# Lubrication of a Porous Exponential Slider Bearing by Ferrofluid with Slip Velocity

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#### Abstract

A bearing with its slider in exponential form and stator with a porous facing of uniform thickness was analysed using a ferrofluid lubricant and considering slip velocity. Expressions for dimensionless pressure, load capacity, friction on the slider, coefficient of friction and for the position of the centre of pressure were obtained. Computed values of load capacity, friction, coefficient of friction and the position of the centre of pressure are displayed in graphical form. The load capacity, friction and coefficient of friction decreased for increasing values of the slip parameter. However, load capacity and friction decreased and the coefficient of friction increased when the permeability parameter increased.

Key words: Ferrofluid, Lubrication, Porous stator, Exponential slider, Slip velocity

### Introduction

Agrawal (1986) analysed an inclined plane slider bearing with a porous-faced stator with a ferrofluid lubricant and found that its performance was better than the corresponding bearing with a conventional lubricant. In practice, the slider was found to bend owing to elastic, thermal or uneven wear effects. Therefore Cameron (1987) suggested an exponential form of the slider to be nearest the true shape. Hence, Bhat and Patel (1991) considered an exponential slider with a porous-faced stator with a ferrofluid lubricant. Such a lubricant caused an increase in load capacity without altering the friction on the slider. Recently Shah and Bhat (2000a, 2000b) considered the effect of ferrofluid lubricant on the squeeze film between curved porous rotating circular plates and two curved annular plates. All the above researchers assumed that there was no slip at the interface of the lubricant film and the porous matrix.

Beavers and Joseph (1967) showed that such an assumption might not hold at the nominal boundary of a naturally permeable material. Sparrow *et al.* (1972) gave simplified boundary conditions in the above case.

In this paper, a porous exponential slider bearing with a ferrofluid lubricant considering slip velocity at the interface of the film region and the porous region is studied.

#### Analysis

The bearing consists of a slider in an exponential form moving with a uniform velocity U in the xdirection and a stator with a porous matrix of uni-

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form thickness  $H^*as$  in Figure 1. A is the length and B is the breadth of the bearing with A<<B.



Figure 1. Porous exponential slider bearing.

The film thickness **h** is defined as

$$h = h_2 e^{-(x \ln a)/A} , 0 \le x \le A \tag{1}$$

where  $a = h_2/h_1$ ,  $h_2$  is the maximum value of h and  $h_1$  is the minimum value of h. The magnetic field  $\overline{H}$  is such that its magnitude H is given by

$$H^2 = Kx(A - x), (2)$$

K is a quantity taken to suit the dimensions of both sides and its inclination is  $\phi$  as determined by Agrawal (1986).

The equation of fluid flow in the film region in Verma (1986) is

$$\frac{\partial^2 u}{\partial z^2} = \frac{1}{\zeta} \frac{\partial}{\partial x} \left( p - \frac{\mu_0 \bar{\mu} H^2}{2} \right) \tag{3}$$

where u is the x-component of the fluid velocity,  $\zeta$  is the fluid viscosity, p is the film pressure,  $\mu_0$  is the permeability of free space and  $\overline{\mu}$  is the magnetic susceptibility.

Solving Equation (3) under the boundary conditions as in Sparrow *et al.* (1972)

$$u = U$$
 when  $z = h$ ,  $u = \left(\frac{1}{s}\frac{\partial u}{\partial z}\right)_{z=0}$  when  $z = 0$ 
  
(4)

with s being the slip constant, substituting the value of u in the integral form of the continuity equation in the film region, using continuity of velocity components of the fluid in the film region and porous matrix across the surface z = 0 and referring to Agrawal (1986) one obtains

$$\frac{d}{dx} \left[ \left\{ 12kH^* + \frac{h^3(4+sh)}{(1+sh)} \right\} \frac{d}{dx} \left( p - \frac{1}{2}\mu_0 \bar{\mu} H^2 \right) \right]$$
$$= 6\zeta U \frac{d}{dx} \left[ \frac{h(2+sh)}{1+sh} \right] \tag{5}$$

k being the permeability of the porous material.

Using Equations (1), (2) and the dimensionless quantities,

$$X = \frac{x}{A} , \Psi = \frac{kH^*}{h_1^3} , \bar{h} = \frac{h}{h_1} , \ \bar{s} = sh_1,$$
  
$$\bar{p} = \frac{h_1^2 p}{\zeta U A} , \mu^* = \frac{\mu_0 \bar{\mu} K A h_1^2}{\zeta U}$$
(6)

