Edge Spalling Formation in a Plate due to Moving Compressive Load

Kubilay ASLANTAŞ, Süleyman TAŞGETİREN
Afyon Kocatepe University, Department of Mechanical Education, Afyon-TURKEY
e-mail: aslantas@aku.edu.tr, tasgetir@aku.edu.tr

Abstract

A subsurface crack subjected to moving compressive and tangential loads grows toward the surface and leads to the formation of sheet-like wear particles. The normal and shear stresses arising at the crack tip due to normal and tangential loads can be characterised by mode I and mode II stress intensity factors.

In this study, the behaviour of an edge crack subjected to moving normal and tangential loads is analysed by the finite element method. The problem is considered under linear elastic fracture mechanics conditions. K_I and K_{II} stress intensity factors at the crack tip are computed for different load positions and different friction coefficients. The problem is considered under 2-D conditions. The analysis applies fracture mechanics to a finite element model to predict spall formation in the elastic state. At the end of the analysis, the effect of the friction coefficient and load position on spalling was examined.

Key words: Edge spalling, Fatigue crack growth, Finite element analysis, Compressive load.

Introduction

The stress intensity factors is a very crucial concepts and the most important magnitudes within fracture mechanics. This factors define the stress field close to the crack tip of a crack and provide fundamental information of how the crack is going to propagate. Spall formation depends on repeatedly acting loads, the friction coefficient and the surface roughness. Models of crack propagation and spall formation based on fracture mechanics have been developed by researchers in wear studies (Suh, 1973; Evans *et al.*, 1984). Crack growth due to cycling loading occurs by a fatigue process, and the crack propagation rate is proportional to a power of the SIF range. Fleming and Suh (1977) obtained the stress intensity factors for a subsurface crack under moving compressive load without considering friction on the crack surface. Dubourg *et al.* studied the interaction problem of two surface cracks. In their study, modified crack modelling in terms of the dislocation theory is first described (Dubourg *et al.*, 1992).

Salehizadeh and Saka (1992) analysed short subsurface cracks with and without a branch lying parallel to the surface and subjected to normal Hertzian loading. In their study, the effect of the crack face friction on plastic deformation around the crack tips were examined (Salehizadeh and Saka, 1992). Aslantaş and Taşgetiren (2000) also attempted to use linear elastic fracture mechanics and the finite element method to compute the stress intensity factors for a subsurface crack. In their study, a computer code was developed for finite element analysis and the displacement correlation method was used to determine the stress intensity factors, K_I and K_{II}, at the crack tips (Aslantaş and Taşgetiren, 2002).

In this study, the behaviour of a subsurface-edge crack in a plate subject to normal and tangential loads is analysed. A spalling model that can be applied to severe sliding situations with low and high friction coefficients is presented. Different friction coefficients and different depths from the free surface are taken as variables. The problem is considered under linear elastic fracture mechanics. Young's

modulus of 169 GPa and Poisson's ratio of 0.25 are assumed since these material properties are representative of austempered ductile iron (Report of Ductile Iron Society, 1998). The analysis applies fracture mechanics to a finite element model to predict spall formation in the elastic state. The FRANC2D finite element code based on the two-dimensional fracture mechanical analysis is used.

Modelling Procedures

Finite element model and boundary conditions

The finite element method is widely employed for solving problems in linear elastic fracture mechanics. The main difficulty in these calculations is related to infinite stresses at the crack tip. In order to deal with mathematical complications, either a fine mesh is constructed near the crack tip, or a special element (singular element) is employed. The finite element mesh of the problem is shown in Figure 1.

In this study, quarter-point elements are used at the crack tip. Numerical modelling is carried out by the FRANC2D finite element program. Eight node isoparametric finite elements are used for modelling the solution domain except those in contact with the crack tip. The mesh size is 100×50 mm and consists of 3281 isoparametric eight node quadrilateral plane stress elements. A compressive point load F_n is applied externally and it moves along the x-axis. The magnitude of the compressive point line load F_n is $100 \, \text{Nmm}^{-1}$. The friction forces are calculated using the coulomb friction theory. The nodes at the bottom boundary of the mesh were constrained against displacement in the vertical direction, whereas the left corner node was constrained against displacement in the horizontal direction. Two different crack

depths are considered (d/h = 0.02 and 0.04). The friction coefficients are $\mu = 0$, 0.1, 0.25, 0.5 and 1.

