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Fuzzy Logic Control of Vibrations of Analytical Multi-Degree-of-Freedom Structural Systems

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Abstract

A fuzzy-logic-based controller and a PD controller are designed for an active control device considering a multi-degree-of-freedom analytical structure against earthquakes. The advantage of the fuzzy logic approach is its robustness and ability to handle the non-linear behavior of the system. The simulated system has five degrees of freedom. An analytical structural system was simulated against the ground motion of the destructive Marmara earthquake ($M_w = 7.4$), which resulted in more than 20,000 deaths in Turkey on 17 August, 1999. In this study, a linear motor is used as the active isolator. At the end of the study, the time history of the storey displacements, control voltages and frequency response of both the uncontrolled and controlled analytical structures are presented and the results are discussed.

Key words: Fuzzy logic control, PD control, Multi-degree-of-freedom structure, Earthquake-induced vibration.

Introduction

A number of studies on structural vibration control have recently been done and practical applications have been realized. Vibration isolation using rubber bearings is one of the most popular methods of passive vibration control. It is known that a seismic isolation rubber bearing, consisting of rubber sheets and steel plates, is effective for an architectural structure whose base is subjected to an earthquake input (Kelly, 1996). In addition, semi-active vibration methods are proposed in the literature. Yoshida and Fujio (1999) applied such a method to a base in which the viscous damping coefficient is changed for vibration control. In recent years, there are studies where active actuators are used for isolation systems in order to isolate earthquake-induced vibrations. Fukushima *et al.* (1996) developed an active-passive composite-tuned mass damper to reduce the wind- and earthquake-induced vibrations of tall buildings. Since there are uncertainties in structural systems and system parameters are not constant, different control methods are offered for the active control of structures (Nishimura *et al.*, 1996). Yagiz (2001) applied sliding mode control for a multi-degree-of-freedom analytical structural system.

In this study, earthquake ground motion is used as input to an analytical building structure. This earthquake motion is obtained using the seismic data of the destructive Marmara earthquake ($M_w = 7.4$), which resulted in more than 20,000 deaths in Turkey on 17 August, 1999 (U.S. Geological Survey, 1999).

Dynamic Model of Five Degrees of Freedom Analytical Structural System

The analytical structure has five degrees of freedom all in a horizontal direction. Since the destructive effect of earthquakes is a result of horizontal vibrations, in this study the degrees of freedom have been assumed only in this direction. The system is modeled including the dynamics of a linear motor, which is used as the active isolator. The analytical system is shown in Figure 1. During an earthquake, the maximum inter-story shear force occurs on the first floor. Assuming equivalent story stiffness and ultimate capacities, the destructive effect of an earthquake is expected to be the largest on the first story. The active control is, therefore, applied on the first story. Where m_1 is movable mass of the ground floor, the mass of each story is m_2, m_3, m_4 and m_5, respectively. x_1, x_2, x_3, x_4 and x_5 are the horizontal displacements and x_0 is the earthquake-induced ground motion disturbance to the analytical structure. These masses cover both those of the floors and walls over them. All springs and dampers are acting in a horizontal direction. The system parameters are presented in the Appendix.

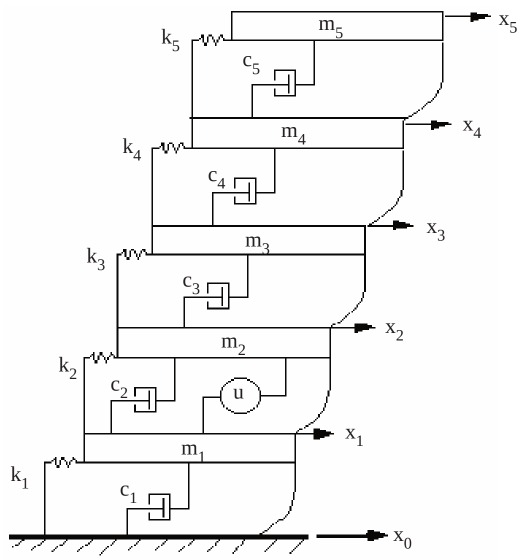

Figure 1. Physical model of an analytical structural system.

The equation of motion of the system is

$$[M]\underline{\ddot{x}} + [C]\underline{\dot{x}} + [K]\underline{x} = \underline{Fd} + \underline{Fu} \tag{1}$$

where $\underline{x} = [x_1\ x_2\ x_3\ x_4\ x_5]^T$, $\underline{F}_d = [-(c_1\dot{x}_0 + k_1 x_0)\ 0\ 0\ 0\ 0]^T$ and $\underline{F}_u = [-F_u\ F_u\ 0\ 0\ 0]^T$. F_d is the force resulting earthquake, F_u is the control force produced by a linear motor, [M], [C] and [K] are the mass, damping and stiffness matrices and these are given in the Appendix. The equation of the linear motor is

$$Ri + K_e(\dot{x}_2 - \dot{x}_1) = u \tag{2}$$

where u and i are the voltage and current of the armature coil, respectively, and u is the control voltage input at the same time. R and K_e are the resistance value and induced voltage constant of the armature coil. The current of the armature coil and control force has the following relation:

$$F_u = K_f i \tag{3}$$

K_f is the thrust constant. The inductance of the armature coil is neglected. By combining equations (1) through (3) and arranging them, it is also possible to obtain the governing equations in state space form. If the system is defined in state space form as

$$\underline{\dot{x}} = \underline{f}(x) + [B] * \underline{Fu} + [W] \tag{4}$$

here $\underline{x} = [x_1\ x_2\ \ldots\ x_{10}]^T$ where $x_6 = \dot{x}_1$, $x_7 = \dot{x}_2$, $x_8 = \dot{x}_3$, $x_9 = \dot{x}_4$, and $x_{10} = \dot{x}_5$. $\underline{f}$(x) is vector functions composed of first order differential equations, [B] is the controller force matrix and [W] is the disturbance force matrix. $\underline{f}$(x), [B] and [W] are given in the Appendix with a nomenclature of structural parameters used in the analytical model.