Equation (5) transforms to

$$\frac{d}{dX} \left[ G \frac{d}{dX} \left\{ \bar{p} - \frac{1}{2} \mu^* X (1 - X) \right\} \right] = \frac{dE}{dX}$$
(7)

where

$$G = 12\Psi + \frac{\bar{h}^3(4 + \bar{s}\bar{h})}{(1 + \bar{s}\bar{h})} , \quad E = \frac{6\bar{h}(2 + \bar{s}\bar{h})}{1 + \bar{s}\bar{h}} \quad (8)$$

$$\bar{h} = ae^{-Xlna} \tag{9}$$

### Solutions

Solving Equation (7) under the boundary conditions

$$\overline{p} = 0 \quad \text{when} \quad X = 0, 1, \tag{10}$$

one can obtain

$$\bar{p} = \frac{1}{2}\mu^* X \left(1 - X\right) + \int_{1}^{X} \frac{E - Q}{G} dX \qquad (11)$$

where

$$Q = \frac{\int\limits_{0}^{1} \frac{E}{G} dX}{\int\limits_{0}^{1} \frac{1}{G} dX}$$
(12)

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The non-dimensional forms of load capacity W, friction force F on the slider, the coefficient of friction f and the x-coordinate  $\overline{Y}$  of the centre of pressure can be expressed as

$$\overline{W} = \frac{h_1^2 W}{\zeta U A^2 B} = \frac{\mu^*}{12} - \int_0^1 X \frac{E - Q}{G} dX$$
(13)

$$\overline{F} = \frac{h_1 F}{\zeta U A B} = \int_0^1 \left[ \frac{\overline{s}}{1 + \overline{s}\overline{h}} + \frac{\overline{h}(2 + \overline{s}\overline{h}) (E - Q)}{2G (1 + \overline{s}\overline{h})} \right] dX$$
(14)

$$\overline{f} = \frac{Af}{h_1} = \frac{\overline{F}}{\overline{W}} \tag{15}$$

$$\overline{Y} = \frac{\overline{X}}{A} = \frac{1}{\overline{W}} \left[ \frac{\mu^*}{24} - \frac{1}{2} \int_0^1 X^2 \frac{E - Q}{G} \, dX \right]$$
(16)

#### **Results and Discussion**

The present analysis reduces to the no-slip case of Bhat and Patel (1991) by setting the slip parameter  $1/\overline{s} = 0$ .



Figure 2. Values of dimensionless load capacity for different values of the slip parameter and the permeability parameter.



Figure 3. Values of dimensionless friction force for different values of the slip parameter and the permeability parameter.



Figure 4. Values of dimensionless coefficient of friction for different values of the slip parameter and the permeability parameter.



Figure 5. Values of dimensionless position of the centre of pressure for different values of the slip parameter and the permeability parameter.

Equations (11), and (13)-(16) give expressions for the bearing characteristics. Values of dimensionless load capacity  $\overline{W}$ , friction force  $\overline{F}$ , coefficient of friction  $\overline{f}$ , and position  $\overline{Y}$  of the centre of pressure are computed as shown in graphical form in Figures 2-5 for different values of the slip parameter  $1/\overline{s}$  and the permeability parameter  $\psi$  with  $\mu^* = 0.5$  and a = 2.

Figures 2-3 show that  $\overline{W}$  and  $\overline{F}$  decrease when  $1/\overline{s}$  or  $\psi$  increases. It can be seen from Figure 4 that  $\overline{f}$  decreases or increases according as  $1/\overline{s}$  or  $\psi$  increases. From Figure 5, the position of the centre of pressure does not change significantly when  $1/\overline{s}$  increases. However, the centre of pressure shifts slightly towards the inlet for large values of  $\psi$ .

Thus a decrease in load capacity owing to slip velocity can be compensated for by increasing the magnetization of the fluid particles.

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#### Nomenclature

- a  $h_2/h_1$
- A bearing length
- B bearing breadth
- E defined in Eq. (8)

References

f	coefficient of friction
F	friction force on the slider
G	defined in Eq. $(8)$
h	film thickness
$h_{1,h_2}$	minimum and maximum values of h
Η	magnitude of applied field
$\mathrm{H}^*$	thickness of porous matrix
k	permeability of porous matrix
Κ	defined in Eq. $(2)$
р	film pressure
Q	defined in Eq. $(12)$
$\mathbf{S}$	slip constant
u	x component of film fluid velocity
U	velocity of slider
W	load capacity
Х	x/A

- fluid viscosity ζ free space permeability  $\mu_0$ defined in Eq. (15)f  $\bar{F}$ defined in Eq. (14) $\bar{h}$ defined in Eq. (6) $\bar{H}$ applied magnetic field  $\bar{p}$ defined in Eq. (6) $\bar{s}$  $sh_1$  $\overline{W}$ defined in Eq. (13) $\overline{X}$ x coordinate of the centre of pressure  $\overline{Y}$ X/Amagnetic susceptibility  $\bar{\mu}$ inclination of  $\bar{H}$  with the x-axis  $\phi$  $\mu^*$ defined in Eq. (6)
- $\psi$  defined in Eq. (6)

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