Dynamic Reliability in Bridge Pier Scouring

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Abstract

Many bridges crossing wide rivers fail due to excessive local scour around piers and abutments during heavy floods. Because of the random nature and complexity of the overall scouring phenomenon through the bridge opening, there exist uncertainties leading to an unavoidable risk in bridge foundation design. A reliability-based assessment of bridge pier scour is required to examine the relationship between safety factors and reliability, which are key parameters for decision making in design. Herein, a dynamic reliability model based on resistance-loading methodology with random independent loading following a Poisson process and random fixed resistance is applied to assess the reliability of local scouring around single cylindrical piers under various combinations of decision variables. Variation of reliability with respect to safety factor, service life, return period and pier size is examined in an integrated manner in a practical application. The results of dynamic reliability are also compared with the findings of the static reliability model.

Key words: Dynamic reliability, Bridge pier, Scour, Return period, Service life, Safety factor

Introduction

Excessive scour around bridge piers and abutments during floods can lead to considerable damage to bridges at river crossings. The overall scouring mechanism is relatively complex because of the combined effects of three-dimensional riverbed degradation, localized scour due to channel constriction at the bridge opening, local scour around bridge piers and abutments due to the accelerated flow and generation of vortices at the bridge opening, and human interference, such as channel mining upstream of a bridge site (Yanmaz and Çiçekdağ, 2000). Most of the parameters characterizing the overall phenomenon are of a stochastic nature. Although the aforementioned processes are normally interdependent to a certain extent, they are usually treated as independent events because of the difficulty in the simulation of the overall phenomenon.

A bridge designed without considering the mutual interaction between hydraulic and structural aspects can lead to the generation of undesirable hydraulic conditions during floods, such as formation of considerable backwater, increased scouring potential around bridge piers and abutments, debris accumulation at the bridge opening, and formation of a hydraulic jump through the bridge opening (Yanmaz and Kürkçüoğlu, 2000). Lack of relevant information and simplicity of deterministic models may lead to considerable uncertainty in bridge pier design. Depending on the severity of flow conditions and degree of erosion in the close vicinity of the bridge opening, an underdesigned bridge may be subject to a recoverable damage or complete failure during a flood (Yanmaz and Coşkun, 1995). In a survey of 823 bridge failures in the USA since 1950, Shirhole and Holt (1991) found that 60% of the failures were associated with the effects of hydraulics, including both channel bed scour around bridge foundations and channel instability.

Decision-making in bridge-pier footing design should be based on the assessment of various levels of reliability and corresponding safety factors. The overall performance of a bridge can be assessed with

respect to time concerning hydraulic, geotechnical and structural aspects, such as hydraulic conformity of a particular type of bridge opening, aging and deterioration of the structural system, or formation of intolerable slab deflections due to excessive structural loading. Therefore, time-dependent interpretation of the probability of occurrence of any one or a combination of the aforementioned factors is needed. As it is extremely difficult to account for the temporal effects of each failure mode of a bridge jointly, a fault tree analysis can be carried out for the worst static (time-independent) events that are likely to occur during a specified service life. In previous works, static reliability models were developed (Yanmaz and Çiçekdağ, 2001; Yanmaz and Üstün, 2001). The objective of this study is to investigate dynamic analysis of reliability of a bridge with respect to failure induced by local scour around bridge piers. This approach was selected specifically as it yields a relationship between reliability, safety factors, service period, and return period that are of importance for decision-making in bridge pier design. An application is presented to examine the effects of the aforementioned parameters on scouring reliability. In the application, a static reliability model is also applied to compare its results with the findings of a dynamic reliability model.

