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Regulation of Irrigation Canals Using a Two-Stage Linear Quadratic Reliable Control

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Abstract

A 2-stage discrete-time linear quadratic reliable control technique is applied to the regulation of irrigation canals. The Saint-Venant equations of open-channel flow are linearized using the Taylor series and a finite difference approximation of the original nonlinear, partial differential equations. The concepts of linear optimal control theory are applied to derive a feedback control algorithm for constant level control of an irrigation canal. Two-step linear quadratic update equations and a sequential gain updating scheme are used to drive a linear quadratic reliable formulation. An example problem with a single pool is considered for evaluating the performance of the reliable control technique used to design an optimal control for irrigation canals. The results from the 2-stage reliable control technique are compared to the results from a standard linear quadratic regulator (LQR). The 2-stage reliable control formulation provides both good stability and performance gain margins in the canal operation. The results of this study show that a 2-stage linear quadratic reliable control for irrigation canals offers an alternative to the standard optimal control formulation if there is a lack of flow depth and flow rate data at some measurement points in the irrigation canal.

Key words: Irrigation canals, Two-stage reliable control, Linear quadratic control.

Introduction

Water is becoming a scarce resource and irrigation water districts are under pressure to use water more judiciously. Improved operation of water resource facilities, such as canals and reservoirs, has been touted as necessary for making proper use of these limited water supplies. With the ever-increasing demand for water, the need for improved management of available water resources is of utmost importance, particularly when the development of new water resources is prohibitively expensive. Irrigated agriculture generally uses large volumes of water compared to municipalities and industry, and competition for good quality water is at an all time high in many regions around the world. Thus it is recognized that improved water management practices in agriculture can lead to substantial benefits in terms of water availability for expanded agricultural activity and for other uses, and can directly address many environmental concerns. Irrigation water delivery systems are designed and managed to receive water from a source and distribute it among farms, where it is used to meet agricultural demands. Water management improvement in irrigation canal systems is widely recognized as an important step in attaining better management at the farm level. Improvements in the operation and maintenance of an irrigation delivery system can be translated into better overall water management in an irrigation project, and automation of irrigation canal gate structures can be an effective way to achieve such improvements. Conveyance and distribution of irrigation canals can be improved to better meet the requirements of farmers by providing modern methods of canal control. The demand for irrigation water varies with time, among other factors, due to weather conditions. Therefore, to avoid overflows and always be able to satisfy the demand the canal system must be controlled to maintain desired flow rates and water surface elevations. The use of flexible irrigation deliveries is necessary for efficient on-farm irrigation water management. Thus, the conversion from rigid to flexible delivery schedules will require better canal control to provide good uniform deliveries.

In the past, the concepts of standard optimal linear quadratic regulator control theory have been applied for driving feedback control algorithms for real-time irrigation canals (Reddy *et al.,* 1992; Malaterre, 1997; Reddy, 1999). However, most of these papers assumed that all of the data are available for the canal system. The outages of measurement devices and a lack of flow depth measurements or flow rates at some measurement points were not taken into account in the design of the controller for the canal system. The standard solution of a linear quadratic problem cannot handle the lack of flow depths and flow rate data at some measurement points. Reddy (1999) demonstrated the application of a stochastic optimal control algorithm for an irrigation canal with 5 pools but the possibility of a lack of flow depths and flow rates at some measurement points was not considered in the study. In recent years, the concepts of reliable linear quadratic design approaches were developed for driving the optimal control algorithms for real-time control (e.g., Siljak, 1980; Yang *et al.,* 1998; Veillette, 1995; Paz and Medanic, 1991; Hsieh, 2003). It should be noted that previous reliable control designs were focused on a H_∞ framework and not on the usual quadratic one. H_∞ optimization is used to shape the singular values of specified transfer functions over frequency. The main advantage of using reliable linear quadratic control is good stability, performance and flexibility when dealing with measurement device outages and a lack of data in the irrigation canal. The objectives of the present paper are to present a 2-stage linear quadratic reliable controller for the operation of irrigation canals in the absence of flow depth or flow rate data at some measurement points and to evaluate the performance of the controller in comparison with a standard linear quadratic regulator (LQR) controller.