Calculation of the stress intensity factors

A mixed mode (mode I and II) stress intensity factor solution is developed for general crack geometry using finite element methodology. In this study, the displacement correlation method is used in the calculation of the stress intensity factors. This method is appropriate for numerical solutions based on the finite element method. After finite element for the cracked structure are obtained, nodal displacement values of nodes 2, 3, 4 and 5 (Fig. 2) are determined. The crack face displacements in both opening and sliding modes are related to the stress intensity factors for mode I and mode II fracture. The opening mode K_I and the shear mode K_{II} are calculated by (Tan and Gao, 1990)

$$K_I = \frac{E}{2(1+\nu)(\kappa+1)} \sqrt{\frac{2\pi}{L}} \left[4\left(v_2 - v_4\right) + \left(v_5 - v_3\right) \right] \tag{1}$$

$$K_{II} = \frac{E}{2(1+\nu)(\kappa+1)} \sqrt{\frac{2\pi}{L}} \left[4\left(u_2 - u_4\right) + \left(u_5 - u_3\right) \right] \tag{2}$$

$$\kappa = \begin{vmatrix} 3 - 4\nu & \text{for plane strain} \\ \frac{3-\nu}{1+\nu} & \text{for plane stress} \end{vmatrix} \tag{3}$$

where E is the Young's modulus, L is the element length at the crack tip, ν is the Poisson's ratio, and u_i, and v_i are the nodal displacements in the x and y directions, respectively.
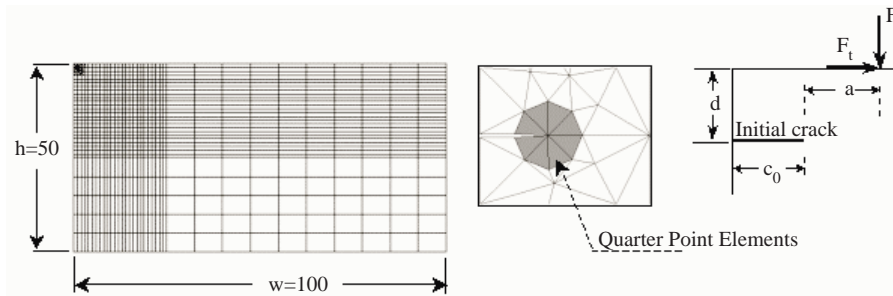

Figure 1. Finite element mesh of plate with subsurface-edge crack and crack tip elements.
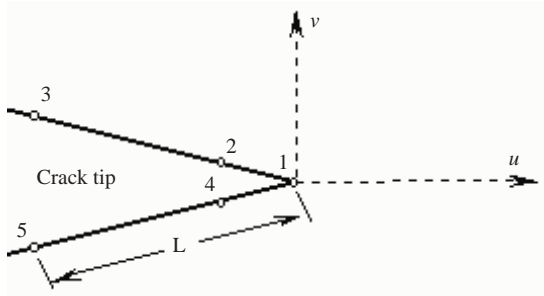
Figure 2. The location of the nodes used to calculate the stress intensity factors K_I and K_{II}.

