

Steady Three-Dimensional Flow of a Second Grade Fluid towards a Stagnation Point at a Moving Flat Plate

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Abstract

The problem considered here is the steady three-dimensional flow of a second grade fluid near the stagnation point of an infinite flat plate moving parallel to itself with constant velocity. The basic equations governing flow and heat transfer are reduced to a set of ordinary differential equations by using the appropriate transformations for the velocity components and temperature. These equations have been solved approximately subject to the relevant boundary conditions by employing a numerical technique. The effect of a nondimensional elastic parameter, S , on the velocity field and temperature are carefully examined.

Key words: Second grade fluid, Stagnation point, Viscoelasticity, Boundary layer.

Introduction

There is a class of two- and three-dimensional flows involving stagnation points that may be computed by solving systems of ordinary differential equations for certain skillfully chosen functions and independent variables. The analysis of such flows is very important in both theory and practice. From a theoretical point of view, flows of this type are fundamental in fluid mechanics and forced convective heat transfer. From a practical point of view, these flows have applications in forced convection cooling processes where a coolant is impinged on a continuously moving plate.

The two-dimensional stagnation point flow against a stationary flat plate was first studied by Hiemenz (1911) for the case of orthogonal flow. Two-dimensional oblique stagnation flow was solved by Stuart (1959) and later by Tamada (1979) and Dorrepaal (1986). Goldstein (1938, p.140) notes that Hiemenz's solution can be obtained without invoking the simplifications of boundary layer theory; that is to say, it does in fact satisfy the full Navier-Stokes equations and not just the boundary layer

equations. The axisymmetric case was studied by Homann (1936). Both two-dimensional and axisymmetric flows were extended to three dimensions by Howarth (1951) and Davey (1961).

Authors like Stuart (1955), Rott (1956), and Glauert (1956) analyzed the two-dimensional stagnation point flow against a plate that is oscillating in its own plane. Yang (1958) investigated the two-dimensional unsteady stagnation flow towards a plate. Yang's work was extended by Williams (1968) for the case of axisymmetric flow and then Cheng *et al.* (1971) for the case of three-dimensional flow. The three-dimensional stagnation flow on a moving plate was considered by Wang (1973) and Libby (1974). Wang (1985) studied the unsteady oblique stagnation point flow.

All the above mentioned studies deal with flows of Newtonian fluids. However, within the past 50 years many new fluids not obeying Newtonian laws are being studied by scientists because of their technological significance. That non-Newtonian fluids are finding increasing application in industry has given impetus to many researchers. Srivastava (1958) has obtained an approximate solution for an axisym-

metric flow of a Reiner-Rivlin fluid near a stagnation point adopting the Karman-Pohlhausen method used for the study of boundary layer equations in Newtonian fluids. Maiti (1965) re-examined the same flow problem by replacing Reiner-Rivlin fluid by power-law fluid. For second order Rivlin-Ericksen fluid, Rajeswari and Rathna (1962) studied the two-dimensional and axisymmetric flows near a stagnation point by using an extension of the Karman-Pohlhausen technique. The Prandtl boundary layer theory has been extended by Beard and Walters (1964) for an idealized elastico-viscous fluid, more specifically such a fluid is called a Walters' B' fluid, and then by Sarpkaya and Rainey (1971) for a second order viscoelastic fluid. They obtained the approximate solution valid for sufficiently small values of the elastic parameter by employing a perturbation procedure, using the coefficient that multiplies the highest order term in the equation as the perturbation parameter, thereby lowering the order of the equation. Soundalgekar and Vighnesam (1980) used the perturbation scheme, which is similar to the scheme employed by Beard and Walters (1964), in order to obtain a solution to the heat transfer problem related to the two-dimensional stagnation point flow of Walters' B' fluid. Garg and Rajagopal (1990) considered the two-dimensional stagnation point flow of thermodynamically compatible second-order fluid, where only the velocity field was studied. The heat transfer aspect of this problem has been investigated Massoudi and Ramezan (1992) and Garg (1994). Garg and Rajagopal (1990) and Garg (1994) have obtained solutions valid for all values of an elastic parameter by using an additional boundary condition at infinity, whereas Massoudi and Ramezan's work is confined to small values of elastic parameter. Labropulu *et al.* (1993) studied the orthogonal and oblique flows of a second grade fluid impinging on a wall with suction or blowing. Ariel (1994) has examined the generalized three-dimensional stagnation point flow of a Walters' B' fluid against a stationary flat plate by using the transformations proposed by Howarth (1951) for the velocity components. He has demonstrated on the basis of his exact numerical solutions that the solutions can be obtained only up to some critical value of the elastic parameter, and that for values less than this critical value dual solutions exist. In his subsequent study, he investigated the laminar, steady stagnation point flow of a Walters' B' fluid towards a moving plate by considering both the cases of two-dimensional and axisymmetric flow

(Ariel, 1995).