The PD Controller

Since the integral controller causes an additional vibration mode, PD control will be used as a traditional example. PD control has been used in industry widely. A general closed loop diagram of the feedback system is shown in Figure 2.

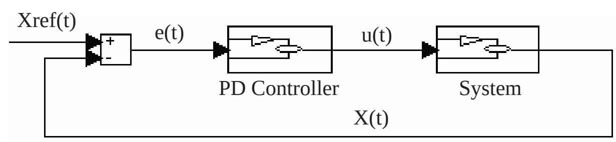

Figure 2. Closed loop block diagram with a PD controller.

Here x_{ref}(t) is the desired value for the output of the system. x(t) is the output, e(t) is the error and u(t) is the control signal. The control input u(t) is obtained as follows:

$$u(t) = K[e(t) + \tau_d \quad \frac{de(t)}{dt}] \tag{5}$$

K and τ_d are proportionality constant and derivative time respectively. These values are obtained using the Ziegler-Nichols method (Ogata, 1990) and are given in the Appendix.

The Fuzzy Logic Controller

The aim of this study was to apply fuzzy logic control to analytical structural systems. Improvements in electromagnetic force sources and sensors have made this application possible (Dan Cho, 1993; Rao and Prahlad, 1997). Fuzzy logic has come a long way since it was first presented to technical society, when Zadeh (1965) published his seminal work "Fuzzy Sets" in the Journal of Information and Control. Since that time, the subject has been the focus of much independent research. The attention currently being paid to fuzzy logic is most likely the result of present popular consumer products employing fuzzy logic (Ross, 1995). The superior qualities of this method include its simplicity, satisfactory performance and robust character.

Linguistic variables, such as Small, Medium, and Big are used to represent the domain knowledge, with their membership values lying between 0 and 1. Basically, a fuzzy logic controller has the following components:

(i) The fuzzification interface to scale and map the measured variables to suitable linguistic variables (fuzzifier).

(ii) A knowledge base comprising a linguistic control rule base.

(iii) A decision-making logic to infer the fuzzy logic control action based on the measured variables, which resembles human decision making (fuzzy reasoning engine).

(iv) A defuzzification interface to scale and map the linguistic control actions inferred to yield a nonfuzzy control input to the plant being controlled (defuzzifier).

The fuzzifier converts each input variable value into the relevant fuzzy variable using its own set of linguistic variables (fuzzy sets) and their pertinent membership functions. For example (Figure 3a), for a generic input variable y_i the fuzzy sets Negative Big, Negative Small, Zero, Positive Small, and Positive Big (nb, ns, z, ps, and pb) are defined in the universe of discourse of y_i. Any value of y_i in its universe of discourse belongs at the same time to different fuzzy sets with a different degree of membership, by the related membership functions (the most commonly used kinds of membership function are bell shaped, trapezoidal and triangular). The value 0.5 of y_3 is both ps with a membership tag 0.6 and zo with a membership tag of 0.17, while it is nb, ns and pb with a membership tag of 0. The fuzzy reasoning engine converts the values of fuzzy input variables into the fuzzy sets of output variables.

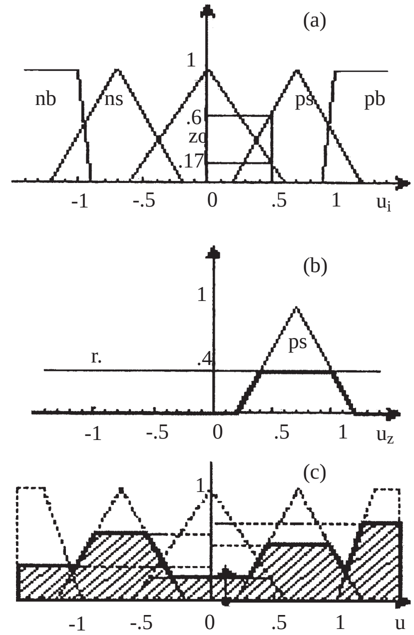

Figure 3. A basic fuzzy logic action.

It consists of a set of fuzzy logic rules of the kind IF {Rule Premise} THEN {Rule Consequence}. The {Rule Premise} block is a set of fuzzy logic operations, whose result, different from a set of Boolean logic operations, is any real value between 0 and 1. The basic operators of fuzzy logic are fuzzy intersection (AND), fuzzy union or disjunction (OR) and fuzzy complement (NOT); their operands are fuzzy sets. The result of the AND (OR) operation is the minimum (maximum) of the membership functions of its two fuzzy set operands; the result of the NOT operation is the complement of the membership function of its fuzzy set operand. The {Rule Consequence} provides a linguistic value for each output variable; its truth value is the numeric result (between 0 and 1) of the {Rule Premise}. Fuzzy sets and their pertinent membership functions have to be defined for each output. In the example of Figure 3b, supposing that in the k^{th} rule the premise result is 0.4, the consequence, in the universe of discourse of the output u_2 (by its own pb fuzzy set), is the evidenced curve. The defuzzifier is responsible for the translation of the fuzzy reasoning engine results into a crisp set of output values. A variety of methods are used to perform defuzzification, the most common being:

i) The Mamdani method that returns the centroid of the output fuzzy region as the crisp output of the fuzzy interface system (Figure 3c).

ii) The TVFI (Truth Value Flow Inference) method that returns a weighted average as the crisp output of the fuzzy interface system (De Falco *et al.*, 1998).