The basis of the model for crack growth and spall formation used in this study is that the relative magnitudes of K_I and K_{II} change for each cycle. When the mode I and mode II stress intensity factors are known, the crack propagation angle can also be estimated. Positive and negative values of K_{II} are important for the determination of crack growth direction. Fatigue crack growth is assumed to occur either in the plane of maximum shear stress intensity factor range or that of the maximum tensile stress intensity range (Komvopoulos and Cho, 1997). Erdogan and Sih developed a theory for prediction of the crack development direction (Erdoğan and Sih, 1963). The theory is based on two assumptions: first, the crack will propagate radially from the crack tip, and second, the crack will propagate in a direction perpendicular to the maximum tangential stress. In the present study, the angle of crack growth associated with each load position is found by the mentioned theory using stress intensity factors at the current load step as

$$\theta = 2\tan^{-1}\left[\frac{1}{4}\left(\frac{K_I}{K_{II}} \pm \sqrt{\left(\frac{K_I}{K_{II}}\right)^2 + 8}\right)\right] \quad (4)$$

Crack Propagation Simulation

FRANC2D is a general-purpose two-dimensional finite element program for cracked structures. Among its variety of capabilities, a unique feature of FRANC2D is the ability to model a crack in the structure. For this purpose the user would first define an intial crack by identifying the node of the crack mouth and coordinates of the crack tip. FRANC2D will then delete the elements in the vicinity of the crack tip and then insert a rosette of quarter point, six-node triangular elements around the crack tip. Finally, the remaining area between the rosette and original mesh is filled with normal six node triangular elements (Lewicki and Ballarini, 1996). Figure 3 shows the crack-modelling scheme of FRANC2D.

Results and Discussion

Variations in stress intensity factors around the crack tip

Variations in K_I and K_{II} at the crack tip are analysed for different load positions and different friction coefficients. 14 different positions from $a/c_0 = 0$ to $a/c_0 = 6.5$ are selected for the load.

Absolute values of K_I at the crack tip are greater than K_{II} values. As the loading applied on the surface is always compressive, K_I takes negative values for most load positions. However, it also takes positive values depending on the friction coefficient (Figure 4). The absolute maximum K_I value occurs at the $a/c_0 = 0$ load position. A negative K_I value shows that the crack faces are close to each other. When the friction coefficient is increased, a positive K_I value is more likely to occur, and the crack faces are open. In particular, K_I takes its maximum value at $a/c_0 = 1$ for $\mu = 1$ (Figure 4).

K_{II} takes a positive value at $a/c_0 = 0$ for μ ¡ 0.25. However, for $\mu \geqslant 0.5$ K_{II} takes a negative value at the $a/c_0 = 0$ load position (Figure 4). Absolute maximum K_{II} values occur at the $a/c_0 = 0.5$ load position. K_{II} takes a positive value once the friction coefficient is increased, and takes its maximum value at $a/c_0 = 2$ for $\mu = 1$.
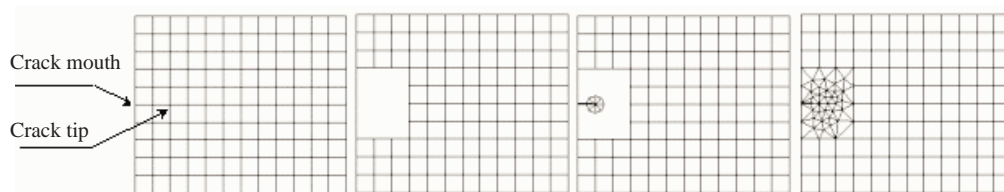

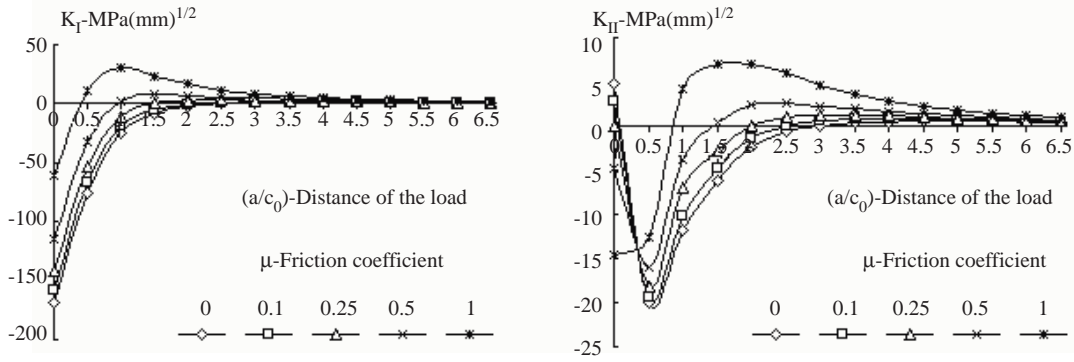**Figure 3.** Crack modelling scheme of FRANC2D finite element program.

