\rho_1\alpha_L\alpha_G}{d_b}|u_r| \tag{8}$$

where u_r is the slip velocity vector between two phases and c_d is the drag coefficient. There are extensive works on the drag coefficient in the literature. The "Dirty water" model of Kuo and Wallis (1988) is employed here. In this model,

$$c_d=\left\{\begin{array}{ll} 6.3/Re_b^{0.385} & Re_b>100, We\le 8 \\ 2.67 & Re_b>100, We>8 \\ We/3.0 & Re_b>2065.1/We^{2.6} \end{array}\right\} \tag{9}$$

where Re_b is the Reynolds number based on the gas bubble diameter,

$$Re_b=\frac{\rho_1|u_r|d_b}{\mu_1} \tag{10}$$

and We is the Weber number defined as,

$$We=\frac{\rho_1|u_r|^2 d_b}{\gamma} \tag{11}$$

whereγ is the interfacial tension between the phases.

S_{i-int} is the interphase heat transfer term can be expressed as,

$$S_{i-int}=\lambda_i A_{int}\left(\mathrm{T}_i-\mathrm{T}_{sat}\right)+\dot{M}_{i-int}\left(h_i-h_{sat}\right) \tag{12}$$

where λ_i is the heat transfer coefficient and M_{i-int} is the rate of mass transfer at the water-steam interface.

Turbulence model

Flow under the conditions considered in this study is turbulent in all cases. It is crucial to choose a turbulence model that is suitable for this system. Therefore, a modified $k-\varepsilon$ model that accounts for turbulence production due to bubble motion is employed here.

Effective viscosity includes both laminar and turbulent contribution in this problem, thus,

$$\mu_{eff} = \mu_t + \mu_l \tag{13}$$

Turbulent viscosity is calculated from a modified version of the $k-\varepsilon$ model that accounts for turbulence induced by the bubble motions. Turbulent (eddy) viscosity is calculated from,

$$\mu_t = \frac{c_\mu \rho_1 k^2}{\varepsilon} \tag{14}$$

where c_μ is an empirical constant and the parameters k and ε represent the turbulence production and dissipation mechanism.

The transport equations governing the k and ε terms are,

k-equation

$$\begin{aligned} &\tfrac{\partial}{\partial z}\left(\rho_L \alpha_L w_L k\right) + \tfrac{1}{r}\tfrac{\partial}{\partial r}\left(r\rho_L \alpha_L v_L k\right) = \\ &\tfrac{1}{r}\tfrac{\partial}{\partial r}\left(r\alpha_L \Gamma_L \tfrac{\partial k}{\partial r}\right) + S_k \end{aligned} \tag{15}$$

ε equation

$$\begin{aligned} &\tfrac{\partial}{\partial z}\left(\rho_L \alpha_L w_L \varepsilon\right) + \tfrac{1}{r}\tfrac{\partial}{\partial r}\left(r\rho_L \alpha_L v_L \varepsilon\right) = \\ &\tfrac{1}{r}\tfrac{\partial}{\partial r}\left(r\alpha_L \Gamma_\varepsilon \tfrac{\partial \varepsilon}{\partial r}\right) + S_\varepsilon \end{aligned} \tag{16}$$

Γ_k and Γ_ε are diffusion coefficients and expressed as

$$\Gamma_k = \mu_l + \frac{\mu_t}{\sigma_k} \tag{17}$$

$$\Gamma_\varepsilon = \mu_l + \frac{\mu_t}{\sigma_\varepsilon} \tag{18}$$

where σ_k and σ_ε are Schmidt numbers for k and ε respectively. S_k and S_ε are source terms and are given as

$$S_k = \rho r_L \alpha_L (G_k - \varepsilon) + \alpha_L G_{kb} \tag{19}$$

$$S_\varepsilon = \rho_L \alpha_L \frac{\varepsilon}{k}\left(C_L G_k - C_2 \varepsilon\right) + \alpha_L c_L G_{kb} \frac{\varepsilon}{k} \tag{20}$$