Earthquake Excitation and the Response of the Analytical Structure

In this study, Matlab Simulink with Fuzzy Toolbox is used. The aim of the fuzzy logic control system for the analytical structural system uses the errors in the second story motion (e = $x_{r2} - x_2$) and the derivatives (de/dt) of its as the input variable while the control force (u) are their outputs. Reference values (x_{r2}, $\dot{x}_{r2}$) are considered to be zero in Figure 4.

A model of the two similar rule bases developed by heuristics with errors in body bounce motion, pitch motion and velocity as input variables are given in Table 1. P, N, Z, B, M, and S represent Positive, Negative, Zero, Big, Medium and Small, respectively. A trial and error approach with triangular membership functions was used to achieve a good controller performance. The limits of displacement of the error (e) are ∓0.015 m, and limits of velocity of the error (de/dt) are ∓1.5 m/s, whereas limits of the control force (u) are ∓2.10^8 N (Figure 5).

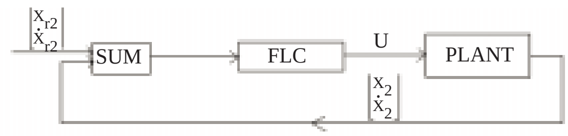

Figure 4. Closed loop model with a fuzzy logic controller.

The first rule in Table 1 is given below:

IF e is XNB and de/dt is VN THEN u is UNB.

All the rules are written similarly to apply to fuzzification in Figure 5d. In this study, the centroid method is used in defuzzification.

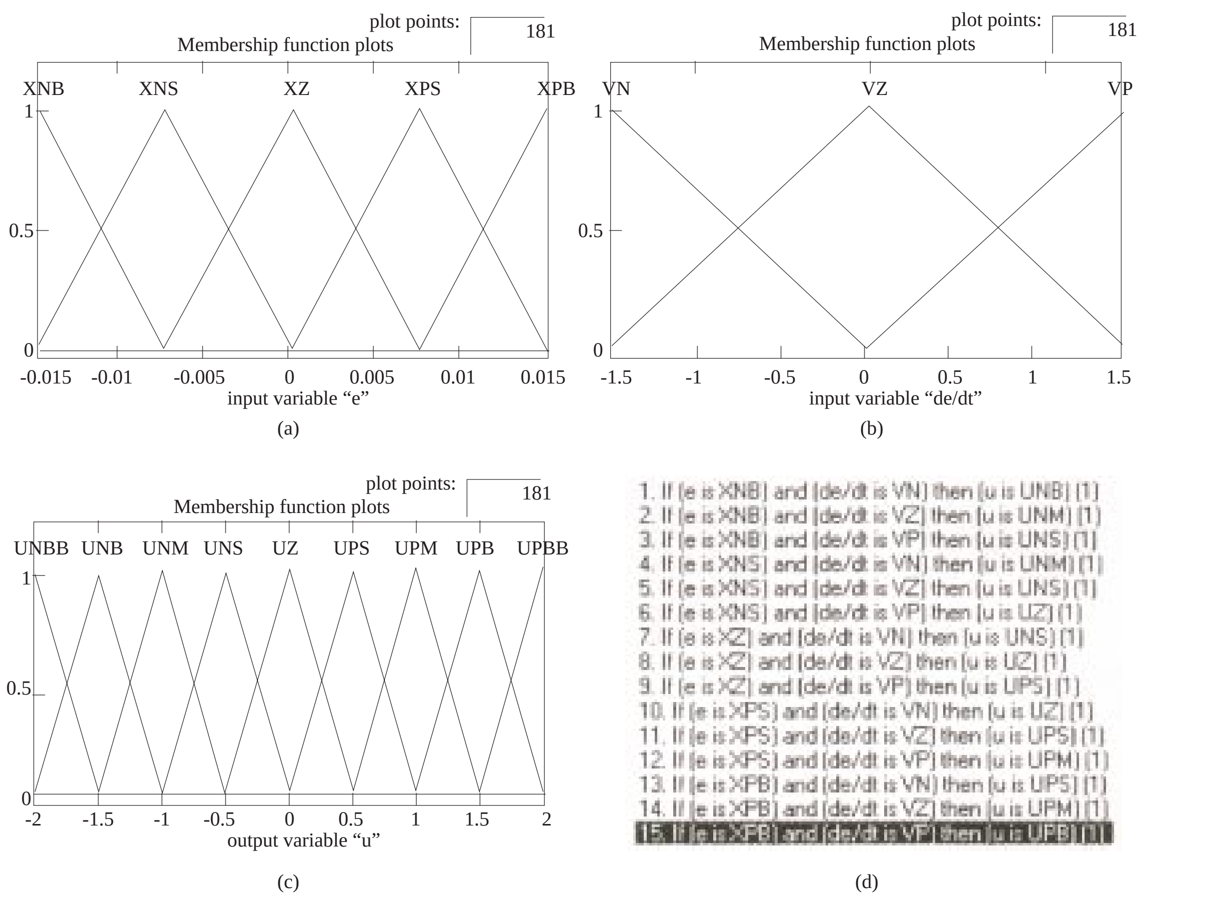

Figure 5. Input variables and rules.