Static and Dynamic Reliability Models for Bridge Pier Scouring

In the composite risk analysis of a hydrosystem using resistance-loading interference, the risks resulting from various sources of uncertainty can be incorporated to produce an overall risk assessment for the design of the system (Chow *et al.*, 1988). The loading, x, on a system is the measure of the impact of external events. Overall loading can be taken as a linear combination of n independent loads, $x = x_1 + x_2 \ldots + x_n$ (Yen *et al.*, 1986). The resistance, y, is the measure of the ability of the system to withstand the loading. Therefore, the reliability, α, of a system can be expressed as the probability that the resistance of the system equals or exceeds the loading,

$$\alpha = P(x \leqslant y) \qquad (1)$$

where P is the probability. Determination of the reliability of a system using Equation (1) requires knowledge of the probability distributions of resistance and loading. If the resistance and loading are depen-

dent variables, system reliability can be expressed as (Mays and Tung, 1992)

$$\alpha = \int\limits_0^\infty \int\limits_0^y f_{x,y}(x, y)dxdy \qquad (2)$$

where $f_{x,y}(x,y)$ is the joint probability density function of resistance and loading. In a static reliability model, system performance should be checked under a single worst loading condition. The level of reliability may be assessed by a safety factor, SF = y/x (Yanmaz and Çiçekdağ, 2001). In a previous study carried out by Yanmaz and Çiçekdağ (2001), a static reliability model based on resistance-loading interference was developed for bridge pier scouring. In that model, system loading was taken as the linear combination of Froude number and the relative approach flow depth, d_0/b, where d_0 is the approach flow depth and b is the diameter of a circular pier. This combination considers dominant variables that are involved in pressure, body, and inertia forces at bridge openings. The system resistance was accepted as the relative pier footing depth, d_f/b, where d_f is the depth of the bottom of the pier footing below the mean bed level, which can be taken as the maximum possible depth of scour around the bridge pier plus a certain safety margin. Through an uncertainty analysis using extensive laboratory data reported in the literature, Yanmaz and Çiçekdağ (2001) observed that resistance and loading were dependent variables having a bivariate (joint) lognormal probability density function. Developmental details of the model can be seen in Yanmaz and Çiçekdağ (2001) and Yanmaz and Üstün (2001). In the present study, different definitions are used for the system loading and resistance so that they can be treated as independent variables. A framework for the development of static and dynamic reliability models incorporating independent variables is given in the following sections. When loading and resistance are independent, as in the case of the present study, Equation (2) is changed to

$$\alpha = \int\limits_0^\infty f_y(y) \left[\int\limits_0^y f_x(x)dx \right] dy \qquad (3)$$

A dynamic reliability model can be used to estimate risk with respect to the service life of bridge foundations. Studies on the application of this model to levee, culvert and river diversion facility design have been performed by Ang (1973); Mays (1979);

Tung and Mays (1980), (1981); Lee and Mays (1983); Mays and Tung (1992); and Yanmaz (2000). The present paper is based on the application of a model presented by Tung and Mays (1980). The model uses random independent loading and random fixed resistance with the assumption that the occurrence of flood events follows a Poisson process. The time-dependent reliability expression is

$$\alpha(t) = \int\limits_{0}^{\infty} f(Q_c) \exp\left[-\alpha_m t \left(1 - F_{Q_d}(Q_c)\right)\right] dQ_c \quad (4)$$

where $\alpha(t)$ is the time-dependent reliability, $f(Q_c)$ is the probability density function of the resistance or capacity Q_c, α_m is the mean rate of occurrence of the loading that may be estimated from historical data, t is the expected service life time of the hydrosystem, and $F_{Q_d}(Q_c)$ is the cumulative density of the loading Q_d, evaluated at a fixed capacity, Q_c. With the above formulation, the relationship between dynamic and static reliabilities is given by

$$\alpha(t) = e^{-\alpha_m t(1-\alpha)} \quad (5)$$

The preliminary step in the application of the model is the identification of relevant loading and resistance parameters imposed on the system. In this paper, it is assumed that a bridge fails when the maximum depth of scour d_s around a single cylindrical bridge pier of diameter b reaches and exceeds the depth of pier footing, d_f. System resistance is then defined as the maximum discharge or capacity that leads to development of maximum scour depth around a bridge pier under design flow. Any discharge having live-bed characteristics, which is smaller than the system resistance, is then not destructive. System loading is the river discharge, Q_d, having a particular return period.