G_k is the rate of production of turbulent energy and is expressed as

$$G_k = \mu_t \left\{ \left(\frac{\partial w_1}{\partial r} + \frac{\partial v_1}{\partial z}\right)^2 + 2\left[\left(\frac{\partial w_1}{\partial z}\right)^2 + \left(\frac{\partial v_1}{\partial r}\right)^2\right] \right\} \tag{21}$$

The second terms in equations (19) and (20) are the turbulence production due to the motion of bubbles. Lopez de Bertodano [13] proposed following equations for G_{kb}:

$$G_{kb} = 0.75 \frac{c_b c_d \rho_1 \alpha_1 \alpha_2}{d_b} |u_r|^3 \tag{22}$$

The values of the constant employed in this study are given in Table 1.

Table1. The values of model constants used in computations

C_b	C_p	C_1	C_2	C_k	C_ε
0.04	0.09	1.44	1.92	1.0	1.3

Boundary conditions

The numerical simulation exactly mirrors the experiment in Bartolemei and Chanturiya (1967) and the numerical study of Lai and Farouk (1993). The subcooled water enters the pipe and is heated from the wall along the pipe. The effects of heat flux and outlet pressure on the void fraction profile, temperature distribution and velocity field are examined. The cases considered are summarized in Table 2.

Table 2. Cases considered

Case #	Inlet temp. (°C)	Inlet subcooling (°C)	Outlet pressure (MPa)	Mass flux kg/ms	Heat flux kW/m^2
Case 1	177.4	22.6	1.5	890	380
Case 2	149.1	50.9	1.5	890	790
Case 3	210	25	3	890	380
Case 4	186.9	48.1	3	890	790
Case 5	231	24	4.5	890	380
Case 6	205	50	4.5	890	790

The boundary wall is fixed and a log-law is employed to calculate the velocity component parallel to the wall. Following Launder and Spalding (1974),

$$\frac{u_1}{u_\tau} = \left(\frac{1}{\kappa}\right) \ln y^+ + 5.4 \tag{23}$$

where κ (=0.435) is the von-Karman constant, u_τ is the shear velocity and y^+ is the dimensionless distance of the node from the wall and is expressed as

$$y^+ = \frac{y u_\tau}{\mu_1} \tag{24}$$

$$u_\tau = \left(\frac{\tau_w}{\rho_1}\right)^{0.5} \tag{25}$$

where τ_w is the wall sheer stress.
k and ε near the wall are calculated from the following relations (Launder and Spalding (1974)).

$$k = 4.2 u_\tau^2 \tag{26}$$

$$\varepsilon = \frac{u_\tau^3}{\kappa y} \tag{27}$$

Numerical method

The set of transport equations presented above is integrated over finite control volume and solved using the PHOENICS computer code (Rosten and Spalding, 1986). The code employs a fully implicit scheme that uses the IPSA algorithm (Singhal and Spalding, 1979). Only half the pipe is solved due to symmetry at the pipe axis. Ten grids in the radial direction and 50 grids in the axial direction are employed after a grid refinement test. A typical CPU time on a Pentium 200 PC was 20 minutes.

Results

Boiling water may exhibit different flow patterns depending on the processing conditions and geometry. Here only heat transfer to subcooled water and bubbly boiling are considered.

Figure 2 shows the average void fraction distribution along the pipe axis. A general characteristic of the boiling is to transfer heat into continuous liquid phase up to around z=0.5 m After subcooled water reaches the saturation temperature, boiling is initiated. Boiling curve exhibits a parabolic increase after the onset of boiling. As shown in Fig. 3, boiling is concentrated near the heated wall for the flow regime considered here. It is seen that the void fraction is substantially higher adjacent to the wall and decreases towards the center.

Figure 4 shows the axial temperature profiles at three radial locations along the pipe. Temperature increases in the pipe due to the heating from the pipe wall and remains constant after the saturation temperature is reached. However, a slight superheating is evident adjacent to wall after the saturation temperature.