Table 1. Rule base for the fuzzy logic controllers.

		VN	VZ	VP
	XNB	UNB	UNM	UNS
Error (e)	XNS	UNM	UNS	UZ
	XZ	UNS	UZ	UPS
	XPS	UZ	UPS	UPM
	XPB	UPS	UPM	UPB

An analytical structural system has been simulated against the earthquake ground motion of the destructive Marmara earthquake ($M_w = 7.4$), which resulted in more than 20,000 deaths in Turkey on 17 August, 1999. Earthquake ground motion is used as the input to an analytical building structure. The accelerations were recorded at the Kandilli Observatory and Earthquake Research Institute strong motion station at the Küçükçekmece Nuclear Research Center in İstanbul, Turkey, during the 17 August, 1999 main shock (Figure 6).

The displacements of the related storys are estimated through accelerometers on them after online integration. Figure 7 shows the uncontrolled time responses of all storys.

Figure 8 shows the controlled and uncontrolled time responses of the storys. It is observed that there is an important improvement with the fuzzy logic controller when the horizontal displacements of the analytical structure are considered. Calculated maximum interstory displacements are shown in Table 2.

Figure 9 demonstrates the change in control voltage inputs.

Figures 10 and 11 show the frequency responses of the second and fifth story displacements, velocities and accelerations respectively for both controlled and uncontrolled cases. Since the system has five degrees of freedom, there are five resonance values at 0.5, 2.9, 5.3, 7.6 and 9.1 Hz.

As expected, the lower curves belong to the controlled systems. When the response plots of the analytical structural systems with PD and fuzzy logic controllers are compared, a superior improvement in terms of magnitudes with fuzzy logic was witnessed (Figures 10 and 11). Therefore, at the resonance values of the response of the storys with a fuzzy logic controller, satisfactory results were obtained.

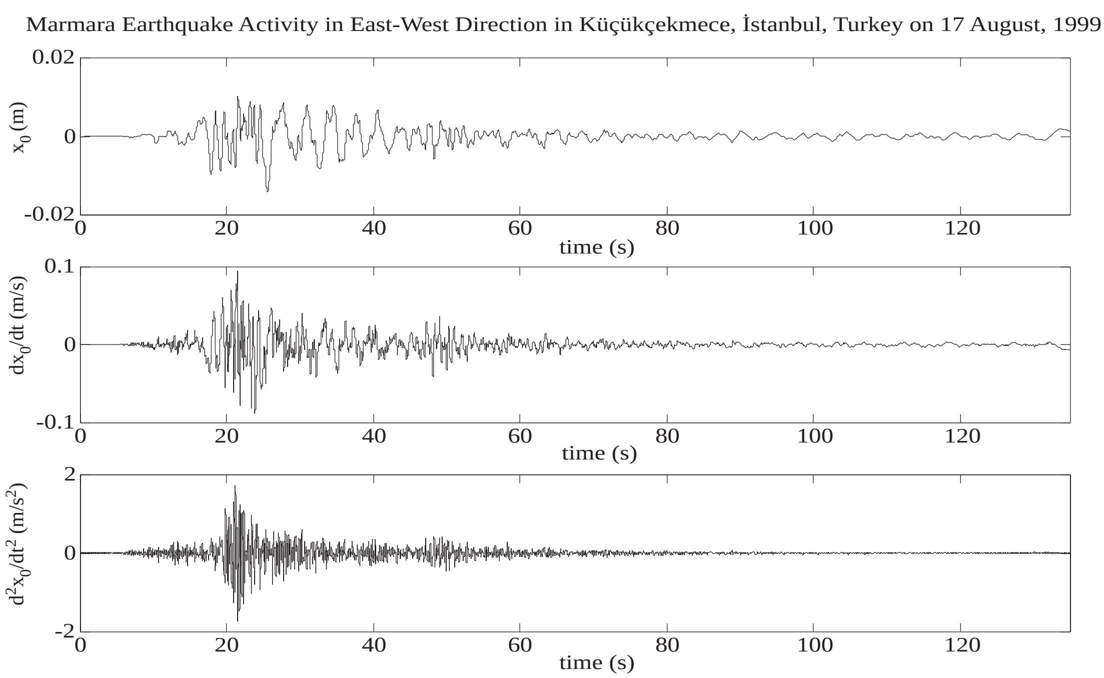

Figure 6. Marmara earthquake excitation input to the analytical model.