To formulate the system resistance, the mechanism of local scour around bridge piers should be interpreted. The relative scour depth, d_s/b, can be expressed by the following functional form under the conditions of steady, non-cohesive uniform bed material, single cylindrical pier, long flow duration, wide and straight river, and flow velocities at or above the threshold conditions (Yanmaz and Çiçekdağ, 2001)

$$\frac{d_s}{b} = f(\frac{d_0}{b}, F_r) \quad (6)$$

where d_0 is the depth of approach flow, $F_r = u/\sqrt{gd_0}$ is the Froude number, u is the mean approach flow velocity, and g is the gravitational acceleration. Equation (6) is valid for a large ratio of pier diameter to sediment size that reflects actual field conditions. According to Raudkivi (1986) and Breusers and Raudkivi (1991), the local scour depth is independent of sediment size for $b/D_{50} \geqslant 50$ where D_{50} is the median sediment size. However, for $b/D_{50} < 50$, the grains are large enough relative to the width of the groove excavated by downflow that impedes the scouring process. Several deterministic scour equations of the form of Equation (6) have been reported in the literature, e.g. Inglis (1949), Shen et al. (1969), Jain and Fischer (1980), Richardson (1987), and Yanmaz (2001). Although the forms of these equations are similar, their results differ widely from each other when they are applied to a specific case. As there is no single, universally accepted equation reported in the literature to date because of the complexity of the phenomenon, the choice of a scour prediction equation is also subject to an unknown level of uncertainty (Yanmaz, 2001). The present study is, therefore, based on the statistical randomness of the governing scouring parameters. Herein, the scour equation based on the best fit of experimental data for the case of single cylindrical piers proposed by Richardson (1987) will be used

$$\frac{d_s}{b} = 2.0 \left(\frac{d_0}{b}\right)^{0.35} F_r^{0.43} \quad (7)$$

The scour depth given by Equation (7) can also be expressed in terms of b, d_0 and u in the SI unit system as

$$d_s = 1.224 b^{0.65} d_0^{0.135} u^{0.43} \quad (8)$$

Solving for u and taking $d_s = d_f$, for the maximum system resistance, the mean capacity, $\overline{Q}_c$, for a rectangular channel of width of B can be determined from the continuity equation. Using the first order approximation of Taylor's series expansion, the mean capacity is obtained as

$$\overline{Q}_c = \frac{0.224 \overline{B} \, \overline{d}_f^{7.41}}{\overline{b}^{4.82} \overline{u}^{2.19}} \quad (9)$$

where the over bar sign stands for the mean values. The uncertainty analysis of the system resistance is

based on the examination of the dependence of resistance variables and their variation. If the contraction ratio, Γ, which is the ratio of the contracted width at the bridge opening to the uncontracted width, is greater than a limiting value, Γ_c, the contraction has no pronounced effects on the flow conditions. In other words, no choked flow conditions arise upstream of the bridge opening and the scouring potential due to the constriction effect is not as critical as the local scouring around bridge piers and abutments. For a rectangular channel, the value of Γ_c is (Chow, 1959)

$$\Gamma_c = \sqrt{\frac{27\varepsilon^3 F_{r3}^2}{(2 + F_{r3}^2)^3}} \qquad (10)$$

where ε is an energy loss factor, and F_{r3} is the Froude number at the downstream face of the channel. Therefore, with the selection of a reasonably small pier and abutment widths relative to the uncontracted river width, which satisfies structural requirements under a given design loading, the mean approach flow velocity can be assumed to be independent of the contraction ratio, Γ. In this study, the effects of degradation scour, which takes place over the long term according to the morphological regime of the river concerned, and contraction or localized scour are ignored. A selection of depth of pier footing is based on the maximum possible depth of scour around the pier that is influenced by b and u.

The total uncertainty, i.e. model and parameter uncertainty associated with a hydraulic phenomenon, cannot be quantified precisely because of the lack of relevant information. Identification of the dependence of parameters involved in a hydraulic phenomenon is based on the interpretation of the cross-correlations among these parameters. Because of the complexity and random nature of the hydraulic phenomena, such information is not available. Even for the capacity determination using Manning's equation, which is valid for uniform flow conditions, the dependence of the parameters, e.g. the dependence of roughness coefficient on flow depth, cross-sectional geometry, and bed slope is ignored, and the parameters are considered independent, as in Mays and Tung (1992) and Yanmaz (2000). Since the hydraulic conditions for the scouring case are more complicated compared to the case of capacity determination based on Manning's equation, the assumption of independent variables has also been made in this study.