Mathematical modeling of open-channel flow

In the operation of irrigation canals, decisions regarding the changes in gate opening in response to arbitrary (random) changes in the water withdrawal rates into lateral or branch canals are required to maintain the flow rate into the laterals close to the desired value. This is accomplished by maintaining the depth of flow or the volume of water in a given pool at a target value. This problem is similar to the process control problem in which the state of the system is maintained close to the desired value by using real-time feedback control. Linear control theory is well developed and is easier to apply than nonlinear control theory. The Saint-Venant equations, presented below, are used to model flow in a canal:

$$\partial A/\partial t + \partial Q/\partial x = q_l \tag{1}$$

$$\partial Q/\partial t + \partial(Q^2/A)/\partial x + gA(\partial y/\partial x - S_0 + S_f) = 0 \tag{2}$$

in which A = wetted area, m^2; q_l= lateral flow, m^2/s; y = water depth, m; t = time, s; x = longitudinal direction of channel, m; g = gravitational acceleration, m^2/s; S_0 = canal bottom slope (m/m); R = hydraulic radius, A/P (m); P = wetted perimeter (m); and S_f = the friction slope, m/m, and is defined as

$$S_f = Q|Q|/K^2 \tag{3}$$

in which K = hydraulic conveyance of canal = $AR^{2/3}/n$; R = hydraulic radius, m; and n = Manning roughness coefficient, s/m$^{1/3}$. In deriving Eq. (2), the effect of the net acceleration terms stemming from removal of a fraction of the surface stream was assumed negligible. Lateral canals in the main canal are usually scattered throughout the length of the supply canal. Manually controlled discharge regulators are used at the head of lateral canals. The mathematical representation of flow through these structures is given as follows:

$$q_l = C_d b_l w_l (2g(Z - Z_l)^{1/2} \text{ for submerged flow} \tag{4}$$

$$q_l = C_d b_l w_l (2g(Z - E_s)^{1/2} \text{ for free flow} \tag{5}$$

in which q_l = lateral discharge rate, m^3/s; C_d = outlet discharge coefficient; b_l = width of outlet structure, m; w_l = height of gate opening of outlet structure, m; Z= water surface elevation in supply canal, m; and Z_l = water surface elevation in lateral canal, m; and E_s = sill elevation of head regulator, m. Obviously the flow rate through a head regulator depends upon the water surface elevation in the supply canal. The water surface elevation in the lateral canal is a function of the discharge rate through the head regulator. Therefore, this equation is an implicit equation. In the case of free flow, the discharge rate through the head regulator is independent of the water surface elevation in the lateral canal. Therefore, once the required discharge into a lateral is specified, then the gate opening is adjusted to get the required flow rate through the head regulator, assuming that the water surface elevation in the supply canal is maintained constant at the target level. When a manually controlled head regulator is used, for simulation purposes the gate opening or the variation in gate opening is specified as a function of time. Conversely, when an automated discharge rate regulator is used, for simulation purposes the lateral discharge rate as a function of time is specified as a known input, i.e. $q_{1=}f_q(t)$. In the regulation of the main canal, decisions regarding the opening of gates in response to random changes in water withdrawal rates into lateral canals are required to maintain the flow rate into laterals close to the desired value. This is accomplished by either maintaining the depth of flow in the immediate vicinity of the turnout structures in the supply canal constant or by maintaining the volume of water in the canal pools at the target value. When the latter option is used, the outlets are often fitted with discharge rate regulators. The water levels or the volumes of water stored in the canal pools are regulated using a series of spatially distributed gates (control elements). Hence, irrigation canals are modeled as distributed control systems. Therefore, in the solution of Eqs. (1) and (2), additional boundary conditions are specified at the control structures in terms of the flow continuity and the gate discharge equations, which are given by

$$Q_{i-1,N}r = Q_{gi} = Q_{i,1} \text{ (continuity)} \tag{6}$$

$$Q_{gi} = C_{di}b_i u_i(2g(Z_{i-1,N} - Z_{i,1})^{1/2} \text{ (gate discharge)} \tag{7}$$

in which $Q_{i-1,N}$ = flow rate through downstream gate (or node N) of pool $i-1$, m^3/s; Q_{gi} = flow rate through upstream gate of pool i, m^3/s; $Q_{i,1}$ = flow rate through upstream gate (or node 1) of pool i, m^3/s; C_{di} = discharge coefficient of gate i; b_i = width of gate i, m; u_i = opening of gate i, m; $Z_{i-1,N}$ = water surface elevation at node N of pool $i-1$, m; $Z_{i,1}$ = water surface elevation at node 1 of pool i, m; and i = pool index ($i = 0$ refers to the upstream constant level reservoir).