Figure 4. Variations in the stress intensity factors (K_I and K_{II}) with respect to load position and friction coefficient for d/h = 0.02.

The load positions at which the stress intensity factors have their maximum values change with the depth of the crack from the free surface. For K_I, these locations are $a/c_0 = 1$ and 2 for d/h = 0.02 and 0.04 respectively. For all conditions, maximum values are obtained for $\mu = 1$ (Figures 4 and 5). While the maximum K_I value occurs at $a/c_0 = 1$ for d/h = 0.02 and $\mu = 1$, it is $a/c_0 = 2$ for d/h = 0.04. However, the maximum and minimum K_{II} values decrease for all locations and friction coefficients for the deeper crack locations.

Fatigue crack growth and spall formation

The sign of the K_{II} stress intensity factor is important for determination crack growth direction. Erdoğan and Sih have shown that a crack continues to advance in its own plane when it is only subjected to mode I. The presence of positive K_{II} at the crack tip deflects the direction of the crack away from the free surface, and negative K_{II} causes the crack to deviate toward the surface. Spall formation on the surface is based on fatigue crack mechanics. The majority of investigations in fracture mechanics are in the area in which it is assumed that the dependence between log (dc/dN) and log ΔK is linear and can be described by the Paris-Erdoğan equation (Paris and Erdoğan, 1961);

$$\frac{dc}{dN} = C[\Delta K]^m \qquad (5)$$

where C and m are material coefficients. In this study, $C = 2.4 \times 10^{-8}$ mm/(cycle*Mpa(mm)$^{0.5}$) and $m = 3.3$ are taken for the austempered ductile iron (Report of Ductile Iron Society, 1998). In Figure 6, the crack growth path and critical crack lengths at

$a/c_0 = 1$ and 2 are given for d/h = 0.02 and d/h = 0.04 respectively. As mentioned, K_I takes its maximum values at these locations (see Figures 4 and 5).

For a growing subsurface-edge crack under constant amplitude compressive loads the conditions at the crack tip are defined by the current value of ΔK. For crack growth due to mixed mode cyclic loading, ΔK is defined by Tanaka 1974) as

$$\Delta K = \left(\Delta K_I^4 + 8\Delta K_{II}^4\right)^{0.25} \qquad (6)$$

where $\Delta K_i = K_{i(\max)} - K_{i(\min)} \qquad i = I, II$

In the majority of studies about subsurface cracks, negative K_I is not considered because it has no effect on crack growth and $K_{I(\min)}$ is assumed to be zero (Komvopoulos, 1996; Salehizadeh and Saka, 1992; Komvopoulos and Cho, 1997). Therefore ΔK_I is equal to $K_{I(\max)}$ and in this study all simulations of crack growth are carried out for $K_{I(\max)}$ (Fig. 6).

In order to calculate the time for the crack reach to the surface, the following procedure carried out for each loading cycle. Firstly, K_I and K_{II} are calculated for the initial crack length, and then ΔK is calculated using Eq. 6 for the current load position. Secondly, dc is determined by the consideration of C and m parameters. This is added to the original crack length to obtain the new crack condition thereby giving the next incremental crack growth direction. If the crack tip reaches free surface, the procedure is ended. Otherwise the number of the cycle is recorded and the procedure is repeated.