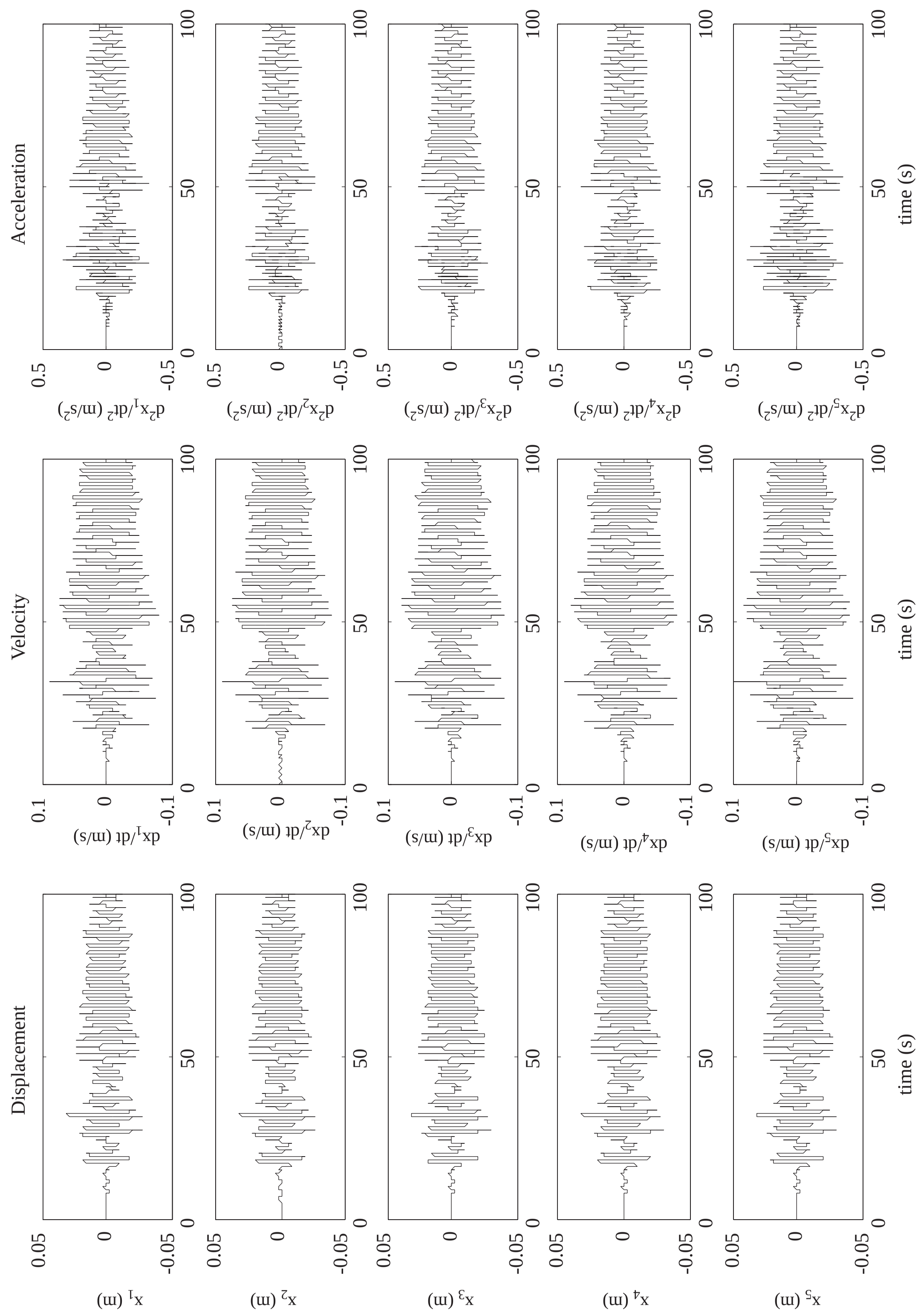

Figure 7. Uncontrolled time responses of all storys.