Linearization and discretization of system equations: Linear control theory is well developed and is easier to apply than nonlinear control theory. The Saint-Venant open-channel equations are linearized about an average operating condition of the canal to apply the linear control theory concepts to the problem. After applying a finite-difference approximation and the Taylor series expansions to Eqs. (1) and (2), a set of linear, ordinary differential equations are obtained for the canal with control gates and turnouts:

$$A_{11}\delta Q_j^+ + A_{12}\delta z_j^+ + A_{13}\delta Q_{j+1}^+ + A_{14}\delta z_{j+1}^+ = A'_{11}\delta Q_j + A'_{12}\delta z_j + A'_{13}\delta Q_{j+1} + A'_{14}\delta z_{j+1} + C_1 \tag{8}$$

$$A_{21}\delta Q^+ j + A_{22}\delta z_j^+ + A_{23}\delta Q_{j+1}^+ + A_{24}\delta z_{j+1}^+ = A'_{21}\delta Q_j + A'_{22}\delta z_j + A'_{23}\delta Q_{j+1} + A'_{24}\delta z_{j+1} + C_2 \tag{9}$$

where δQ_j^+ and δz_j^+ = discharge and water-level increments from time level $n+1$ at node j; δQ_j and δz_j = discharge and water-level increments from time level n at node j; C_1 and C_2 are distribution matrices; and A_{11}, A'_{21}, ... A_{12}, A_{22} are the coefficients of the continuity and momentum equations, respectively, computed with known values at time level n. Similar equations are derived for channel segments that contain a gate structure, a weir or some other type of hydraulic structure. The matrix form of the above equations for the canal can be defined as follows:

$$\underbrace{\begin{pmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ & -(\partial f/\partial z_j)_e 1 - (\partial f/\partial z_{j+2})_e & & \end{pmatrix}}_{A_L} \begin{pmatrix} \delta Q_j^+ \\ \delta z_j^+ \\ \delta Q_{j+1}^+ \\ \delta z_{j+1}^+ \\ \delta Q_{j+2}^+ \\ \delta z_{j+2}^+ \end{pmatrix} =$$

$$\underbrace{\begin{pmatrix} A'_{11} & A'_{12} & A'_{13} & A'_{14} \\ A'_{21} & A'_{22} & A'_{23} & A'_{24} \\ & -(\partial f/\partial z_j)_e 1 - (\partial f/\partial z_{j+2})_e & & \end{pmatrix}}_{A_R} \begin{pmatrix} \delta Q_j \\ \delta z_j \\ \delta Q_{j+1} \\ \delta z_{j+1} \\ \delta Q_{j+2} \\ \delta z_{j+2} \end{pmatrix} +$$

$$\underbrace{\begin{pmatrix} 0 \\ 0 \\ -(\partial f/\partial z_j)_e \end{pmatrix}}_{B} \tag{10}$$

From the matrix form of the equations above, the state of system equation at any sampling interval k can be written in a compact form as follows:

$$A_L \delta x(k+1) = A_R \delta x(k) + B \delta u(k) \tag{11}$$

where $A = lxl$ system feedback matrix, $B= lxm$ control distribution matrix, and k = time increment, s. The elements of the matrices A and B depend upon the initial condition. Equation (11) can be written in a state-variable form along with the output equations as follows:

$$\delta x(k+1) = \Phi \delta x(k) + \Gamma \delta u(k) \tag{12}$$

$$\delta y(k) = H \delta x(k) \tag{13}$$

where $\Phi = (A_L)^{-1} * A_R$, $\Gamma = (A_L)^{-1} * B$,and $\delta x(k)$ = $lx1$ state vector, $\delta u(k) = mx1$ control vector, $\delta y(k)$ = $rx1$ vector of output (measured variables), H = rxloutput matrix, l = number of dependent (state) variables in the system, m = number of controls (gates) in the canal, p = number of outlets in the canal, and r = number of outputs. The elements of the matrices Φ, Γ, and Ψ depend upon the canal parameters, the sampling interval, and the assumed average operating condition of the canal. In Eq. (11), the vector of state variables is defined as follows:

$$\delta x = (\delta Q_{i,1}, \delta Z_{i,2}, \delta Q_{i,2}, \ldots \delta Z_{i,N-1}, \delta Q_{i,n-1}, \delta Q_{i,N}) \tag{14}$$