The literature survey clearly indicates that little attention has been paid to the three-dimensional flows of non-Newtonian fluids near the stagnation point of a moving plate. Therefore, the present paper aims to solve such a problem by introducing a second grade fluid and examining qualitatively the effect of the elasticity of fluid on velocity and temperature distributions.

Formulation of the Problem

In this paper we consider the three-dimensional stagnation point flow of a non-Newtonian fluid, namely the second grade fluid, against a moving flat plate. The Cauchy stress tensor \mathbf{T} in such a fluid is related to the motion in the following manner (Truesdell and Noll, 1965):

$$\mathbf{T} = -p\mathbf{I} + \mu\mathbf{A}_1 + \alpha_1\mathbf{A}_2 + \alpha_2\mathbf{A}_1^2, \quad (1)$$

where μ is the coefficient of viscosity, and α_1 and α_2 material moduli which are usually referred to as the normal stress coefficient. In the foregoing equation, p is the pressure, \mathbf{I} is the identity tensor and kinematical tensors \mathbf{A}_1 and \mathbf{A}_2 are defined through (Rivlin and Ericksen, 1955)

$$\mathbf{A}_1 = \mathbf{L} + \mathbf{L}^T, \quad (2)$$

$$\mathbf{A}_2 = \frac{D}{Dt}\mathbf{A}_1 + \mathbf{L} \cdot \mathbf{A}_1 + \mathbf{A}_1 \cdot \mathbf{L}^T, \quad (3)$$

$$\mathbf{L} = \nabla\mathbf{v}(L_{ij} = v_{j;i}), \quad (4)$$

where \mathbf{v} is the velocity vector, ∇ is the gradient operator, the semicolon stands for covariant differentiation and D / Dt is the material time derivative which is defined as follows:

$$\frac{D}{Dt}[\cdot] = \frac{\partial}{\partial t}[\cdot] + \mathbf{v} \cdot \nabla[\cdot], \quad (5)$$

where $\partial/\partial t$ is the partial time derivative. We notice that $\alpha_1 = \alpha_2 = 0$, the model (1) reduces to the classical linearly viscous fluid model.

If the fluid modelled by Eq. (1) is to be compatible with thermodynamics in the sense that all motions of the fluid meet the Clausius-Duhem inequality and the assumption that the specific Helmholtz

free energy of the fluid is a minimum in equilibrium, then (Dunn and Fosdick, 1974)

$$\mu \geq 0, \alpha_1 \geq 0, \alpha_1 + \alpha_2 = 0. \quad (6)$$

The fluids characterized by the above restrictions have come to be known in the literature as second grade fluids as opposed to second order fluids (Ariel, 2001). The restrictions in Eq. (6) have been the subject of much controversy. Experimental results made under the assumption that the fluid being tested is a second order fluid contradict the restrictions in Eq. (6)_{2,3}. On the other hand, Dunn and Fosdick (1974) demonstrated that if $\alpha_1 < 0$ while the other two restrictions hold, the fluid exhibits unacceptable instability characteristics. Later Fosdick and Rajagopal (1979) showed that if $\alpha_1 < 0$, the fluid exhibited anomalous behaviour not expected in materials of rheological interest. A thorough discussion of the issues involved can be found in the recent critical review of Dunn and Rajagopal (1995). We shall not discuss these issues further. In this study we shall assume that the model under consideration meets Eq. (6), and is compatible with the present literature. For this case the constitutive equation (1) can be written as

$$\mathbf{T} = -p\mathbf{I} + \mu\mathbf{A}_1 + \alpha(\mathbf{A}_2 - \mathbf{A}_1^2), \alpha = \alpha_1 = -\alpha_2. \quad (7)$$

The orthogonal three-dimensional stagnation point flow against an infinite flat plate at $z = 0$ moving with constant velocity U in the x direction is illustrated in Figure 1. A non-Newtonian fluid flowing in the direction of the negative z -axis approaches a moving plane at $z = 0$, and divides into streams proceeding away from the stagnation point at the origin.