The time for the crack to reach the surface increases as the depth of the crack from the surface increases. However, the spall size also increases with increasing load cycle for spall formation. The load

position also affects the direction of crack propagation and the number of load cycles required for spall formation. In Figure 7, the propagation direction of the subsurface-edge crack is given for different load positions.
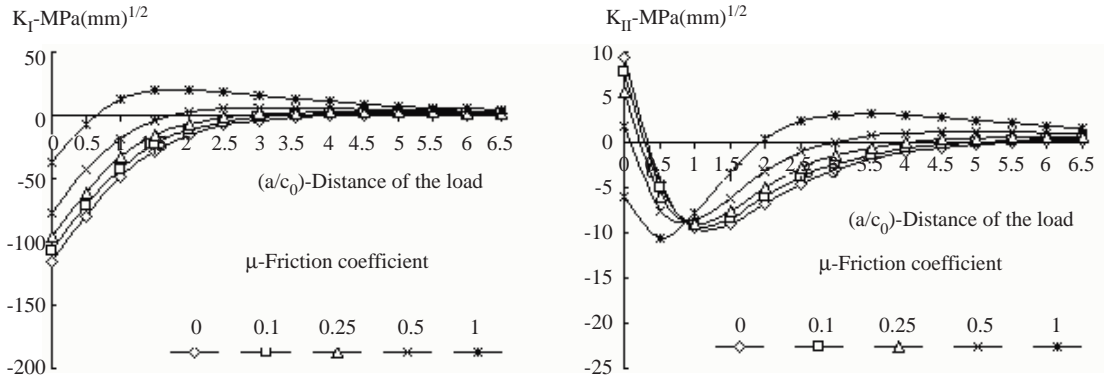
Figure 5. Variations in the stress intensity factors (K_I and K_{II}) with respect to load position and friction coefficient for d/h = 0.04.
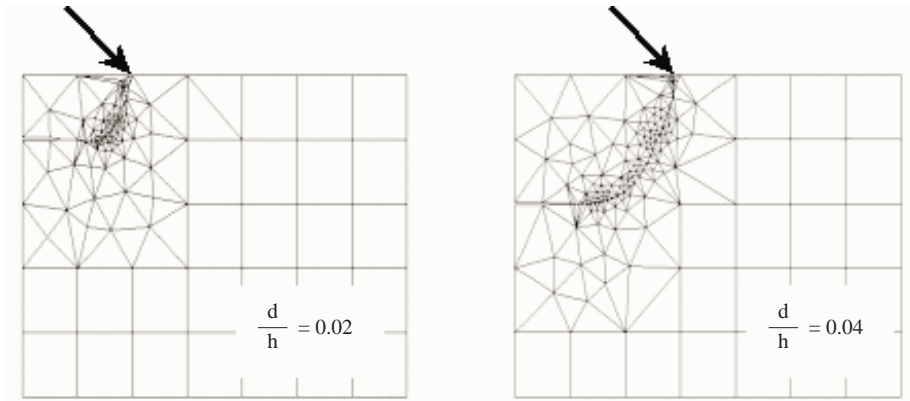
Figure 6. The propagation directions of the subsurface-edge crack for $\mu = 1$
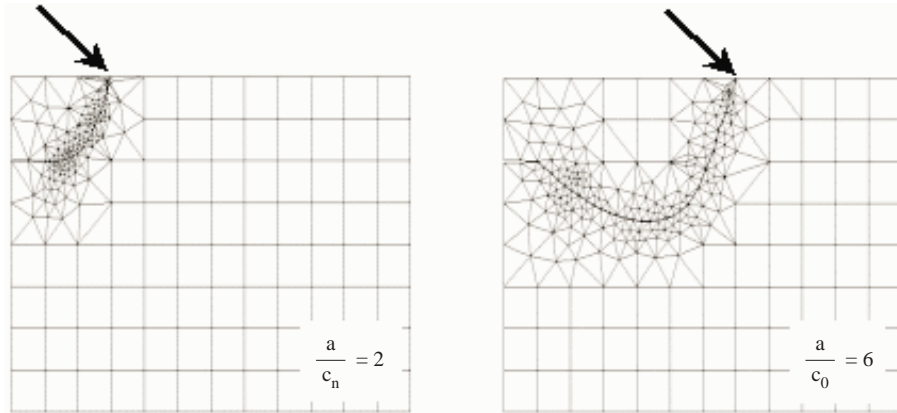
Figure 7. Effect of the load position on the propagation directions of the subsurface-edge crack for d/h = 0.04 and $\mu = 1$.