Standard optimal linear quadratic regulator (LQR)

In the control literature, much attention has been devoted to linear quadratic regulator design problems, largely as a result of their elegant problem formulation, solution tractability, and robust properties with respect to fairly large variations of system parameters. The problem of designing a linear feedback control system minimizing a quadratic performance index can be reduced to the problem of obtaining a positive definite solution of a matrix Riccati equation. An important characteristic of transient performance of an open canal is its stability. Once a canal is disturbed from its original equilibrium condition, the responses to the disturbance will be stable, neutral, or unstable. The stability requirement of the considered system is defined in terms of the eigenvalues, which are the roots of the characteristic equation of matrix Φ and must have values less than unity. The oscillatory behavior of a canal water surface is associated with the presence of complex roots in the solution of the characteristic equation of the system. The response amplitude grows continuously if the absolute value of the complex roots is greater than unity, decays to zero if the absolute value is less than unity, and oscillates at a constant amplitude if the real part of the roots is zero. Furthermore, because of inertia, it is almost impossible to derive the deviation in water surface elevation (error) instantaneously to zero. Thus the output of the system lags the desired input and results in overshoot or oscillation of the water level about its equilibrium position. The objective of control theory is to find a control law that will bring an initially disturbed water surface to the desired target water level in the presence of external disturbances acting on the canal. This can be accomplished by applying a large proportional control in which the change in gate opening is proportional to the changes in flow depths and flow rates; this has the following form:

$$\delta u(k) = -K(k) \delta x(k) \tag{15}$$

where $K(k)$ =controller gain matrix. Controllability ensures the stability of the system and maintains the water level at any desired value by suppressing the influence of external disturbances. A canal is said to be controllable if it is possible to derive it from any initial water level to any specified water level (state) within a finite number of steps. Equation (15), which was used throughout the study, is called the discrete state equation and control law. This equation describes the condition or evolution of the basic internal variables of the system. The variables in the equation (i.e. δx) are called the state variables. In optimal control theory, the elements of gain matrix K can be obtained by formulating the control problem as an optimization problem in which the cost function to be minimized is given as follows:

$$J = \sum_{i=1}^{K_\infty} [\delta x(k)^T Qx_{lxl} \delta x(k) + \delta u(k)^T R_{mxm} \delta u(k)] \tag{16}$$

subject to the constraint that

$$-\delta x(k+1) + \Phi \delta x(k) + \Gamma \delta u(k) = 0 k = 0, \ldots, K\infty \tag{17}$$

where K_∞= number of sampling intervals considered to derive the steady state controller; Qx_{lxl} = state cost weighting matrix; and R_{mxm} = control cost weighting matrix. The matrices Qx and R are symmetric, and to satisfy the non-negative definite condition, they are usually selected to be diagonal with all diagonal elements positive or zero. The first term in Eq. (16) represents the penalty on the deviation of the state variables from the average operating (or target) condition, where the second term represents the cost of control. This term is included in an attempt to limit the magnitude of the control signal $\delta u(k)$. Unless a cost is imposed for use of control, the design that emerges is liable to generate control signals that cannot be achieved by the actuator. In this case the saturation of the control signal will occur, resulting in a system behavior that is different from the closed loop system behavior that was predicted assuming that saturation will not occur. Therefore, the control signal weighting matrix elements are selected to be large enough to avoid saturation of the control signal under normal operating conditions. Equations (16) and (17) constitute a constrained-minimization problem that can be solved using Lagrange multipliers. This produces a set of coupled difference equations that must be solved recursively backwards in time. However, since irrigation canals run for a long time, and the dynamics of the canals are usually very slow, a steady state controller is more desirable. For the steady state case, the solution for $\delta u(k)$ is the same form as Eq. (15), except that K is given by

$$K = [R + \Gamma^T P \Gamma]^{-1} \Gamma^T P \Phi \tag{18}$$

P is a solution of the discrete algebraic Riccati equation (DARE):