The velocity components corresponding to the x , y and z directions are respectively denoted by u , v and w . Far from the plate, as z tends to infinity, the velocity distribution in the frictionless potential flow is given by

$$u_\infty = ax, \quad v_\infty = ay, \quad w_\infty = -2az, \quad (8)$$

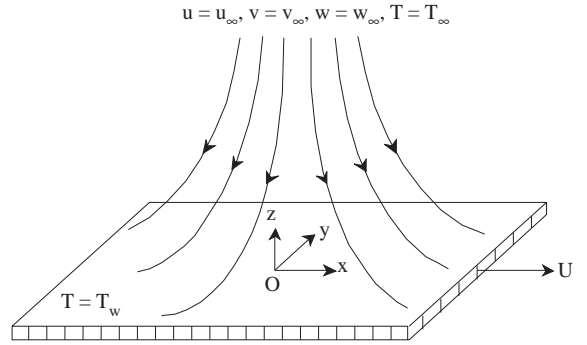


Figure 1. Sketch of flow geometry and coordinate system

where a is a physical constant with dimensions of T^{-1} , depending on the velocity in potential motion. Then, from the Euler equation the pressure distribution will be

$$\begin{aligned} p - p_0 &= -\frac{\rho}{2} (u_\infty^2 + v_\infty^2 + w_\infty^2) \\ &= -\frac{\rho a^2}{2} (x^2 + y^2 + 4z^2), \end{aligned} \quad (9)$$

where ρ is the density and p_0 is a constant which corresponds to the pressure at the stagnation point. Since the velocity field given in Eq. (8) does not satisfy the no-slip conditions at the plate, it is not an acceptable solution of the equations of viscous flow. The problem is to obtain a solution that satisfies the no-slip boundary conditions and agrees with the outer solution far from the stagnation point. This is why we shall seek a velocity field compatible with the continuity equation of the form (Rott, 1956; Glauert, 1956; Wang, 1973)

$$\begin{aligned} u &= Uf(\eta) + axh'(\eta), \quad v = ayh'(\eta), \\ w &= -2\sqrt{a\nu}h(\eta), \end{aligned} \quad (10)$$

where $\nu = \mu/\rho$ is called the kinematic viscosity, $\eta = \sqrt{a/\nu}z$ and the prime denotes the differentiation with respect to η . It is important to note that the function $f(\eta)$ represents velocity profile due to the translation of the plate at $z = 0$.

The boundary conditions for the velocity field are

$$\begin{aligned} z = 0 : \quad & u = U, \quad v = 0, \quad w = 0, \\ z \rightarrow \infty : \quad & \lim_{z \rightarrow \infty} u \rightarrow u_\infty = ax, \quad \lim_{z \rightarrow \infty} v \rightarrow v_\infty = ay, \\ & \lim_{z \rightarrow \infty} w \rightarrow w_\infty = -2az \end{aligned} \quad (11)$$

The assumptions made in this analysis are as follows: (a) the flow is steady and laminar; (b) the

fluid is incompressible; (c) the body forces are negligible; (d) all the physical properties, e.g. the viscosity, specific heat and thermal conductivity of the fluid, remain invariable throughout the fluid; (e) the heat flux vector can be represented by Fourier's law; and (f) the effects of radiant heating and viscous dissipation are negligible. Under the above stated assumptions, the basic equations of the problem are as follows:

Continuity equation:

$$\nabla \cdot \mathbf{v} = 0, \quad (12)$$

Equations of motion:

$$\rho(\mathbf{v} \cdot \nabla \mathbf{v}) = \nabla \cdot \mathbf{T}, \quad (13)$$

Energy equation:

$$\rho c_p(\mathbf{v} \cdot \nabla T) = k\Delta T, \quad (14)$$

where T is the temperature, c_p the specific heat at constant pressure, k the thermal conductivity and Δ is the Laplacian operator.