Radial temperature profiles at three locations are shown in Fig. 5. Thermal non-equilibrium is evident in this figure. There are large temperature differences between the center and wall of the pipe at z=0.5 m, where bubble production starts close to wall while the center temperature is below the saturation temperature. Non-equilibrium conditions decrease along the pipe and a radially uniform temperature distribution is reached around z=1.5 m. There is a slight superheating close to the wall.

Figure 6 shows the radial distribution of mean axial fluid velocity at three locations along the pipe.

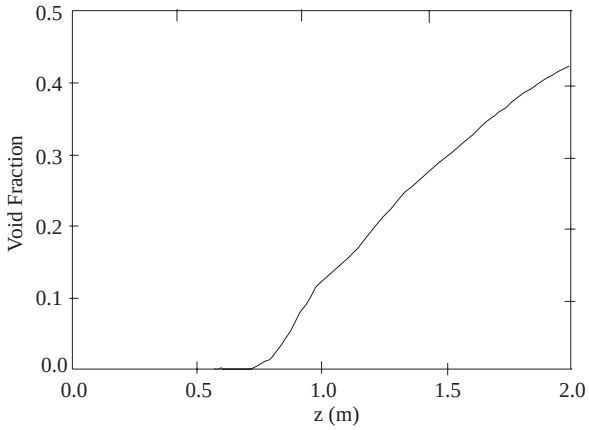

Figure 2. Average void fraction distribution along the pipe axis (case 1).

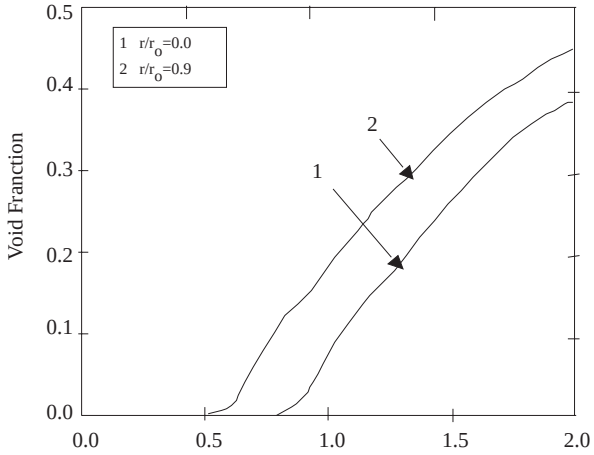

Figure 3. Axial void fraction distribution at two locations in the pipe (case 1).

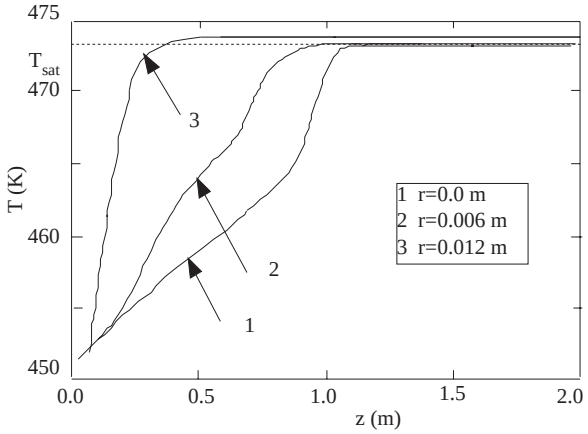

Figure 4. Estimated radial temperature profile at three axial locations (case 1).

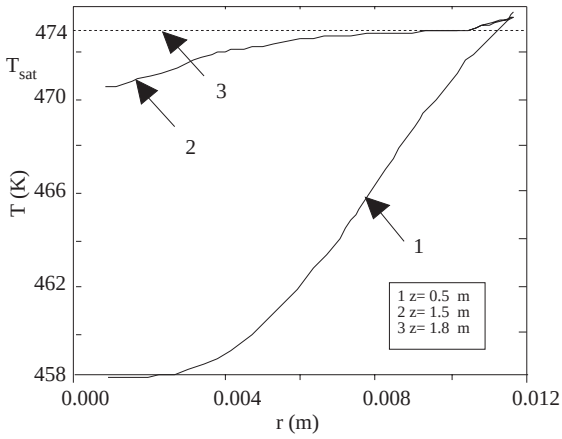

Figure 5. Estimated axial temperature profiles at three radial locations (case 1).