$$\Phi^T P \Phi - \Phi^T P \Gamma S^{-1} \Gamma^T P \Phi + Qx = P \tag{19}$$

where S = R + Γ^TS Γ; R = $R^T > 0$ and Qx = Qx^T = $H^T H \geq 0$. The solution of the discrete algebraic Riccati equation is fundamental for the implementation of optimal control. The control law defined by Eq. (15) brings an initially disturbed system to an equilibrium condition in the absence of any external disturbances acting on the system. In hydraulic engineering problems, the depth of flow, flow rate, and velocity as a function of distance can be considered the state or internal variables. Sometimes, the volume of water in a given reach of a canal can also be considered a state variable. In this paper, the water surface elevation and flow rate were considered the state variables. Given initial conditions $[\delta x(0)]$, δu, and δq, Eq. (16) can be solved for variations in flow depth and flow rate as a function of time. If the system is really at equilibrium [i.e. δx $(0) = 0$ at time $t = 0$] and there is no change in the lateral withdrawal rates (disturbances), the system would continue to be at equilibrium forever; then there is no need for any control action. Conversely, in the presence of disturbances (known or random), the system would deviate from the equilibrium condition. The actual condition of the system may be either above or below the equilibrium condition, depending upon the sign and magnitude of the disturbances. If the system deviates significantly from the equilibrium condition, the discharge rates into the laterals will be different (either more or less) than the desired values. But in canal operations, the main objective is to keep these deviations to a minimum so that a nearly constant rate of flow is maintained through the turnouts.

Two-stage linear quadratic reliable controller

If there are outages of measurement devices or a lack of flow depth measurements or flow rates at some measurement points in an irrigation canal, the above standard linear quadratic optimal control may not be the most suitable one. Instead, the 2-stage linear quadratic reliable control seems to be an attractive means to guarantee the canal system's stability and performance. A control system designed to tolerate outages of measurement devices and a lack of data, while retaining desired control system properties, will be called a reliable control system. The major objective of reliable control design is to synthesize a control structure so that the system performs satisfactorily under outages of measurement sensors or unavailability of data while maintaining a good performance in the nominal condition. To begin with, the following 2-step linear quadratic optimal control update equations, which are inspired by the well-known 2-step Kalman filter update equations, are presented: (1) Control weighting update equations:

$$\delta u(k) = -\bar{K}\delta\bar{K}(k) \quad (20)$$

$$\bar{K} = [\Gamma' P \Gamma + R]^{-1}\Gamma' P \quad (21)$$

$$\bar{P} = P[I - \Gamma\bar{K}] \quad (22)$$

and (2) State weighting update equations:

$$\delta\hat{x}(k) = \Phi\delta x(k) \quad (23)$$

$$P = \Phi'\bar{P}\Phi + Qx \quad (24)$$

Then, using a dual of the sequential measurement update equations (Singer and Sea, 1971) in the control weighting update equations, Eq. (19) can be reformulated as follows

$$\Phi^T P(I - \Gamma^{\Omega x}\bar{K}^{\Omega x})(I - \Gamma^{\Omega}\bar{K}^{\Omega})\Phi - Q_{rel} = P \quad (25)$$

where

$$Q_{rel} = Qx + (\bar{K}^{\Omega}\Phi)'[(\Gamma^{\Omega})'\bar{P}\Gamma^{\Omega} + R^{\Omega})]\bar{K}^{\Omega}\Phi \quad (26)$$

$$\bar{K}^{\Omega x} = (R^{\Omega})^{-1}(\Gamma^{\Omega x})' P(I - \Gamma^{\Omega x}\bar{K}^{\Omega x}) \quad (27)$$

$$\bar{K}^{\Omega} = (R^{\Omega})^{-1}(\Gamma^{\Omega})' P(I - \Gamma^{\Omega x}\bar{K}^{\Omega x})(I - \Gamma^{\Omega x}\bar{K}^{\Omega x}) \quad (28)$$

in which Ω denotes the selected nodes where data are available and Ωx denotes the selected nodes where data are not available because of outages of measurement sensors or a lack of data. As seen in the above equations, in fact the 2-stage linear quadratic reliable control can be seen as a standard linear quadratic regulator with a modified state weighting matrix (Hsieh, 2003). In other words, the reliable linear quadratic control can be obtained by a standard LQR design with a modified state-weighting matrix, i.e. Qx is replaced by Q_{rel}. Bitmead and Gevers (1991) show that if Φ,Γ^{Ω} is a stabilizable pair and Φ, Qx is a detectable pair, then there exists a unique and positive definite symmetric solution P for Eq. (25).