Substituting Eq. (10) into the equations of motion (13) for the second grade fluid given in Eq. (7) and using the conditions of integrability, we get

$$h''' + 2hh'' - h'^2 + 1 + S(2h'h''' - 2hh^{IV} - h''^2) = 0, \quad (15)$$

$$f'' + 2hf' - h'f + S(-2hf''' + h'f'' - h''f' + h'''f) = 0, \quad (16)$$

where the nondimensional elastic parameter $S = \alpha a/\mu$.

The boundary conditions (11) are re-written as

$$\begin{aligned} \eta = 0 : \quad & h(0) = 0, \quad h'(0) = 0, \quad f(0) = 1, \\ \eta \rightarrow \infty : \quad & \lim_{\eta \rightarrow \infty} h'(\eta) \rightarrow 1, \quad \lim_{\eta \rightarrow \infty} f(\eta) \rightarrow 0. \end{aligned} \quad (17)$$

The terms in Eqs. (15) and (16) having the S factor represent the non-Newtonian character of the fluid. It is noticed that the system of equations characterizing the flow has an order of seven, but there are only five boundary conditions. To obtain a solution we need two extra boundary conditions.

One of the possible methods that overcomes this requirement of additional conditions is the perturbation technique. However, we are treating a singular perturbation as if it were regular. In the absence of a means for prescribing additional boundary conditions, this is perhaps the best that can be done. Thus, we seek a solution of Eqs. (15) and (16) in the form

$$h = h_0 + Sh_1 + O(S^2), \quad (18)$$

$$f = f_0 + Sf_1 + O(S^2), \quad (19)$$

valid for a sufficiently small S . Inserting Eqs. (18) and (19) into Eqs. (15) and (16), and equating the corresponding coefficient of S up to first order, the following set of ordinary differential equations is obtained

$$h_0''' + 2h_0h_0'' - h_0'^2 + 1 = 0, \quad (20)$$

$$\begin{aligned} h_1''' + 2h_0h_1'' - 2h_0'h_1' + 2h_0''h_1 &= -2h_0'h_0''' \\ + 2h_0h_0^{IV} + h_0''^2, \end{aligned} \quad (21)$$

$$f_0'' + 2h_0f_0' - h_0'f_0 = 0, \quad (22)$$

$$\begin{aligned} f_1'' + 2h_0f_1' - h_0'f_1 &= f_0h_1' - 2h_1f_0' - h_0'f_0'' - f_0h_0''' \\ + f_0'h_0'' + 2h_0f_0'', \end{aligned} \quad (23)$$

In a similar manner the higher order terms can be obtained. However, the calculations will become complicated. Moreover, the solutions considered are valid for small values of S . Therefore, we retain up to first order terms. From Eqs. (17) - (19) it follows that the boundary conditions for Eqs. (20) - (23) are

$$h_0(0) = 0, \quad h_0'(0) = 0, \quad \lim_{\eta \rightarrow \infty} h_0'(\eta) \rightarrow 1, \quad (24)$$

$$h_1(0) = 0, \quad h_1'(0) = 0, \quad \lim_{\eta \rightarrow \infty} h_1'(\eta) \rightarrow 0, \quad (25)$$

$$f_0(0) = 1, \quad \lim_{\eta \rightarrow \infty} f_0(\eta) \rightarrow 0, \quad (26)$$

$$f_1(0) = 0, \quad \lim_{\eta \rightarrow \infty} f_1(\eta) \rightarrow 0, \quad (27)$$

It is recorded that for Newtonian fluid ($S = 0$) Eqs. (20) and (22) together with the associated boundary conditions (24) and (26) are the same as those obtained by Wang (1973). In addition, Eq. (20) with the boundary conditions (24) represents the axisymmetric stagnation point flow against a stationary plate (Homann, 1936). The integration of Eqs. (20) - (23) subject to the related boundary conditions (24) - (27) has been performed numerically.

Having computed the velocity field, substituting the results back into equations of motion and then integrating, it can be shown that the general expression for the pressure distribution is

$$P(x, y, \eta) = -\frac{\rho}{2} (4\nu a \{h' + h^2 - 2Shh'' - 3Sh'^2\} + a^2 \{x^2 + y^2\} \{1 - 2Sh''^2\} - 2USf' \{Uf' + 2axh''\}) + P_0, \quad (28)$$

where P_0 is a constant reference pressure. In the absence of S , Eq. (28) is the same as that obtained by Wang (1973).