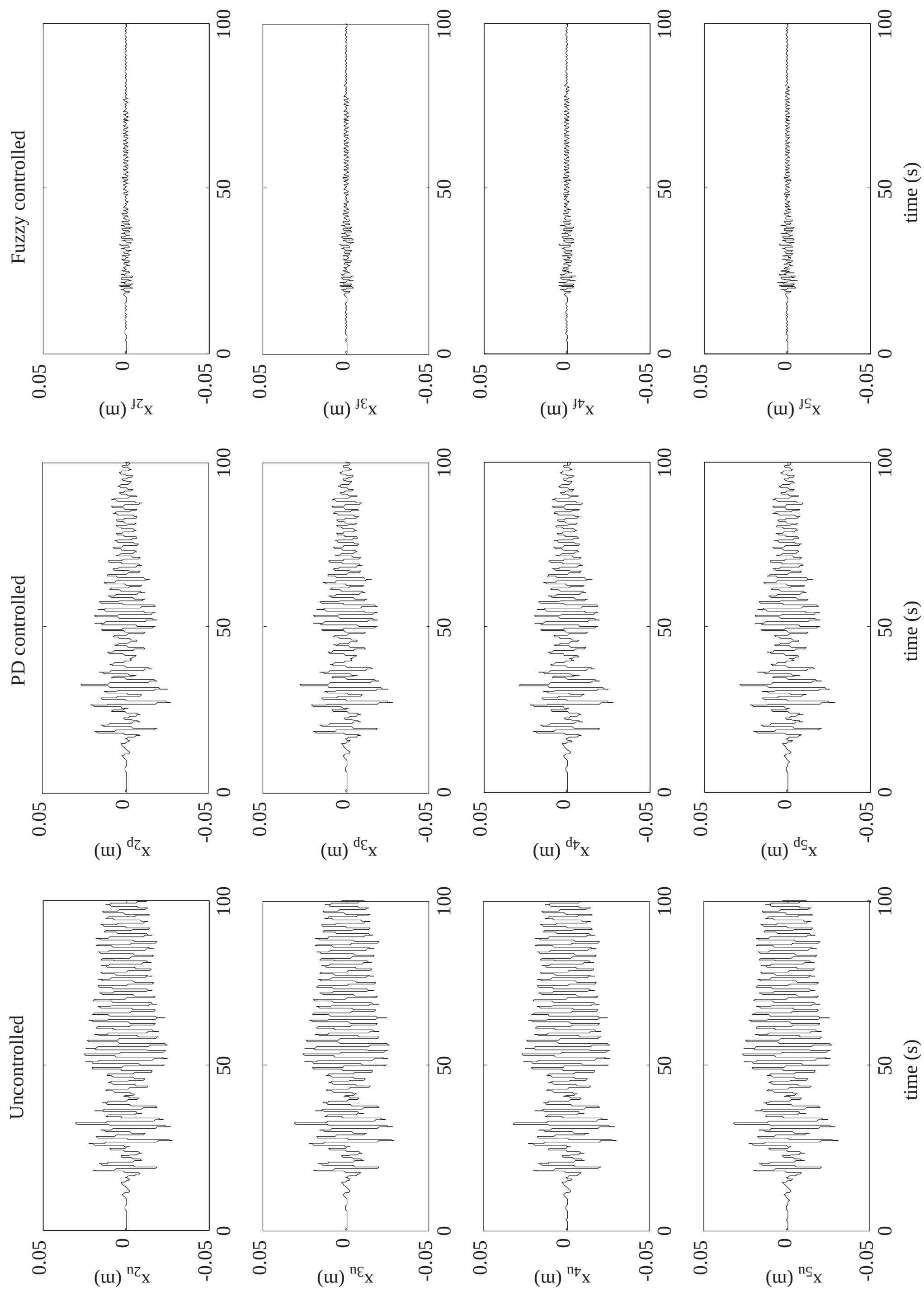

Figure 8. Controlled and uncontrolled time responses of the storys.

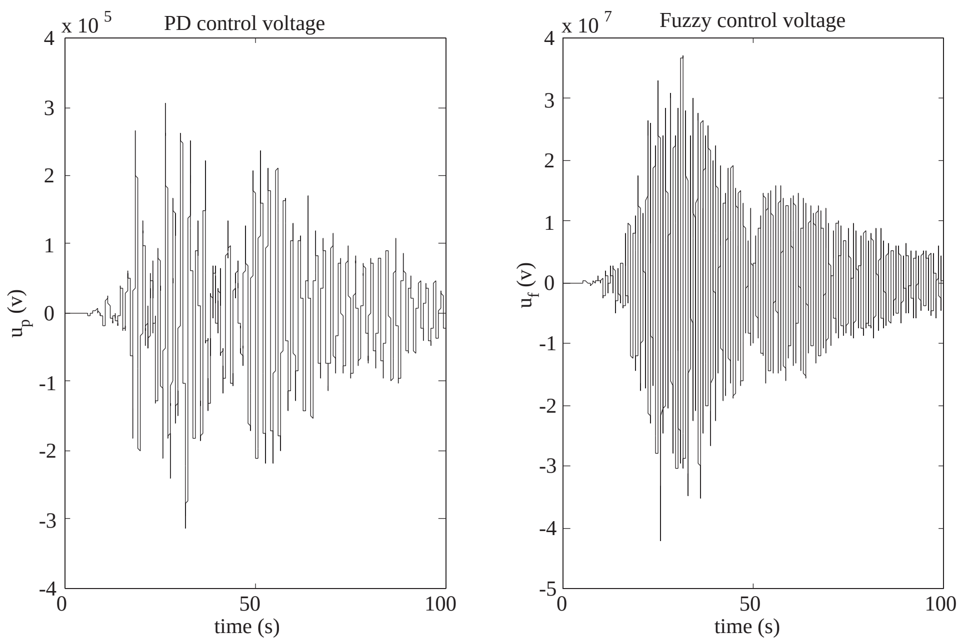

Figure 9. Time history of control voltages.