Using the first order analysis of uncertainty (Tung and Mays, 1980), the parameter uncertainty of the capacity is determined from the following relation

$$
\begin{aligned}
\Omega_{Q_c}^2 &= \left(\frac{\partial Q_c}{\partial B}\right)_{Q_C = \overline{Q}_C}^2 \left(\frac{\overline{B}}{\overline{Q}_C}\right)^2 \Omega_B^2 + \\
&\quad \left(\frac{\partial Q_C}{\partial d_f}\right)_{Q_C = \overline{Q}_C}^2 \left(\frac{\overline{d_f}}{\overline{Q}_C}\right)^2 \Omega_{d_f}^2 + \\
&\quad \left(\frac{\partial Q_C}{\partial b}\right)_{Q_C = \overline{Q}_C}^2 \left(\frac{\overline{b}}{\overline{Q}_C}\right)^2 \Omega_b^2 + \\
&\quad \left(\frac{\partial Q_C}{\partial u}\right)_{Q_C = \overline{Q}_C}^2 \left(\frac{\overline{u}}{\overline{Q}_C}\right)^2 \Omega_u^2 \qquad (11)
\end{aligned}
$$

where Ω_i is the coefficient of variation. Using Equations (9) and (11), the total parameter uncertainty in the system resistance is obtained as

$$\Omega_{Q_c} = \left(\Omega_B^2 + 54.9\Omega_{d_f}^2 + 23.2\Omega_b^2 + 4.8\Omega_u^2\right)^{1/2} \qquad (12)$$

where the total uncertainty in the mean approach flow velocity for a wide channel can be obtained from the Manning equation as

$$\Omega_u = \left(\Omega_n^2 + 0.44\Omega_{d_0}^2 + 0.25\Omega_{S_f}^2\right)^{1/2} \qquad (13)$$

in which n and S_f are Manning's roughness coefficient and friction slope, respectively.

Application

A bridge will be constructed at a hypothetical site where the river has a rectangular cross-section of width of B = 40 m. There is a nearby stream-gaging station that has the rating curve relation defined by $Q = 60d_0^{2.2}$ where d_0 is in m and Q is the discharge in m^3/s. The river is wide and its bed is composed of poorly graded quartz sand having $D_{50} = 1$ mm, relative density of $\Delta = 1.65$, and a mean bed slope of $S_0 = 0.0006$. The annual series of the flows is assumed to follow the lognormal distribution with the logarithmic mean and standard deviation of $\mu_{lnQ} = 3.8$ and $\sigma_{lnQ} = 0.6$, respectively. Single cylindrical piers are placed at the mid-span of the bridge in a row aligned with the flow direction. Interference of scour holes as a result of the group effect of piers is ignored and the most upstream pier is taken into account. For decision-making, information is

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required concerning the interrelation among reliability, service life, safety factor, and return period. To this end, various service lives, i.e. t = 10, 25, 50, and 100 years under return periods of T_r = 25, 50, and 100 years, will be used in the dynamic reliability analysis. Static reliability computations will also be carried out for these return periods to observe the differences between the two approaches.

To quantify the total parameter uncertainty the system resistance, proper values should be assigned to the coefficients of variations in Equations (12) and (13). With reference to Johnson (1996) and Yanmaz (2000), the following coefficients of variations can be taken: $\Omega_n = 0.10$, $\Omega_{d_0} = 0.05$, $\Omega_{S_f} = 0.15$, $\Omega_B = 0.02$, $\Omega_{d_f} = 0.02$, and $\Omega_b = 0.01$. The total uncertainty in u is obtained from Equation (13) as $\Omega_u = 0.129$. However, the influence of channel resistance on the mean flow velocity under live bed conditions is more complicated, so the channel resistance can-

not be expressed by a constant Manning's roughness coefficient. Therefore, to account for the additional uncertainties on u for loose channel boundaries, Ω_u is assumed to attain a somewhat greater value, such as 0.20. Hence, the total uncertainty in the system capacity is determined from Equation (12) as 0.465. The probability density function for the resistance in case of conveyance systems can be expressed by a lognormal distribution (Tang and Yen, 1972; Mays, 1979; Tung and Mays, 1980; Tung and Mays, 1981; and Yanmaz, 2000). The standard deviation of the lognormally distributed resistance, $\sigma_{\ln Q_c}$, is determined from (Mays and Tung, 1992)