Results and Discussion

To demonstrate and compare the feasibility of the 2-stage linear quadratic reliable controller, an optimal regulation problem for a discrete-time single pool irrigation canal was simulated (Figure 1). An example problem obtained from Reddy (1990) was used in the study. The data used were as follows: length of canal reach = 5000 m, number of nodes = 7, number of subreaches used = 4, Δx = 1250 m, channel slope = 0.0003, side slope = 1.0, bottom width = 1.7 m, turnout demand = 2.5 m^3/s, discharge required at the end of the canal = 0.52 m^3/s, upstream reservoir elevation = 103.2 m, downstream reservoir elevation = 101.14 m, target depth at downstream end = 1.2 m, gate width = 1.7 m, and gate discharge coefficient = 0.75. First the data were used to calculate the steady state values, which in turn were used to compute the initial gate openings and the elements of the Φ, Γ, H matrices using a sampling interval of 30 s. The analysis was started by evaluating the system stability. All the eigenvalues of the feedback matrix were positive and had values less than one. The system was also found to be both controllable and observable. In the derivation of the control matrix, Γ, elements, it was assumed that both the upstream and downstream gates of each reach could be manipulated to control the system dynamics. The downstream-end gate position was frozen at the original steady state value, and only the upstream-end gate of the given reach was controlled to maintain the system at the equilibrium condition. The effect of variations in the opening of the downstream gate must be taken into account through real-time feedback of the actual depths immediately upstream and downstream of the downstream gate (node N). In the derivation of the feedback gain matrix, R was set equal to 100,000, whereas Qx was set equal to an identity matrix of dimension 10 (the dimensions of the system). In the absence of a well-defined procedure for selecting the elements of these matrices, these values were selected based upon trial and error.

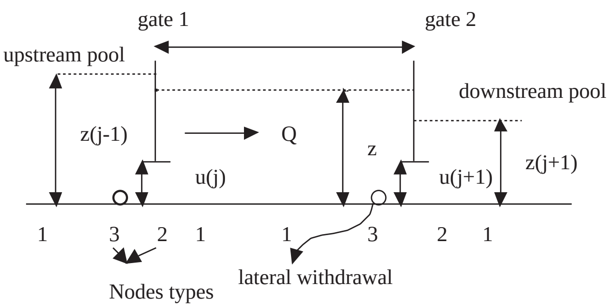

Figure 1. Schematic of an irrigation canal pool.

Figure 2 illustrates the variations in downstream depth of flow in the pool for both 2-stage linear quadratic reliable and standard linear quadratic regulator (LQR) controllers. At 7500 s, the variation in downstream flow depth in the pool is 0.04 m for the 2-stage reliable controller, whereas it is 0.0534 m for the standard LQR controller. It is obvious that there is no significant difference in flow depth variations between the 2-stage reliable controller and standard LQR controller. However, the former holds less flow depth variations at the downstream of the canal. As shown in Figure 3, along the simulation, the incremental gate openings for the 2-stage reliable controller have values less than those of the standard LQR controller's incremental gate openings; but there are no significant differences between the 2 controllers' incremental gate openings. In Figure 4, the cumulative gate opening is 0.019 m for the two-stage reliable controller and 0.0398 m for the standard LQR controller at the end of the simulation. Cumulative gate openings for the 2-stage reliable controller have variations less than those of the standard one. Figure 4 shows that the 2-stage controller cumulative gate openings will reach the equilibrium condition quicker than the standard LQR controller. Finally, the final gate openings for both the standard LQR and 2-stage reliable controllers are 0.4199 m and 0.3991 m, respectively (Figure 5). There are no significant differences between the final gate openings of the 2 controllers. As shown in the above numerical example, 2-stage linear quadratic reliable design can provide better stability than the standard one. The overall results of this study show that the proposed 2-stage linear quadratic reliable controller offers an efficient alternative to the standard LQR controller when dealing with the absence of flow depth measurements or flow rates at some measurement points.