It is also of interest to determine the effect of elasticity (S) on the shear stress on the plate in the x direction. From the constitutive Equation (7), we obtain

$$\tau_w = \frac{T^{xz}|_{z=0}}{\mu U \sqrt{\frac{a}{\nu}}} = f'(0) + \left(\frac{ax}{U} + S\right) h''(0) \quad (29)$$

Next, we introduce a temperature field of the form

$$T = T_\infty + (T_w - T_\infty)\theta(\eta), \quad (30)$$

where T_∞ is the temperature of the fluid at infinity and T_w is the temperature of the plate, respectively. Substituting Eqs. (10) and (30) into Eq. (14) leads to the ordinary differential equation

$$\theta'' + 2Prh\theta' = 0, \quad (31)$$

where $Pr = \mu c_p/k$ is the Prandtl number. Equation (31) is to be solved subject to the boundary conditions

$$\theta(0) = 1, \quad \lim_{\eta \rightarrow \infty} \theta(\eta) \rightarrow 0. \quad (32)$$

In order to solve Eq. (31), h_0 and h_1 functions are first determined from Eqs. (20) and (21) and it can then be solved numerically.

The heat transfer rate per unit area on the plate may be written by Fourier's law as follows:

$$q_w = -k \left(\frac{dT}{dz} \right)_{z=0} = -k \sqrt{\frac{a}{\nu}} (T_w - T_\infty) \theta'(0) \quad (33)$$

Numerical Results and Discussion

The two-point boundary value problem represented by Eqs. (20) - (23) under the relevant conditions given in Eqs. (24) - (27) was solved numerically using the shooting method. Having found $h(= h_0 + Sh_1)$, the solution for Eq. (31) subject to the boundary conditions (32) is obtained by a similar shooting method. The values of $h_n''(0)$ and $f_n'(0)$ are estimated, and the differential equations are then integrated numerically by using the fourth-order Runge-Kutta procedure as though we had an initial value problem from $\eta = 0$ to η_∞ , where η_∞ is a sufficiently large number; in practice, setting η_∞ as low as 5.0 yields satisfactory accuracy for the present problem. The accuracy of the assumed missing initial conditions are then checked by comparing the calculated values of $h_n'(\eta_\infty)$ and $f_n(\eta_\infty)$ with their given values at $\eta = \eta_\infty$. If a difference exists, the computations with new and improved values for $h_n''(0)$ and $f_n'(0)$ are repeated. The systematic way used here for finding the new values of missing initial conditions is equivalent to a modified Newton's method for finding the roots of equations in several variables. This process is continued until agreement between the calculated and given values at $\eta = \eta_\infty$ is within a preset tolerance. The accuracy of missing initial conditions which yield the known values at the terminal point is 10^{-6} at least. Table 1 gives some of the resulting values of $h_n(\eta)$, $h_n'(\eta)$, $h_n''(\eta)$, and Table 2 gives those of $f_n(\eta)$, $f_n'(\eta)$.

Table 1. Numerical values of h_n, h'_n and h''_n

η	h_0	h_1	h'_0	h'_1	h''_0	h''_1
0	0.000000	0.000000	0.000000	0.000000	1.31193769	-1.45219720
0.2	0.024906	-0.026750	0.242394	-0.256033	1.112107	-1.108386
0.4	0.094303	-0.097850	0.444987	-0.443675	0.914547	-0.769543
0.6	0.200306	-0.199780	0.608710	-0.564781	0.724509	-0.444694
0.8	0.335339	-0.319578	0.735773	-0.623180	0.549215	-0.144310
1	0.492414	-0.445271	0.829868	-0.624961	0.395936	0.119091
1.5	0.944105	-0.721951	0.955221	-0.449159	0.135655	0.506668
2	1.433033	-0.882425	0.991892	-0.200165	0.030954	0.423937
2.5	1.931293	-0.940425	0.999025	-0.053504	0.004546	0.169366
3	2.431111	-0.953142	0.999924	-0.008420	0.000422	0.035634
3.5	2.931099	-0.954813	0.999996	-0.000777	0.000024	0.004118
4	3.431098	-0.954944	0.999999	-0.000042	0.000001	0.000268
4.5	3.931098	-0.954950	1.000000	-0.000001	0.000000	0.000010
5	4.431098	-0.954951	1.000000	-0.000000	0.000000	0.000000