Spall formation is delayed and pitting chips are formed when the load position is away from the crack tip. According to the results, the crack grows to the surface directly for $a/c_0 < 3$. On the later load positions the crack grows away from the surface and then turns to grow towards the surface (Figure 7).

Conclusions

In this study, a subsurface-edge crack subjected to moving loads was analysed by the finite element method. The problem is considered under two-dimensional linear elastic fracture mechanics and conditions. FRANC2D was used for numerical solutions. K_I and K_{II} stress intensity factors at the crack tip were computed for different load positions and different crack depths. Displacement correlation was used to calculate the stress intensity factors. In order to determine the crack growth direction, the maximum cleavage stress theory was used.

The absolute value of K_I at the crack tip is greater than K_{II} values. K_I takes negative values because the load applied on the surface always compressive. The absolute maximum K_I value always occurs at the $a/c_0 = 0$ load position. However, K_I takes positive values when the friction coefficient increases. After some point, K_I also takes positive values for all crack depths and friction coefficients.

The load position also affects the direction of crack propagation and load cycle required for spall formation. Spall formation is delayed but larger chips form when the load is applied at a farther position.

Nomenclature

d	crack depth from the free surface
c_0	initial crack length
a	distance between load and crack tip
C, m	coefficients of the Paris equation
N	number of cycle
ΔK	stress intensity factor range
L	element length at the crack tip
x, y	Cartesian coordinates
F_n, F_t	normal and tangential load
K_I, K_{II}	mode I and mode II stress intensity factors
E	Young's modulus
ν	Poisson's ratio
μ	friction coefficient
θ	crack kink angle

References

Aslantaş, K. and Taşgetiren, S., "Analysis of Propagation Behaviour of a Subsurface Crack by the Finite Element Method", (in Turkish) Turkish J. Eng. Env. Sci., 26, 134-145, 2002.

Dubourg, M.C. and Villechaise, B., "Analysis of Multiple Fatigue Crack-Part II: Result", Journal of Tribology, 114, 462-468, 1992.

Ductile Iron Data, Online publication of Ductile Iron Society, www.ductile.org, 1998.

Erdogan, F. and Sih, G.C., "On the Crack Extension in Plates under Plane Loading and Transverse Shear", J. Basic Eng., 85, 519-527, 1963.

Evans, A.G. and Hutchinson, J.W., "On the Mechanics of Delamination and Spalling in the Compressed Films", Int. J. Solid Struct., 20, 455-466, 1984.

Fleming, J.R. and Suh, N.P., "The Relationship between Crack Propagation Rates and Wear Rates", Wear, 44, 57-64, 1977.

FRANC2D User's Manual. Available from: www.cfg.cornell.edu

Komvopoulos, K. and Cho, S.S., "Finite Element Analysis of Subsurface Crack Propagation in a Half-Space due to Moving Asperity Contact", Wear, 209, 57-68, 1997.

Komvopoulos, K., "Subsurface Crack Mechanisms under Indentation Loading", Wear, 199, 9-23, 1996.

Lewicki, D.G. and Ballarini, R., "Effect of Rim Thickness on Gear Crack Propagation Path", NASA technical report ARL-TR-1110, 1996.

Paris, P.C. and Erdogan, F., "A Critical Analysis of Crack Propagation Laws", J. of Basic Eng., 85, 528-534, 1961.

Salehizadeh, H. and Saka, N., "Crack Propagation in Rolling Line Contact", Journal of Tribology, 114, 690-697, 1992.

Suh, N. P., "The Delamination Theory of Wear", Wear, 25, 111-124, 1973.

Tan, C.L. and Gao, Y.L., "Treatment of Bimaterial Interface Crack Problems Using the Boundary Element Method", Eng. Fracture Mec., 36, 919-932, 1990.

Tanaka, K., "Fatigue crack propagation from a crack inclined to the cyclic tensile axis", Engineering Fracture Mechanics, 6, 493-507, 1974.