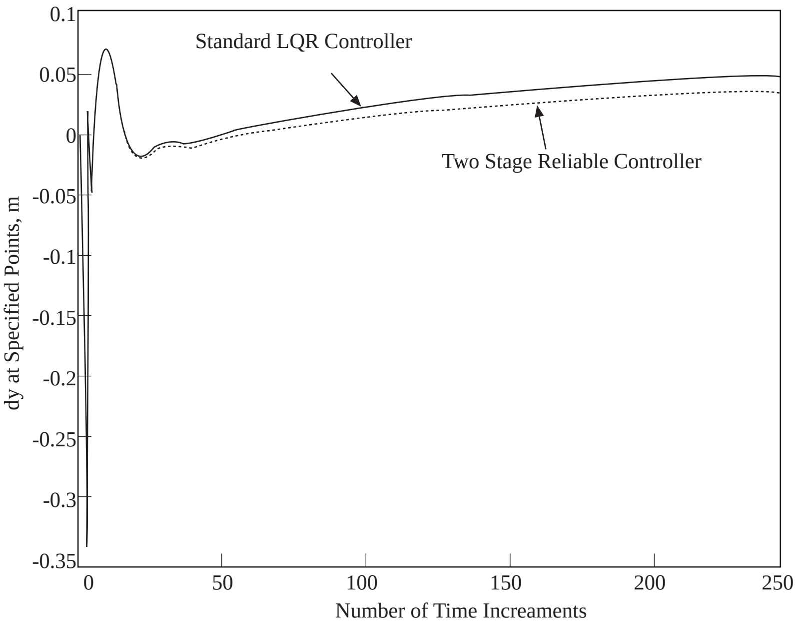

Figure 2. Flow depth variations for standard LQR and 2-stage reliable controllers.

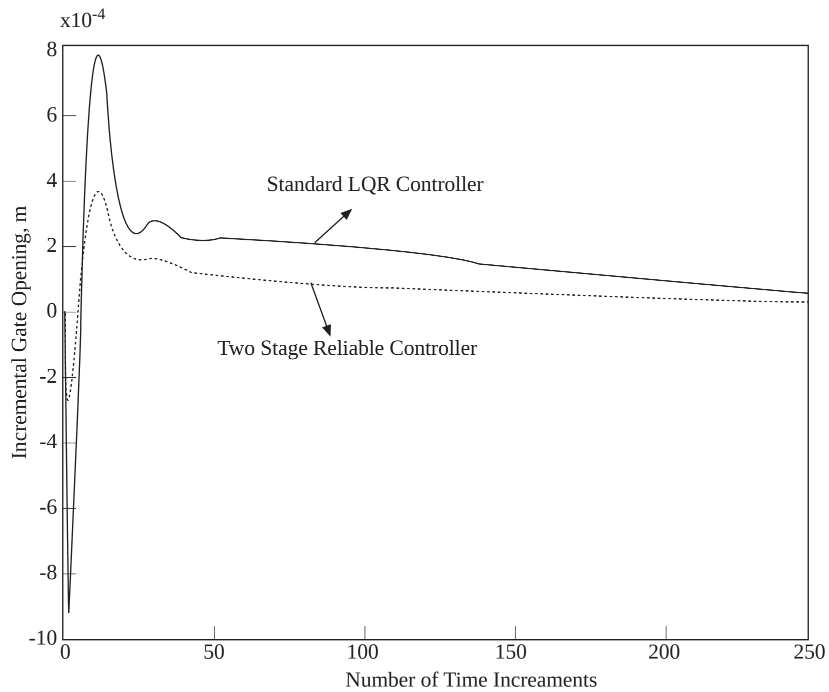

Figure 3. Incremental gate openings for standard LQR and 2-stage reliable controllers.

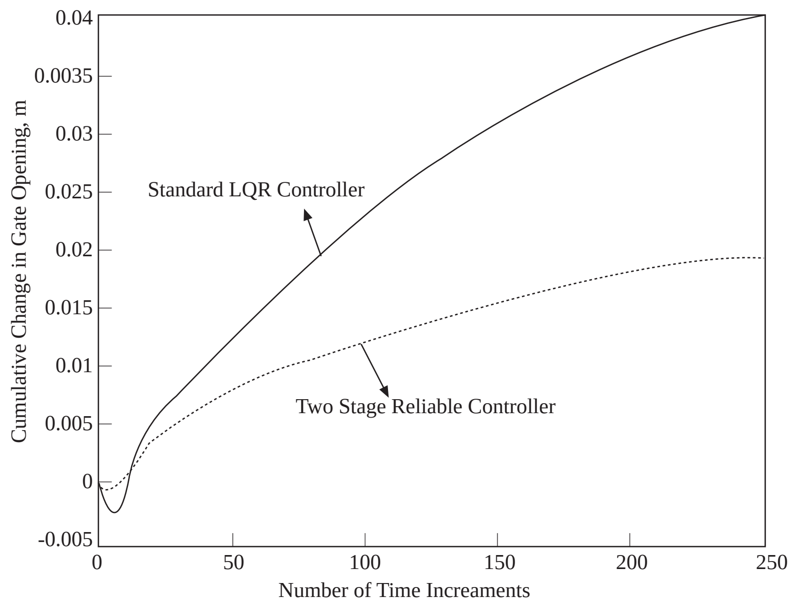

Figure 4. Cumulative gate openings for standard LQR and 2-stage reliable controllers.