Table 2. Numerical values of f_n and f'_n

η	f_0	f_1	f'_0	f'_1
0	1.000000	0.000000	-0.93873274	0.82664147
0.2	0.813937	0.158966	-0.913798	0.755277
0.4	0.637447	0.299602	-0.844428	0.645946
0.6	0.478490	0.415544	-0.740306	0.509282
0.8	0.342815	0.501602	-0.613899	0.346795
1	0.233450	0.552418	-0.479684	0.156977
1.5	0.069787	0.502124	-0.194842	-0.340516
2	0.014117	0.274869	-0.050979	-0.480566
2.5	0.001869	0.086614	-0.008380	-0.248936
3	0.000158	0.015526	-0.000855	-0.062095
3.5	0.000009	0.001592	-0.000054	-0.008131
4	0.000000	0.000094	-0.000002	-0.000584
4.5	0.000000	0.000003	-0.000000	-0.000024
5	0.000000	0.000000	-0.000000	-0.000000

The predictions based on the foregoing analysis are displayed graphically for various values of nondimensional elastic parameter S in Figures 2-5. Since our perturbation analysis is valid only for small values of elastic parameter S , the variation of S is limited to a range from 0.0 to 0.2. In addition, u_∞ and U are assumed to be in the same direction, i.e. $a x / U > 0$. Figures 2 to 4 show the velocity profiles corresponding to the x , y and z directions, respectively. We observe from these figures that the main effect of elasticity (S) on the three-dimensional stagnation point flow against a moving flat plate is to increase in velocity component in the x direction, whereas it is to decrease the velocity components in the y and z directions.

The temperature profiles are presented in Figure 5. It is apparent from Figure 5 that the temperature profiles slightly increase with the increase in elasticity of the fluid. Again from Figure 5, we arrive at the conclusion that the thermal boundary layer thickness becomes small for the increase in Prandtl number, as expected. It is also noted the temperature distribution is independent of plate translation.

The values of shear stress on the plate in the x direction (τ_w) are tabulated in Table 3 for several different values of the parameters $a x / U$ and S . We conclude from Table 3 that an increase in elastic parameter S leads to a reduction in the value of wall shear stress τ_w . Table 4 illustrates the effect of elastic parameter S on the heat transfer rate per

unit area on a plate for a selection of values of the Prandtl number. From Table 4, we note that with an increase in elastic parameter S , the heat loss per unit area from the plate decreases. This change in heat transfer is more pronounced for a large Prandtl number.

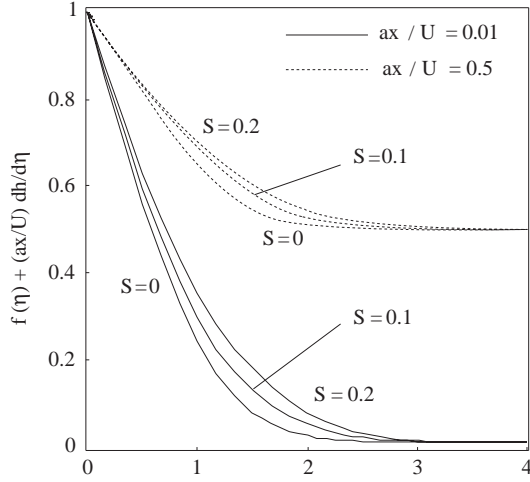


Figure 2. The velocity component in the x direction

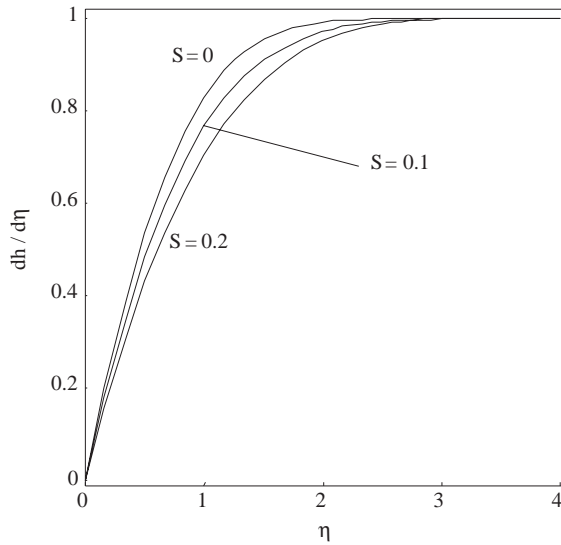


Figure 3. The velocity component in the y direction

Table 3. Values of wall shear stress τ_w

S	$a x / U = 0.01$	$a x / U = 0.2$	$a x / U = 0.5$
0	-0.925614	-0.676345	-0.282764
0.1	-0.727730	-0.506053	-0.156038
0.2	-0.558890	-0.364805	-0.058356