$$\sigma_{\ln Q_C} = \left[\ln\left(\Omega_{Q_C}^2 + 1\right)\right]^{1/2} \tag{14}$$

to result in 0.442 using the previous information. With lognormally distributed loading and resistance, Equation (4) is expressed as

$$\alpha(t) = \frac{1}{\sqrt{2\pi}\sigma_{\ln Q_c}} \int_0^\infty \frac{1}{Q_C} \exp\left[-\frac{1}{2}\left(\frac{\ln Q_C - \mu_{\ln Q_C}}{\sigma_{\ln Q_C}}\right)^2\right] \exp\left\{-\alpha_m t[1 - F_{Q_d}(Q_C)]\right\} dQ_C \tag{15}$$

in which

$$F_{Q_d}(Q_C) = \frac{1}{\sqrt{2\pi}\sigma_{\ln Q_d}} \int_0^{Q_C} \frac{1}{Q_d} \exp\left[-\frac{1}{2}\left(\frac{\ln Q_d - \mu_{\ln Q_d}}{\sigma_{\ln Q_d}}\right)^2\right] dQ_d \tag{16}$$

For the annual series, the value of α_m can be taken as $1/T_r$ where T_r is the return period (Tung and Mays, 1980). Using the frequency factors for the lognormal distribution (Chow et al., 1988), the river discharges corresponding to 25- 50- and 100-year return periods are obtained as 128, 153 and 181 m³/s, respectively. In the analysis, the value of depth of pier footing, d_f, is taken as 3.0 m. The safety factor, SF, is then defined as the ratio of d_f to the maximum value of the scour depth, d_s, under a loading (discharge) having a particular return period. For decision-making, successive pier widths of 0.25 m increments have been considered in the range of 1.0 m ≤ b ≤ 2.0 m. The average velocity for live bed conditions has been determined using Engelund's method (see Breusers and Raudkivi, 1991). The critical shear velocity, u_{*c}, is 0.021 m/s using Shields criterion. For B = 40 m and d_f = 3.0 m, a relation is obtained between u and d_0 for successive values of pier widths

in the range 1.0 m ≤ b ≤ 2.0 m. For two unknowns, an additional relationship is required, i.e. the velocity relation of the Engelund equation (Breusers and Raudkivi, 1991)

$$\frac{u}{\sqrt{gd_0'S_0}} = 5.75\log\left(\frac{D_{65}}{2d_0'}\right) + 6 \tag{17}$$

where d_0' is the flow depth corresponding to grain resistance and D_{65} is the characteristic grain size for which 65% is finer. In this application, $D_{65} \approx D_{50}$ since the bed material is poorly graded or uniform. In case of low flow regime, i.e. ∼ 0.20 ≤ F_r ≤∼ 0.65, the relation between d_0' and d_0 is given by Engelund's method as (Breusers and Raudkivi, 1991)

$$\frac{d_0'S_0}{\Delta D_{50}} = 0.06 + 0.4\left(\frac{d_0S_0}{\Delta D_{50}}\right)^2 \tag{18}$$

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Solving for d_0' from Equation (18) and inserting it into Equation (17), the unknowns d_0 and u are determined simultaneously for each pier size. For the information given, low flow conditions are obtained (see Table 1). System capacity is determined from Equation (9). The shear velocity, u_*, is obtained from $(gd_0S_0)^{0.5}$. It is observed that live-bed conditions prevail for each case (Table 1). The maximum depth of scour is determined from Equation (8) for each pier size under the loadings having the return periods of $T_r = 25$, 50 and 100 years. The corresponding safety factors are then computed (Table 2). The dynamic system reliability is computed from Equation (15) for various service lives of t = 10, 25, 50 and 100 years under the given return periods. In the numerical solution of Equation (15), Simpson's one-third rule of numerical integration was used. The results of the reliability computations are shown in Figures 1 through 3. The reliability thus increases with increasing safety factor and return period under a particular value of service life. On the other hand, reliability decreases with increasing service life under a constant return period and safety factor. For this application, the increase in reliability is negligibly small for the safety factors greater than 1.30, 1.25 and 1.20 under the return periods of 25, 50 and 100 years, respectively. In this application, the static reliability computations are also carried out. As the system resistance, Q_c, and loading, Q_d, are considered independent, Equation (5) is used in static reliability computations by taking lognormal distributions for both resistance and loading. The results of this analysis are plotted together with the findings of the dynamic reliability approach in Figures 1 through 3. Based on the interpretation of Equation (5) it can be stated that the static and dynamic reliabilities at a particular safety factor are almost the same when $t = T_r$. For $t > T_r$, the dynamic reliability is less than static reliability. In contrast, dynamic reliability becomes greater than static reliability for $t < T_r$ (see Figures 1 through 3). The results are based on the fact that as service life decreases, scour-induced reliability increases. This result also agrees with classical risk analysis based on Bernoulli trials.