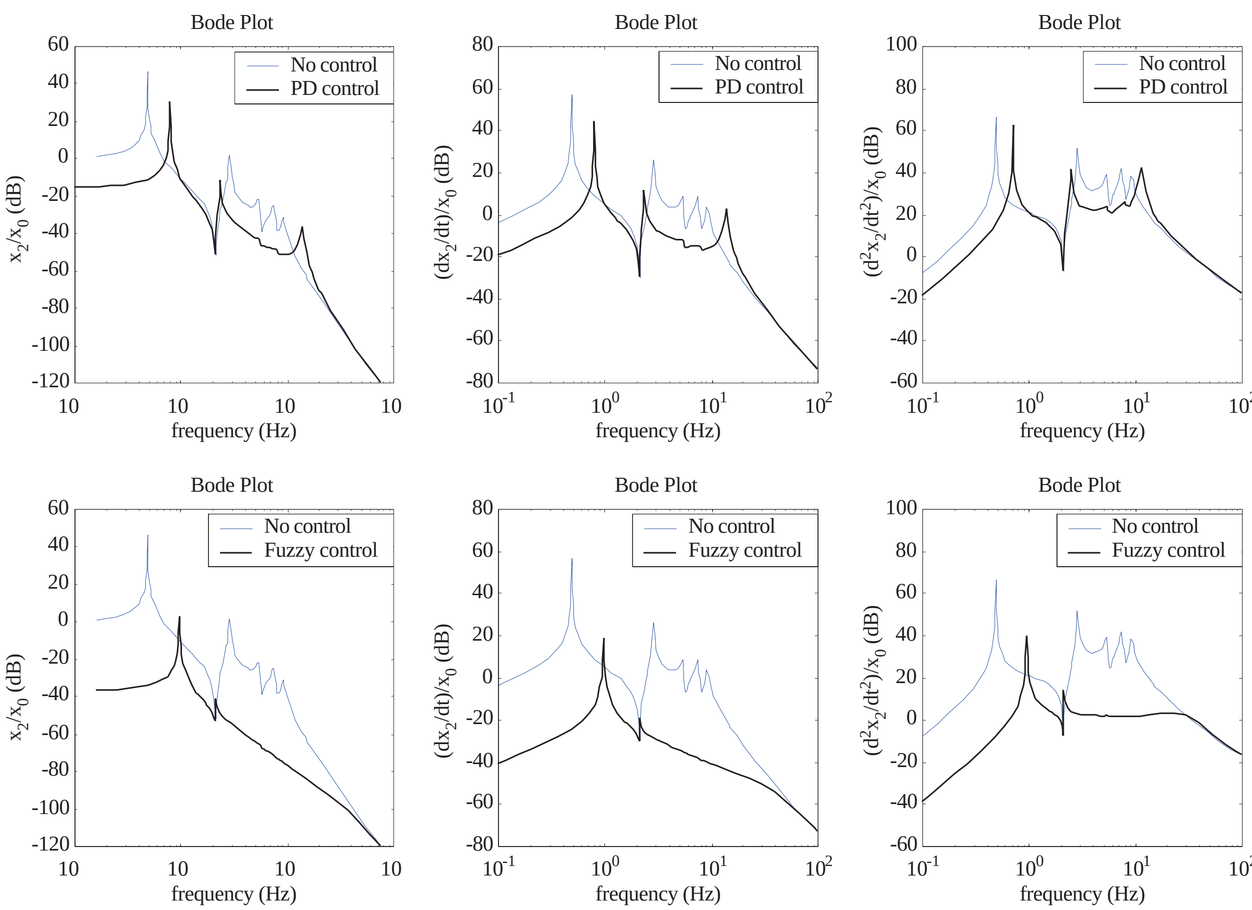

Figure 10. Controlled and uncontrolled frequency responses of the second story.

Table 2. Calculated maximum interstory displacements ($.10^{-3}$ m).

Storys	Uncontrolled	PD controlled	Fuzzy controlled
First story (x_1-x_0)	33.9	34.5	50.8
Second story (x_2-x_1)	1.14	1.24	39.9
Third story (x_3-x_2)	1.01	1.09	1.41
Fourth story (x_4-x_3)	0.86	1.02	1.32
Fifth story (x_5-x_4)	0.55	0.67	0.86