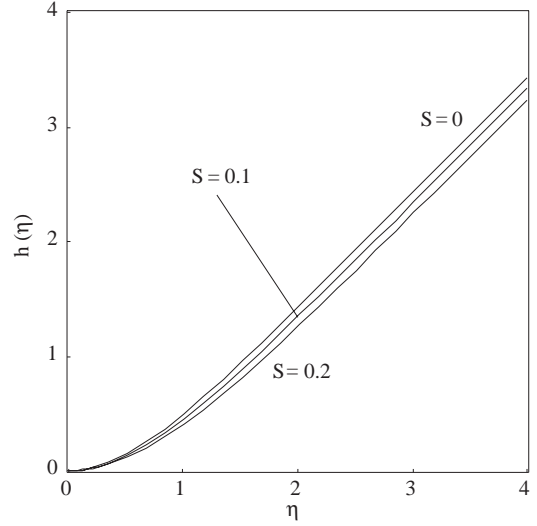


Figure 4. The velocity component in the z direction

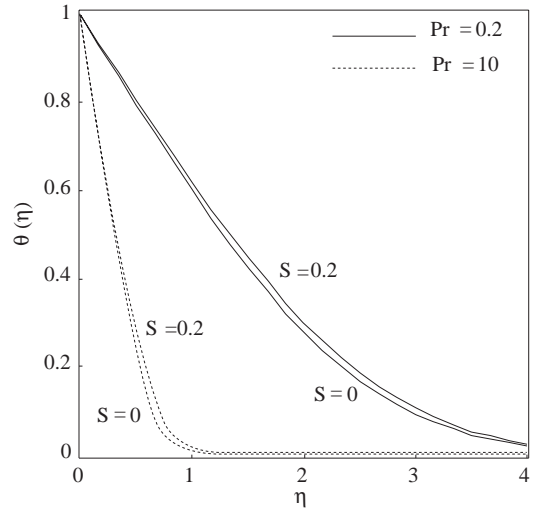


Figure 5. Temperature distribution

Table 4. Values of heat transfer parameter $-\theta'(0)$

S	Pr = 0.2	Pr = 10	Pr = 50
0	0.405419	1.752083	3.055577
0.1	0.395240	1.687631	2.940866
0.2	0.384962	1.618727	2.817269

Nomenclature

\mathbf{A}_n	Rivlin-Ericksen tensor of rank n , T^{-1}	T_∞	temperature of fluid at infinity, ϑ
c_p	specific heat at constant pressure, $L^2T^{-2}\vartheta^{-1}$	t	time, T
\mathbf{I}	identity tensor, dimensionless	\mathbf{U}	translation velocity of plate, LT^{-1}
k	thermal conductivity, $MLT^{-3}\vartheta^{-1}$	u, v, w	components of the velocity vector, LT^{-1}
Pr	Prandtl number, dimensionless	$u_\infty, v_\infty, w_\infty$	velocity components at infinity, LT^{-1}
p	pressure, $ML^{-1}T^{-2}$	\mathbf{v}	velocity vector, LT^{-1}
q_w	heat transfer rate per unit area on plate, MT^{-3}	x, y, z	cartesian coordinates, L
S	elastic number, dimensionless	α	normal stress modulus, ML^{-1}
T	temperature, ϑ	α_1, α_2	normal stress moduli, ML^{-1}
\mathbf{T}	Cauchy stress tensor, $ML^{-1}T^{-2}$	μ	coefficient of viscosity, $ML^{-1}T^{-1}$
T_w	temperature of plate, ϑ	ν	kinematic viscosity, L^2T^{-1}
		ρ	density, ML^{-3}
		τ_w	shear stress on plate, dimensionless

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