The variation of pier width with respect to safety factor under these return periods is shown in Figure 4. Pier width is inversely proportional to safety factor under a particular return period. For constant pier size, safety factor increases with decreasing return period. Although a thick pier may be required

from the structural design procedure, it would yield a smaller safety factor because of the development of a greater scour depth. The designer is supposed to select a proper pier width that would satisfy structural requirements and would lead to a reasonable depth of pier footing compatible with local foundation conditions. Yanmaz and Kürkçüoğlu (2000) and Yanmaz and Bulut (2001) give guidelines for the joint consideration of hydraulic and structural requirements in bridge design.
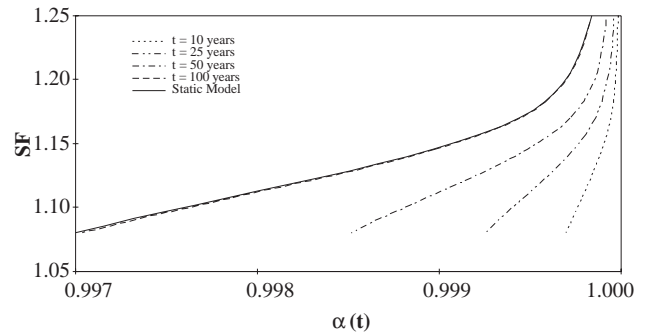

Figure 1. Variation of reliability against safety factor and service life under $T_r = 100$ years
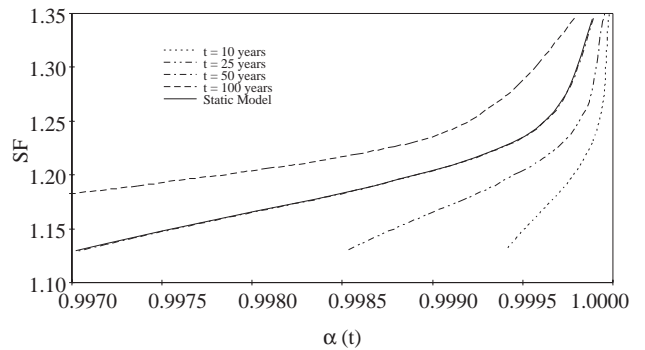

Figure 2. Variation of reliability against safety factor and service life under $T_r = 50$ years

Table 1. System capacity for various pier sizes for the application.

b (m) (1)	d_0 (m) (2)	u (m/s) (3)	u_* (m/s) (4)	F_r (5)	Q_c (m³/s) (6)
1.00	6.00	4.57	0.188	0.60	1097
1.25	4.75	3.50	0.167	0.51	665
1.5	3.94	2.83	0.152	0.46	446
1.75	3.35	2.36	0.140	0.41	316
2.20	2.90	2.01	0.131	0.38	233

Table 2. Safety factors for various return periods

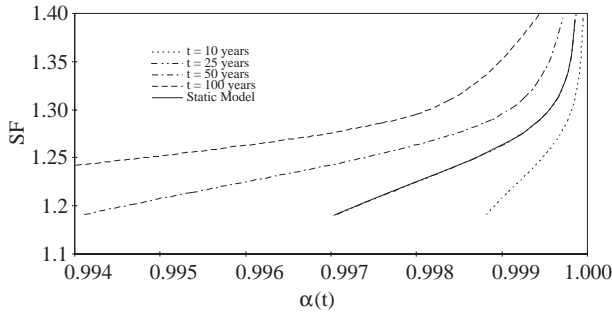| b (m)	d_s (m)			SF		
	$T_r = 25$ y	$T_r = 50$ y	$T_r = 100$ y	$T_r = 25$ y	$T_r = 50$ y	$T_r = 100$ y
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1.00	1.61	1.69	1.78	1.86	1.78	1.69
1.25	1.86	1.95	2.06	1.61	1.54	1.46
1.5	2.10	2.20	2.32	1.43	1.36	1.29
1.75	2.32	2.43	2.56	1.29	1.23	1.17