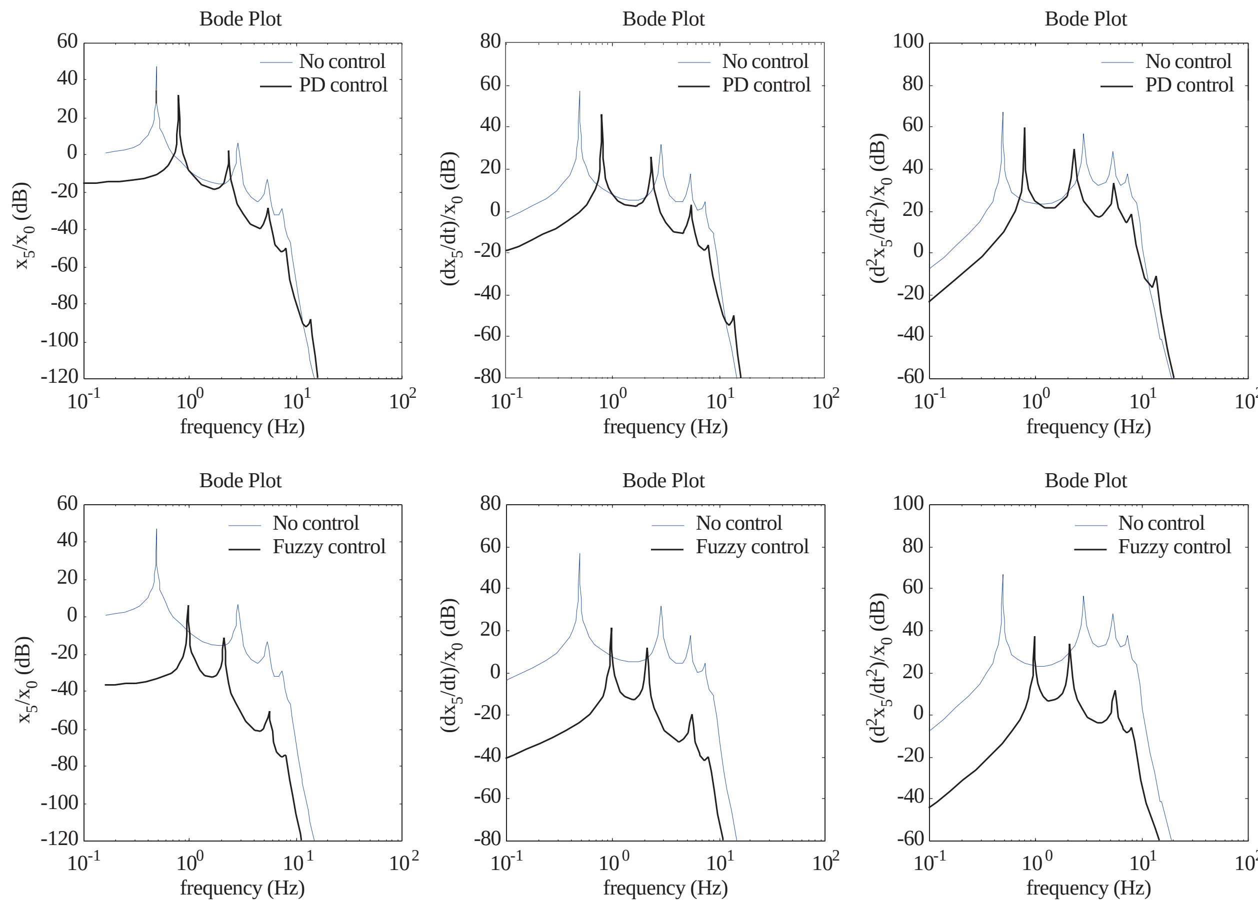

Figure 11. Controlled and uncontrolled frequency responses of the fifth story.

Conclusion

In this study, fuzzy logic and PD controllers were designed for a multi-degree-of-freedom analytical system having the parameters of a real structure and simulation results were presented. The main idea behind proposing a fuzzy logic controller is its success and the ability of using these types of controllers on structural systems.

Integrated circuits were selected for use in hardware controls. They have very fast processing speeds, and therefore they provide the quick response times necessary to reduce vibrations effectively. This also minimizes the time between sensor measurements and actuator responses. The time lag between the controller and actuator is very small and therefore it is assumed to be zero. These circuits can last a lifetime under normal environmental conditions, and can operate over a broad range of temperatures.

Using a UPS with active control systems will prevent the disadvantage of losing main electric power. In addition, using the fuzzy logic control together with robust control methods such as a sliding mode control would improve the success against different earthquakes and changing system parameters.

Since the destructive effect of earthquakes is a result of horizontal vibrations, in this study the degrees of freedom were assumed only in this direction. The system is modeled including the dynamics of a linear motor, which is used as the active isolator. Against earthquake excitation, it shown that a designed fuzzy logic controller brought better active control performance than a PD controller. The improvement in resonance values and decrease in vibration amplitudes support this result.

Appendix

Mass matrix.

$$[M] = \begin{bmatrix} m_1 & 0 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 & 0 \\ 0 & 0 & m_3 & 0 & 0 \\ 0 & 0 & 0 & m_4 & 0 \\ 0 & 0 & 0 & 0 & m_5 \end{bmatrix}$$

Stiffness matrix.

$$[K] = \begin{bmatrix} k_1+k_2 & -k_2 & 0 & 0 & 0 \\ -k_2 & k_2+k_3 & -k_3 & 0 & 0 \\ 0 & -k_3 & k_3+k_4 & -k_4 & 0 \\ 0 & 0 & -k_4 & k_4+k_5 & -k_5 \\ 0 & 0 & 0 & -k_5 & k_5 \end{bmatrix}$$

Damping matrix.

$$[C] = \begin{bmatrix} c_1+c_2 & -c_2 & 0 & 0 & 0 \\ -c_2 & c_2+c_3 & -c_3 & 0 & 0 \\ 0 & -c_3 & c_3+c_4 & -c_4 & 0 \\ 0 & 0 & -c_4 & c_4+c_5 & -c_5 \\ 0 & 0 & 0 & -c_5 & c_5 \end{bmatrix}$$

Parameters of the five degrees of freedom of a realistic structural system.

m_1	450,000 kg	c_1	26,170 Ns/m
$m_2 = m_3 = m_4 = m_5$	345,000 kg	c_2	490,000 Ns/m
k_1	18,050,000 N/m	c_3	467,000 Ns/m
k_2	340,000,000 N/m	c_4	410,000 Ns/m
k_3	326,000,000 N/m	c_5	350,000 Ns/m
k_4	285,000,000 N/m	K_f	2 N/A
k_5	250,000,000 N/m	K_e	2 Volt
R	4.2 Ω		

The controller force and the disturbance force matrices.

$$[B] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -\frac{1}{m_1} \\ \frac{1}{m_2} \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad [W] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{(c_1\dot{x}_0+k_1x_0)}{m_1} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

PD controller parameters.

$$K = 2,580,000N/m, \qquad \tau_d = 1.5s$$

State equations excluding control inputs;

$$f_1(x) = x_6, \quad f_2(x) = x_7, \quad f_3(x) = x_8, \quad f_4(x) = x_9, \quad f_5(x) = x_{10}$$

$$f_6(x) = 1/m_1[-(c_1 + c_2)\ x_6 + c_2 x_7 - (k_1 + k_2)\ x_1 + k_2 x_2]$$

$$f_7(x) = 1/m_2[-(c_2 + c_3)\ x_7 + c_2 x_6 + c_3 x_8 - (k_2 + k_3)\ x_2 + k_2 x_1 + k_3 x_3]$$

$$f_8(x) = 1/m_3[-(c_3 + c_4)\ x_8 + c_3 x_7 + c_4 x_9 - (k_3 + k_4)\ x_3 + k_3 x_2 + k_4 x_4]$$

$$f_9(x) = 1/m_4[-(c_4 + c_5)\ x_9 + c_4 x_8 + c_5 x_{10} - (k_4 + k_5)\ x_4 + k_4 x_3 + k_5 x_5]$$

$$f_{10}(x) = 1/m_5[-c_5\, x_{10} + c_5 x_9 - k_5 x_5 + k_5 x_4]$$

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