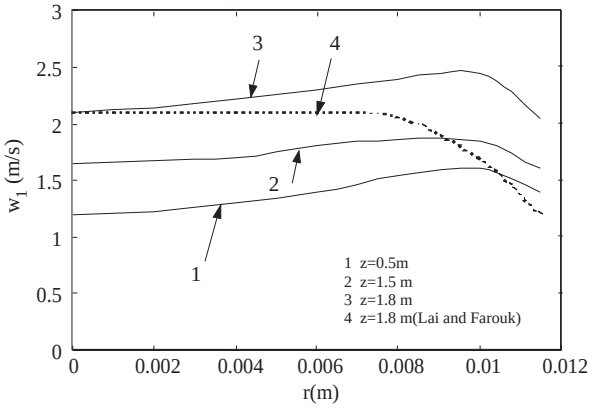

Figure 6. Radial distribution of axial liquid velocity at three locations along the pipe

It is seen that liquid velocity increases along the pipe. This is the result of increased steam fraction as boiling occurs and lower density of the steam with respect to the continuous phase (water). The estimated results are compared with those obtained by Lai and Farouk (1993) at z=1.8 m While the numerical results of Lai and Farouk (1993) exhibit a parabolic velocity distribution where the velocity is maximum at the center and minimum at the vicinity of the wall, the present model predicts maximum liquid velocity at the vicinity of the wall where the void fraction is higher. Recent experimental findings of Roy *et al.* (1997) support the predicted trend in the liquid velocity.

Figure 7 shows the radial turbulence intensity profile in the pipe. The turbulence intensity is calculated as $u_t = \left(\frac{2}{3}k\right)^{0.5}$ which is the average fluctuation two directions. Turbulence intensity increases

as the void fraction increases. However, this increase is not proportional to the increase in the void fraction. This phenomenon was also observed in an experimental study by Seriwaza *et al.* (1975). The decrease in the turbulence production at the high void fraction may be attributed to energy dissipation associated with the lateral relative motion of the bubbles. The radial distribution of the Reynolds shear stress ($u^{'}v^{'}$) is shown in Fig. 8. It is seen that the magnitude of the shear stress increases in the bubbly region with the void fraction. However, a decrease is obtained after this region; this may be attributed to a more flat velocity distribution as the void fraction increases.