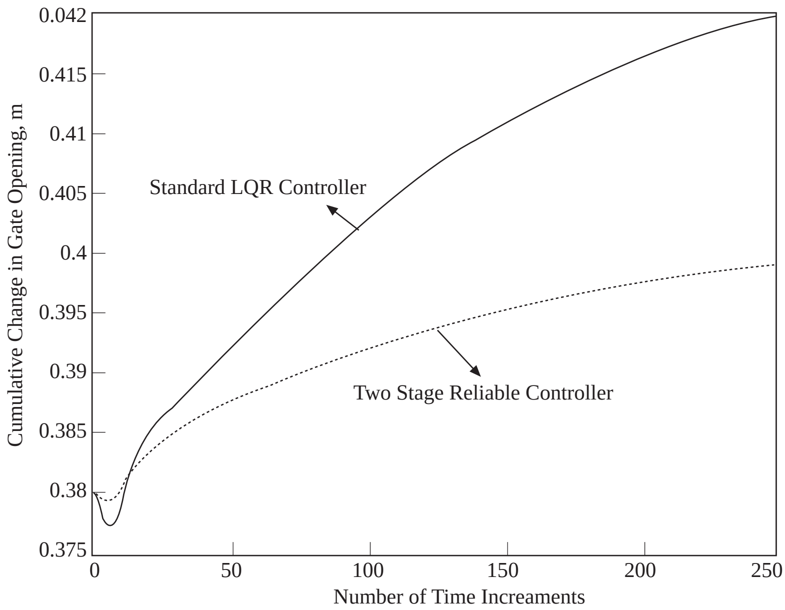

Figure 5. Final gate openings for standard LQR and two-stage reliable controllers.

Conclusions

The Saint-Venant equations of open-channel flow were linearized around the average condition of an example single pool canal. Two-step linear quadratic update equations and a sequential gain updating scheme were used to drive a linear quadratic reliable controller formulation. This formulation was applied to drive a control algorithm for constant-level control of an irrigation canal. The performance of 2-stage linear quadratic reliable control was compared with the performance of the standard LQR controller. The 2-stage linear quadratic reliable controller provides both good stability and performance under outages of measurement devices or unavailability of data in the canal. The results showed that the 2-stage linear quadratic reliable formulation offers an efficient alternative to standard LQR control when dealing with the lack of flow depth measurements or flow rates at some measurement points.

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Notations

$\delta\hat{x}$	estimated values of the state variables	-
θ	weighting coefficient	-
A_b	horizontal water surface area of the basin width m	
B	width of the water surface	m
b_l	width of outlet structure	m
C_d	outlet discharge coefficient	-
D	hydraulic depth	m
E_s	sill elevation of head regulator	m
g	acceleration due to gravity	m/s^2
I	identity matrix of appropriate dimension	-
J	cost function used in optimal control	-
k	numbers or sampling interval	-
K(k)	controller gain matrix	-
K_∞	number of sampling intervals considered to derive	-
$M(\omega)$	magnitude ratio	-
P	solution of the discrete algebraic Riccati equation	-
Q	discharge in the channel	m^3/s
q	discharge per unit	m^2/s
q_l	lateral in or out flow	m^2/s
Qx	state weighting matrix	-
S_f	slope of energy line	-
S_o	canal bottom slope	-
t	time	s
T	top width of the canal	m
V	mean velocity	m/s
V	state matrix in the RDNN P	-
w_l	height of gate opening of outlet structure	m
x	length of the canal reach	m
y	depth of water in the canal	m
$Y_d(z)$	desired output of control system	-
z	water level as referred to a horizontal datum	m
γ	unit weight of water	N/m^3
Δ	smallest structured perturbation	-
δ	q vector of system disturbances	m^3/s
δ	u vector of system inputs (gate openings)	m
Δ	x length of subsurface	m
δ	y vector of system outputs	m
Φ	system transition matrix	-
Ψ	disturbance distribution matrix	-
Ω	the selected nodes where data available	-
Ω x	the selected nodes were data is not available	-
Γ	control distribution matrix	-