Figure 3. Variation of reliability against safety factor and service life under $T_r = 25$ years
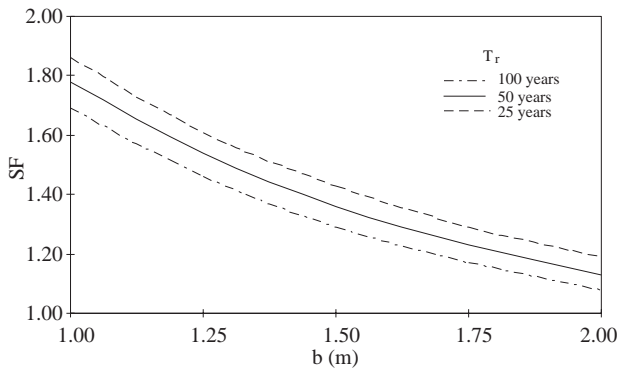

Figure 4. Variation of safety factor against pier width under various return periods

In the selection of an appropriate service life for a bridge with respect to the overall performance of the bridge, hydraulic, geotechnical and structural conformities should be jointly taken into account. Considering maintenance and repair costs, which are influenced by the type of structural system, intensity of traffic, and severity of flow conditions affecting bridge sub-structure, a reasonable value may be taken for service life. Selection of the design return period is based on the location of the bridge. For bridges located in urban areas serving intense traffic, a large return period, e.g. $T_r \geqslant 100$ years may be taken. A set of scour countermeasures should be implemented to further increase the system safety.

Conclusions

The high degree of uncertainty involved in estimating bridge-scouring mechanisms necessitates a reliability-based assessment for pier footing design. The assessment method proposed herein has a versatility to use scour equations different from Equation (7) and to consider the bed material gradation effect. The dynamic reliability method enables a designer to assess the level of local scouring reliability under several combinations of decision variables, i.e. pier width, footing depth and service life, under various return periods. Through an application it is observed that dynamic reliability increases with increasing safety factor and return period under a particular value of service life. On the other hand, reliability decreases with increasing service life under a constant return period and safety factor. Static reliability is slightly smaller than that of dynamic reliability under a particular safety factor. With the application of the model, a designer can select appropriate values for decision variables, namely pier size and pier footing depth. The value of service life, which corresponds to desired reliability and safety levels, should be selected concerning the system performance with respect to hydraulic, structural, geotechnical and material aspects.

Nomenclature

B   =   width of the river;
b   =   pier width;
d_0 =   depth of approach flow;
d_f =   depth of pier footing;
d_s =   depth of maximum scour around a bridge pier;
F_r =   Froude number;

F_{Q_d}	=	cumulative density of loading;
$f_x(x)$	=	probability density function of loading;
$f_y(y)$	=	probability density function of resistance;
$f_{x,y}(x,y)$	=	joint probability density function of resistance and loading;
g	=	gravitational acceleration;
Q_c	=	capacity;
Q_d	=	loading;
$\overline{S}_f$	=	mean friction slope;
SF	=	safety factor;
T_r	=	return period;
t	=	service period;
u	=	mean velocity of approach flow;
α (t)	=	time dependent reliability;

α_m	=	mean rate of occurrence of loading;
Δ	=	relative density;
μ_{lnQ}	=	logarithmic mean of loading;
σ_{lnQ}	=	logarithmic standard deviation of loading;
Γ	=	contraction ratio;
Γ_c	=	critical contraction ratio;
Ω_B	=	coefficient of variation of channel width;
Ω_b	=	coefficient of variation of pier width;
Ω_{d_f}	=	coefficient of variation of pier footing depth;
Ω_{d_0}	=	coefficient of variation of approach flow depth;
Ω_u	=	coefficient of variation of mean flow velocity

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