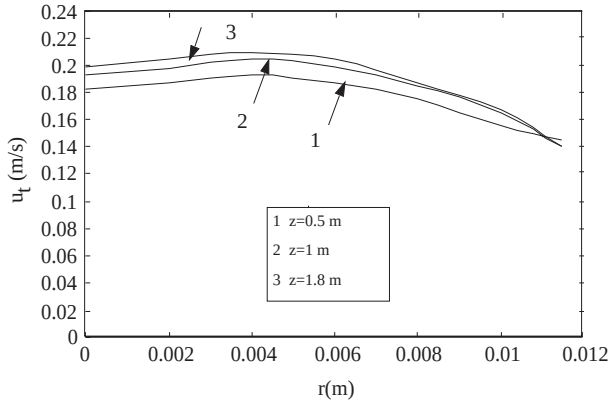

Figure 7. Radial turbulence energy distribution at three axial locations

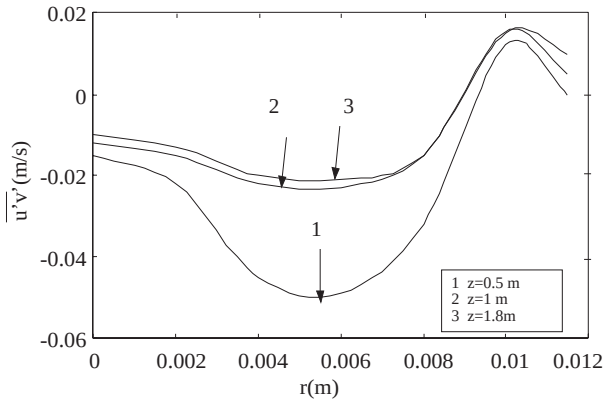

Figure 8. Radial Reynolds stress distribution at three axial locations

Figure 9 shows radial void fractions at two locations. It is also seen that at z=0.5 m there are bubbles only very close to the wall while the void fraction at the center increases at a higher location (z=1.5 m). The estimated results were also compared with numerical data of Lai and Farouk (1993). The present study agrees very well with that of Lai and Farouk (1993) at z=0.5 m, while the present study gives a better void fraction profile at z=1.5 m, which agrees with previous experimental observations (Levy, 1967; Pierre and Bankoff, 1967).

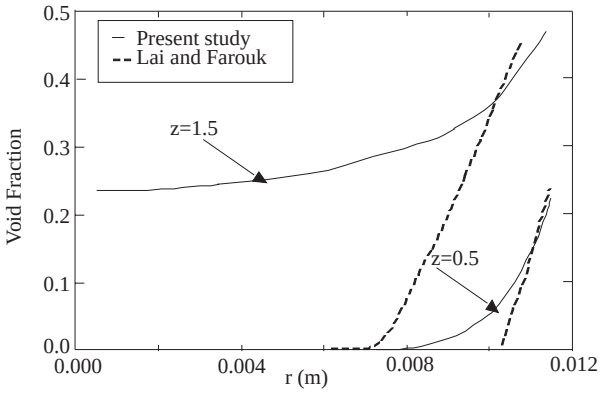

Figure 9. Comparison of estimated radial void fraction with that of Lai and Farouk (1993).

The estimated axial void fraction distribution is compared with the experimental data of Bertolemei and Chanturiya (1967) and the numerical data of Lai and Farouk (1993) in Fig. 10. It is seen that the present study and the data of Bertolemei and Chanturiya (1967) and Lai and Farouk (1993) are in very good agreement at high void fractions; however, Lai and Farouk's study slightly underestimated the void distribution at low void fractions and the onset of boiling. Specifically, the present study predicts the initial point of net vapor generation where the void fraction increases rapidly with the heated length better than did Lai and Farouk's study. Figures 11 and 12 compare the present results and those of Bartolemei and Chanturiya (1967) and the numerical result of Lai and Farouk (1993) for the flow conditions summarized in case 2 and 3. It is seen that similar to Fig. 10, again the present results and Lai and Farouk's (1993) data are in agreement with experiment, while present results estimate the initial point of vapor generation better in both cases. Similar results are obtained for the remaining cases given in Table 2. Therefore, they are not presented here for reasons of brevity.

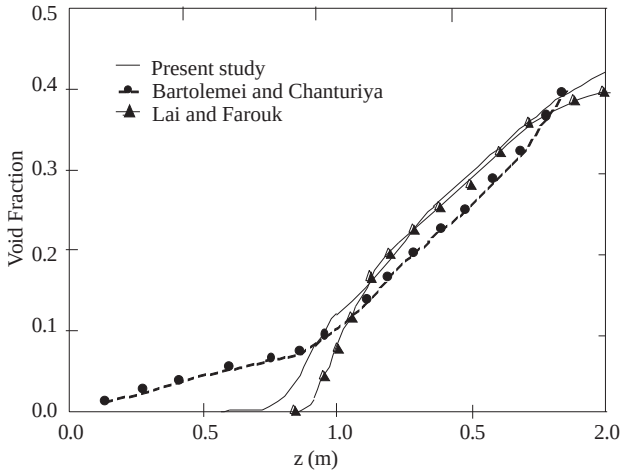

Figure 10. Comparison of estimated axial void fraction (case 1).

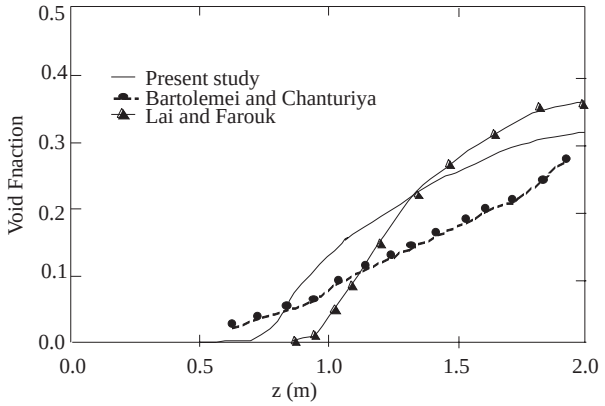

Figure 11. Comparison of estimated axial void fraction (case 2).

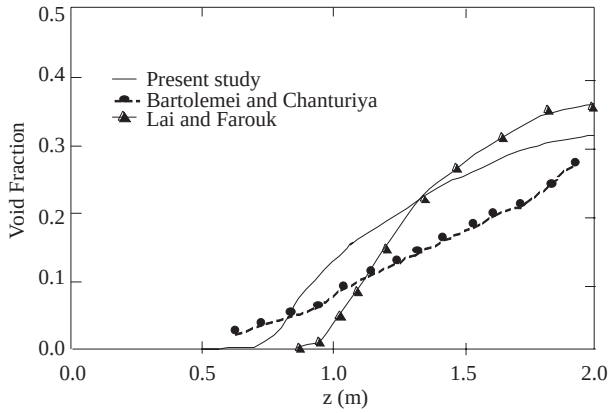

Figure 12. Comparison of estimated axial void fraction (case 3).

Conclusions

The void fraction profile, temperature distribution and velocity fields for a subcooled boiling of water in a vertical pipe are calculated using a two-fluid model. This model involves solutions of transport equations for the variables of each phase with allowance for interphase transfer of momentum and energy. A modified $k-\varepsilon$ turbulence model is adapted to estimate the turbulence characteristics of the flow. The turbulence model considers the turbulence production due to bubble motion. The estimated results are compared with experimental and numerical data in the literature.

It is found that subcooled water enters the pipe and subsequently boils due to the uniform heating from the pipe wall. Bubbles form adjacent to the heated wall and then move towards the center. The liquid velocity is found to increase along the pipe as the void fraction increases. The maximum liquid velocity occurs near the heated wall.

The estimated void fraction profile agrees satisfactorily with those in the literature. It is found that the present study improves on the results of a previous numerical effort, especially at a lower void fraction. Specifically, the onset of boiling and void distribution are estimated better in this study.

Nomenclature

A_{int}	interfacial area for unit volume, m^2
c_d	drag coefficient
d_b	bubble diameter, m
F	volumetric inter-fluid friction,kg/m^3sn
F_b	buoyancy forces, kg/m^2s^2
g	gravity vector, m/s^2
G_k	production of turbulence energy
h	entalphy, kj/kg
k	turbulence energy, m^2/s^2
p	static pressure, pa
r	radial coordinate
r_o	radius
Pr	Prandtl number
Re	Reynolds number
T	temperature, °C, °K
u	velocity vector, m/s
u_r	slip velocity, m/s
v	radial velocity component, m/s
We	Weber number
w	axial velocity component, m/s
ΔH	latent heat of vaporization, kj/kg

y^+	dimensionless distance
z	axial coordinate

Greek Letters

α	volume fraction
ρ	density, kg/m^3
σ	Schmidt number
ε	rate of dissipation of turbulent energy, m^2/s^3
γ	interfacial tension, kg/s^2
κ	Von Karman constant
μ	viscosity, Ns/m^2
τ	shear stress, N/m^2
λ	heat transfer coefficient, W/m^2 °C

Subscripts

b	bubble
eff	effective
G	gas phase
i	gas or Liquid phase
int	interphase
j	gas or liquid phase
l	laminar
L	liquid phase
sat	saturation
t	turbulent

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